On the classical equivalence of superstring field theories

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ABSTRACT: We construct mappings that send solutions of the cubic and non-polynomial open superstring field theories to each other. We prove that the action is invariant under the maps and that gauge orbits are mapped into gauge orbits. It follows that the perturbative spectrum around solutions is the same in both theories. The mappings also preserve the string field reality condition.

We generalize to the cases of a non-BPS D-brane and of multi-D-brane systems. We analyze the recently found analytical solutions of the cubic action, both in the BPS sector and the non-BPS sector and show that they span a one parameter family of solutions with empty cohomology and identical action, which suggests that they are gauge equivalent. We write the gauge transformations relating these solutions explicitly. This seems to suggest that open superstring field theory is able to describe a vacuum solution even around a BPS D-brane.

KEYWORDS: String Field Theory.

Contents

1. Introduction

Covariant open superstring field theory was first constructed by Witten [1], following his construction of the bosonic theory [2]. The formal structure of the theories is very similar. They both rely on the algebraic structures obeyed by the star-product, the BRST operator Q and the integration over the space of string fields. There are two new features that arise in the case of the superstring. One is the existence of several sectors, namely GSO($\pm$) and NS/R, that should be separately handled. The other is the notion of picture number [3].

Picture number is the expression of the existence of zero modes for the superghosts of the RNS formalism. Just as the zero modes of the fermionic bc ghost system imply a two-fold degeneracy in the description of vertex operators, namely integrated/unintegrated vertex, the zero modes of the bosonic $\beta\gamma$ system imply the existence of an infinite number of equivalent vertices, distinguished by a new quantum number, called picture number. Picture number is most easily dealt with using fermionization [3], where the β, γ system is expressed in terms of the η, ξ, ϕ variables,

$$\beta = e^{-\phi}\partial\xi, \qquad \gamma = \eta e^{\phi}. \tag{1.1}$$

The operation of integration over the space of string fields is realized using a CFT expectation value, which is zero for fields of zero picture number. This implies that one has either to assign non-zero picture number to the string field or to append it to the operations of star-product/derivation/integration.

Witten chose to define the (NS+) string field in the natural (-1) picture and to append a picture changing operator to the cubic term in the action. This picture changing operator had to be inserted at the interaction (mid-string) point $z = i$ in order to preserve the associativity of the star product. Soon thereafter it was noticed that in calculations of scattering amplitudes and of iterated gauge transformations picture changing operators formally collide [4, 5]. This is an immediate consequence of inserting the operator at $z = i$, which is invariant under the star-product. The collisions of picture changing operators produce singularities, since the OPE of the picture changing operator with itself is divergent. It was shown that the theory can be regularized to leading order by adding counter terms (already at the classical level), but it was also shown that the problems persist at higher order and it was not understood if and how could one resolve these problems systematically.

Resolutions of these problems can be achieved by leaving the string fields at ghost number one, while modifying their picture number to zero [6, 7, 8]. The modified cubic action (in the NS+ sector) reads,

$$S_{\text{cub}} = -\frac{1}{g_o^2} \int Y_{-2}\left(\frac{1}{2}\Psi Q\Psi + \frac{1}{3}\Psi^3\right), \tag{1.2}$$

where g_o is the open string coupling constant, Y_{-2} is the double-step inverse picture changing operator and the string fields are multiplied in the action using Witten's star product, which we leave implicit throughout the paper.

There are two possible definitions for Y_{-2}, namely the chiral the non-chiral operators. The chiral is a local operator, whose exact form will not be of interest for us, while the non-chiral is a non-local operator, defined using the doubling trick as

$$Y_{-2}(z) = Y(z)Y(\bar{z}). \tag{1.3}$$

This definition cannot be used on the real line, where picture changing operators and vertex operators are usually inserted, due to the singular OPE of Y with itself. Nonetheless, it makes perfect sense when inserted in the string mid-point, $z = i$. In this formalism there are no problems of collisions, since while the interaction vertex comes with a Y_{-2} factor, the propagator comes with a factor of its inverse, which is well defined. These operators are canceled in pairs and scattering amplitudes can be calculated and give (at least on-shell and for tree diagrams) the expected results [9].

The two different choices define two a-priori inequivalent theories. The theory that uses the chiral operator was shown to have some strange features in level truncation [10]. Also, being chiral it does not respect the twist symmetry that one expects it to obey. For these reasons the non-chiral theory was used in most studies of cubic superstring field theory and is considered the more reliable one. This is the version of the theory we shall be using here. The non-chiral Y_{-2} was recently used also in the construction of boundary superstring field theory [11].

From the action (1.2) one derives the equation of motion

$$Y_{-2}(i)\big(Q\Psi + \Psi^2\big) = 0\,. \tag{1.4}$$

The operator $Y_{-2}(i)$ has a non-trivial kernel. However, this kernel consists of states of a somewhat singular nature that are localized at $z = \pm i$. It is not clear if it has a non-trivial intersection with the correct space of open string fields, especially since it is not clear what should this space be. Assuming that this kernel poses no problem, the equation of motion gets the desired form

$$Q\Psi + \Psi^2 = 0\,. \tag{1.5}$$

The action is invariant under the gauge transformation

$$\Psi \to e^{-\Lambda}(\Psi + Q)e^{\Lambda}\,. \tag{1.6}$$

Another resolution of the problems, not relying on picture changing operators, was obtained by Berkovits [12], who developed another form of superstring field theory. This theory is non-polynomial, resembling a WZW-theory, although (unlike closed/heterotic string field theory [13, 14, 15]) string fields are still multiplied using only Witten's star-product. The non-polynomial theory uses ghost number zero, picture number zero string fields that reside in the large Hilbert space. The large Hilbert space consists of two copies of the small (standard) Hilbert space, the usual one and another one, which is multiplied by ξ_0. This redundancy is taken care of by a novel gauge symmetry, which implies that only the copy proportional to ξ_0 is physical. The non-polynomial action reads,

$$S_{\mathrm{np}} = \frac{1}{2g_o^2} \oint \Big(e^{-\Phi}Q(e^{\Phi})e^{-\Phi}\eta_0(e^{\Phi}) - \int_0^1 dt\, e^{-t\Phi}\partial_t e^{t\Phi}[e^{-t\Phi}\eta_0 e^{t\Phi}, e^{-t\Phi}Qe^{t\Phi}]\Big), \tag{1.7}$$

and the gauge symmetry is

$$e^{\Phi} \to e^{-Q\Lambda_Q}e^{\Phi}e^{\eta_0\Lambda_\eta}\,. \tag{1.8}$$

In (1.7) and elsewhere we use the square brackets to denote the graded commutator, meaning that $[A, B]$ is an anti-commutator if A and B are odd (as it is here) and a commutator otherwise. The integral over string fields corresponds to a CFT expectation value. For the non-polynomial theory this should be calculated in the large Hilbert space. To distinguish this case we define

$$\oint A \equiv \langle A \rangle_{\mathrm{Large}}\,, \qquad \int A \equiv \langle A \rangle_{\mathrm{Small}}\,. \tag{1.9}$$

