Peeling-Ballooning Stability of the Tokamak Plasma with 3D Magnetic Perturbations: Towards ELITE-3D

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Background Future tokamaks must avoid large edge localized modes (ELMs). A promising solution is the application of 3D magnetic perturbations (MPs), which under certain conditions can mitigate, or even suppress, ELMs. However, quantifying the optimal magnetic geometry and plasma conditions to achieve this on future devices is uncertain in the absence of a rigorous theoretical model. Key to such a model is a predictive capability for peeling-ballooning mode stability in the presence of non-axisymmetric MPs. Such calculations are possible (eg [1]) but have great demands on computational resources, so detailed convergence tests and parameter scans are prohibitive. This motivates our project to develop an efficient code to quantify the impact of MPs on ideal MHD stability of the plasma edge. Here, we describe the methodology and some of the foundations on which we shall build our full capability in the future.

Methodology The methodology is based on that developed in [2], aiming to exploit a feature of high toroidal mode number, n, ballooning modes to develop a variational principle. A ballooning mode consists of a large number of coupled poloidal Fourier harmonics, each labeled by their poloidal mode number, m, and localized about their rational surface where $m=nq$ with $q(\psi)$ being the safety factor and ψ the poloidal flux. Each of the Fourier coefficients has approximately the same shape in ψ, and this is described by the well-known $n=\infty$ ballooning equation. They have an amplitude that is slowly varying from one value of m to another, which is set by the weak radial (ψ) variation of the equilibrium. This structure is illustrated in Fig 1, which shows the output from a particular ELITE calculation for a peeling-ballooning mode. Each curve shows the radial dependence of a different poloidal Fourier mode, with two of them highlighted in red for clarity. Note their similarity in shape, but differing amplitude, consistent with the description above. The region where $x<0$ (approximately) corresponds to the vacuum region, and this plasma-vacuum interface interrupts the "ballooning symmetry". In particular, the Fourier modes resonant in the vacuum (i.e. $m\geq m_0$) have a different shape, and enable the energy of the kink-mode to be tapped into (see example highlighted red).

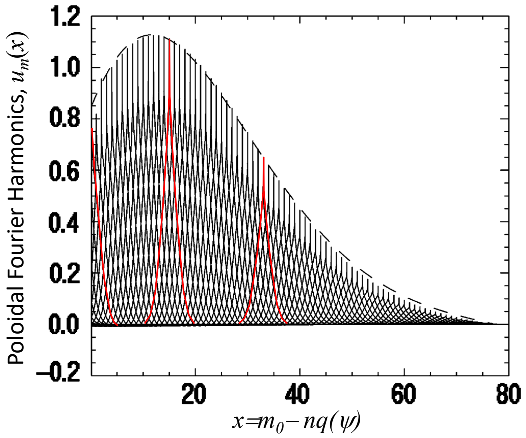

Figure 1: Classic peeling-ballooning mode structure predicted by ELITE, showing the similarity in shape of the different core Fourier harmonics.

For our approach, we conjecture that the effect of weak toroidal variation associated with a 3D MP will act similarly to the radial variation, primarily affecting how poloidal and toroidal Fourier modes couple while having little influence on their shape. Thus we adopt a trial function for the displacement, ξ, of the form:

$$\boldsymbol{\xi} = \sum_{m,n} C_{m,n}\boldsymbol{\xi}_{m,n}(\psi)e^{-im\theta}e^{in\varphi} \qquad (1)$$

where θ is the straight field line poloidal angle and φ is the toroidal angle. The functions $\xi_{m,n}(\psi)$ are provided by ELITE stability calculations for the toroidally averaged equilibrium and the coefficients $C_{m,n}$ are the variational parameters to be obtained by minimization of the energy functional δW. Assuming an applied MP with a single toroidal mode number, N, the energy functional can be written in a schematic form:

$$\delta W = \sum_{k,m,l,n} C^*_{k,l}\, C_{m,n}\{\delta_{n,l}\langle\boldsymbol{\xi}^*_{k,l}\cdot F_0(\boldsymbol{\xi}_{m,n})\rangle + \delta_{n,l\pm N}\langle\boldsymbol{\xi}^*_{k,l}\cdot F_N(\boldsymbol{\xi}_{m,n})\rangle\} \qquad (2)$$

Angled brackets denote Jacobian-weighted integrals over ψ and θ, and the ideal MHD force operators for the axisymmetric and non-axisymmetric field contributions are represented by $\boldsymbol{F}_0$ and $\boldsymbol{F}_N$, respectively. ELITE will provide the harmonics $\xi_{m,n}$ so that once the force operators are derived, minimization of Eq (2) with respect to $C^*_{k,l}$ provides an eigenmode equation for the $C_{m,n}$, with the new growth rate provided as an eigenvalue (note that $\boldsymbol{F}_0$ and $\boldsymbol{F}_N$ include inertia and the vacuum energy). The challenge is then to derive forms for the force operators that are numerically tractable. We first consider the axisymmetric piece, which is the largest and best understood of the two. Nevertheless, we shall find that it is not trivial to reconstruct numerically when one is considering intermediate-to-high n modes because of the cancellations that must be captured. In the rest of this paper, we describe our approach which then sets the foundations for introducing the non-axisymmetric force operator in a next stage of our project.

