Numerical modelling of sawteeth and sawtooth-free regime

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1. Introduction

Sawteeth oscillations are often observed in tokamak experiments. In contrast, the hybrid scenario discharges are sawtooth-free (*SF*), featured by flat profiles of the safety factor q with $q \approx 1$ in the central region [1]. Due to its better stability and confinement compared to that of standard H-mode, the hybrid scenario is a possible candidate for a fusion reactor.

To better understand the sawteeth physics and the *SF regime*, numerical calculations up to quasi-steady state have been carried out, basing on two-fluid equations and including the bootstrap current perturbation with the large aspect ratio approximation [2]. It is found that: (1) The sawtooth crash is caused by the non-ideal double kink mode for a finite bootstrap current fraction (>10%); (2) The *SF regime* exists for a relatively lower ion density.

The input parameters for calculations are based on ASDEX Upgrade experiments: the toroidal field $B_t = 2.5T$, the plasma minor (major) radius $a = 0.5m$ ($R = 1.75m$), a parabolic profile for the original equilibrium electron temperature T_e with $T_e = 2.1keV$ at the magnetic axis, the perpendicular plasma viscosity, particle diffusivity and heat conductivity $1m^2/s$, and the ratio between the parallel and perpendicular heat conductivity 10^9. If not mentioned elsewhere, a constant equilibrium ion density of $\mathrm{n_{i0}} = 8.6 \times 10^{19}\mathrm{m^{-3}}$ is assumed, leading to the Lundquist number $S = \tau_R/\tau_A = 1.4 \times 10^8$ for deuterium plasma, where $\tau_R = a^2\mu_0/\eta = 17s$ is the resistive diffusion time, $\tau_A = a/V_A$, and V_A is the Alfvén velocity. The plasma resistivity is calculated from $\eta \sim \mathrm{T_{e,0/0}^{-3/2}}$ in simulations, where $\mathrm{T_{e,0/0}}$ is the calculated electron temperature of the $\mathrm{m/n} = 0/0$ component (m/n is the poloidal/toroidal mode number).

2. Sawtooth crash caused by non-ideal double kink mode

The time evolution of the $\mathrm{T_{e,0/0}}$ at $\mathrm{r} = 0.15\mathrm{a}$ (normalized to the original equilibrium one at $\mathrm{r} = 0$) is shown in figure 1(a) for $\mathrm{f_b} = 0$ (black curve) and $\mathrm{f_b} = 0.23$ (red), where $\mathrm{f_b}$ is the original equilibrium bootstrap current density fraction at the original equilibrium $\mathrm{q} = 1$ surfaces, being at $\mathrm{r_1/a} = 0.229$ and 0.34, respectively. Sawteeth are found in both cases. For $f_b = 0$, the crash is caused by the usual resistive internal kink mode, as expected [3]. The crash time is tens of microseconds in two-fluid simulations, in agreement with that found in single sawtooth simulations and in experiments [2,4].