The integrals are related in a simple way

$$\oint \xi_0 A = \int A\,. \tag{1.10}$$

From the action (1.7) the following equation of motion is derived,

$$\eta_0(G^{-1}QG) = 0\,, \tag{1.11}$$

where we define

$$G = e^{\Phi}\,. \tag{1.12}$$

Equation (1.11) states that $G^{-1}QG$ lives in the small Hilbert space, where the string fields of the cubic theory reside. It also has the correct quantum numbers to be identified with Ψ. This expression takes exactly the form of a gauge solution of the cubic theory (1.6). It is not a genuine gauge solution, since the "gauge field" G lives in the large Hilbert space. It is nevertheless a solution of the cubic theory. Thus, there exists a simple map of solutions of the non-polynomial theory to solutions of the cubic theory [16, 17],

$$\Psi = G^{-1}QG\,. \tag{1.13}$$

The purpose of this paper is to show that (at least classically and in the NS sector) the cubic superstring field theory and Berkovits' non-polynomial superstring field theory are equivalent. We prove this in section 2 by adding to the map (1.13) also a map that works in the other direction. This map also sends solutions to solutions and leaves the action invariant. We further show that both maps respect the gauge symmetries, so we obtain a one to one mapping of gauge orbits of solutions in both directions. In section 3 we show how to write the solutions of the cubic theory as formal gauge solutions. In this form our map takes the form of a mapping of gauge fields. We also comment on bosonic string field theory. In section 4 we describe how the GSO(−) sector is added to the theory and to our mapping. This enables us to generalize the mapping to non-BPS D-branes and to multi D-brane systems.

In section 5 we describe analytical solutions in both theories on BPS and non-BPS D-branes that seem to describe the vacuum without the original D-brane. These solutions are based on the recent solutions of [18, 19, 20]. While such solutions are to be expected for the non-BPS D-brane, where a tachyon is present, it may seem strange to have them on the BPS D-brane, where a physical tachyon is absent. Moreover, a BPS D-brane carries a RR charge, which is absent in the vacuum without the D-brane. Thus, such solutions lead to a change in the closed string background. We claim, that it seems to be the case that such solutions are nevertheless realized. We comment on this issue and offer some other conclusive remarks in section 6.

2. Mapping the cubic theory to the non-polynomial one

In this section we construct a map of string fields from the cubic superstring field theory to string fields in the non-polynomial theory. We construct the map in 2.1 and show that it sends solutions to solution in 2.2. We compose this map and the map (1.13), which goes in the opposite direction and is defined only for solutions. The composition in one direction gives the identity transformation on solutions of the cubic theory and in the other direction it gives a gauge transformation on the space of solutions of the non-polynomial theory. Further, we prove in 2.3 that both maps send gauge equivalent solutions to gauge equivalent solutions and that all gauge orbits are accessible. Thus, the two maps define a one to one correspondence of gauge orbits of solutions. A direct consequence of this fact is that the cohomologies around solutions in both theories agree. Finally, in 2.4 we study the action of solutions and prove that it has the same value in both theories.

2.1 The map

A map of Ψ to Φ should contain multiplication by a $\xi_0 = \xi(0)$ factor, which reduces the ghost number by one, while increasing the picture number by one. In order to get the correct quantum numbers, we need also an operator that decreases the picture number by one, without modifying the other quantum numbers, such as ghost number and conformal weight. Such an operator exists, namely, the inverse picture changing operator,

$$Y(z) = (c\partial\xi e^{-2\phi})(z)\,. \tag{2.1}$$

Combining the two operators we get the operator

$$P(z) = (\xi Y)(z) = -(c\xi\partial\xi e^{-2\phi})(z)\,, \tag{2.2}$$

that can be used to define the desired mapping. So far we did not fix z. In fact, we can define a map

$$\Phi = P\Psi\,, \tag{2.3}$$

using a linear combination

$$P = \sum_i k_i P(z_i)\,. \tag{2.4}$$

The operator P contains ξ_0 as long as $\sum k_i \neq 0$. It is also possible to define P using different insertion points for $Y(z)$ and $\xi(z)$. This will become useful in 2.4.

An important property of P is that it is nilpotent,

$$P^2 = 0\,. \tag{2.5}$$

This follows from the OPE

$$P(z)P(w) = -\frac{z-w}{12}(cc'\xi\xi'\xi''\xi'''e^{-4\phi})(w) + \mathcal{O}\big((z-w)^2\big)\,. \tag{2.6}$$

This is in contrast to the OPE of the inverse picture changing operator Y with itself,

$$Y(z)Y(w) = -\frac{(cc'\xi'\xi''e^{-4\phi})(w)}{(z-w)^2} + \mathcal{O}(\frac{1}{z-w})\,. \tag{2.7}$$

The picture changing operator

$$X(z) = [Q, \xi(z)] \tag{2.8}$$

suffers from a similar singularity. The appearance of these singularities in Witten's theory were the source of a need for its modification.

We demand that the P operator be nilpotent also under the star-product

$$(P\Psi_1)(P\Psi_2) = 0\,. \tag{2.9}$$

We achieve this by inserting $P(z)$ at the string mid-point

$$P = k_1 P(i) + k_2 P(-i)\,, \tag{2.10}$$

which is invariant under the star-product.

The operator $P(z)$ is the inverse of the BRST charge Q in the sense that

$$[Q, P(z)] = 1, \qquad \forall z. \tag{2.11}$$

We can define a string field

$$A = P(z) |1\rangle, \tag{2.12}$$

where $|1\rangle$ is the identity string field. This state obeys,

$$QA = |1\rangle. \tag{2.13}$$

From the above it follows that the cohomology of Q is trivial [21, 22] (in the large Hilbert space). The same also holds for η_0 that obeys

$$[\eta_0, \xi(z)] = 1, \qquad \forall z. \tag{2.14}$$

The property (2.11) is crucial for our construction. Thus, we shall demand that the coefficients in (2.10) sum to unity,

$$P(k) = kP(i) + (1-k)P(-i). \tag{2.15}$$

Similarly to $P(k)$ we define

$$Y(k) = kY(i) + (1-k)Y(-i). \tag{2.16}$$

2.2 Equations of motion

Suppose that Φ is given by (2.3), where P is given by (2.15) for some k. The nilpotency of P under the star-product (2.9) implies,

$$G = 1 + P\Psi, \qquad G^{-1} = 1 - P\Psi. \tag{2.17}$$

Let us map G back to Ψ' using (1.13). For G of the form (2.17), we find,

$$\Psi' = G^{-1}QG = (1 - P\Psi)(\Psi - PQ\Psi) = (1 - P\Psi)(\Psi + P\Psi\Psi) = \Psi. \tag{2.18}$$

Here, we used (2.11), then we used the equation of motion for Ψ (1.5) and the nilpotency of P under the star-product (2.9). We conclude that the transformation (1.13) is the inverse of (2.3) in this direction. This is not true when the transformations are composed in the other order.

