Derivation of the q=1 dynamic profile from the 1/1 perturbation amplitude

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Introduction

The importance of the $q = 1$ profile derivation is linked to the $1/1$ perturbation developing in central plasma either as a reconnection process triggered by a $q_0 < 1$ plasma central safety factor [1] or as a $1/1$ interchange perturbation [2] when $q \gtrsim 1$ near the magnetic axis, both flattening the central temperature and driving a restored sawtooth preventing $q_0 \gtrsim 1$ profile afterwards. The latter profile is possible to be kept in specific plasma scenarios, the sawtooth being absent due to a self regulating mechanism such as the magnetic flux pumping [3]-[5]. A way of deriving the $q = 1$ profile is presented here via the determination $1/1$ mode location. This is an alternate method when an unclear inversion radius is found from the associated electron cyclotron emission temperature measurements or soft X-ray emission profiles.

The perturbed solution

In this regard, the following perturbed solution for the m/n magnetic flux [6]

$$\begin{aligned}\Psi_s^{mn}(t) &= \mathrm{i}(m/q_s - n) \\ &\times \left[\left.\frac{\Delta^l}{(\tau+\mathrm{i}\Omega_{MP})\Delta}\right|_{\tau=0} + \left.\frac{\Delta^l}{\tau\Delta}\right|_{\tau=-\mathrm{i}\Omega_{MP}} \exp(-\mathrm{i}\Omega_{MP}t) + \sum_{p=1}^{7L} \left.\frac{(\tau-\tau_p)\Delta^l}{\tau(\tau+\mathrm{i}\Omega_{MP})\Delta}\right|_{\tau=\tau_p} \exp(\tau_p t)\right]\end{aligned} \tag{1}$$

is used for deriving the $q = 1$ profile via our modes localization technique [7]. Δ is the determinant of the Laplace transformed system of linearized perturbed plasma and outer plasma equations having Ψ_s^{mn} and its radial derivative at the m/n magnetic surface radial coordinate r_s as unknowns. τ_p are the roots of $\Delta(\tau) = 0$ with τ the Laplace transform variable. Δ^l is simply the numerator determinant according to the Cramer's rule applied to the mentioned system of equations in order to find the solution associated to every considered mode. $l =$

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[*] See the author list of "Overview of T and D-T results in JET with ITER-like wall" by C.F. Maggi et al., to be published in Nuclear Fusion Special Issue from the 29th Fusion Energy Conference (London, UK, 2023)

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$m-m_1+1+(n-n_1)(m_2-m_1+1)$ where m and n span $m_1,n_1 \leq m,n \leq m_2,n_2$ and $L = \max\limits_{m,n}(l)$. q_s and Ω_{MP} are the safety factor at r_s and the outer coils signal rotation frequency, respectively. It has been demonstrated that the modes amplitude $b_{\theta f}^{mn} = (mr_s^m/r_f^{m+1})|\Psi_s^{mn}|$ and frequency $f^{mn} = \mathrm{Im}[(\partial\Psi_s^{mn}/\partial t)/\Psi_s^{mn}]$ derived based on our solution are good matches for the experimental corresponding quantities [6] provided by the JET MHD data analysis code [8]. The perturbation amplitude is derived by means of the perturbed poloidal magnetic field measured at the JET Fast Magnetic Acquisition System diagnostic coils, disposed at r_f. Our method basically consists in inverting our model and, starting from the experimental amplitude as input data and based on the validity of the proposed model, in finally retrieving the dynamic profile of the $1/1$ mode via the $1/1$ mode localization derivation. The latter technique has been presented in Ref. [7] and is simply based on the incremental spanning of r_s between 0 and 1 m and on choosing the appropriate mode location when the derived amplitude best matches the experimental one.

Modelled vs experimental results

The JET shot to be analyzed is the EUROfusion flux-pumping campaign shot no. 103110 showing a reported clear, highest unstable $1/1$ mode amplitude in figure 1(b) along with its neighboring modes and a (red color) frequency in the spectrogram from figure 1(c). A potential signature of the $1/1$ flux pumping is discovered, as reported. The $1/1$ mode experimental amplitude and frequency provided by the JET data analysis MHD python code are shown in figure 2(a) and 2(b). Our technique in deriving the mode location is based on the suitable choice of the initial conditions as to most accurately retrieve the mode amplitude and frequency via our

