

# Space-time correlations in monitored kinetically constrained discrete-time quantum dynamics

Marcel Cech,<sup>1</sup> María Cea,<sup>2,3</sup> Mari Carmen Bañuls,<sup>2,3</sup> Igor Lesanovsky,<sup>4,5</sup> and Federico Carollo<sup>1</sup>

<sup>1</sup>*Institut für Theoretische Physik, Universität Tübingen,  
Auf der Morgenstelle 14, 72076 Tübingen, Germany*

<sup>2</sup>*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, D-85748 Garching, Germany*

<sup>3</sup>*Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, D-80799 München, Germany*

<sup>4</sup>*Institut für Theoretische Physik and Center for Integrated Quantum Science and Technology,  
Universität Tübingen, Auf der Morgenstelle 14, 72076 Tübingen, Germany*

<sup>5</sup>*School of Physics and Astronomy and Centre for the Mathematics  
and Theoretical Physics of Quantum Non-Equilibrium Systems,  
The University of Nottingham, Nottingham, NG7 2RD, United Kingdom*

State-of-the-art quantum simulators permit local temporal control of interactions and midcircuit readout. These capabilities open the way towards the exploration of intriguing nonequilibrium phenomena. We illustrate this with a kinetically constrained many-body quantum system that has a natural implementation on Rydberg quantum simulators. The evolution proceeds in discrete time and is generated by repeatedly entangling the system with an auxiliary environment that is monitored and reset after each time-step. Despite featuring an uncorrelated infinite-temperature average stationary state, the dynamics displays coexistence of fast and slow space-time regions in stochastic realizations of the system state. The time-record of measurement outcomes on the environment serves as natural probe for such dynamical heterogeneity, which we characterize using tools from large deviation theory. Our work establishes the large deviation framework for discrete-time open quantum many-body systems as a means to characterize complex dynamics and collective phenomena in quantum processors and simulators.

**Introduction.** — Statistical mechanics is typically concerned with the analysis of equilibrium and nonequilibrium stationary properties of physical systems [1–3]. Certain many-body systems may however be characterized by “trivial” stationary states while displaying intricate collective dynamical behavior [4–10]. Kinetically constrained models provide a paradigmatic example of such interesting (typically slow) dynamics towards structureless thermodynamic equilibrium states [11, 12]. These many-body systems are described by simple dynamical rules that only allow for transitions between single-particle states if a given condition, e.g., the absence or presence of particles in their close neighborhood, is satisfied. This reduction in the connectivity between configurations of the system can result in highly correlated and heterogeneous dynamics [13–19]. Within the framework of classical stochastic systems, methods from large deviation theory have been employed to investigate collective dynamical behavior in kinetically constrained models, leading, for instance, to new insights into the glass transition [20–29]. Extending these models to the quantum realm holds the promise to reveal new dynamical behavior and nonequilibrium phases [19, 30–36]. Interestingly, this undertaking also constitutes a daunting task, in particular due to the intrinsic complexity of many-body quantum systems.

Currently available quantum simulation platforms are promising candidates to address this open challenge [37–44]. Improvements in both the fidelity and the versatility of implementable quantum operations already allowed for a deeper understanding of quantum many-

body dynamics [45–59]. Most notably, in-circuit measurements and a high level of control over interactions in neutral atom platforms [60–64], ionic quantum processors [65–69] and superconducting quantum computers [70–74] allow for the direct implementation of engineered discrete-time open-system dynamics [75–82]. In these settings, open quantum dynamics are realized by sequentially coupling the system with auxiliary degrees of freedom [cf. Fig. 1(a)], which are subsequently measured and reinitialized. The time-record of measurements on these *ancillary* (or environmental) units provides so-called quantum trajectories, revealing *in-situ* information on the stochastic dynamics [5, 79–94]. Characterizing the quantum state in single stochastic realizations of the dynamics [cf. Fig. 1(b)] is, however, not possible for many-body systems and for long observation times [93–95].

In this paper, we propose an approach capable to uncover and systematically characterize distinct dynamical phases in discrete-time many-body quantum dynamics. Our framework is tailored for open systems and solely utilizes information that is efficiently accessible in noisy intermediate scale quantum (NISQ) devices. The analysis is indeed based on so-called “dynamical order parameters”, which can be defined from measurement time-records [cf. Fig. 1(c)] in single dynamical realizations. To illustrate our ideas, we consider a model, inspired by the physics of driven-dissipative Rydberg gases [4–7, 96, 97], which features effectively kinetically constrained dynamics in certain parameter regimes [98–102]. Despite the average evolution being characterized by a “trivial” stationary state, stochastic realizations

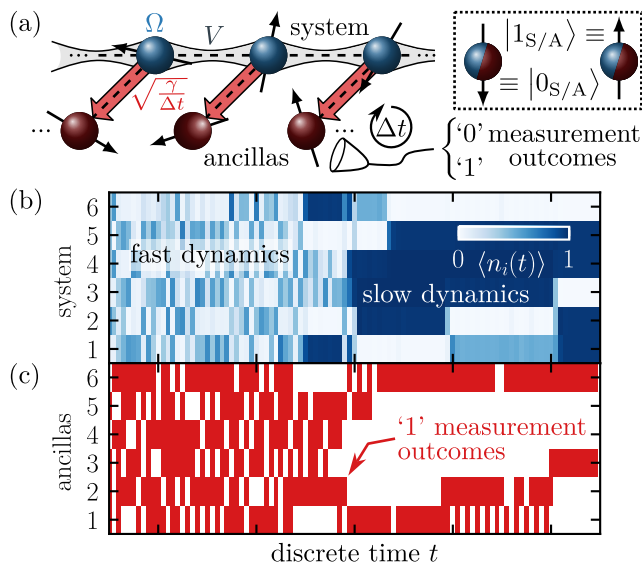


FIG. 1. **Discrete-time dynamics in the Rydberg collision model.** (a) Collision model setup described by Eqs. (1,2). The interacting quantum system consisting of  $L$  two-level systems (blue qubits) is coupled to an ancillary chain (red qubits) of the same length. After the collision time  $\Delta t$ , the ancillas are measured in their computational basis  $\{|0_A\rangle, |1_A\rangle\}$  and reset to  $|0_A\rangle$ . (b) Stochastic system dynamics for  $L = 6$ ,  $\Delta t = 1.25/\Omega$ ,  $V = 5.875\Omega$  and  $\gamma = 3\Omega$ . Iterating the collision model dynamics, the quantum system can show coexistence of fast and slow dynamical regimes, visible, for instance, in the probability of observing system qubits in  $|1_S\rangle$ ,  $\langle n_i(t) \rangle$ . (c) Corresponding quantum trajectory. Remarkably, also the ancillary measurement time-record, here associated with the stochastic realization in panel (b), shows signatures of the dynamical heterogeneity by displaying distinct space-time regions, e.g., the absence of ‘1’ measurements.

display complex correlated and heterogeneous dynamics, which also becomes apparent in the corresponding ancillary measurement time-record [see Fig. 1(b,c)]. The frequency and correlations of the measurement outcomes, laid out in space and time, provide suitable dynamical order parameters that show large fluctuations over individual realizations. Exploiting a thermodynamic analogy [80, 81, 103–108], we uncover the presence of macroscopically distinct dynamical phases, whose coexistence in space and time is responsible for the observed large fluctuations. Our findings showcase the potential of this approach for analyzing complex open-system dynamics on quantum computers.

**Collision model setup.** — We consider discrete-time open quantum dynamics emerging from repeated interactions of the system of interest with ancillary degrees of freedom. The latter encode an effective environment and undergo measurements and resets at stroboscopic times [75–82]. Both the system (S) and the ancillas (A) are described by a chain of  $L$  two-level systems, or qubits, with local computational basis  $\{|0_{S/A}\rangle, |1_{S/A}\rangle\}$

[see Fig. 1(a)]. A discrete time-step is realized in the following way. The system unitarily collides with the auxiliary chain, initialized in  $|0_A\rangle = \bigotimes_{i=1}^L |0_A\rangle_i$ . During the interaction, the unitary operator  $U = e^{-iH_{\text{CM}}\Delta t}$ , implemented by the Hamiltonian  $H_{\text{CM}}$ , entangles system and ancillas. Afterwards, a projective measurement onto the computational basis is performed on the ancilla qubits. The measurement outcome is collected in a binary string  $\mathbf{k} = (k_1, \dots, k_L)$ , reporting the outcomes  $k_i$  for each qubit, and the ancillary chain is then reset to its initial state in preparation for the next collision. The system, for instance initialized in  $|\psi(0)\rangle = |0_S\rangle$  as well, thus evolves by one discrete-time step according to the relation  $|\psi\rangle \rightarrow |\psi'\rangle = K_{\mathbf{k}}|\psi\rangle / \|K_{\mathbf{k}}|\psi\rangle\|$ , with probability  $\pi(\mathbf{k}) = \|K_{\mathbf{k}}|\psi\rangle\|^2$ . Here, the Kraus operator  $K_{\mathbf{k}} = \langle \mathbf{k}_A | U | 0_A \rangle$  describes the reduced evolution of the system conditional to the measurement outcome  $\mathbf{k} = (k_1, \dots, k_L)$  for the ancillary qubits. A single realization of the stochastic dynamics,  $|\psi(0)\rangle \rightarrow \dots \rightarrow |\psi(T)\rangle$ , is thereby conditioned on the quantum trajectory  $\eta(T) = [\mathbf{k}(t)]_{t=1}^T$ , i.e., the space-time record of all ancillary measurements [see Fig. 1(b,c)], and is observed with a probability of  $\pi(\eta(T)) = \|K_{\mathbf{k}(T)} \dots K_{\mathbf{k}(1)} |\psi(0)\rangle\|^2$ . In contrast, the single time-step unconditional average evolution is governed by the Kraus map  $\rho \rightarrow \rho' = \mathcal{E}[\rho]$ , with  $\mathcal{E}[\rho] := \sum_{\mathbf{k}} K_{\mathbf{k}} \rho K_{\mathbf{k}}^\dagger$  [109–111].