Proving that G defined by our map is a solution is now straightforward, as

$$\eta_0(G^{-1}QG) = \eta_0\Psi = 0, \tag{2.19}$$

since Ψ resides in the small Hilbert space and so is annihilated by η_0. We conclude that given our map, the equation of motion for Φ follows from that of Ψ.

2.3 Gauge equivalence of solutions and cohomology

Not all solutions of the non-polynomial theory can be written in the form (2.3). Here, we prove that all of them are gauge equivalent to ones that can be so written. Then, we prove that the two maps (2.3) and (1.13) map gauge equivalent solutions to gauge equivalent solutions. Thus, the maps define a one to one and onto mapping of gauge orbits of solutions of both theories.

Parametrize an arbitrary solution G as

$$G = \frac{1}{1 - \Phi} \,. \tag{2.20}$$

In this scheme (1.13) takes the form,

$$\Psi = G^{-1}QG = -QG^{-1}G = Q\Phi \frac{1}{1 - \Phi} \,. \tag{2.21}$$

Map it now back to the non-polynomial theory using (2.3),

$$\tilde{G} = 1 + PQ\Phi \frac{1}{1 - \Phi} = 1 + \big((1 - QP)\Phi\big)\frac{1}{1 - \Phi} = \big(1 - Q(P\Phi)\big)G \,. \tag{2.22}$$

We conclude that the state to which Ψ is mapped is gauge equivalent to the original G. Important immediate corollaries are the existence of a solution of the form (2.3) in all gauge orbits, the gauge equivalence of any two solutions that produce the same Ψ via (1.13) and the fact that the various choices of k in (2.15) result in gauge equivalent solutions.

It may seem strange that the gauge transformation relating two solutions, which give rise to the same solution in the cubic theory, is the one based on Q, present also in the cubic theory. One should remember, though, that in this equation, Q acts on string fields residing in the large Hilbert space, which includes more degrees of freedom. This extra amount of degrees of freedom is fixed by the other, η_0-based gauge transformation, which we did not fix.

Let there be two gauge equivalent solutions of the non-polynomial theory,

$$\tilde{G} = e^{-Q\Lambda_Q} G e^{\eta_0 \Lambda_\eta} \,. \tag{2.23}$$

It follows that

$$\begin{aligned}
\tilde{G}^{-1}Q\tilde{G} &= e^{-\eta_0 \Lambda_\eta} G^{-1} e^{Q\Lambda_Q} Q\big(e^{-Q\Lambda_Q} G e^{\eta_0 \Lambda_\eta}\big) \\
&= e^{-\eta_0 \Lambda_\eta} G^{-1}\big(QG e^{\eta_0 \Lambda_\eta} + GQ e^{\eta_0 \Lambda_\eta}\big) = e^{-\Lambda}\big(\Psi + Q\big)e^{\Lambda} \,,
\end{aligned} \tag{2.24}$$

where we defined

$$\Lambda = \eta_0 \Lambda_\eta \,. \tag{2.25}$$

We see that gauge equivalent states are mapped under (1.13) to gauge equivalent states.

Now, let there be two gauge equivalent string fields of the cubic theory,

$$\tilde{\Psi} = e^{-\Lambda}\big(\Psi + Q\big)e^{\Lambda} \,. \tag{2.26}$$

We get,

$$1 + P\tilde{\Psi} = 1 + P\big(-Qe^{-\Lambda} + e^{-\Lambda}\Psi\big)e^{\Lambda} = \big(Q(Pe^{-\Lambda}) + Pe^{-\Lambda}\Psi\big)e^{\Lambda} = Q(Pe^{-\Lambda})\big(1 + P\Psi\big)e^{\Lambda}$$
$$= Q(Pe^{-QP\Lambda})\big(1 + P\Psi\big)e^{\Lambda} = e^{-Q\Lambda_Q}\big(1 + P\Psi\big)e^{\eta_0\Lambda_\eta}\,, \tag{2.27}$$

where we used (2.11), (2.14) and defined

$$\Lambda_\eta = \xi\Lambda\,, \qquad \Lambda_Q = P\Lambda\,, \tag{2.28}$$

and the location of the ξ insertion is arbitrary. The result (2.27) takes the form of a gauge transformation (1.8) of $(1 + P\Psi)$. We conclude that in this direction gauge equivalent states are mapped to gauge equivalent states even off-shell.

Let us now consider linearizing the theories around two equivalent solutions. The linearized equations of motion of both theories coincide under the maps, since the exact equations coincide and similarly for the linearized gauge transformations. Solutions of the linearized equation of motion modulo linearized gauge transformations define the cohomology around the solution. We conclude that the linearized map around a solution can be interpreted as a map between the cohomology spaces of the solutions, which is one to one and onto. This suggests that solutions are mapped to solutions with the same physical content.

2.4 The action

Another physical property of a solution is its action. The action of the non-polynomial theory (1.7) can be put in the form,

$$S_{\mathrm{np}} = \frac{1}{2g_o^2}\oint\Big(-Qe^{\Phi}\eta_0e^{-\Phi} + \int_0^1 dt\,\Phi[\eta_0e^{-t\Phi}, Qe^{t\Phi}]\Big). \tag{2.29}$$

Assume that Φ is given by (2.3). We want to prove that the non-polynomial action coincides with the cubic action of Ψ.

We begin by explicitly writing down,

$$e^{t\Phi} = 1 + tP\Psi\,, \qquad e^{-t\Phi} = 1 - tP\Psi\,. \tag{2.30}$$

We use the map to write the first term as

$$S_1 \equiv -\frac{1}{2g_o^2}\oint QG\eta_0G^{-1} = \frac{1}{2g_o^2}\oint(\Psi - PQ\Psi)Y\Psi = -\frac{1}{2g_o^2}\int(YQ\Psi)(Y\Psi)\,. \tag{2.31}$$

Here we used the fact that in the resulting expression ξ_0 appears only in P, the relation between P and Y and the relation between the integration in the large and small Hilbert spaces (1.10).