Reconstructing the energy functional We decompose our displacement, $\boldsymbol{\xi}$, and magnetic fluctuation (ie that associated with the instability), $\boldsymbol{\delta B}$, in the forms:

$$\boldsymbol{\xi} = \frac{X}{R^2B_p^2}\nabla\psi + R^2U\nabla\varphi + Z\boldsymbol{B} \qquad \boldsymbol{\delta B} = \frac{Q_\psi}{RB_p}\nabla\psi + \frac{Q_\wedge}{RB_pB}\boldsymbol{B}\times\nabla\psi + \frac{Q_b}{B}\boldsymbol{B} \qquad (3)$$

Assuming an incompressible plasma model in this first study, we eliminate Z from δW. We then follow standard procedures to derive the contributions to the potential energy, δW_p:

$$\delta W_1 = \frac{1}{2}\int dV\,|Q_\psi|^2 \qquad\qquad Q_\psi = \frac{1}{RB_p}(\boldsymbol{B}\cdot\nabla)X$$

$$\delta W_2 = \frac{1}{2}\int dV\, |Q_\wedge|^2 \qquad\qquad Q_\wedge = -\left[\frac{RB_p}{B}(\boldsymbol{B}\cdot\nabla)U - \frac{B_\varphi B_p}{B}\frac{\nu'}{\nu}X\right]$$

$$\delta W_3 = \frac{1}{2}\int dV\, B^2|\nabla\cdot\boldsymbol{\xi}_\perp + 2\kappa\cdot\xi_\perp|^2 = \frac{1}{2}\int dV\, B^2\left|inU + \frac{\partial X}{\partial\psi} + \frac{p'}{B^2}X + \frac{B_\varphi}{BB_p}Q_\wedge\right|^2$$

$$\delta W_4 = \frac{1}{2}\int dV\left(-2(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp^*)(\boldsymbol{\xi}_\perp\cdot\nabla p)\right) = \frac{1}{2}\int dV\left(-\frac{2p'}{B^2}\frac{\partial}{\partial\psi}\left(p+\frac{B^2}{2}\right)|X|^2\right)$$

$$\delta W_5 = -\frac{1}{2}\int dV\frac{J_\parallel}{B}(\boldsymbol{\xi}_\perp^*\times\boldsymbol{B})\cdot\delta\boldsymbol{B} = \frac{1}{2}\int dV\,\sigma\left[X^*(B\cdot\nabla)U + X(B\cdot\nabla)U^* - \frac{B_\varphi}{R}\frac{\nu'}{\nu}|X|^2\right]$$

A prime denotes derivative with respect to ψ, ν' is the local shear, $J_\parallel$ is the parallel current density, $\sigma = -J_\parallel/B$ and other parameters have their usual meanings. Note that δW_1 and δW_2 are field line bending, δW_3 is magnetic compression, δW_4 is the curvature drive and δW_5 is the kink drive. We have dropped two complex terms which cancel between δW_4 and δW_5.

Minimization of δW with respect to U^* provides the Euler equation relating U to X, and by employing a high n expansion of this equation one can readily show that the magnetic compression is reduced to $O(n^{-2})$. However, this Euler equation is only valid for the toroidally symmetric equilibrium, and we want to retain the possibility that the MPs could change it. This is captured in our formalism by using the axisymmetric ELITE calculations to derive the Fourier amplitudes for X, then using the Euler equation relating U and X to provide the Fourier amplitudes for U. We then allow these Fourier amplitudes to vary relative to each other, with scaling coefficients that differ from those used to adjust X. Thus, when we introduce the 3D equilibrium contributions to δW we will perform two minimizations to derive both sets of coefficients. To do this reliably, we must develop a formalism that captures the cancellations we expect in the axisymmetric limit, but has freedom to adjust in the non-axisymmetric case.