The model we consider is specified by the collision Hamiltonian

$$H_{\text{CM}} = H_S \otimes \mathbb{1} + \sqrt{\frac{\gamma}{\Delta t}} \sum_{i=1}^L P_i \otimes \tau_i^x, \quad (1)$$

with  $P = |0_S\rangle\langle 0_S|$  and  $\tau^x = |0_A\rangle\langle 1_A| + \text{h.c.}$ , where  $\gamma$  represents a dephasing rate in the continuous-time limit  $\Delta t \rightarrow 0$  (see Supplemental Material [112] for details). The system Hamiltonian  $H_S$  furthermore models a coherent drive with Rabi frequency  $\Omega$  and nearest-neighbor interaction with strength  $V$ , when both qubits are in the (Rydberg) state  $|1_S\rangle$  [48, 116]. We thus define (assuming periodic boundary conditions for the chain)

$$H_S = \Omega \sum_{i=1}^L \sigma_i^x + V \sum_{i=1}^L n_i n_{i+1}, \quad (2)$$

with  $\sigma^x = |0_S\rangle\langle 1_S| + \text{h.c.}$  and  $n = |1_S\rangle\langle 1_S| = 1 - P$ . For large interaction strengths  $V$ , the coherent drive of a qubit located near a qubit in state  $|1_S\rangle$  is highly detuned. In the limit of infinitely strong interactions,  $V \rightarrow \infty$ , this results in the so-called Rydberg-blockade mechanism [117–120], for which sites close to a qubit in state  $|1_S\rangle$  are temporarily frozen. This effect leads to a strict separation of the Hilbert space into sectors labeled by the set of frozen sites with adjacent qubits in state  $|1_S\rangle$  [98–102]. In such a limit, the Hamiltonian  $H_S$  can be mapped onto the so-called PXP Hamiltonian  $H_{\text{PXP}} = \sum_{i=1}^L P_{i-1} \sigma_i^x P_{i+1}$ , which realizes a kinetically constrained model. For finite interaction strengths  $V$ , the Rydberg blockade is not

perfect. As such, on short to intermediate time-scales, the Hamiltonian  $H_S$  is well approximated by the PXP Hamiltonian, but the system eventually explores the full Hilbert space at long enough times.

In Fig. 1(b), we consider a stochastic realization of the collision model dynamics. Specifically, we show the local probability  $\langle n_i(t) \rangle$  of qubits in the state  $|1_S\rangle$ , which immediately allows one to identify two dynamical regimes: a fast regime, which is characterized by rapid changes of  $\langle n_i(t) \rangle$ , and a slow one where adjacent qubits are close to being in state  $|1_S\rangle$ ,  $\langle n_i(t) \rangle \approx 1$ . Due to the Rydberg-blockade effect, these space-time regions are preserved over several time-steps and substantially slow down the evolution of neighboring qubits as well. It is important to note, that the expectation values  $\langle n_i(t) \rangle$ , as shown in Fig. 1(b), are not directly accessible on a quantum computer. The reason is that for estimating  $\langle n_i(t) \rangle$  of a stochastic realization, repeated measurements of the operator  $n_i(t)$  have to be taken on identical quantum trajectories [cf. Fig. 1(c)]. This comes with a post-selection overhead that grows exponentially in time and system size [93–95]. However, signatures of the above-mentioned dynamical regimes are already clearly visible in the quantum trajectories themselves, i.e., in the space-time records of the measurement outcomes on the ancillary qubits shown in Fig. 1(c). The fast regime is then related to ancillas being frequently observed in  $|1_A\rangle$ . On the other hand, space-time regions containing adjacent system qubits with  $\langle n_i(t) \rangle \approx 1$ , which are found in the slow regime, are reflected in a lack of ‘1’ ancillary measurement outcomes. This connection is a consequence of the structure of the system-ancilla interaction  $\propto P_i \otimes \tau_i^x$  in Eq. (1), solely allowing for a rotation of the  $i$ th ancilla into the state  $|1_A\rangle$  if the corresponding system qubit features a finite overlap with the state  $|0_S\rangle$ . The heterogeneous dynamics shown in Figs. 1(b,c) is even more intriguing when considering that the average stationary state  $\rho_{ss}$ , i.e.,  $\mathcal{E}[\rho_{ss}] = \rho_{ss}$ , is the fully mixed state  $\mathbb{1}/2^L$  [112].

**Space-time correlations in quantum trajectories.** — We characterize dynamical behavior in quantum trajectories by means of so-called dynamical order parameters. For the purpose of this work these are the *activity*

$$a(t) = \frac{1}{L} \sum_{i=1}^L \overline{k_i(t)} \quad (3)$$

and the *space-time correlations* defined through the spatially averaged covariance

$$c_\delta(t) = \frac{1}{L} \sum_{i=1}^L \left[ \overline{k_i(t-\delta_t) k_{i+\delta_i}(t)} - \overline{k_i(t-\delta_t)} \overline{k_{i+\delta_i}(t)} \right]. \quad (4)$$

The former observable keeps track of the frequency of  $k_i(t) = 1$  measurement outcomes, while the latter one

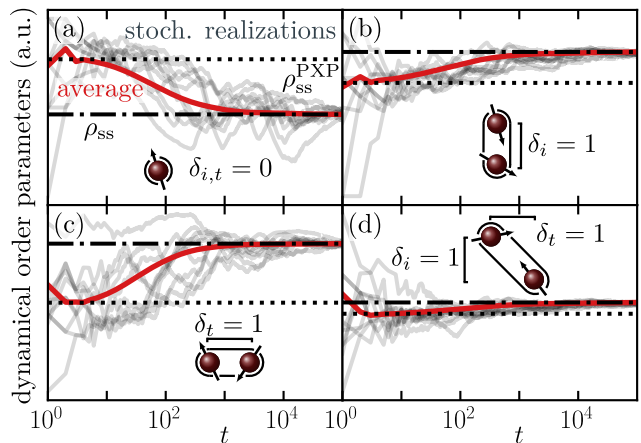


FIG. 2. **Dynamical order parameters for  $L = 6$ ,  $\Delta t = 1.25/\Omega$ ,  $V = 5.875\Omega$  and  $\gamma = 3\Omega$ .** (a) Activity. (b-d) Space-time correlations with space-time distance  $\delta$ . We compare the dynamical order parameters of the ensemble (red lines) with the corresponding observables (see main text) extracted from 10 quantum trajectories (light gray lines). Using Eqs. (5,6), the dash-dotted black line represents the stationary value obtained on  $\rho_{ss} = \mathbb{1}/2^L$ , while the dotted black line contains the prediction of the projection onto the free sector of the PXP model  $\rho_{ss}^{\text{PXP}} \propto \mathcal{P}_{\text{PXP}}[\rho_{ss}]$  with no adjacent qubits in state  $|1_S\rangle$ .

of their correlations when outcomes are considered at the space-time distance  $\delta = (\delta_i, \delta_t)$ . The average  $\bar{\cdot}$  in Eqs. (3,4) is an ensemble average over all possible quantum trajectories, weighted with their respective probabilities. In terms of the average state  $\rho(t) = \mathcal{E}^t[\rho(0)]$ , the relevant probabilities for computing the ensemble averages in Eqs. (3,4) are

$$p(\mathbf{k}(t) = \mathbf{k}) = \text{Tr} \left\{ K_{\mathbf{k}} \rho(t-1) K_{\mathbf{k}}^\dagger \right\}, \quad (5)$$

for single-time and

$$p(\mathbf{k}(t) = \mathbf{k}, \mathbf{k}(t-1) = \mathbf{k}') = \text{Tr} \left\{ K_{\mathbf{k}} K_{\mathbf{k}'} \rho(t-2) K_{\mathbf{k}'}^\dagger K_{\mathbf{k}}^\dagger \right\}, \quad (6)$$

for two-time functions [93]. The above equations show that, asymptotically in time, dynamical order parameters are completely described by the stationary state  $\rho_{ss}$ .

The calculation of the above quantities still requires sampling multiple dynamical realizations on quantum processors. Interestingly, we can define closely related observables, displaying analogous behavior, which are solely defined on individual quantum trajectories  $\eta(T) = [\mathbf{k}(t)]_{t=1}^T$ . These time-integrated observables are of the following form

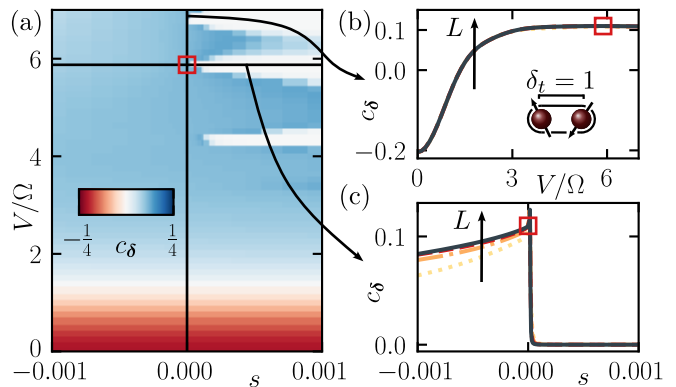
$$\mathcal{O}_\delta(\eta(T)) = \sum_{t=1+\delta_t}^T \sum_{i=1}^L k_i(t-\delta_t) k_{i+\delta_i}(t). \quad (7)$$

The rationale is that time averages and ensemble averages coincide at stationarity. This allows us

to compare  $\mathcal{O}_0(\eta(T))/(LT)$  and  $\mathcal{O}_\delta(\eta(T))/(LT') - [\mathcal{O}_0(\eta(T))/(LT)]^2$ , where  $T' = (T - \delta_t)$  denotes the extensivity of the first sum in Eq. (7), with the activity and the space-time correlations in Eqs. (3,4), respectively.

In Fig. 2, we test this understanding by comparing the activity and space-time correlations with the corresponding observables in Eq. (7) calculated for 10 quantum trajectories. We furthermore show the prediction of the stationary state  $\rho_{ss} = \mathbb{1}/2^L$  and of its projection onto the free PXP sector,  $\rho_{ss}^{\text{PXP}} \propto \mathcal{P}_{\text{PXP}}[\rho_{ss}]$ , with no adjacent qubits in state  $|1_S\rangle$ , as selected by the initial state  $|\psi(0)\rangle = |\mathbf{0}_S\rangle$  [99, 121]. We observe that, for a long (metastable) timescale, dynamical order parameters are initially described by the prediction obtained with  $\rho_{ss}^{\text{PXP}}$ . After this transient regime, the system overcomes the blockade effect, which is not perfect due to finite values of  $V$ , and the average state eventually approaches  $\rho_{ss} = \mathbb{1}/2^L$ . The sampled quantum trajectories and their time-integrated observables follow the same trend and converge towards the same stationary values. We observe two important differences. First, we note a delay in the transient behavior of the quantum trajectories over the instantaneous ensemble-averaged dynamical order parameters. This delay is expected since time-integrated observables have some ‘‘inertia’’ due to their dependence on the past. Additionally, these quantities show a large variance over long time-periods. Recalling the dynamical heterogeneity observed in Figs. 1(b,c), these fluctuations are due to the intermittent exploration of different PXP sectors, based on the position of adjacent qubits in state  $|1_S\rangle$ . With the system being approximately confined in one sector over an extended period of time, observations dominantly account for the single-sector contribution. For very long times, the system eventually explores the Hilbert space uniformly (see  $\rho_{ss} = \mathbb{1}/2^L$ ) and also dynamical order parameters of single quantum trajectories approach the stationary prediction.