We now face a problem. In the above equation we have a collision of Y's, which is singular. Let us assume that some sort of a regularization was employed such that all the expressions are well defined and proceed with the evaluation of the cubic term (we comment below on the nature of the needed regularization),

$$S_2 \equiv \frac{1}{2g_o^2}\oint\int_0^1 dt\,P\Psi[-tY\Psi, t\Psi - tPQ\Psi] = -\frac{1}{6g_o^2}\oint P\Psi[Y\Psi, \Psi] = -\frac{1}{3g_o^2}\int(Y\Psi)(Y\Psi)\Psi\,. \tag{2.32}$$

Here we used (2.9) and evaluated the simple t integral. Then, we moved to the small Hilbert space and expanded the commutator giving two identical terms. Summing up the two terms gives,

$$S_{\mathrm{np}}(k) = S_1 + S_2 = \int Y(k)^2 L \,, \qquad L \equiv -\frac{1}{2g_o^2}\left(\frac{1}{2}\Psi Q\Psi + \frac{1}{3}\Psi^3\right). \tag{2.33}$$

The action $S_{\mathrm{np}}(k)$ is formally dependent on the choice of k in (2.15). However, as we showed in the previous subsection, the choice of k corresponds to a gauge transformation. Hence, the action is actually k independent. Notice that the cubic action (1.2) is given by

$$S_{\mathrm{cub}} = \int Y_{-2} L \,. \tag{2.34}$$

Consider now an arbitrary $k \neq 0, 1$. Evaluating the action one gets,

$$S_{\mathrm{np}}(k) = k^2 S_{\mathrm{np}}(1) + (1-k)^2 S_{\mathrm{np}}(0) + 2k(1-k)S_{\mathrm{cub}} \,. \tag{2.35}$$

Using the k-independence of the action the above implies,

$$S_{\mathrm{np}} = S_{\mathrm{cub}} \,, \tag{2.36}$$

which ends our proof.

One can get directly S_{cub} from the non-polynomial theory (plus two terms that cancel each other) by choosing

$$k = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}} \,. \tag{2.37}$$

An additional property this k has is that it obeys

$$1 - k = \bar{k} \qquad \Longleftrightarrow \qquad \Re(k) = \frac{1}{2} \,, \tag{2.38}$$

which ensures that the map respects the string field reality condition.

Before closing this section, let us comment on the needed regularization. The points $z = \pm i$ are singular from several points of view: they are located on the boundary of the local coordinate patch and they are also invariant under the star product. It is clear that a regularization would have to move z slightly away from the local coordinate patch. One can think of such a regularization as a regularization of the star product itself, in the spirit of Witten's original construction. This interpretation was used in various contexts in recent developments [23, 18, 24], where it was represented using

$$\mathcal{O} \to s^{-L_0} \mathcal{O} s^{L_0} \,. \tag{2.39}$$

A proper regularization should always keep some relations intact, such as those which originate from symmetry principles. In the case at hand there are two relations that should continue to hold. First, the nilpotency of P should hold, at least in the limit in which the regularization is removed, even upon a contraction with two regularized Y insertions. A direct calculation reveals that a symmetric limit of colliding two P insertions even with

a single Y insertion is divergent. One way to remedy this problem would be to use a regularization in which the Y and ξ ingredients of P approach $\pm i$ at a different pace. It is possible then to arrange the needed nilpotency in the limit. Of course, such a regularization can no longer be considered as related to the star product and we interpret it as defining a family of regularized solutions.

The other property that the regularization should respect is the gauge equivalence of regularized solutions with different k values as well as different ϵ values, where ϵ is the regularization parameter. It is clear that for $z \neq \pm i$, more terms should be added to G, G^{-1} as compared with (2.17). We believe that such a regularization exists. We leave finding its explicit form to future work.

3. Formal gauge form of solutions in the cubic theories

In this section we discuss the formal gauge representation of solutions. In 3.1 we give the formal gauge form of solutions of the cubic superstring field theory and describe our map as a map of gauge fields. Then, in 3.2 we discuss the problems in obtaining the same formal representation for solutions of the bosonic theory and give some directions towards this end.

3.1 Gauge form of the solutions

In [18] a mapping of bosonic solutions, given in a formal gauge form, to the non-polynomial theory, was considered. That mapping could also be used to map superstring fields from the cubic theory, as we are doing here. Let Λ be a formal gauge field defining a solution to a cubic (bosonic or supersymmetric) string field theory via

$$\Psi = e^{-\Lambda} Q e^{\Lambda} \,. \tag{3.1}$$

Define a corresponding solution in the non-polynomial theory by defining the gauge fields,

$$\Lambda_\eta = \xi(z)\Lambda \,, \qquad \Lambda_Q = P(z)\Lambda \,, \tag{3.2}$$

for some z. A pure gauge solution of the non-polynomial theory is given by

$$e^{\Phi} = e^{-Q\Lambda_Q} e^{\eta_0 \Lambda_\eta} = e^{P(z)Q\Lambda - \Lambda} e^{\Lambda} \,, \tag{3.3}$$

and we see that the choice of z for ξ drops out of the expression for the solution, but remains for P. Expansion in powers of Λ gives a simple leading order behaviour,

$$\Phi_1 = P(z)Q\Lambda = P(z)\Psi_1 \,, \tag{3.4}$$

which has a canonical form (up to the choice of z) and agrees with the first order of (2.3) (with the same choice of z).

We can change our scheme [25, 18] and redefine

$$1 + \Phi = (1 - Q\Lambda_Q)\frac{1}{1 - \eta_0\Lambda_\eta} = (1 + P(z)Q\Lambda - \Lambda)\frac{1}{1 - \Lambda} = 1 + (P(z)Q\Lambda)\frac{1}{1 - \Lambda} \,. \tag{3.5}$$

If we choose to work with a P of the form (2.15), we can write that as

$$1 + \Phi = 1 + P(Q\Lambda\frac{1}{1-\Lambda}) = 1 + P\Psi\,, \tag{3.6}$$

which is exactly the map (2.3).

An important question raised in [25, 18] is how general is the representation of solutions as formal gauge solutions. We can now give a simple answer for the case of the cubic superstring field theory: all its solutions can be formally represented as gauge solutions via (1.13), where G is defined by (2.17).

One may wonder what happens when this gauge representation is used in order to obtain a general formal gauge form of solutions of the non-polynomial theory via (3.2). Now, the gauge field of the cubic theory is given by

$$\Lambda = \log G = P\Psi\,. \tag{3.7}$$

Plugging this Λ into (3.2) results in an expression, which generally differs from what one would like to obtain and in particular is trivial,

$$\tilde{G} = 1\,, \tag{3.8}$$

if the same z is used in the definition of P and in (3.2). This implies that we cannot use our construction to obtain a general formal gauge representation for the non-polynomial theory. This should have been expected, since we did not extend the space in any way that would allow the introduction of formal gauge fields.