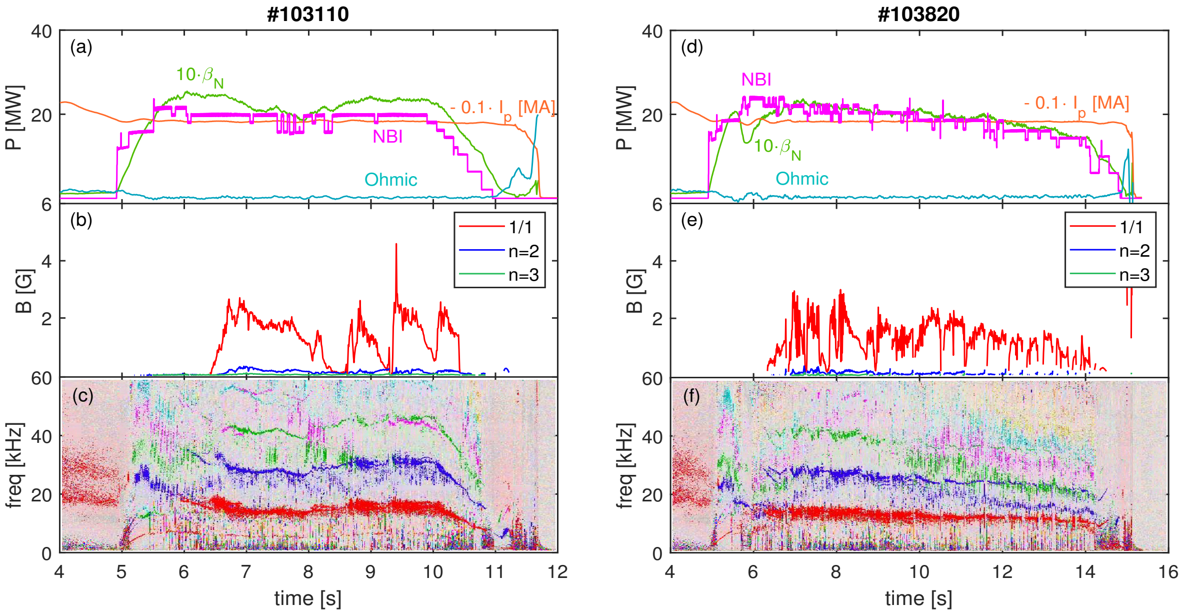

Figure 1: *JET 103110 and 103820 shots* $1/1$ *mode (a,d) normalized beta, plasma current, NBI and ohmic power, (b,e) modes amplitudes and (c,f) modes spectrogram of frequencies.*

calculated location when the experimental mode amplitude and frequency play the role of the input data. It can be clearly seen that the match between the retrieved quantities by means of the