**Unraveling dynamical phases.** — The observed dynamical heterogeneity and large fluctuations are reminiscent of first-order phase transitions and phase coexistence [1]. In what follows, we make this connection concrete by means of a large deviation approach [3]. We consider the ensemble of quantum trajectories generated by the collision model,  $\{\eta(T), \pi(\eta(T))\}$ , and interpret this as a microcanonical ensemble. We further introduce a counting field  $s$ , playing the role of an inverse temperature, associated with a time-integrated observable  $\mathcal{O}$ , embodying instead an energy function [80–82, 104, 108]. Through these quantities, we define a canonical ensemble with probabilities  $\pi(\eta(T), s) \propto e^{-s\mathcal{O}(\eta(T))}\pi(\eta(T))$ . By varying the counting field  $s$ , we can thus control the probability over quantum trajectories. As we will show, this thermodynamic-like construction allows us to compute a suitable phase diagram for the stochastic process, demonstrating that the observed phenomenology is related to



**FIG. 3. Dynamical phase diagram.** (a) Temporal correlations  $c_\delta$ , with  $\delta = (0, 1)$ , for  $L = 6$ ,  $\Delta t = 1.25/\Omega$ ,  $\gamma = 3\Omega$ . The phase diagram in the interaction strength  $V$  and counting field  $s$  displays dynamical phases with decreased activity ( $s > 0$ ) that potentially come with vanishing temporal correlations. For  $s = 0$  and  $L = 4$  to  $L = 7$ , panel (b) shows that these distinct dynamical phases do not affect the bare average of dynamical observables. However, the sharp change with  $s$  displayed in panel (c) for  $V = 5.875\Omega$ , reveals nearby dynamical phases with drastically different temporal correlations that result in the large fluctuations observed Fig. 2.

the coexistence of dynamical phases.

Based on the observation in Fig. 1 that slow dynamics are accompanied by a space-time region of primarily ‘‘0’’ ancillary measurement outcomes, we take the time-integrated activity  $\mathcal{O}_0$  as observable associated with the counting field. Therefore, positive (negative) values of  $s$  will decrease (increase) the typical activity in the canonical ensemble. We consider the temporal correlations  $c_\delta$ , with  $\delta = (0, 1)$ , as our dynamical order parameter and investigate its stationary behavior upon variations of the counting field  $s$  and of the interaction strength  $V$ . The resulting phase diagram is shown in Fig. 3(a). At first, we observe temporal correlations that slowly increase with the interaction strength. Furthermore, for  $s > 0$ , i.e., for decreasing activities, we observe a drastic decrease of temporal correlations for certain interaction strengths  $V$ . Taking a closer look at  $s = 0$ , see Fig. 3(b), we see that the dynamical order parameter changes smoothly with  $V$ . This fact can be understood by recalling Eqs. (5,6) and considering that the stationary state, for  $s = 0$ , is the fully mixed state for any interaction strength  $V$ . Conversely, Fig. 3(c) shows that temporal correlations can suddenly drop for certain values of  $V$ , upon considering low-activity regimes ( $s > 0$ ). This abrupt change in the behavior of the dynamical order parameter in the vicinity of  $s \approx 0$ , which is the regime associated with collision model dynamics, explains the observed heterogeneity [see Fig. 1(b,c)] as stemming from the coexistence of different dynamical phases. This thus connects the observed phenomenology to the emergence of a first-order transition in the dynamical phase diagram of the collision model. It

further demonstrates how large deviation methods allow to unravel complex dynamical behavior in discrete-time open quantum dynamics.

Note that the phase diagram in Fig. 3(a) can also be computed on a real quantum processor. This can be achieved by deriving auxiliary collision models, whose probabilities over quantum trajectories are given by the canonical ensembles introduced via the counting field  $s$  [80, 81, 112].

**Conclusions and outlook.** — We have investigated discrete-time open-system dynamics inspired by the dissipative evolution of driven Rydberg atoms. Despite having a trivial stationary state, the model we have considered displays dynamical heterogeneity, stemming from the coexistence of fast and slow dynamics in single stochastic realizations. We have shown how such collective behavior can be characterized by means of dynamical order parameters, such as the frequency and the correlations of measurement outcomes on the environmental ancillary qubits. The investigation performed here is limited to relatively small system sizes, which however already show clear signatures of many-body phenomena. In order to address significantly larger system sizes, it would be necessary to devise suitable tensor-network approaches [91, 122–124]. Such larger systems should be, however, already accessible on state-of-the-art quantum processors, offering an intriguing perspective for the exploration of dynamical heterogeneity in glassy relaxation.

The code and the data that support the findings of this work are available on Zenodo [125].

**Acknowledgements.** — We thank Dominik Sulz, Cecilia De Fazio and Farokh Mivehvar for fruitful discussions. We acknowledge funding from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy – EXC-2111 – 390814868, through the Research Unit FOR 5413/1, Grant No. 465199066, and through the Research Unit FOR 5522/1, Grant No. 499180199. This project has also received funding from the European Union’s Horizon Europe research and innovation program under Grant Agreement No. 101046968 (BRISQ). F.C. is indebted to the Baden-Württemberg Stiftung for the financial support of this research project by the Eliteprogramme for Postdocs.

- 
- [1] N. Goldenfeld, *Lectures on phase transitions and the renormalization group* (CRC Press, 1992).
  - [2] D. Ruelle, *Thermodynamic Formalism: The Mathematical Structure of Equilibrium Statistical Mechanics*, Vol. 2 (Cambridge University Press, 2004).
  - [3] H. Touchette, The large deviation approach to statistical mechanics, *Phys. Rep.* **478**, 1 (2009).

- [4] I. Lesanovsky and J. P. Garrahan, Kinetic Constraints, Hierarchical Relaxation, and Onset of Glassiness in Strongly Interacting and Dissipative Rydberg Gases, *Phys. Rev. Lett.* **111**, 215305 (2013).
- [5] I. Lesanovsky, M. van Horsen, M. Guță, and J. P. Garrahan, Characterization of Dynamical Phase Transitions in Quantum Jump Trajectories Beyond the Properties of the Stationary State, *Phys. Rev. Lett.* **110**, 150401 (2013).
- [6] I. Lesanovsky and J. P. Garrahan, Out-of-equilibrium structures in strongly interacting Rydberg gases with dissipation, *Phys. Rev. A* **90**, 011603 (2014).
- [7] M. M. Valado, C. Simonelli, M. D. Hoogerland, I. Lesanovsky, J. P. Garrahan, E. Arimondo, D. Ciampini, and O. Morsch, Experimental observation of controllable kinetic constraints in a cold atomic gas, *Phys. Rev. A* **93**, 040701 (2016).
- [8] B. Olmos, I. Lesanovsky, and J. P. Garrahan, Out-of-equilibrium evolution of kinetically constrained many-body quantum systems under purely dissipative dynamics, *Phys. Rev. E* **90**, 042147 (2014).
- [9] G. Ferioli, A. Glicenstein, I. Ferrier-Barbut, and A. Browaeys, A non-equilibrium superradiant phase transition in free space, *Nat. Phys.* **19**, 1345 (2023).
- [10] C. De Fazio, J. P. Garrahan, and K. Klobas, Exact results on the dynamics of the stochastic Floquet-East model, [arXiv:2406.17464](https://arxiv.org/abs/2406.17464) (2024).
- [11] V. Lecomte, C. Appert-Rolland, and F. van Wijland, Thermodynamic Formalism for Systems with Markov Dynamics, *J. Stat. Phys.* **127**, 51 (2007).
- [12] J. P. Garrahan, R. L. Jack, V. Lecomte, E. Pitard, K. v. Duijvendijk, and F. v. Wijland, First-order dynamical phase transition in models of glasses: an approach based on ensembles of histories, *J. Phys. A Math. Theor.* **42**, 075007 (2009).
- [13] N. Cancrini, F. Martinelli, C. Roberto, and C. Toninelli, Kinetically constrained spin models, *Probab. Theory Relat. Fields* **140**, 459 (2008).
- [14] R. Gutiérrez, J. P. Garrahan, and I. Lesanovsky, Self-similar nonequilibrium dynamics of a many-body system with power-law interactions, *Phys. Rev. E* **92**, 062144 (2015).
- [15] J. W. P. Wilkinson, K. Klobas, T. Prosen, and J. P. Garrahan, Exact solution of the Floquet-PXP cellular automaton, *Phys. Rev. E* **102**, 062107 (2020).
- [16] G. Chen, W. Huang, and Y. Yao, Dynamics of entangled domain walls in the PXP model under driving: Crossover from prethermalization to localization, *Phys. Rev. B* **106**, 174302 (2022).
- [17] B. Bertini, P. Kos, and T. Prosen, Localized Dynamics in the Floquet Quantum East Model, *Phys. Rev. Lett.* **132**, 080401 (2024).
- [18] C. Gavazzoni, C. Brito, and M. Wyart, Testing Theories of the Glass Transition with the Same Liquid but Many Kinetic Rules, *Phys. Rev. Lett.* **132**, 248201 (2024).
- [19] T. Zhang and Z. Cai, Quantum Slush State in Rydberg Atom Arrays, *Phys. Rev. Lett.* **132**, 206503 (2024).
- [20] F. Ritort and P. Sollich, Glassy dynamics of kinetically constrained models, *Adv. Phys.* **52**, 219 (2003).
- [21] M. Merolle, J. P. Garrahan, and D. Chandler, Space-time thermodynamics of the glass transition, *Proc. Natl. Acad. Sci.* **102**, 10837 (2005).
- [22] J. P. Garrahan, R. L. Jack, V. Lecomte, E. Pitard, K. van Duijvendijk, and F. van Wijland, Dynamical

