THE MICROSCOPIC DYNAMICS OF QUANTUM SPACE AS A GROUP FIELD THEORY

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Max Planck Institute for Gravitational Physics (Albert Einstein Institute)

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MAIN AIM(S) OF THIS TALK

- highlight some issues that are relevant to any tentative quantum theory of gravity, and show room for a QFT formalism
- introduce the Group Field Theory approach to QG (focus on 3d case)
- discuss some recent results in GFT and point out what still needs to be done

QUANTUM GRAVITY

- Main lesson from current theory of gravity (GR): "Gravity is spacetime geometry", thus spacetime is itself a physical (and dynamical) system
- so, maybe Quantum Gravity is not so much a quantization of GR, but a microscopic quantum theory of spacetime structure (atomic theory of space)?
- background independence:
  - no spacetime geometry can be taken as fixed reference for processes
  - it should allow transitions between different backgrounds (e.g. topological BF theory not good enough): theory should be rich
  - still, above leaves room for presence of several "background structures"(see later)
- so, first QG questions:
  - what do space and time emerge from, at quantum level?
  - can we define a quantum theory *of* space & time, thus in absence of space and time?
  - if QFT framework, what are the fundamental quanta? ....quanta *of* space itself....
  - but can it be a QFT?
- note: most of above relevant even if QG is "just"quantum GR

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FAILURE OF PERTURBATIVE QUANTIZATION AROUND FLAT SPACE

Quantum gravity is not a quantum field theory of gravitons on flat space:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \rightarrow S(h_{\mu\nu}) \rightarrow Z = \int \mathcal{D}h_{\mu\nu}\, e^{-S(h_{\mu\nu})}$$

such theory is perturbatively non-renormalizable (no more than effective field theory)

- missing ingredients?
  - new symmetries? (supergravity?)
  - unification? only gravity+matter can be quantized as above?
  - non-local fundamental structure? beyond point-like objects? (strings,...)
  - degrees of freedom? metric not correct variable?
  - GR itself only effective field theory (not to be quantized as such)?
- background independence!
  - cannot fix spacetime geometry as background
  - ok, are there other background structures (also in GR)?
- above does not rule out QFT as framework.......
  ....but QFT needs *some* background....

Background structures in General Relativity

What are the background structures in GR?

- continuum and local (field-theoretic) picture of space(time)
- dimensionality & signature
- local symmetry group (Lorentz)
- spatial topology
- spacetime topology
- space of geometries on given topology (Wheeler's superspace)

which of them is our quantum (field) theory of gravity to be based on?

which of them are turned into dynamical features of the world (thus, new d.o.f.)?

GFT keeps dimensionality, signature and local symmetry group, in discrete setting

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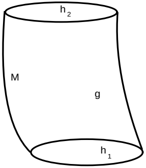
TRADITIONAL/CONSERVATIVE FRAMEWORKS (FORMAL)

The most conservative option is to retain all the background structures of GR, consider spacetime and geometry as fundamental, and "just quantize GR"

canonical approach
kinematics $\longrightarrow \Psi(h_{ij}(x)) \quad \hat{O}(h_{ij}(x))$
dynamics $\longrightarrow \widehat{\mathcal{H}}_{WdW}\, \Psi(h_{ij}) = 0$



covariant approach
$\langle h^F_{ij} | h^I_{ij} \rangle = \int_{h^I}^{h^F} \mathcal{D}g\, e^{-S(g)}$, e.g. $S(g) = \int d^4x \sqrt{g}\, R(g)$

having made sense of the above, "only problem" is semi-classical limit

making sense of it: discretize = divide S, M into chunks $\longrightarrow \Delta$
$\Rightarrow h_{ij}, g_{\mu\nu} \longrightarrow$ finite variables $\{L_e\}$, $S(g) \longrightarrow S_\Delta(L_e)$ (discrete QG)

$\longrightarrow$ quantum Regge calculus, dynamical triangulations......
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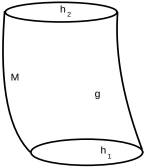

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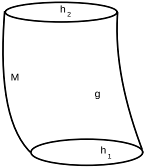

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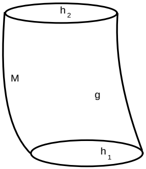

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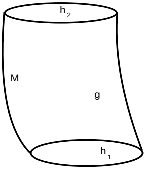

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LESS CONSERVATIVE (EVEN MORE FORMAL): DYNAMICAL TOPOLOGY?