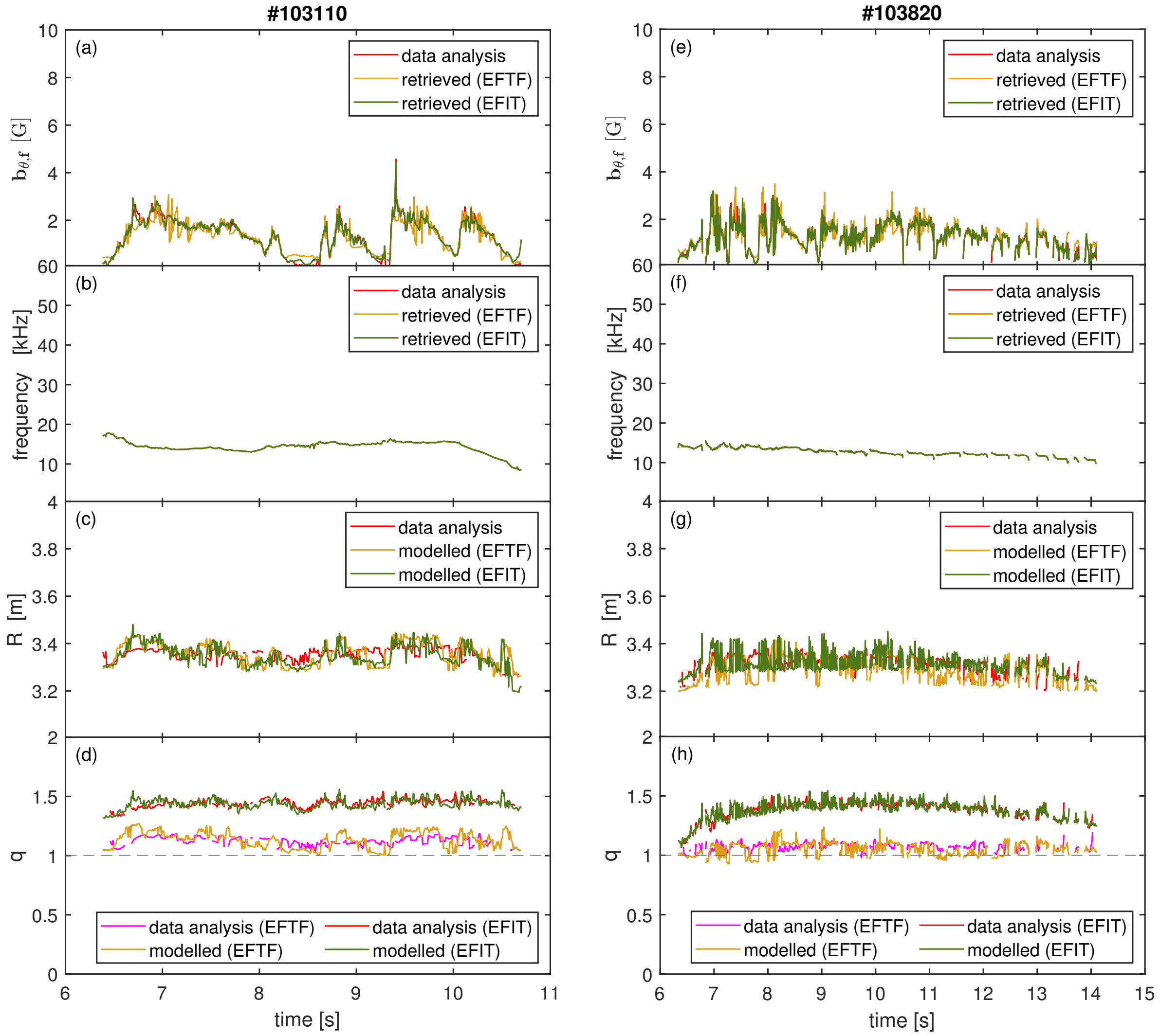

Figure 2: *JET 103110, 103820 shots* $1/1$ *mode experimental vs. modelled/retrieved (a,e) amplitude, (b,f) frequency, (c,g) location and (d,h) safety factor profile.*

calculated location and the experimental ones is pretty good for the both EFTF and EFIT equilibrium reconstruction data profiles used into calculus. The best retrieval ensures the obtaining of the sought after $1/1$ mode location. Figure 2(c) shows a reasonable good inversion major radius match. Based on the obtained inversion radius, the corresponding safety factor $q = 1$ in figure 2(d) is simply found from the spatial profile of the EFIT/EFTF q_{mag} midplane safety factor data by interpolating with respect to our derived location. It can be clearly observed that the calculated $q = 1$ matches pretty good the experimental safety factor for the case of the EFIT data profile. However, the quality of the EFTF data is lower, the match being less accurate. This also can be seen from a slightly less precise amplitude retrieval. On the other hand, the EFIT and EFTF based inversion radii and safety factor profiles are quite different. By looking at its spanned range, the EFTF shows a possible sawtooth activity whereas the EFIT data rather indicates a sawtooth free and a possible flux pumping dynamics. A similar analysis leading to the same conclusions is performed for the JET shot no. 103820 (involving sawtooth activity), shown in figures 1(d-f) and 2(e-h). The $1/1$ mode is locking when the NBI is off. Again, a good $q = 1$ calculated vs experimental match is obtained. From the $q = 1$ profiles derived in figure 2, by using the following Wesson type safety factor formula experiencing plasma shaping corrections [9]

$$q(r) = \frac{q_a \rho^2}{1 - [1-\rho^2]^{(q_a/q_0)} + \varepsilon_0^2 \lambda \ \sin(\pi \rho^4)} \tag{2}$$

$$\lambda = (a/4\pi)(3/2 + \Delta_a'^2 + 2\Delta_a' + E_a'^2 + 6E_aE_a' - 3E_a^2 + T_a'^2 + 16T_aT_a' - 8T_a^2) \tag{3}$$

the q_0 profile is determined in figure 3. $\rho = r/a$, $\varepsilon_0 = a/R_0$ where a, R_0, q_a, $a\varepsilon_0\Delta_a$, $a\varepsilon_0 E_a$, $a\varepsilon_0 T_a$ are the minor and the major radius and the safety factor, Shafranov shift, ellipticity and triangularity on the boundary, respectively. ‘ means the r derivation. Therefore the $q = 1$ profile accuracy is additionally checked by the subsequently good match between our derived plasma central safety factor q_0 and the experimental one, no matter the used diagnostic data.

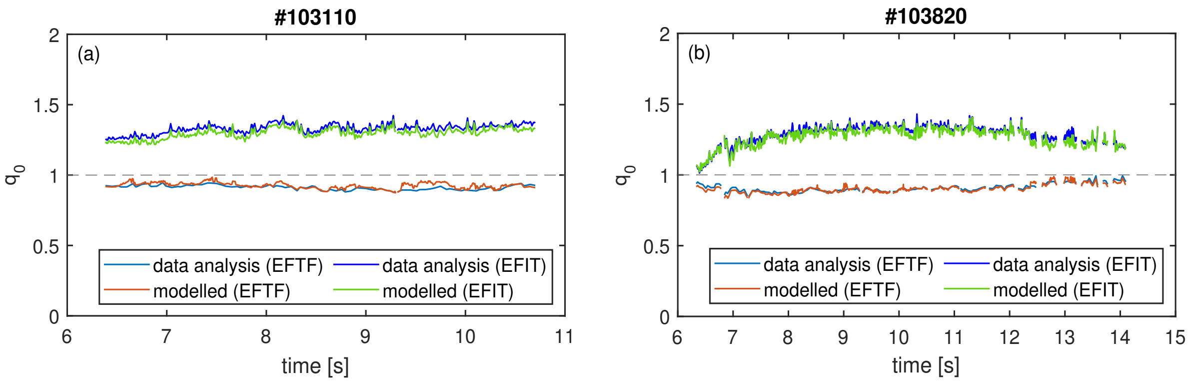

Figure 3: *JET shots no. (a) 103110 and (b) 103820 experimental vs. modelled q_0 safety factors.*

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