- First-Order Phase Transition in Kinetically Constrained Models of Glasses, *Phys. Rev. Lett.* **98**, 195702 (2007).
- [23] K. Binder and W. Kob, *Glassy Materials and Disordered Solids: An Introduction to Their Statistical Mechanics*, revised ed. (World Scientific, Singapore, 2011).
- [24] J. Sanders, M. Jonckheere, and S. Kokkelmans, Sub-Poissonian Statistics of Jamming Limits in Ultracold Rydberg Gases, *Phys. Rev. Lett.* **115**, 043002 (2015).
- [25] J. P. Garrahan, Aspects of non-equilibrium in classical and quantum systems: Slow relaxation and glasses, dynamical large deviations, quantum non-ergodicity, and open quantum dynamics, *Physica A* **504**, 130 (2018).
- [26] M. C. Bañuls and J. P. Garrahan, Using Matrix Product States to Study the Dynamical Large Deviations of Kinetically Constrained Models, *Phys. Rev. Lett.* **123**, 200601 (2019).
- [27] L. Causser, I. Lesanovsky, M. C. Bañuls, and J. P. Garrahan, Dynamics and large deviation transitions of the XOR-Fredrickson-Andersen kinetically constrained model, *Phys. Rev. E* **102**, 052132 (2020).
- [28] L. Causser, M. C. Bañuls, and J. P. Garrahan, Optimal sampling of dynamical large deviations via matrix product states, *Phys. Rev. E* **103**, 062144 (2021).
- [29] L. Causser, M. C. Bañuls, and J. P. Garrahan, Finite Time Large Deviations via Matrix Product States, *Phys. Rev. Lett.* **128**, 090605 (2022).
- [30] C. Ates, B. Olmos, J. P. Garrahan, and I. Lesanovsky, Dynamical phases and intermittency of the dissipative quantum Ising model, *Phys. Rev. A* **85**, 043620 (2012).
- [31] M. Marcuzzi, J. Minář, D. Barredo, S. de Léséleuc, H. Labuhn, T. Lahaye, A. Browaeys, E. Levi, and I. Lesanovsky, Facilitation Dynamics and Localization Phenomena in Rydberg Lattice Gases with Position Disorder, *Phys. Rev. Lett.* **118**, 063606 (2017).
- [32] M. van Horssen, E. Levi, and J. P. Garrahan, Dynamics of many-body localization in a translation-invariant quantum glass model, *Phys. Rev. B* **92**, 100305 (2015).
- [33] Z. Lan, M. van Horssen, S. Powell, and J. P. Garrahan, Quantum Slow Relaxation and Metastability due to Dynamical Constraints, *Phys. Rev. Lett.* **121**, 040603 (2018).
- [34] E. Gillman, F. Carollo, and I. Lesanovsky, Nonequilibrium Phase Transitions in (1+1)-Dimensional Quantum Cellular Automata with Controllable Quantum Correlations, *Phys. Rev. Lett.* **125**, 100403 (2020).
- [35] N. Pancotti, G. Giudice, J. I. Cirac, J. P. Garrahan, and M. C. Bañuls, Quantum East Model: Localization, Non-thermal Eigenstates, and Slow Dynamics, *Phys. Rev. X* **10**, 021051 (2020).
- [36] M. Magoni, C. Nill, and I. Lesanovsky, Coherent Spin-Phonon Scattering in Facilitated Rydberg Lattices, *Phys. Rev. Lett.* **132**, 133401 (2024).
- [37] R. P. Feynman, Simulating physics with computers, *Internat. J. Theoret. Phys.* **21**, 467 (1982).
- [38] J. Preskill, Quantum Computing in the NISQ era and beyond, *Quantum* **2**, 79 (2018).
- [39] J. Preskill, Quantum computing 40 years later, [arXiv:2106.10522](https://arxiv.org/abs/2106.10522) (2023).
- [40] R. J. Valencia-Tortora, N. Pancotti, and J. Marino, Kinetically Constrained Quantum Dynamics in Superconducting Circuits, *PRX Quantum* **3**, 020346 (2022).
- [41] N. U. Köyliüoğlu, N. Maskara, J. Feldmeier, and M. D. Lukin, Floquet engineering of interactions and entanglement in periodically driven Rydberg chains, [arXiv:2408.02741](https://arxiv.org/abs/2408.02741) (2024).
- [42] J. Feldmeier, N. Maskara, N. U. Köyliüoğlu, and M. D. Lukin, Quantum simulation of dynamical gauge theories in periodically driven Rydberg atom arrays, [arXiv:2408.02733](https://arxiv.org/abs/2408.02733) (2024).
- [43] R. J. Valencia-Tortora, N. Pancotti, M. Fleischhauer, H. Bernien, and J. Marino, Rydberg Platform for Nonergodic Chiral Quantum Dynamics, *Phys. Rev. Lett.* **132**, 223201 (2024).
- [44] L. Zhao, P. R. Datla, W. Tian, M. M. Aliyu, and H. Loh, Observation of quantum thermalization restricted to Hilbert space fragments, [arXiv:2403.09517](https://arxiv.org/abs/2403.09517) (2024).
- [45] P. Richerme, Z.-X. Gong, A. Lee, C. Senko, J. Smith, M. Foss-Feig, S. Michalakis, A. V. Gorshkov, and C. Monroe, Non-local propagation of correlations in quantum systems with long-range interactions, *Nature* **511**, 198 (2014).
- [46] R. Islam, R. Ma, P. M. Preiss, M. Eric Tai, A. Lukin, M. Rispoli, and M. Greiner, Measuring entanglement entropy in a quantum many-body system, *Nature* **528**, 77 (2015).
- [47] C. Gross and I. Bloch, Quantum simulations with ultracold atoms in optical lattices, *Science* **357**, 995 (2017).
- [48] H. Bernien, S. Schwartz, A. Keesling, H. Levine, A. Omran, H. Pichler, S. Choi, A. S. Zibrov, M. Endres, M. Greiner, *et al.*, Probing many-body dynamics on a 51-atom quantum simulator, *Nature* **551**, 579 (2017).
- [49] C. Gross and W. S. Bakr, Quantum gas microscopy for single atom and spin detection, *Nat. Phys.* **17**, 1316 (2021).
- [50] M. Rodriguez-Vega, E. Carlander, A. Bahri, Z.-X. Lin, N. A. Sinitsyn, and G. A. Fiete, Real-time simulation of light-driven spin chains on quantum computers, *Phys. Rev. Research* **4**, 013196 (2022).
- [51] W. J. Huggins, B. A. O’Gorman, N. C. Rubin, D. R. Reichman, R. Babbush, and J. Lee, Unbiasing fermionic quantum Monte Carlo with a quantum computer, *Nature* **603**, 416 (2022).
- [52] Y. Yang, A. Christianen, S. Coll-Vinent, V. Smelyanskiy, M. C. Bañuls, T. E. O’Brien, D. S. Wild, and J. I. Cirac, Simulating Prethermalization Using Near-Term Quantum Computers, *PRX Quantum* **4**, 030320 (2023).
- [53] E. Chertkov, Z. Cheng, A. C. Potter, S. Gopalakrishnan, T. M. Gatterman, J. A. Gerber, K. Gilmore, D. Gresh, A. Hall, A. Hankin, *et al.*, Characterizing a non-equilibrium phase transition on a quantum computer, *Nat. Phys.* **19**, 1 (2023).
- [54] J. D. Guimarães, J. Lim, M. I. Vasilevskiy, S. F. Huelga, and M. B. Plenio, Noise-Assisted Digital Quantum Simulation of Open Systems Using Partial Probabilistic Error Cancellation, *PRX Quantum* **4**, 040329 (2023).
- [55] Y. Kim, A. Eddins, S. Anand, K. X. Wei, E. van den Berg, S. Rosenblatt, H. Nayfeh, Y. Wu, M. Zaletel, K. Temme, and A. Kandala, Evidence for the utility of quantum computing before fault tolerance, *Nature* **618**, 500 (2023).
- [56] X. Mi, A. A. Michailidis, S. Shabani, K. C. Miao, P. V. Klimov, J. Lloyd, E. Rosenberg, R. Acharya, I. Aleiner, T. I. Andersen, *et al.*, Stable quantum-correlated many-body states through engineered dissipation, *Science* **383**, 1332 (2024).
- [57] B. Fauseweh, Quantum many-body simulations on digital quantum computers: State-of-the-art and future challenges, *Nat. Commun.* **15**, 2123 (2024).