2nd (3rd?) quantization of gravity? (Giddings, Strominger, Banks, Coleman, Hawking, Kuchar, Isham, McGuigan,...)
a) field on space of geometries (say, on S^3);
b) all possible interactions (creation/annihilation) of universes (topology change)?

$\Psi(h_{ij}) \longrightarrow \phi(h_{ij})$ on (super-)space of geometries (Giulini, '09) on S^3

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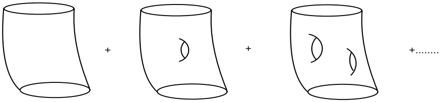

$$Z = \int \mathcal{D}\phi\, e^{-S(\phi)} = \sum_M \lambda^V Z_M = \sum_M \lambda^V \int \mathcal{D}g\, e^{i\,S(g;M)}$$

"impossible" to define in proper mathematical way + conceptual issues

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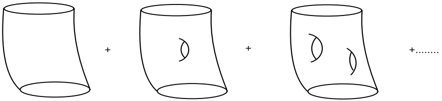

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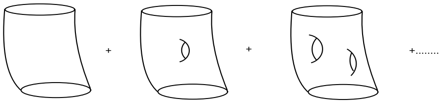
Feynman diagrams M: 

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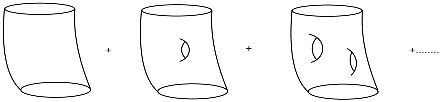

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GRAVITY/GEOMETRY: FUNDAMENTAL OR EMERGENT? GOING DISCRETE?

is the notion of gravity and/or geometry fundamental?

if not, what are the pre-geometric data defining the ‘substance”(kinematics) of QG?

and then,

what is the quantum dynamics of the quantum (pre-)geometric data?

Using dynamical lattices (or any discrete structure) is highly non-trivial step:

- it means dropping *all background structures* of GR, together with continuum
- all have to be recovered in continuum approx.; non-trivial!!!

discrete, finite sets of data (classical or quantum), even if coming from discretizing a smooth geometry, can be understood as “pre-geometric/pre-spacetime”data, from which spacetime and geometry are *emergent*

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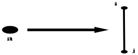
TOWARD GFT: MATRIX MODELS FOR 2D QUANTUM GRAVITY

- general idea: generalise combinatorics of Feynman diagrams from 1d to 2d, from graphs to discrete surfaces, from point particles to 1d objects



- $M^i{}_j \quad i,j = 1,...,N \quad N \times N$ hermitian matrix
- action:

$$S(M,g) \quad = \quad \frac{1}{2} trM^2 - \frac{g}{\sqrt{N}} trM^3 = \frac{1}{2} M^i{}_j K_{jkli} M^k{}_l - \frac{g}{\sqrt{N}} M^i{}_j M^m{}_n M^k{}_l V_{jmknli}$$

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- transition amplitudes defined in terms of Feynman diagrams

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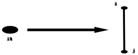

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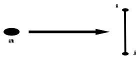

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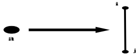

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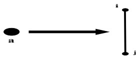
TOWARD GFT: MATRIX MODELS FOR 2D QUANTUM GRAVITY

- general idea: generalise combinatorics of Feynman diagrams from 1d to 2d, from graphs to discrete surfaces, from point particles to 1d objects



- $M^i{}_j \quad i, j = 1, ..., N \quad N \times N$ hermitian matrix
- action:

$$S(M, g) \;=\; \frac{1}{2} trM^2 - \frac{g}{\sqrt{N}} trM^3 = \frac{1}{2} M^i{}_j K_{jkli} M^k{}_l - \frac{g}{\sqrt{N}} M^i{}_j M^m{}_n M^k{}_l \, V_{jmknli}$$