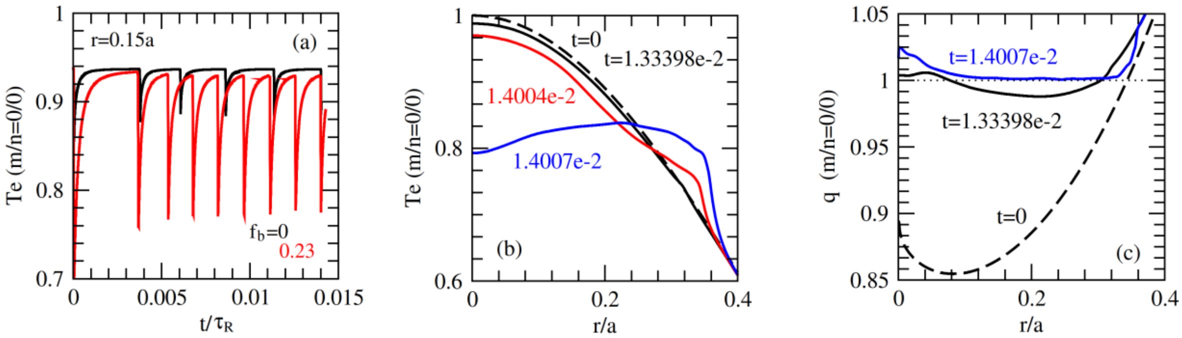

Figure 1 (a) Time evolution of the $T_{e,0/0}$ *at* $r = 0.15a$ *for* $f_b = 0$ *(black curve) and* 0.23 *(red); (b) Corresponding to the last crash shown by the red curve in (a), radial profiles of* $T_{e,0/0}$ *at* $t = 0$ *(dashed black curve),* $1.33398 \times$ *(solid black),* $1.4004 \times$ *(red), and* $1.4007 \times 10^{-2}\tau_R$ *(blue); (c) The corresponding* $q_{0/0}$ *profiles at* $t = 0$ *(dashed),* $1.33398 \times$ *(solid black) and* $1.4007 \times 10^{-2}\tau_R$ *(blue). The horizontal dotted line marks* $q_{0/0} = 1$ *value.*

Corresponding to the last crash shown by the red curve in figure 1(a) for $f_b = 0.23$, radial profiles of $T_{e,0/0}$ are shown at different times in figure 1 (b). The crash time is about $3 \times 10^{-6}\tau_R$ (51μs). The corresponding radial profiles of the averaged safety factor $q_{0/0}$, calculated by using only the m/n=0/0 component poloidal magnetic field, are shown in figure 1 (c). Before the crash at $t = 1.33398 \times 10^{-2}\tau_R$ (solid black), two $q_{0/0} = 1$ surfaces emerge in the central region, caused by the bootstrap current density and plasma resistivity perturbations during sawteeth. This differs from that of the original equilibrium q profile (dashed). The central $q_{0/0}$ value becomes slightly above unity after the crash (blue).

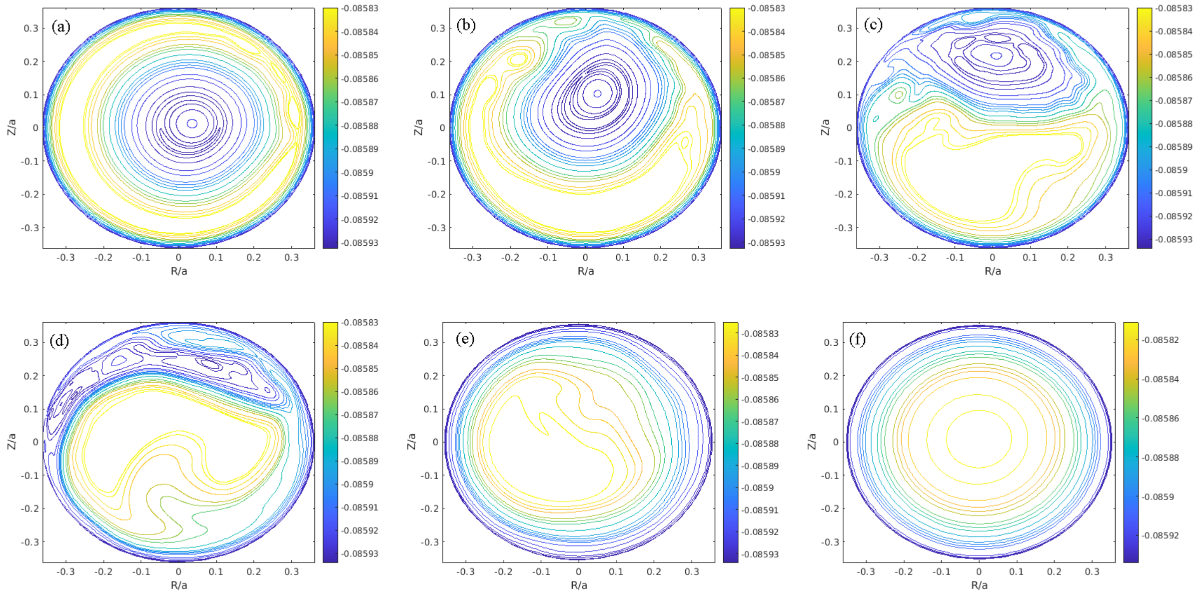

Figure 2 Corresponding to figures 1 (b) and (c), the evolution of magnetic surfaces at $t = 1.33398 \times$ (a), $1.4004 \times$ (b), $1.4007 \times$ (c), $1.4010 \times$ (d), $1.4025 \times$ (e), *and* $1.4085 \times 10^{-2}\tau_R$ (f).