- [58] S. L. Cornish, M. R. Tarbutt, and K. R. A. Hazzard, Quantum computation and quantum simulation with ultracold molecules, *Nat. Phys.* **20**, 1 (2024).
- [59] G. Pichard, D. Lim, E. Bloch, J. Vaneecclo, L. Bourachot, G.-J. Both, G. Mériaux, S. Dutartre, R. Hostein, J. Paris, *et al.*, Rearrangement of single atoms in a 2000-site optical tweezers array at cryogenic temperatures, [arXiv:2405.19503](https://arxiv.org/abs/2405.19503) (2024).
- [60] F. Cesa and H. Pichler, Universal Quantum Computation in Globally Driven Rydberg Atom Arrays, *Phys. Rev. Lett.* **131**, 170601 (2023).
- [61] G. Bornet, G. Emperauger, C. Chen, F. Machado, S. Chern, L. Leclerc, B. Gély, Y. T. Chew, D. Barredo, T. Lahaye, *et al.*, Enhancing a Many-Body Dipolar Rydberg Tweezer Array with Arbitrary Local Controls, *Phys. Rev. Lett.* **132**, 263601 (2024).
- [62] D. Bluvstein, S. J. Evered, A. A. Geim, S. H. Li, H. Zhou, T. Manovitz, S. Ebadi, M. Cain, M. Kalinowski, D. Hangleiter, *et al.*, Logical quantum processor based on reconfigurable atom arrays, *Nature* **626**, 58 (2024).
- [63] P. L. Ocola, I. Dimitrova, B. Grinkemeyer, E. Guardado-Sanchez, T. Dordević, P. Samutpraphoot, V. Vuletić, and M. D. Lukin, Control and Entanglement of Individual Rydberg Atoms near a Nanoscale Device, *Phys. Rev. Lett.* **132**, 113601 (2024).
- [64] S. Anand, C. E. Bradley, R. White, V. Ramesh, K. Singh, and H. Bernien, A dual-species Rydberg array, [arXiv:2401.10325](https://arxiv.org/abs/2401.10325) (2024).
- [65] C. D. Bruzewicz, J. Chiaverini, R. McConnell, and J. M. Sage, Trapped-ion quantum computing: Progress and challenges, *Appl. Phys. Rev.* **6**, 021314 (2019).
- [66] J. M. Pino, J. M. Dreiling, C. Figgatt, J. P. Gaebler, S. A. Moses, M. S. Allman, C. H. Baldwin, M. Foss-Feig, D. Hayes, K. Mayer, C. Ryan-Anderson, and B. Neyenhuis, Demonstration of the trapped-ion quantum CCD computer architecture, *Nature* **592**, 209 (2021).
- [67] S. Heußen, L. Postler, M. Rispler, I. Pogorelov, C. D. Marciniak, T. Monz, P. Schindler, and M. Müller, Strategies for a practical advantage of fault-tolerant circuit design in noisy trapped-ion quantum computers, *Phys. Rev. A* **107**, 042422 (2023).
- [68] S.-A. Guo, Y.-K. Wu, J. Ye, L. Zhang, W.-Q. Lian, R. Yao, Y. Wang, R.-Y. Yan, Y.-J. Yi, Y.-L. Xu, *et al.*, A site-resolved two-dimensional quantum simulator with hundreds of trapped ions, *Nature* **630**, 1 (2024).
- [69] M. Iqbal, N. Tantivasadakarn, T. M. Gatterman, J. A. Gerber, K. Gilmore, D. Gresh, A. Hankin, N. Hewitt, C. V. Horst, M. Matheny, *et al.*, Topological order from measurements and feed-forward on a trapped ion quantum computer, *Commun. Phys.* **7**, 1 (2024).
- [70] D. C. McKay, S. Filipp, A. Mezzacapo, E. Magesan, J. M. Chow, and J. M. Gambetta, Universal Gate for Fixed-Frequency Qubits via a Tunable Bus, *Phys. Rev. Appl.* **6**, 064007 (2016).
- [71] P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver, A quantum engineer's guide to superconducting qubits, *Appl. Phys. Rev.* **6**, 021318 (2019).
- [72] T. Alexander, N. Kanazawa, D. J. Egger, L. Capelluto, C. J. Wood, *et al.*, Qiskit pulse: programming quantum computers through the cloud with pulses, *Quantum Sci. Technol.* **5**, 044006 (2020).
- [73] J. M. Koh, S.-N. Sun, M. Motta, and A. J. Minnich, Measurement-induced entanglement phase transition on a superconducting quantum processor with mid-circuit readout, *Nat. Phys.* **19**, 1 (2023).
- [74] T. Kehrer, T. Nadolny, and C. Bruder, Improving transmon qubit measurement on IBM Quantum hardware, *Phys. Rev. Research* **6**, 013050 (2024).
- [75] G. García-Pérez, M. A. C. Rossi, and S. Maniscalco, IBM Q Experience as a versatile experimental testbed for simulating open quantum systems, *NPJ Quantum Inf.* **6**, 1 (2020).
- [76] M. Cattaneo, G. De Chiara, S. Maniscalco, R. Zambrini, and G. L. Giorgi, Collision Models Can Efficiently Simulate Any Multipartite Markovian Quantum Dynamics, *Phys. Rev. Lett.* **126**, 130403 (2021).
- [77] M. Cattaneo, G. L. Giorgi, R. Zambrini, and S. Maniscalco, A brief journey through collision models for multipartite open quantum dynamics, *Open Syst. Inf. Dyn.* **29**, 2250015 (2022).
- [78] P. Strasberg, G. Schaller, T. Brandes, and M. Esposito, Quantum and Information Thermodynamics: A Unifying Framework Based on Repeated Interactions, *Phys. Rev. X* **7**, 021003 (2017).
- [79] F. Ciccarello, S. Lorenzo, V. Giovannetti, and G. M. Palma, Quantum collision models: open system dynamics from repeated interactions, *Phys. Rep.* **954**, 1 (2022).
- [80] D. Cilluffo, G. Buonaiuto, I. Lesanovsky, A. Carollo, S. Lorenzo, G. M. Palma, F. Ciccarello, and F. Carollo, Microscopic biasing of discrete-time quantum trajectories, *Quantum Sci. Technol.* **6**, 045011 (2021).
- [81] M. Cech, I. Lesanovsky, and F. Carollo, Thermodynamics of Quantum Trajectories on a Quantum Computer, *Phys. Rev. Lett.* **131**, 120401 (2023).
- [82] M. van Horssen and M. Guță, Sanov and central limit theorems for output statistics of quantum Markov chains, *J. Math. Phys.* **56**, 022109 (2015).
- [83] C. Pérez-Espigares and P. I. Hurtado, Sampling rare events across dynamical phase transitions, *Chaos* **29**, 083106 (2019).
- [84] D. Cilluffo, G. Buonaiuto, S. Lorenzo, G. M. Palma, F. Ciccarello, F. Carollo, and I. Lesanovsky, Witnessing nonclassicality through large deviations in quantum optics, *Phys. Rev. Research* **2**, 023078 (2020).
- [85] F. Carollo, R. L. Jack, and J. P. Garrahan, Unraveling the Large Deviation Statistics of Markovian Open Quantum Systems, *Phys. Rev. Lett.* **122**, 130605 (2019).
- [86] T. Vovk and H. Pichler, Entanglement-Optimal Trajectories of Many-Body Quantum Markov Processes, *Phys. Rev. Lett.* **128**, 243601 (2022).
- [87] A. Cabot, L. S. Muhle, F. Carollo, and I. Lesanovsky, Quantum trajectories of dissipative time crystals, *Phys. Rev. A* **108**, L041303 (2023).
- [88] P. J. Paulino, I. Lesanovsky, and F. Carollo, Large Deviation Full Counting Statistics in Adiabatic Open Quantum Dynamics, *Phys. Rev. Lett.* **132**, 260402 (2024).
- [89] C. A. Brown, K. Macieszczak, and R. L. Jack, Unraveling metastable Markovian open quantum systems, *Phys. Rev. A* **109**, 022244 (2024).
- [90] M. Eissler, I. Lesanovsky, and F. Carollo, Unraveling-induced entanglement phase transition in diffusive trajectories of continuously monitored noninteracting fermionic systems, [arXiv:2406.04869](https://arxiv.org/abs/2406.04869) (2024).
- [91] L. Causer, M. C. Bañuls, and J. P. Garrahan, Dynamical heterogeneity and large deviations in the

- open quantum East glass model from tensor networks, [arXiv:2404.03750](#) (2024).
- [92] Z.-K. Liu, K.-H. Sun, A. Cabot, F. Carollo, J. Zhang, Z.-Y. Zhang, L.-H. Zhang, B. Liu, T.-Y. Han, Q. Li, *et al.*, Microwave control of collective quantum jump statistics of a dissipative Rydberg gas, [arXiv:2402.04815](#) (2024).
- [93] G. T. Landi, Patterns in the jump-channel statistics of open quantum systems, [arXiv:2305.07957](#) (2023).
- [94] G. Passarelli, X. Turkeshi, A. Russomanno, P. Lucignano, M. Schirò, and R. Fazio, Many-Body Dynamics in Monitored Atomic Gases without Postselection Barrier, *Phys. Rev. Lett.* **132**, 163401 (2024).
- [95] H. Breuer, P. Breuer, F. Petruccione, and S. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, New York, 2002).
- [96] C. Pérez-Espigares, I. Lesanovsky, J. P. Garrahan, and R. Gutiérrez, Glassy dynamics due to a trajectory phase transition in dissipative Rydberg gases, *Phys. Rev. A* **98**, 021804 (2018).
- [97] I. Lesanovsky, K. Macieszczak, and J. P. Garrahan, Non-equilibrium absorbing state phase transitions in discrete-time quantum cellular automaton dynamics on spin lattices, *Quantum Sci. Technol.* **4**, 02LT02 (2019).
- [98] I. Lesanovsky and H. Katsura, Interacting Fibonacci anyons in a Rydberg gas, *Phys. Rev. A* **86**, 041601 (2012).
- [99] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, Weak ergodicity breaking from quantum many-body scars, *Nat. Phys.* **14**, 745 (2018).
- [100] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, Quantum scarred eigenstates in a Rydberg atom chain: Entanglement, breakdown of thermalization, and stability to perturbations, *Phys. Rev. B* **98**, 155134 (2018).
- [101] M. Serbyn, D. A. Abanin, and Z. Papić, Quantum many-body scars and weak breaking of ergodicity, *Nat. Phys.* **17**, 675 (2021).
- [102] S. Moudgalya, B. A. Bernevig, and N. Regnault, Quantum many-body scars and Hilbert space fragmentation: a review of exact results, *Rep. Prog. Phys.* **85**, 086501 (2022).
- [103] R. L. Jack and P. Sollich, Large Deviations and Ensembles of Trajectories in Stochastic Models, *Prog. Theor. Phys. Suppl.* **184**, 304 (2010).
- [104] J. P. Garrahan and I. Lesanovsky, Thermodynamics of Quantum Jump Trajectories, *Phys. Rev. Lett.* **104**, 160601 (2010).
- [105] R. Chetrite and S. Gupta, Two Refreshing Views of Fluctuation Theorems Through Kinematics Elements and Exponential Martingale, *J. Stat. Phys.* **143**, 543 (2011).
- [106] R. Chetrite and H. Touchette, Nonequilibrium Microcanonical and Canonical Ensembles and Their Equivalence, *Phys. Rev. Lett.* **111**, 120601 (2013).
- [107] R. Chetrite and H. Touchette, Variational and optimal control representations of conditioned and driven processes, *J. Stat. Mech. Theory Exp.* **2015**, P12001 (2015).
- [108] F. Carollo, J. P. Garrahan, I. Lesanovsky, and C. Pérez-Espigares, Making rare events typical in Markovian open quantum systems, *Phys. Rev. A* **98**, 010103 (2018).
- [109] K. Kraus, A. Böhm, J. Dollard, and W. Wootters, *States, Effects, and Operations: Fundamental Notions of Quantum Theory*, Lecture Notes in Physics (Springer Berlin Heidelberg, 1983).
- [110] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition* (Cambridge University Press, 2010).
- [111] G. Cemin, M. Cech, E. Weiss, S. Soltan, D. Braun, I. Lesanovsky, and F. Carollo, Machine learning of quantum channels on NISQ devices, [arXiv:2405.12598](#) (2024).
- [112] See Supplemental Material, which further contains Refs. [113–115], for details.
- [113] R. Löw, H. Weimer, J. Nipper, J. B. Balewski, B. Butscher, H. P. Büchler, and T. Pfau, An experimental and theoretical guide to strongly interacting Rydberg gases, *J. Phys. B: At., Mol. Opt. Phys.* **45**, 113001 (2012).
- [114] J. R. Johansson, P. D. Nation, and F. Nori, QuTiP 2: A Python framework for the dynamics of open quantum systems, *Comput. Phys. Commun.* **184**, 1234 (2013).
- [115] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, 2009).
- [116] M. Saffman, T. G. Walker, and K. Mølmer, Quantum information with Rydberg atoms, *Rev. Mod. Phys.* **82**, 2313 (2010).
- [117] A. Gaëtan, Y. Miroshnychenko, T. Wilk, A. Chotia, M. Viteau, D. Comparat, P. Pillet, A. Browaeys, and P. Grangier, Observation of collective excitation of two individual atoms in the Rydberg blockade regime, *Nat. Phys.* **5**, 115 (2009).
- [118] T. Wilk, A. Gaëtan, C. Evellin, J. Wolters, Y. Miroshnychenko, P. Grangier, and A. Browaeys, Entanglement of Two Individual Neutral Atoms Using Rydberg Blockade, *Phys. Rev. Lett.* **104**, 010502 (2010).
- [119] L. Isenhower, E. Urban, X. L. Zhang, A. T. Gill, T. Henage, T. A. Johnson, T. G. Walker, and M. Saffman, Demonstration of a Neutral Atom Controlled-NOT Quantum Gate, *Phys. Rev. Lett.* **104**, 010503 (2010).
- [120] X. L. Zhang, L. Isenhower, A. T. Gill, T. G. Walker, and M. Saffman, Deterministic entanglement of two neutral atoms via Rydberg blockade, *Phys. Rev. A* **82**, 030306 (2010).
- [121] Y. Li, P. Sala, and F. Pollmann, Hilbert space fragmentation in open quantum systems, *Phys. Rev. Research* **5**, 043239 (2023).
- [122] Y. Yang, S. Iblisdir, J. I. Cirac, and M. C. Bañuls, Probing Thermalization through Spectral Analysis with Matrix Product Operators, *Phys. Rev. Lett.* **124**, 100602 (2020).
- [123] E. Gillman, F. Carollo, and I. Lesanovsky, Quantum and Classical Temporal Correlations in (1+1)D Quantum Cellular Automata, *Phys. Rev. Lett.* **127**, 230502 (2021).
- [124] G. E. Fux, D. Kilda, B. W. Lovett, and J. Keeling, Tensor network simulation of chains of non-Markovian open quantum systems, *Phys. Rev. Research* **5**, 033078 (2023).
- [125] M. Cech, M. Cea, M. C. Bañuls, I. Lesanovsky, and F. Carollo, [Data and code for “Space-time correlations in monitored kinetically constrained discrete-time quantum dynamics”](#) (2024).