3.2 Formal gauge representation for the bosonic solutions

The map of gauge fields to the non-polynomial theory (3.2), (3.3) can also be used to map bosonic string fields to the non-polynomial theory[1]. On the other hand, the mapping (2.3) cannot be generally used to map bosonic solutions to analogous ones in the non-polynomial theory. The reason being that the supersymmetric BRST charge contains terms that do not exist in the bosonic theory,

$$Q = Q_0 + Q_1 + Q_2\,. \tag{3.9}$$

Here, the subscript of Q counts its ϕ charge. The bosonic BRST charge is contained in Q_0. The difference $Q_0 - Q_{bos}$ is zero when acting upon bosonic states not containing factors of the b ghost. The Q_2 term, on the other hand, gives zero upon terms not containing the c ghost. The Q_1 term is zero on terms not containing the ∂X^μ oscillators. Thus, bosonic solutions are generally modified when translated to the analogous supersymmetric ones. This implies that (2.18) does not simply generalize to the bosonic case, since the use there

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[1]The idea of mapping bosonic solutions to supersymmetric ones may seem strange, since the two theories are obviously different. However, it seems natural to assume that a bosonic string field, without an explicit dependence on more than ten space-time directions, can be mapped to some sort of a supersymmetric string field, whose physical meaning is similar. Using such a map we would be able to learn about the superstring from its simpler bosonic counterpart. For another relation of the two theories see [26].

of the supersymmetric Q is important for its derivation. We do not have a general from for a formal gauge representation for bosonic solutions.

One can still map bosonic solutions to supersymmetric ones, in those cases where a formal gauge representation of the solution exists,

$$\Psi_{bos} = e^{-\Lambda_{bos}} Q_{bos} e^{\Lambda_{bos}} . \tag{3.10}$$

One can seek a formal gauge representation for bosonic solutions by enlarging the bosonic Hilbert space using a couple of canonically conjugated anti-commuting variables $\tilde{b}_0, \tilde{c}_0$. Define their conformal weight to be zero and their ghost number to be -1 and 1 respectively. Also, define

$$\tilde{c}_0 \left|0\right\rangle = 0 . \tag{3.11}$$

Then, the redefinition

$$\tilde{Q}_{bos} = Q_{bos} + \tilde{c}_0 , \tag{3.12}$$

does not change the cohomology problem when restricted to the subspace without the $\tilde{b}_0$ mode. In the enlarged space, on the other hand, $\tilde{Q}_{bos}$ has a trivial cohomology,

$$\tilde{Q}_{bos}\Psi = 0 \quad \Rightarrow \quad \Psi = Q(\tilde{b}_0\Psi) . \tag{3.13}$$

We also define that these operators simply multiply with respect to the star product. Then, we can write all bosonic solutions as formal gauge solutions,

$$\Psi = G^{-1}\tilde{Q}_{bos}G , \qquad G = 1 + \tilde{b}_0\Psi . \tag{3.14}$$

While the above may be useful for the bosonic theory, it does not help us with mapping bosonic solutions to supersymmetric ones. A redefinition of the supersymmetric Q according to

$$\tilde{Q} = Q + \tilde{c}_0 , \tag{3.15}$$

may be used to define a supersymmetric string field $G^{-1}\tilde{Q}G$. Unfortunately, this string field contains in general a $\tilde{b}_0$ piece, due to the $Q - Q_{bos}$ term. So it cannot be used for deriving supersymmetric solutions from the bosonic ones.

4. Incorporating the GSO($-$) sector

Our construction can be generalized to the cases of non-BPS D-branes and D-brane systems. Recall first that in order to describe the GSO($-$) sector one has to introduce into the theory the so called "internal Chan-Paton indices" [27, 28]. For the non-polynomial theory one has to tensor the GSO($+$) sector with the two dimensional unit matrix $\mathbf{1}$ and the GSO($-$) sector with the Pauli matrix σ_1. The two gauge fields of a GSO($+$) sector are to be tensored with σ_3, while the gauge fields of the GSO($-$) sectors get a factor of $i\sigma_2$. The operators Q, η_0 are also tensored with σ_3 and in the action a trace over the new matrix space should be performed and the result should be normalized by $2 = \text{tr}(\mathbf{1})$. With these simple rules all the algebraic properties hold just as in the case of a single GSO($+$) sector. The non-BPS D-brane can be described by introducing one GSO($+$) and one GSO($-$) sector, while for

more complicated brane systems one has to introduce in addition to the internal space also the genuine Chan-Paton space, with each factor tensored also in the internal space according to its GSO parity. The same story works for the cubic theory. Now, however, the gauge fields are the ones that get the factors of $\mathbf{1}, \sigma_1$ and the string fields are tensored with $\sigma_3, i\sigma_2$. The only other modification is the inclusion of a factor of σ_3 also for Y_{-2}.

The algebraic properties that are important for our construction are the commutation relations (2.11,2.14). To ensure that they continue to hold we have to assign a factor of σ_3 to $P(z)$ and $\xi(z)$, which implies that $Y(z)$ carries a factor of $\mathbf{1}$. Then, upon moving from the large to the small Hilbert space by integrating the ξ_0, the leftover σ_3 factor combines with the two Y's and the equality of the action also follows. With these simple assignments all the proofs of section 2 carry over to the general brane system without any further modification. We conclude that there is no obstruction for generalizing our map to an arbitrary D-brane system.

5. Vacuum solutions

In this section we discuss vacuum solutions in superstring field theory. In 5.1 we describe the preliminary proposal for a vacuum solution in the non-polynomial theory on a BPS D-brane given in [18]. We explain the problem of the proposal and correct it. Then, we discuss the analogous construction, by Erler, in the cubic theory [19] and show that the two are related by (2.3). This proves that the solution of the non-polynomial theory shares the properties of correct action and zero cohomology derived in [19] for the cubic solution.

The solution on the BPS D-brane was criticized in [20], where a solution living on a non-BPS D-brane was constructed. In 5.2 we describe the criticism and the new solution and prove that the cohomology of the solution is trivial as expected. We further study the one parameter family of solutions interpolating Erler's solution on the non-BPS D-brane and the new solution and show that all the members in this family are gauge equivalent.

5.1 On the BPS D-brane

In [18], it was suggested that the map of gauge solutions (3.2),(3.3) from the bosonic to the non-polynomial theory can be used to find a "tachyon vacuum" in the supersymmetric theory. Since there is no tachyon on the BPS D-brane the new vacuum cannot represent tachyon condensation. It can still, potentially, represent a state where the D-brane is absent.