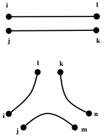
$$K_{jkli} = \delta^j{}_k \, \delta^l{}_i \qquad V_{jmknli} = \delta^j{}_m \, \delta^n{}_k \, \delta^l{}_i \qquad \left(K^{-1}\right)_{jkli} = K_{jkli}$$

- fundamental building blocks are 1d simplices with no additional data; microscopic dynamics: no GR, pure 2d combinatorics

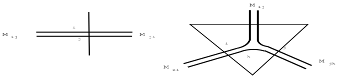
- transition amplitudes defined in terms of Feynman diagrams

MATRIX MODELS - FEYNMAN DIAGRAMS AND SIMPLICIAL COMPLEXES

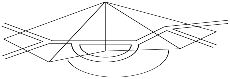
building blocks for Feynman diagrams: $\left(K^{-1}\right)_{jkli}$ V_{jmknli}



- simplicial intepretation:



$\Gamma \simeq$ 2d simplicial complex Δ (triangulation)
$\simeq$ 2d discrete spacetime



Feynman amplitudes: join vertices with propagators and sum over common variables (indices) i

$$Z = \sum_{\Gamma} g^{V_\Gamma} N^{\chi(\Gamma)}$$

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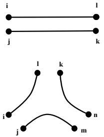

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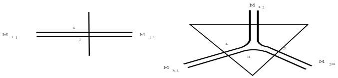

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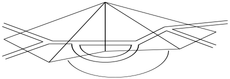

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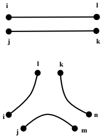

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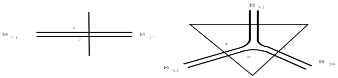

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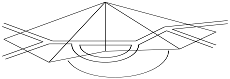

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MATRIX MODELS AND SIMPLICIAL 2D GRAVITY

- continuum (Riemannian) 2d GR: $\int_S d^2x \sqrt{g}\,(-R(g) + \Lambda) = -4\pi\,\chi + \Lambda A_S$
- discrete 2d GR: chop surface S into equilateral triangles of area a:
  $\frac{1}{G}\int_S d^2x \sqrt{g}\,(-R(g) + \Lambda) \rightarrow -\frac{4\pi}{G}\chi + \frac{\Lambda a}{G} t$
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  (trivial) sum over histories of discrete GR on given 2d complex
  plus sum over all possible 2d complexes of all topologies
- discrete 2nd quantization of GR in 2d !!!!!
- question: control over sum over triangulations/topologies?
  - large-N limit - sum governed by topological parameters

$$Z = \sum_\Delta g^{t_\Delta} N^{2-2h} = \sum_h N^{2-2h} Z_h(g) = N^2 Z_0(g) + Z_1(g) + N^{-2} Z_2(g) + \ldots..$$

  - in the limit $N \rightarrow \infty$ (semi-classical approximation of discrete system), only spherical (trivial topology, planar, genus 0) contribute

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MATRIX MODELS AND CONTINUUM 2D GR

- question: does it match results from continuum 2d gravity path integral?
  - task: continuum limit for trivial topology
  - expand $Z_0(g)$ in g: $Z_0(g) = \sum_V V^{\gamma-3}\left(\frac{g}{g_c}\right)^V \simeq_{V\to\infty} (g-g_c)^{2-\gamma} \quad (\gamma > 2)$
  - expectation value of area of surface: $\langle A\rangle = a\langle t_\triangle\rangle = \langle V_\Gamma\rangle = a\frac{\partial}{\partial g}\ln Z_0(g) \simeq \frac{a}{g-g_c}$, for large V
  - thus we can send area of triangle $a \to 0$ and $t = V \to \infty$ (continuum limit), while sending $g \to g_c$, to get finite continuum macroscopic area
  - this defines continuum limit (phase transition of discrete system!)
  - results match those of continuum 2d gravity path integral (GR as effective theory)
- can also define continuum limit with contributions from non-trivial topologies - double scaling limit
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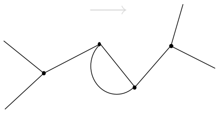
From point particles to fields, from matrices/tensors to GFT

point particles $\longrightarrow$ fields

$$S(X) = \frac{1}{2}X^2 + \frac{\lambda}{3}X^3$$



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matrices

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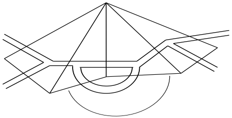