# SUPPLEMENTAL MATERIAL

## Space-time correlations in monitored kinetically constrained discrete-time quantum dynamics

Marcel Cech<sup>1</sup>, María Cea,<sup>2,3</sup> Mari Carmen Bañuls,<sup>2,3</sup> Igor Lesanovsky,<sup>4,5</sup> and Federico Carollo<sup>1</sup>

<sup>1</sup>*Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, 72076 Tübingen, Germany*

<sup>2</sup>*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, D-85748 Garching, Germany*

<sup>3</sup>*Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, D-80799 München, Germany*

<sup>4</sup>*Institut für Theoretische Physik and Center for Integrated Quantum Science and Technology, Universität Tübingen, Auf der Morgenstelle 14, 72076 Tübingen, Germany*

<sup>5</sup>*School of Physics and Astronomy and Centre for the Mathematics and Theoretical Physics of Quantum Non-Equilibrium Systems, The University of Nottingham, Nottingham, NG7 2RD, United Kingdom*

### CONTINUOUS-TIME DYNAMICS

In this section, we connect the continuous-time limit,  $\Delta t \rightarrow 0$ , of the collision model dynamics with a Lindblad master equation describing driven and interacting Rydberg atoms subject to local dephasing. We then study the phase diagram, when we control the activity in the canonical ensemble.

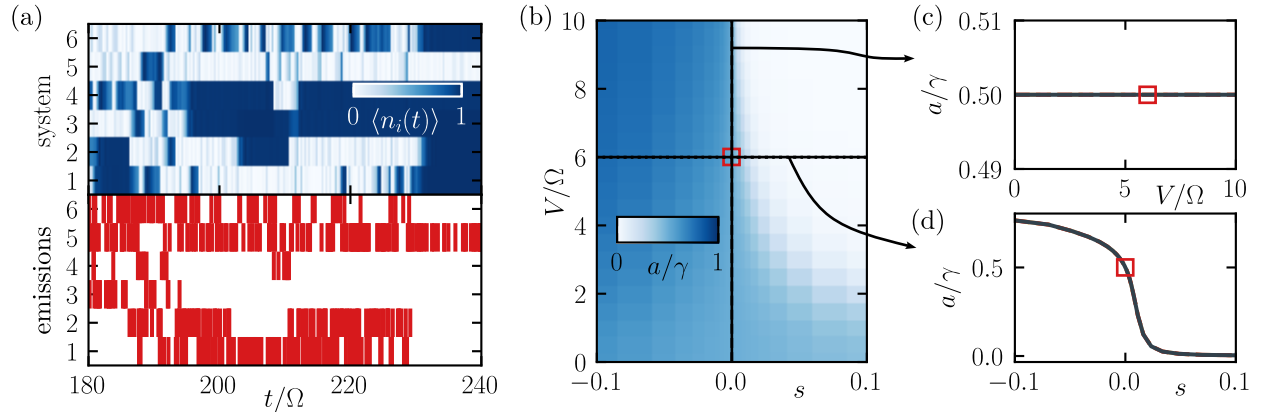


FIG. S1. **Continuous-time dynamics.** (a) Stochastic realization and quantum trajectories for  $L = 6$ ,  $V = 6\Omega$  and  $\gamma = 3\Omega$ . In the top panel, we show the spatially resolved probability  $\langle n_i(t) \rangle$  to find a qubit in state  $|1_S\rangle$ . In the bottom panel, a red stroke at the space-time  $(i, t)$  represents an emission event associated with the action of the jump operator  $J_i = \sqrt{\gamma}P_i$  at time  $t$ . (b) Dynamical phases for  $L = 6$  qubits and  $\gamma = 3\Omega$ . We change the interaction strength  $V$  and the conjugate field  $s$  and plot the stationary activity in the respective ensemble. (c) Cut at  $s = 0$ . As outlined in the text, we observe a constant stationary activity  $a_{ss} = \gamma/2$ , when changing  $V$ . (d) Cut at  $V = 6\Omega$ . We show the dependence of the stationary activity on the counting field  $s$  for  $L = 4$  to  $L = 7$ . The curves for different system sizes overlap, but the separation of dynamical phases is not as sharp as in Fig. 3.

In a first step, we perform the continuous-time limit  $\Delta t \rightarrow 0$ . The average collision model dynamics, with  $U = e^{-i\Delta t(H_S \otimes \mathbb{1} + H_{\text{int}})}$  [see Eq. (1)], of system in state  $\rho(t)$  for small collision times and ancillas initialized in  $\rho_A = |\mathbf{0}_A\rangle\langle\mathbf{0}_A|$  can be written as

$$\frac{\rho(t) - \rho(t-1)}{\Delta t} \approx -i \left[ H_S + \text{Tr}_A \{ H_{\text{int}} \rho_A \}, \rho(t-1) \right] + \sum_{\{\mathbf{k}\}} \left( J_{\mathbf{k}} \rho(t-1) J_{\mathbf{k}}^\dagger - \frac{1}{2} \left\{ J_{\mathbf{k}}^\dagger J_{\mathbf{k}}, \rho(t-1) \right\} \right). \quad (\text{S1})$$

In Eq. (S1),  $\text{Tr}_A$  denotes the trace over all ancillary two-level systems,  $\{X, Y\} = XY + YX$  the anticommutator and  $J_{\mathbf{k}} = \langle \mathbf{k}_A | H_{\text{int}} | \mathbf{0}_A \rangle \Delta t$  the potential jump-operators [79]. Based on Eq. (1), the system-ancilla interaction is

$H_{\text{int}} = \sqrt{\gamma/\Delta t} \left( \sum_{i=1}^L P_i \otimes \tau_i^x \right)$  and thus, we obtain  $\text{Tr}_A \{ H_{\text{int}} \rho_A \} = 0$  and  $J_{\mathbf{k}} = J_i = \sqrt{\gamma} P_i$  if  $\mathbf{k}$  includes a 1 for the  $i$ th ancilla and 0 otherwise. Therefore, the Lindblad master equation

$$\dot{\rho} = \mathcal{L}[\rho] = -i[H_S, \rho] + \sum_{i=1}^L \left( J_i \rho J_i - \frac{1}{2} \{ J_i^2, \rho \} \right), \quad (\text{S2})$$

with Hamiltonian  $H_S = \Omega \sum_{i=1}^L \sigma_i^x + V \sum_{i=1}^L n_i n_{i+1}$  and hermitian jump operators  $J_i = \sqrt{\gamma} P_i$ , describes the continuous-time limit. Here,  $H_S$  models driven Rydberg atoms, whose interactions are approximated by the dominant nearest-neighbor contribution, and  $\{J_i\}$  resembles the intrinsic or engineered dephasing [4, 7, 96, 113].

In Fig. S1(a), we now investigate the dynamics by means of quantum jump trajectories obtained by unraveling the master equation with the quantum Hamiltonian  $H_S$  and jump operators  $\{J_i\}$ . The simulations are performed using the python package QuTiP and its Monte-Carlo solver `mcsolve` [114]. In the top panel, we show the spatially resolved probability  $\langle n_i(t) \rangle$  to find the  $i$ th qubit in  $|1_S\rangle$ . Similar to Fig. 1, we observe that adjacent qubits in this state, if dynamically created, show slower changes of this probability than individual ones. This heterogeneity is also reflected in the lower panel, where we plot the locally resolved emissions, i.e., actions of the respective jump operators  $\{J_i\}$ . Here, the space-time region of adjacent qubits with  $\langle n_i(t) \rangle \approx 1$  in the system is completely inactive.