A first challenge arises from the terms involving $(B.\nabla)U \sim [i(m-nq)/n](\partial X/\partial\psi)$. The $(m-nq)$ factor enhances the contributions of higher m harmonics, and in fact we find that we require $(m-nq)\sim n$. This violates the ordering in ELITE, with the result that the coefficient of the second order derivatives can have false zeros at certain radial locations, which then results in discontinuities in the radial derivatives of the higher m Fourier harmonics. While this does not

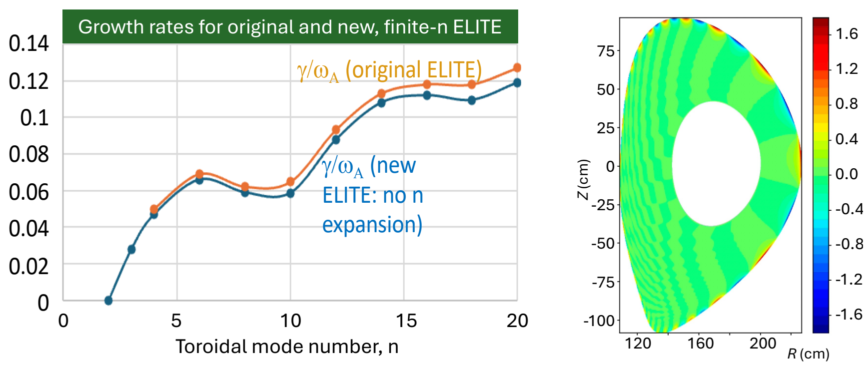

Figure 2: Comparison of growth rates of peeling-ballooning modes in DIII-D shot 170063 [4] calculated using the original high n expansion version of ELITE (orange curve) and the new arbitrary n version (blue curve). The figure on the right shows the mode structure in poloidal cross section for n=6.

influence the ELITE eigenmode solver, it does mean δW does not converge robustly with increasing number of Fourier harmonics. We have fixed this by employing a finite n version of ELITE [3]. This had numerical problems for EFIT equilibria, but we have now resolved these. Figure 2 shows the good agreement between the new finite n version of ELITE and the original ELITE code for DIII-D EFIT equilibrium, shot 170063 [4], even down to quite low n.

A second issue concerns the level of cancellation required to accurately calculate δW_3 in the axisymmetric limit, which is equivalent to three orders of n (i.e. 1 in 1000 for n=10). Actually, there is also a factor of $(B/B_p)^2$ which is a further factor of 100, so cancellation to 1 part in more than 10^5 is required. To facilitate this, we introduce the variable W, related to U through $U=(i/n)[\partial X/\partial\psi+p'X/B^2+W]$. The new version of ELITE works with W rather than U to account for cancellations, and we find this is even more important for constructing δW. A further manipulation is required and that is to combine the terms involving $|Q_\wedge|^2$ in δW_2 and δW_3 before introducing the Fourier expansions for W and X, and then integrating over plasma volume. Only when we adopt all of these do we get a satisfactory agreement between the square of the growth rate, γ^2, calculated by ELITE and that calculated from the reconstructed δW (Fig 3).

Summary As a first step towards developing an efficient variational code for peeling-ballooning stability in toroidal magnetic confinement devices with 3D geometry, we have (1) upgraded ELITE to work reliably for arbitrary n (this may be important for spherical tokamaks), and (2) identified a procedure to capture the cancellations required to accurately reproduce δW for axisymmetric plasmas, needed as a pre-requisite for our variational approach.

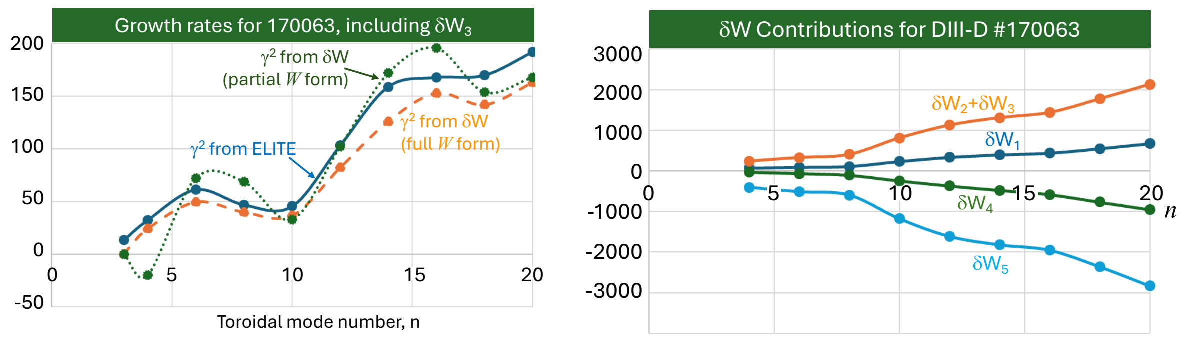

Figure 3: Growth rate, γ^2, calculated as a function of n for (1) ELITE eigenmode approach (blue); (2) from δW only replacing leading order inU terms with W (green) and (3) replacing all U terms with W (orange), showing the importance of capturing the cancellation analytically (left). Contributions to δW versus n, showing the dominance of the kink drive for this DIII-D shot 170063 (δW_3 is negligible and combined with δW_2) (right).

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References [1] J. Puchmayr, et al, NF **64** (2024) 086013; [2] M. Anastopoulos, et al NF **60** (2020) 106003; [3] A. Lunniss, University of York PhD Thesis (2016) [4]; S. Gu et al NF **62** (2022) 076031