Corresponding to figures 1 (b) and (c), the evolution of magnetic surfaces is shown in figure 2. In addition to an outer magnetic island at $r/a \sim 0.34$, an inner island also exists near the magnetic axis, indicating that the non-ideal double kink mode leads to the sawtooth crash. During the crash, the inner island moves outwards together the hot core, and they are elongated along the poloidal direction in a later phase. Eventually, the outer island occupies the core region, reaching a magnetic configuration similar to that before the crash.

3. Sawtooth-free regime

With $\mathrm{f_b} = 0.23$, the time evolution of the $\mathrm{T_{e,0/0}}$ at $\mathrm{r} = 0.15\mathrm{a}$ is shown in figure 3 (a) for $\mathrm{n_{i0}} = 8.6 \times$ (black curve) and $2.15 \times 10^{19}\mathrm{m^{-3}}$ (red). Sawteeth are seen for a higher ion density. With a lower ion density, however, sawteeth disappear, replaced by small fluctuation of the central $\mathrm{T_{e,0/0}}$ in the quasi-steady state, indicating the *SF regime*.

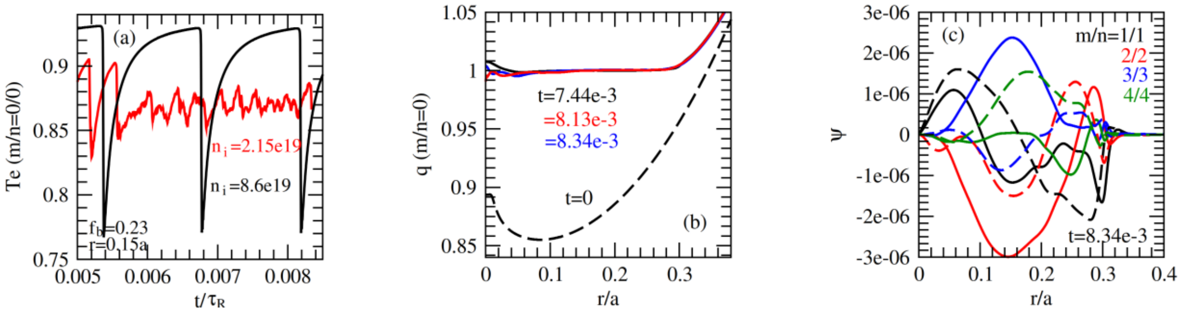

Figure 3 *(a) With* $f_b = 0.23$*, the time evolution of* $T_{e,0/0}$ *at* $r = 0.15a$ *for* $\mathrm{n_{i0}} = 8.6 \times$ *(black curve) and* $2.15 \times 10^{19} m^{-3}$*(red). (b) Corresponding to the red curve in (a), radial profiles of* $q_{0/0}$ *at* $t = 7.44 \times$ *(solid black),* $8{,}13 \times$ *(red) and* $8.34 \times 10^{-3}\tau_R$ *(blue). The original equilibrium q profile at* $t = 0$ *is shown by the dashed curve; (c) Corresponding to (b) at* $t = 8.34 \times 10^{-3}\tau_R$*, radial profiles of the normalized helical magnetic flux of the* $m/n = 1/1$ *(black),* $2/2$ *(red),* $3/3$ *(blue), and* $4/4$ *(green) components. The solid (dashed) curves are the real (imaginary) parts.*