Similar to the approach in the main manuscript, we consider the instantaneous activity  $a(t)$  associated with frequency of abrupt changes per qubit in the quantum jump Monte-Carlo approach. On average, the activity is given by  $a(t) = (\sum_{i=1}^L \text{Tr}\{J_i^2 \rho(t)\})/L$  [95, 115]. Since Eq. (S2) features hermitian jump operators  $J_i$ , the stationary state is again the fully mixed state  $\rho_{\text{ss}} = \mathbb{1}/2^L$ . Moreover, we can directly evaluate the stationary activity and find  $a_{\text{ss}} = \gamma/2$ , since  $\text{Tr}\{J_i^2\} = \gamma 2^{L-1}$ .

In order to access the fluctuations in the activity, we resort again to the thermodynamics of quantum trajectories [104, 108]. Here, we calculate the dominant eigenvalue of the tilted Lindblad master equation

$$\mathcal{L}_s[\rho] = -i[H_S, \rho] + \sum_{i=1}^L \left( e^{-s} J_i \rho J_i - \frac{1}{2} \{ J_i^2, \rho \} \right), \quad (\text{S3})$$

to extract the scaled cumulant generating function  $\theta(s)$  of the activity. Analogously to the approach in the main manuscript, the counting field  $s$  controls the activity of the canonical ensemble that we construct utilizing a Gibbs weight with the time-integrated activity embodying an energy function. Panel (b) shows the first scaled cumulant,  $-\theta'(s)$ , which describes the phase diagram of the stationary activity. Utilizing the cuts at  $s = 0$  in panel (c) and at  $V = 6\Omega$  in panel (d), we observe that the heterogeneity in panel (a) can be understood in terms of two distinct dynamical phases. While for  $s \leq 0$ , the activity is (approximately) constant in  $V$ , it decreases for positive counting fields  $s$  as we increase  $V$ . However, we also observe that the separation between the distinct dynamical phases, i.e., inactive at  $s > 0$  and active at  $s < 0$ , is not as sharp as in the discrete-time dynamics considered in the main manuscript, for similar values of  $V/\Omega$ .

## UNITALITY AND STATIONARY STATE

As anticipated in the main manuscript, the quantum channel representing the average dynamics considered in this work is unital, i.e.,  $\mathcal{E}[\mathbb{1}] = \mathbb{1}$ . In this section, we provide the corresponding arguments for this statement and moreover argue that the fully mixed state  $\mathbb{1}/2^L$  is the stationary state of the average dynamics for finite interaction strengths  $V$ .

We start with the observation that the collision Hamiltonian in Eq. (1) only contains hermitian and commuting operators  $\tau_i^x$  that act on the ancillas. Furthermore, we utilize the freedom of choice in the trace to write

$$\mathcal{E}[\rho] = \text{Tr}_A \{ U(\rho \otimes |\mathbf{0}_A\rangle\langle\mathbf{0}_A|) U^\dagger \} = \sum_{\{m\}} \langle m_A | U | \mathbf{0}_A \rangle \rho \langle \mathbf{0}_A | U^\dagger | m_A \rangle = \sum_{\{m\}} K_m \rho K_m^\dagger, \quad (\text{S4})$$

with potentially different Kraus operators  $K_m = \langle m_A | U | \mathbf{0}_A \rangle$  for the chosen basis enumerated by  $m$ . Note that this simply establishes the unitary freedom of Kraus operators representing a certain quantum channel [110]. Choosing the eigenbasis labeled by  $m$  such that each  $|m_A\rangle$  is an eigenstate of the ancillary part of the system-ancilla interaction, we immediately see that every Kraus operators  $K_m$  becomes proportional to a unitary matrix. Therefore, together with the trace-preservation of the quantum channel  $\mathcal{E}$ , i.e.,  $\mathcal{E}^*[\mathbb{1}] = \sum_{\{m\}} K_m^\dagger K_m = \mathbb{1}$ , we find that  $\mathcal{E}[\mathbb{1}] = \sum_{\{m\}} K_m K_m^\dagger = \mathbb{1}$ .

Finally, we argue that the fully mixed state is the stationary state  $\rho_{\text{ss}}$  of the average dynamics for finite  $V$ . Unlike for the PXP model, the finite interaction strength  $V$  and the transverse driving with  $\Omega$  ensure that there is no sector of the Hilbert space that is dynamically disconnected from any other. The unitality therefore makes the fully mixed state  $\mathbf{1}/2^L$  the (unique) stationary state of the dynamics.

### RESET-FREE IMPLEMENTATION

In this section, we outline the implementation of the discrete-time open-system dynamics of the collision model described by Eq. (1) without resetting the ancillas after every collision. We show how a simple post-processing of the observed quantum trajectory can facilitate the simulation of the considered collision model dynamics on a quantum processor.

In contrast to the arguments to prove that the quantum channel is unital, the unraveling into stochastic realizations depends genuinely on the specific choice of the measurement basis. For the specific collision model in this work however, we notice that the system-ancilla interaction in Eq. (1) is inversion symmetric on the ancillary part, i.e., invariant under the exchange of  $|0_A\rangle$  and  $|1_A\rangle$ . We can exploit this symmetry to investigate closer  $K_{\mathbf{k}|\mathbf{k}'} = \langle \mathbf{k}_A | U | \mathbf{k}'_A \rangle$ , representing the observation of the ancilla measurement  $\mathbf{k}$  after previously observing  $\mathbf{k}'$ . By omitting the reset,  $K_{|\mathbf{k}-\mathbf{k}'|} = \langle \mathbf{k} - \mathbf{k}' |_A | U | \mathbf{0}_A \rangle$  in the usual collision model setup is replaced by the Kraus operator  $K_{\mathbf{k}|\mathbf{k}'}$  in the reset-free implementation. Therefore, reinterpreting a change in the measurement outcome of the  $i$ th ancilla between two time-steps in the reset-free quantum trajectory stands in one-to-one correspondence with the observation of a ‘1’ measurement outcome in the usual collision model setup that resets the ancillas before the collision takes place.

### PROBABILITIES AND DYNAMICAL ORDER PARAMETERS

In this section, we discuss the necessary steps to obtain Eqs. (5,6) and demonstrate their application in deriving the instantaneous activity and space-time correlations in Eqs. (3,4). For most of this discussion, we follow the steps outlined in Ref. [93] that directly apply to our collision model dynamics.

Directly from the definition of the stochastic realizations in discrete-time open quantum dynamics, the probability to observe a specific trajectory is given by

$$\pi(\eta(T) = [\mathbf{k}(1), \dots, \mathbf{k}(T)]) = \|K_{\mathbf{k}(T)} \dots K_{\mathbf{k}(1)} |\psi(0)\rangle\|^2 = \text{Tr} \left\{ K_{\mathbf{k}(T)} \dots K_{\mathbf{k}(1)} \rho(0) K_{\mathbf{k}(1)}^\dagger \dots K_{\mathbf{k}(T)}^\dagger \right\}, \quad (\text{S5})$$

with the initial density matrix  $\rho(0) = |\psi(0)\rangle\langle\psi(0)|$ . To calculate the instantaneous activity and space-time correlations in Eqs. (3,4), we marginalize over measurement outcomes that do not appear in the considered discrete times. For example, the probability to observe the measurement outcomes  $\mathbf{k}$  at the discrete-time  $t$  is

$$\begin{aligned} p(\mathbf{k}(t) = \mathbf{k}) &= \sum_{\{\eta\}} \pi(\eta = [\mathbf{k}(1), \dots, \mathbf{k}(t)]) \delta_{\mathbf{k}(t), \mathbf{k}} \\ &= \sum_{\{\mathbf{k}(t-1)\}} \dots \sum_{\{\mathbf{k}(1)\}} \text{Tr} \left\{ K_{\mathbf{k}} K_{\mathbf{k}(t-1)} \dots K_{\mathbf{k}(1)} \rho(0) K_{\mathbf{k}(1)}^\dagger \dots K_{\mathbf{k}(t-1)}^\dagger K_{\mathbf{k}}^\dagger \right\} \\ &= \text{Tr} \left\{ K_{\mathbf{k}} \rho(t-1) K_{\mathbf{k}}^\dagger \right\}, \end{aligned} \quad (\text{S6})$$

where we utilize the subsequent application of Kraus maps as in  $\rho(1) = \sum_{\{\mathbf{k}(1)\}} K_{\mathbf{k}(1)} \rho(0) K_{\mathbf{k}(1)}^\dagger$ . Analogously, we find the joined probability of  $\mathbf{k}$  and  $\mathbf{k}'$  as  $\mathbf{k}(t)$  and  $\mathbf{k}(t-\delta_t)$  to be

$$p(\mathbf{k}(t) = \mathbf{k}, \mathbf{k}(t-\delta_t) = \mathbf{k}') = \text{Tr} \left\{ K_{\mathbf{k}} \mathcal{E}^{\delta_t-1} \left[ K_{\mathbf{k}'} \rho(t-\delta_t-1) K_{\mathbf{k}'}^\dagger \right] K_{\mathbf{k}}^\dagger \right\}. \quad (\text{S7})$$

In this way, we obtain both Eq. (5) and a generalized expression of Eq. (6) for  $\delta_t \geq 1$ . Note that we introduce a prime in the above notation to better distinguish the measurement results at  $t-\delta_t$  and at  $t$  in Eqs. (6,S7).

We can utilize these probabilities to obtain the instantaneous activity

$$\overline{k_i(t)} = \sum_{\{\mathbf{k}\}} p(\mathbf{k}(t) = \mathbf{k}) k_i, \quad (\text{S8})$$

and space-time correlations

$$\overline{k_i(t-\delta_t)k_{i+\delta_i}(t)} = \sum_{\{\mathbf{k}, \mathbf{k}'\}} p(\mathbf{k}(t)=\mathbf{k}, \mathbf{k}(t-\delta_t)=\mathbf{k}') k_{i+\delta_i} k'_i, \quad (\text{S9})$$

as the weighted sums of the ensemble's probabilities.

## UNRAVELING DYNAMICAL PHASES WITH BIASED MANY-BODY DYNAMICS

In this section, we outline the calculations to obtain the dynamical phase diagram in Fig. 3 of the main manuscript. We therefore connect the canonical ensemble constructed from the counting field  $s$  and the time-integrated activity  $\mathcal{O}_0$  with an auxiliary collision model dynamics [81]. In particular, we focus on the long-time limit, where the dynamics essentially coincides with the time-independent biased dynamics in Ref. [80].