During most of that work the operator $P(0)$ was used. This results in

$$\Lambda_Q = (PBc)(0) \ket{0} = P(0) \ket{0} , \tag{5.1}$$

implying that

$$Q\Lambda_Q = \ket{0} , \tag{5.2}$$

which lives in the small Hilbert space. The gauge transformation $\eta_0 \Lambda_\eta$ always leads, by definition, to a state in the small Hilbert space. The solution resides in the small Hilbert space and is therefore pure gauge. Since the source of illness here is the OPE of $P(z)$ and

$Bc(z)$, an immediate remedy would be to move $P(z)$ to another point. Here, we suggest to move it to $\pm i$, that is, to use P of (2.15), so as to get

$$\Lambda_Q = P\Lambda \,. \tag{5.3}$$

This expression lives in the large Hilbert space.

In order to calculate the coefficients of the various fields we have to expand P around $z = 0$. We choose k of (2.37) and use a derivative, i.e., conformal weight expansion,

$$P = (\cos \partial - \sin \partial)P(0) = P(0) - P'(0) - \frac{1}{2}P''(0) + \dots \,. \tag{5.4}$$

To find the lowest non-trivial field we can use (3.6) and recall that for Ψ the lowest level field with non-zero value is $c(0)\,|0\rangle$ and the next non-zero fields are of two levels higher. Acting with (5.4) on $c(0)\,|0\rangle$ we get

$$Pc(0)\,|0\rangle \approx -P'(0)c(0)\,|0\rangle = -cc'\xi\xi'e^{-2\phi}\,|0\rangle \,. \tag{5.5}$$

The field $cc'\xi\xi'e^{-2\phi}$ lives in the large Hilbert space, so it may give rise to some physical content. It does not obey the Siegel gauge, so it was not studied much in the level-truncation literature. It may seem to describe an unintegrated vertex operator because of the c factor. However, a direct evaluation reveals that it does not correspond to a vertex operator. This field is the only field other than the identity field with all quantum numbers being zero. It follows that there is no way to correct it in order to obtain a vertex operator so it is an auxiliary field.

In [19], Erler suggested to use the trivial mapping of Schnabl's gauge field to the cubic superstring field theory, in order to obtain a non-perturbative vacuum in this theory. He further showed that it gives the expected action of a solution describing the absence of the D-brane and that the cohomology around the solution is empty. The cohomology proof is in fact straightforward. The resulting string field is almost identical to the bosonic one and differs only due to the new terms in Q. This gives,

$$\Psi = \Psi_{bos} + B\gamma^2\,|0\rangle = \Psi_{bos} + B\eta\partial\eta e^{2\phi}\,|0\rangle \,. \tag{5.6}$$

The cohomology proof in the bosonic case [22] used the operator

$$A = B \int_0^1 dt\,\Omega^t \,, \tag{5.7}$$

where $\Omega = |0\rangle$ is the vacuum and its powers are the wedge states $\Omega^t = |t+1\rangle$ [29]. This field obeys the relation

$$Q_\Psi A = QA + \Psi A + A\Psi = 1 \,, \tag{5.8}$$

which implies that the cohomology is empty. Since the only new term in (5.6) is simply proportional to B and contains no c factors, the above equation holds without modifying A for the supersymmetric case as well.

The calculation of the action relies on the fact that in order to get a non-trivial result after the integration, the integrand has to be proportional to $e^{-2\phi}$. The picture changing

operator Y_{-2} supplies a -4 factor to the ϕ momentum in the action, so the string field should supply a factor of two. This can come either from the γ^2 term in (5.6) or from Q acting on c. Finally, all expectation values give linear functions rather than the trigonometric functions one gets in the bosonic case. This fact enables a much simpler evaluation, which results in the correct D-brane tension.

Mapping Erler's solution using (2.3) is equivalent to using the modified gauge representation (5.3). From the discussion of section 2 we conclude that the solution of the non-polynomial theory also has the correct action and cohomology.

5.2 On the Non-BPS D-brane

In [20], Aref'eva, Gorbachev and Medvedev (hereafter AGM), criticized Erler's work. First, it was claimed that a vacuum solution on a non-BPS D-brane should non exist. Then, it was claimed that the condensation of the string field $c(0)\left|0\right\rangle$, which is the leading order in Erler's solution, should not correspond to the tachyon vacuum anyway, since this field, called "the BPS tachyon" is an auxiliary field and not a vertex operator and is an artifact of working in the zero picture. It is easy to see that if one tries to map it to the "natural" -1 picture, it is mapped to zero. In fact, a solution based on this vertex operator (in the theory with chiral Y_{-2}) was found using lowest-level truncation in [30], where it was interpreted as a supersymmetry breaking solution. In AGM it was suggested that the correct resolution of these problems is to define the string field on the non-BPS D-brane using a gauge field with Erler's gauge field in the GSO(+) sector[2] and with

$$\Lambda_- = B\gamma(0)\left|0\right\rangle, \tag{5.9}$$

in the GSO($-$) sector.

It is immediate that the action is the same as in Erler's case. We think of the AGM solution as being expanded around Erler's one and write the difference

$$\tilde{\Psi} = \Psi_{AGM} - \Psi_E. \tag{5.10}$$

From general considerations it follows that

$$S_{AGM} = S_E + \frac{1}{6}\int \frac{1}{2}\operatorname{tr}(Y_{-2}\tilde{\Psi}^3). \tag{5.11}$$

The form of the gauge field (5.9) implies that

$$\tilde{\Psi}_+ \approx \gamma B\gamma, \tag{5.12}$$

carries momentum two, while the GSO($-$) part given by (split-string notations [31, 32, 33])

$$\tilde{\Psi}_- = Fc\frac{KB}{1-\Omega}\gamma F + F\gamma\frac{KB}{1-\Omega}cF, \tag{5.13}$$

carries one unit of ϕ momentum. It follows that the minimal ϕ-momentum power that one can get in the above action is four, which implies

$$S_{AGM} = S_E. \tag{5.14}$$

[2]A GSO(+) component is indispensable, since the action of a purely GSO($-$) solution is trivially zero.