Corresponding to the red curve shown in figure 3 (a), radial profiles of $\mathrm{q_{0/0}}$ at $\mathrm{t} = 7.44 \times$ (solid black), $8.13 \times$ (red) and $8.34 \times 10^{-3}\tau_\mathrm{R}$ (blue) are shown in figure 3 (b), being flattened to be about unity in the core region in the quasi-steady state, similar to that observed in hybrid scenario experiments [1]. Corresponding to figure 3 (b) at $\mathrm{t} = 8.34 \times 10^{-3}\tau_\mathrm{R}$, radial profiles of the normalized helical magnetic flux of the $\mathrm{m/n} = 1/1$ (black curve), $2/2$ (red), $3/3$ (blue), and $4/4$ (green) components are shown in figure 3 (c). Their amplitudes remain in the order of $10^{-6}\mathrm{aB_t}$ in quasi-steady state in the *SF regime*. Together with plasma velocity perturbations, they lead to the required dynamo effect for maintaining flat $\mathrm{q_{0/0}}$ profiles in the core region.

Corresponding to the *SF regime* shown by the red curve in figure 3 (a), an example of the magnetic surfaces and electron temperature contours at $\mathrm{t} = 8.13 \times 10^{-3}\tau_\mathrm{R}$ is shown in figures 4 (a) and (b), respectively. There are always magnetic islands in the core region in the *SF regime*, and their shape change with time.

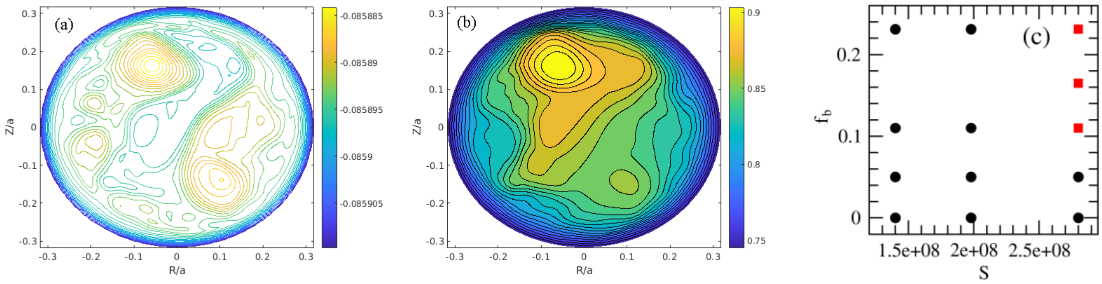

Figure 4 *Corresponding to the SF regime shown by the red curve in figure 3 (a), contours of magnetic surfaces (a) and electron temperature (b) at* $t = 8.13 \times 10^{-3}\tau_R$. *The right figure shows the sawtooth (black circles) and the SF regimes (red squares) in the* $(S - f_b)$ *plane.*

Using different equilibrium plasma density and f_b values in calculations, the obtained diagram for the sawtooth (black circles) and the *SF regime* (red squares) is shown in the $(S - f_b)$ plane in figure 4 (c). The *SF regime* exists for $f_b > 10\%$ and a sufficiently large Lundquist number (due to a low ion density). The sawteeth frequency (amplitude) increases (decreses) when approaching the *SF regime* from the sawtooth regime. On the left hand side of the *SF regime* with lower Lundquist numbers, the crashes in the sawtooth regime are all caused by the non-ideal double kink mode for $f_b \geq 0.1$.

Numerical simulations have also been carried based on single fluid equations. Because there is no diamagnetic drift in this model, the required Lundquist number for entering into the *SF regime* is much smaller than that found from two-fluid simulations.

4. Summary

Numerical simulations basing on two-fluid equations with the large aspect ratio approximation indicate that:

(1) When the bootstrap current density fraction is larger than 10% in the core region, the sawtooth crash, occurring in tens of microseconds, is caused by the non-ideal double kink mode rather than the internal kink mode, in contrary to the conventional understanding. For $f_b = 0$, the crash is caused by the usual resistive internal kink mode.

(2) For relatively lower ion density of middle-size tokamak plasmas and a finite bootstrap current fraction, the *sawtooth-free regime* is found, as observed in experiments [1], in which flat q profiles with $q \approx 1$ in the core region are maintained by the dynamo effect.

References

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