We briefly recall the canonical ensemble of quantum trajectories. Its probabilities are defined as

$$\pi(\eta(T), s) = \frac{1}{Z_T(s)} e^{-s\mathcal{O}_0(\eta(T))} \pi(\eta(T)), \quad (\text{S10})$$

where

$$Z_T(s) = \sum_{\{\eta(T)\}} e^{-s\mathcal{O}_0(\eta(T))} \pi(\eta(T)) \quad (\text{S11})$$

is a normalization that we neglected for brevity in the main manuscript. Since  $e^{-s\mathcal{O}_0(\eta(T))}$  implements a bias of the trajectory's probability that depends on the number of observed '1' measurement outcomes, we notice that auxiliary dynamics are essentially implemented by the tilted Kraus operators  $e^{-s\mathcal{O}_0(\mathbf{k})/2} K_{\mathbf{k}}$ . In this expression,  $\mathcal{O}_0(\mathbf{k})$  counts the amount of '1' measurement outcomes in  $\mathbf{k}$ . Note however that the hereby defined tilted Kraus map  $\mathcal{E}_s[\rho] = \sum_{\{\mathbf{k}\}} e^{-s\mathcal{O}_0(\mathbf{k})} K_{\mathbf{k}} \rho K_{\mathbf{k}}^\dagger$  is no physical quantum channel as it violates trace preservation for  $s \neq 0$ . Therefore, it does not permit consistent probabilities in Eqs. (5,6) to evaluate Eqs. (3,4), but generates the normalization  $Z_T(s) = \text{Tr}\{\mathcal{E}_s^T[\rho(0)]\}$ .

As demonstrated in Ref. [81], we can define a generalized rotation to obtain physical dynamics with the intended properties by means of time-dependent biased Kraus operators

$$\tilde{K}_{\mathbf{k}(t)} = e^{-s\mathcal{O}_0(\mathbf{k})/2} G_t K_{\mathbf{k}} G_{t-1}^{-1}, \quad (\text{S12})$$

with the recursion relation  $G_{t-1} = \sqrt{\mathcal{E}_s^*[(G_t)^2]}$ . Here, the dual tilted Kraus map  $\mathcal{E}_s^*$  acts sequentially on the operator  $G_t$  backpropagating it to earlier times. Asymptotically, these operators are described in terms of the dominant matrix  $l_s$  of the dual tilted Kraus map  $\mathcal{E}_s^*$ . The corresponding eigenvalue equation yields  $\mathcal{E}_s^*[l_s] = \Lambda_s l_s$ . Therefore, the time-independent biased Kraus operators

$$\tilde{K}_{\mathbf{k}} = \frac{e^{-s\mathcal{O}_0(\mathbf{k})/2}}{\Lambda_s^{1/2}} l_s^{1/2} K_{\mathbf{k}} l_s^{-1/2} \quad (\text{S13})$$

emerge [80]. The biased Kraus operators in Eq. (S13) implement *bona-fide* quantum dynamics. In particular, the completely positive trace-preserving quantum channel allows for a proper probabilistic interpretation, while encoding the probabilities in the canonical ensemble at large times. Therefore, we can apply Eqs. (5,6) to calculate the dynamical phase diagram (see Fig. 3).

## SINGLE-BODY DYNAMICS

In this section, we investigate the case of a noninteracting, i.e., single-body, dynamics, achieved by setting  $V = 0$  in Eq. (2). We calculate analytically the activity and temporal correlations at stationarity and before exploring the influence of a static detuning  $\Delta$  in the system Hamiltonian  $H_S$ .

We start by parametrizing the collision model Hamiltonian in  $U = e^{-iH_{\text{CM}}\Delta t}$  for a single qubit as

$$H_{\text{CM}}\Delta t = a(\sigma^x \otimes \mathbb{1}) + b(P \otimes \tau^x), \quad (\text{S14})$$

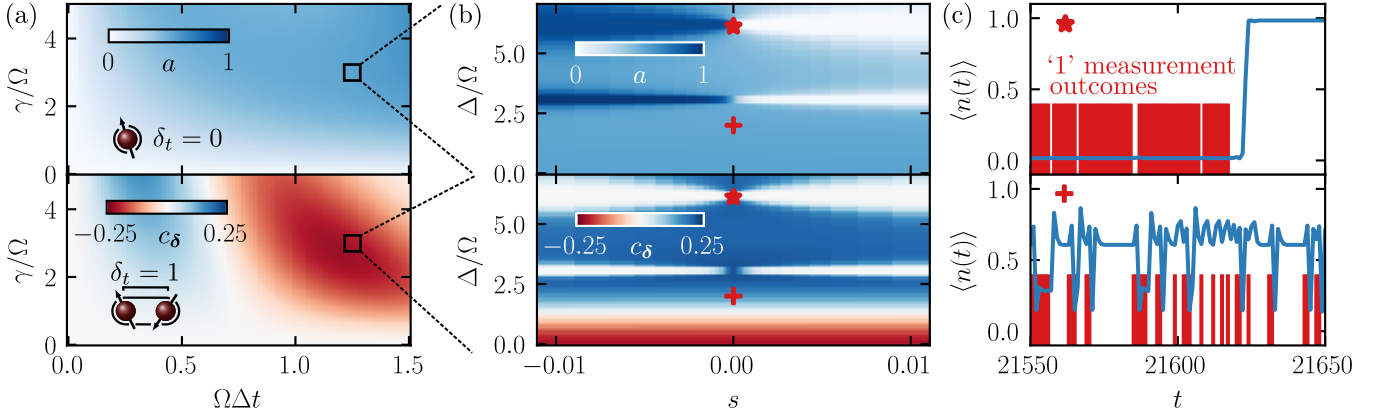


FIG. S2. **Single-body dynamics.** (a) Dynamical order parameters across  $(\Delta t, \gamma)$ -plane. For  $L = 1$ , we show the stationary activity  $a$  (top) and temporal correlations  $c_\delta$ , with  $\delta = (0, 1)$  (bottom). (b) Influence of static detuning. For the parameters  $(\Delta t, \gamma) = (1.25/\Omega, 3\Omega)$ , we plot the dependence of both dynamical order parameters on the static detuning  $\Delta$  and the activity counting field  $s$ . (c) Stochastic realizations. For two chosen detunings  $\Delta$  [see star and cross in panel (b)], we investigate the probability  $\langle n(t) \rangle$  to find the qubit in state  $|1_S\rangle$  (blue line) and the corresponding ancilla measurements (red strokes) of the stochastic realization.

where we define  $a = \Omega\Delta t$  and  $b = \sqrt{\gamma\Delta t}$ . Utilizing the stationary state of the dynamics  $\rho_{ss} = \mathbb{1}/2$ , the stationary probability to observe  $k(t) = 1$  reads

$$p(k(t)=1) \xrightarrow{t \rightarrow \infty} \text{Tr}\{K_1 \rho_{ss} K_1^\dagger\} = \frac{1}{2} \text{Tr}\{K_1 K_1^\dagger\} = \frac{1}{2} - \frac{1}{2c} \cos b (b^2 \cos \sqrt{c} + 4a^2), \quad (\text{S15})$$

with  $c = 4a^2 + b^2$ . Similarly, in order to also resolve temporal correlations, we compute the probability for two consecutive ‘1’ measurements in time, which is given by

$$\begin{aligned} p(k(t)=1, k(t-1)=1) &\xrightarrow{t \rightarrow \infty} \text{Tr}\{(K_1)^2 \rho_{ss} (K_1^\dagger)^2\} = \frac{1}{2} \text{Tr}\{(K_1)^2 (K_1^\dagger)^2\} \\ &= \frac{4a^2 (4b^2 \cos \sqrt{c} + c \cos(2b)) - 4c \cos b (b^2 \cos \sqrt{c} + 4a^2) + b^2 (c \cos(2b) - 4a^2) \cos(2\sqrt{c}) + 3(16a^4 + 4a^2 b^2 + b^4)}{8c^2}. \end{aligned} \quad (\text{S16})$$

Since here  $\overline{k(t)} = p(k(t)=1)$  and  $\overline{k(t)k(t-1)} = p(k(t)=1, k(t-1)=1)$ , the activity at stationarity is thus given by Eq. S15, while the corresponding temporal correlations, with  $\delta = (0, 1)$ , read

$$c_\delta(t) = \overline{k(t)k(t-1)} - \overline{k(t)} \overline{k(t-1)} \xrightarrow{t \rightarrow \infty} \frac{1}{2c^2} \left( b^2 \sin^2 b \sin^2 \left( \frac{1}{2} \sqrt{c} \right) \left( (8a^2 + b^2) \cos \sqrt{c} + b^2 \right) \right). \quad (\text{S17})$$

We show the explicit dependence of both dynamical order parameters on the collision time  $\Delta t$  and dephasing rate  $\gamma$  in Fig. S2(a). We observe both temporal correlations and anti-correlations. In particular, for the collision parameters  $(\Delta t, \gamma) = (1.25/\Omega, 3\Omega)$  that we utilize throughout our investigations, we find temporal anti-correlations [see also  $V = 0$  in Fig. 3].

For these parameters, we now consider a static detuning  $\Delta$  of the  $|1_S\rangle$  state that models an off-resonant Rabi drive. The corresponding system Hamiltonian then yields

$$H_S = \Omega \sigma^x + \Delta n. \quad (\text{S18})$$

In Fig. S2(b), we plot the activity and temporal correlation phase diagrams for canonical ensembles with  $s$  being the counting field with respect to the time-integrated number of ‘1’ measurement outcomes. We observe that both quantities vary smoothly with the detuning  $\Delta$  at  $s = 0$ . This is not surprising, since the fully mixed state  $\rho_{ss} = \mathbb{1}/2$  remains the stationary state of the dynamics. Conversely, both dynamical order parameters show sharp changes, when varying  $s$  at certain recurrent detunings  $\Delta$ . Here, negative counting fields promote a very active phase, reducing temporal correlation to almost zero, while positive  $s$  values lead to very inactive dynamics with also negligible temporal correlations. In Fig. S2(c), we investigate the influence of the sharp changes in dynamical order parameters on

stochastic realizations. We show exemplary dynamics for detunings with and without distinct dynamical phases in their immediate vicinity in  $s$  [corresponding to the star and cross in the phase diagrams]. In the first case, displayed in the upper panel, we observe intermittent periods of the active and the inactive phase alongside the system qubit being in the  $|0_S\rangle$  state or the  $|1_S\rangle$  state. Conversely, in the lower panel at a detuning, where no distinct dynamical phases meet at  $s \approx 0$ , the state of the system is not pinned to one of the computational basis states. Furthermore, the measurements are not bunched over a long period of time.