The proof (not given in AGM) that the cohomology of the AGM solution is empty relies again on the same A (5.7) as in the previous cases, only now it should be tensored with σ_3. For (5.8) to hold in this case, we only have to prove

$$[A, \tilde{\Psi}] = 0 \,. \tag{5.15}$$

This is trivial for the GSO(+) part (5.12), which is proportional to B, without any c insertions. The GSO(−) part (5.13) gives

$$[\tilde{\Psi}, A] = \left(- F\gamma \frac{KB}{1-\Omega} cFB \int_0^1 \Omega^t + \int_0^1 \Omega^t BFc \frac{KB}{1-\Omega} \gamma F \right) \otimes \sigma_1 \,. \tag{5.16}$$

Expanding the denominator we can write the terms in parentheses as

$$KB \int_0^\infty \big(\Omega^t(\gamma(0) \,|0\rangle) - (\gamma(0) \,|0\rangle)\Omega^t \big) = B \int_0^\infty \partial_t \big(\Omega^t(\gamma(0) \,|0\rangle) - (\gamma(0) \,|0\rangle)\Omega^t \big) =$$
$$B\big(\Omega^\infty(\gamma(0) \,|0\rangle) - (\gamma(0) \,|0\rangle)\Omega^\infty \big) = 0 \,,$$

where we used the fact that K acts as a derivation with respect to the strip length and the final identity is correct, upon a contraction with a Fock space state, to order N^{-3}, where we used Ω^N with $N \to \infty$ for regularization. This is the same behaviour as in the original proof of the vanishing of the cohomology [22].

The calculation of the action, the triviality of the cohomology and the existence of γ in the gauge field seem to be in favour of the interpretation of the AGM solution as representing tachyon condensation on the non-BPS D-brane. Nevertheless, one may wonder whether this solution is indeed different from Erler's one, or are they gauge equivalent. Furthermore, the reasoning regarding the form of the solution as well as the evaluation of the action and the cohomology remains unchanged if one multiplies the GSO(−) gauge field Λ_- by an arbitrary parameter ϵ,

$$\Lambda_+ = Bc(0) \,|0\rangle \,, \qquad \Lambda_- = \epsilon B\gamma(0) \,|0\rangle \,. \tag{5.17}$$

In this manner one gets a one parameter family Ψ_ϵ of solutions generated by (5.17). This family interpolates between Erler's solution and the AGM solution. All these solutions have the same action and trivial cohomology so it is natural to expect them to be gauge equivalent. If this is indeed the case, one is to conclude that the $\epsilon = 0$ solution can also be used to describe tachyon condensation on the non-BPS D-brane. It is then very natural to assume that Erler's solution on the BPS D-brane and its counterpart in the non-polynomial theory indeed manage to describe the state without the BPS D-brane despite the fact that there are no tachyons living on it.

Writing the gauge transformation between two solutions in the ϵ-family is (at least formally) immediate. Since these solutions are formally given as gauge solutions one only has to compose these two gauge transformation to get a new one. Then, the question is whether the resulting transformation is a genuine gauge transformation or whether it is singular in any sense.

We call the gauge transformation from Erler's solution, with $\epsilon = 0$ to the ϵ-solution Ψ_ϵ, $e^{\Lambda_E^\epsilon}$. Recall however that the gauge form of the solutions is given in the left scheme, so we can write

$$e^{\Lambda_E^\epsilon} = (1 - \Lambda_E)\frac{1}{1 - \Lambda_\epsilon} = 1 \otimes \mathbf{1} + \epsilon B\gamma(0)\,|0\rangle \otimes \sigma_1\,,$$
$$e^{-\Lambda_E^\epsilon} = (1 - \Lambda_\epsilon)\frac{1}{1 - \Lambda_E} = 1 \otimes \mathbf{1} - \epsilon B\gamma(0)\,|0\rangle \otimes \sigma_1\,,$$

(5.18)

where we used the relation

$$\Lambda_-^2 = \Lambda_- \Lambda_+ = 0\,,$$

(5.19)

to get the final expressions. These transformations form an abelian group with the simple multiplication rule,

$$e^{\Lambda_E^\epsilon} e^{\Lambda_E^{\tilde\epsilon}} = e^{\Lambda_E^{\epsilon+\tilde\epsilon}}\,,$$

(5.20)

where we again used (5.19). We see no sign of a singularity at any value of ϵ.

We can also check that this gauge transformation acts correctly on A

$$A_\epsilon = e^{-\Lambda_E^\epsilon} A\, e^{\Lambda_E^\epsilon} = A\,.$$

(5.21)

A singularity in the gauge transformation (5.18) would probably express itself as a singularity in the above equation. This does not happen. Finally, one may want to calculate some invariants of the Ψ_ϵ solution as in [34, 35] and verify that they are ϵ-independent. We did not try this method, but we believe that if tried it would indeed result in ϵ-independent expressions.

We can map the ϵ-family to the non-polynomial theory. Explicitly, the solution

$$\Psi_\epsilon = \left(Fc\frac{KB}{1-\Omega}cF + F\gamma\big(\frac{\epsilon^2 K}{1-\Omega} + (1-\epsilon^2)\big)B\gamma F\right) \otimes \sigma_3$$
$$+ \epsilon\left(Fc\frac{KB}{1-\Omega}\gamma F + F\gamma\frac{KB}{1-\Omega}cF\right) \otimes (i\sigma_2)\,,$$

(5.22)

is mapped to

$$\Phi_\epsilon = P\left(Fc\frac{KB}{1-\Omega}cF + F\gamma\big(\frac{\epsilon^2 K}{1-\Omega} + (1-\epsilon^2)\big)B\gamma F\right) \otimes \mathbf{1}$$
$$+ \epsilon P\left(Fc\frac{KB}{1-\Omega}\gamma F + F\gamma\frac{KB}{1-\Omega}cF\right) \otimes \sigma_1\,.$$

(5.23)

While this mapping seems to be trivial, it serves us to address one last important point. The B line integral runs from $-i$ to i and the B operator has non-trivial commutation relation with P. It follows that again we have to regularize in order to make sense of the expression. We let B pass to the right of the regularized P insertion, as is implicit in (5.23). Then, one may wonder whether the commutation relation of B and P would invalidate the nilpotency of Φ, which was inherited from that of P. As in 2.4, we can establish the nilpotency by defining a different pace for the regularization of $Y(z)$ and $\xi(z)$. All our treatment remains consistent when such a regularization is used.

When using wedge state algebra in the evaluation of specific expressions, one has to include the so called phantom-terms. The form and origin of these terms is quite clear by now [19]. They originate from replacing terms of the form $(1-\Omega)^{-1}$ with their series form. The form and amount of phantom terms needed depends on the specific solution and on the theory considered. For (5.22), it was found that two phantom terms are needed and their form was explicitly given in [19, 20]. Due to the linear nature of the mapping, one can then use this result, in order to write down also the phantom pieces needed for (5.23). Of course, the phantom pieces are an artifact of a specific regularization. There is no need for the phantom piece, neither in (5.22) nor in (5.23) when using other consistent regularizations, such as level-truncation. The advantage of using the wedge-state based regularization is the ability to obtain analytical results (in some calculations).

6. Conclusions

In this work we presented mappings between gauge orbits of the cubic and the non-polynomial superstring field theories. The mapping in one of the directions depends on a parameter k, which turns out to be a gauge degree of freedom. One can choose k in a way that respects the string reality condition. An obvious question is whether it is possible to extend our construction to the quantum theory. Two important challenges stand in the way: the extension of the maps to general (off-shell) string fields and the inclusion of the Ramond sector.

The space of string fields that one has to map for the off-shell theory is larger than the space of string fields we considered in this work. This stems from the fact that both theories are infinitely reducible gauge theories. Moreover, the reducibility of the gauge symmetry is realized only on-shell. This suggests that quantization should be addressed using the BV formalism [36, 37, 38] (see [39, 40] for reviews), as was done for the bosonic theory in [41, 42, 43][3]. The enlargement of the space of fields comes from the inclusion of the (second quantized) ghosts and antifields. It can happen that the maps can be defined in this enlarged space despite not being defined on the smaller space. These hypothetical maps would have to commute with the superbrackets and would therefore have the interpretation of canonical transformations.

The fact that the gauge symmetries of the two theories are different may be an obstacle for this ambitious plan. A possible resolution is to gauge fix one of the gauge symmetries of the non-polynomial theory. One can find in the literature the gauge fixing

$$\xi_0 \Phi = 0 \,. \tag{6.1}$$

This is not invariant under the star product. A more adequate gauge choice seems to be

$$\xi(i)\Phi = 0 \,, \tag{6.2}$$

or the analogous form in the physical case of a theory with the reality condition included. This gauge fixing is immediately solved by writing

$$\Phi = \xi(i)\tilde{\Phi} \,, \tag{6.3}$$

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[3]A BRST study of the bosonic string was performed in [44, 45].

and our map reduces in this case to

$$\tilde{\Phi} = Y(i)\Psi \,. \tag{6.4}$$

Now, the spaces and gauge symmetries of both theories are quite similar. For the theory with the reality condition imposed, this construction does not work in this way, since P of (2.15) cannot be written as a product of ξ and Y. One possible resolution of this problem is to impose the gauge condition directly with P, with the immediate solution being (2.3). The fact that the transformation is linear implies that at least formally no problematic measure factors would emerge. The question is how to generalize the inverse map (1.13) so as to include also off-shell states and anti-fields.

The inclusion of the Ramond sector is essential, since Ramond states appear in loops even in scattering processes of NS states. This raises the familiar problems related to defining an action for the Ramond sector in superstring field theory [16, 46]. We hope that it will be possible to extend this work to the Ramond sector. Presumably it could also help to resolve some of the problems related to the definition of the Ramond sector action.

Understanding the Ramond sector is desirable already at the classical level. For one thing, one may want to be able to study classical Ramond sector solutions. Another important issue is the evaluation of the complete spectrum of the theory around a classical solution. As we mentioned in 5.2, a preliminary numerical study of a solution analogous to Erler's one performed in [30] resulted in the interpretation of the new vacuum as a supersymmetry breaking one. Understanding the Ramond sector will enable us to study this question analytically. We believe that this solution represents the closed string vacuum and so should have a vanishing cohomology also in the Ramond sector. Not only that the supersymmetry should not be broken around it, but the solution should also be left invariant under a larger supersymmetry. Other aspects of supersymmetry are also of importance.

A common objection to the use of the cubic theory is its usage of the (inverse) picture changing operators $Y(\pm i)$. The problem being that these operators have non-trivial kernels, which may result in the equations of motion (1.4) and (1.5) being inequivalent. It is not clear if this is really a problem, since the problematic states are of a somewhat exotic form, i.e., they have a factor that is localized at the string mid-point and it is not clear if such states belong to the space of string fields that one has to consider. We used just these states in our map to the non-polynomial theory. Hence, we may conclude that the cubic theory on a space not including these states is equivalent to the non-polynomial one on a space where they are included. One may still wonder whether it is possible to define an adequate space of string fields for the cubic theory. It is known for example that the space of finite combinations of local insertions on the vacuum is not closed under the star product. Closeness under the star product is certainly a very desirable property of the looked-for space. Can one define a space closed under the star product, that is large enough for including the expected classical solutions, yet not containing states with insertions at $\pm i$?

In order to get some intuition about the needed space of string fields we may use the remarkable developments in the field during the last couple of years, following Schnabl's analytical solution [47]. Subsequent work [31, 48, 22] enabled the proof of Sen's conjectures [49, 50], analytical solutions describing marginal deformations were found [51,

52, 53, 54, 55, 25, 18, 56, 57, 58], scattering amplitudes and gauge conditions were studied [59, 60, 24, 61] and much more [62, 63, 64, 65, 66, 67, 68, 69, 70, 71]. These developments used to a large extent wedge states [29, 72] with local insertions [73] and with insertions of the line integrals B, K. It seems plausible that the space of string fields should be based on such states, which do form a closed algebra under the star product (similar but extended subalgebras may include general surface states [74, 75] with local insertions and the subalgebra of wedge and butterfly states [76, 77] with local insertions). Of course, infinite linear combinations are still needed. This may result in no restriction at all on the space, since without restricting the coefficients all possible states can be formed from local insertions, even without the introduction of wedge states. Thus, we have to assume that the coefficients should be restricted not to grow, in some sense, faster than in the known cases. If this is the case, the formal problems in our construction can be solved, as was explained at the end of 5.2. Obviously, more research in this direction is needed.

We showed that the vacuum solution on a non-BPS D-brane is gauge equivalent to a solution that can be defined also on the BPS D-brane. This convinced us that the solution for the BPS D-brane should be taken seriously. The main criticism on this solution is that it contradicts our understanding that the BPS D-brane is stable. In other words, a condensation of a BPS D-brane would correspond to a non-continuous change in the Ramond-Ramond charge. Yet, we can think of a continuous dynamical process in which a D-brane disappear. For example, we can start with a system of two D5-branes, send one D5-brane to spatial infinity and remain with only one D5-brane. One might worry that hiding one D-brane at infinity is only a trick and try considering a compact space. However, on a compact space the total RR change has to vanish. This means that the possibility of hiding the D-brane at infinity is correlated with the existence of a non-vanishing RR charge. We could also consider space-time filling D9-branes. In this case tadpole cancellation would constrain the total charge to zero. All the above arguments are related to the close-string sector, which we did not consider in this paper. Still, it is important to show that they work out correctly, otherwise inconsistencies would probably show up at 1-loop calculations. A related issue is the common expectation that open string field theory should describe only changes of open string moduli. It would be very desirable to understand better the realm of open string field theory.

Acknowledgments

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