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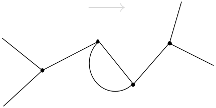

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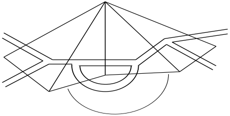

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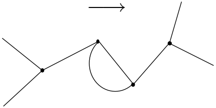

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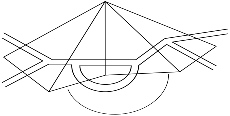

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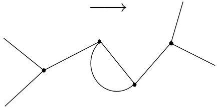

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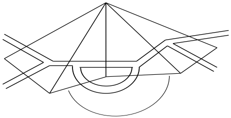
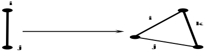


Tensor models

- generalize further in (combinatorial) dimension, from 2d to 3d (and higher) - from 1d objects (edges) to 2d objects (triangles) (and higher) - from 2d simplicial complexes as FD to 3d ones (and higher)



- $M^i{}_j \rightarrow T_{ijk} \quad i,j,k = 1,..,N \qquad N \times N \times N$ tensor
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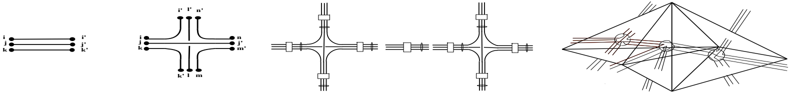
  with combinatorial pattern of edges in tetrahedron
- $Z = \int \mathcal{D}T\, e^{-S(T)} = \sum_\Gamma \lambda^{V_\Gamma} Z_\Gamma$
- Feynman diagrams again formed by vertices, lines and faces, but now 1) also form "bubbles"(3-cells), and 2) are dual to 3d simplicial complexes



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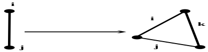

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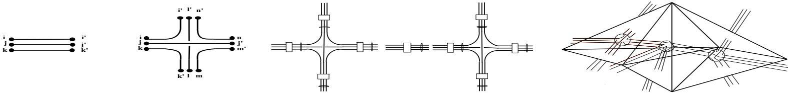

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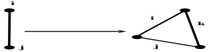

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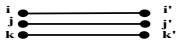
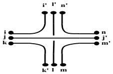
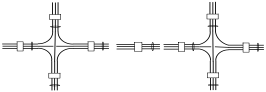
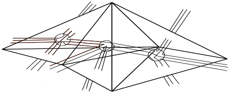
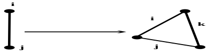





TENSOR MODELS

- generalize further in (combinatorial) dimension, from 2d to 3d (and higher) - from 1d objects (edges) to 2d objects (triangles) (and higher) - from 2d simplicial complexes as FD to 3d ones (and higher)



- $M^i{}_j \rightarrow T_{ijk} \quad i,j,k = 1,..,N \qquad N \times N \times N$ tensor
- action: $S(T) = \frac{1}{2} trT^2 - \lambda \, trT^4 = \frac{1}{2}\sum_{i,j,k} T_{ijk}T_{kji} - \lambda \sum_{ijklmn} T_{ijk}T_{klm}T_{mjn}T_{nli}$
  kinetic term $= K_{ijki'j'k'} = \delta_{ii'}\delta_{jj'}\delta_{kk'} = (K^{-1})_{ijki'j'k'} =$ propagator
  vertex term $= V_{ii'jj'kk'll'mm'nn'} = \delta_{ii'}\delta_{jj'}\delta_{kk'}\delta_{ll'}\delta_{mm'}\delta_{nn'}$
  with combinatorial pattern of edges in tetrahedron
- $Z = \int \mathcal{D}T \, e^{-S(T)} = \sum_\Gamma \lambda^{V_\Gamma} Z_\Gamma$
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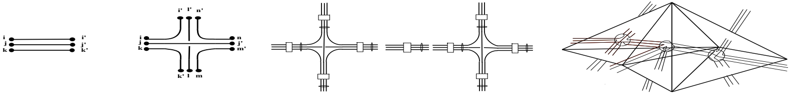

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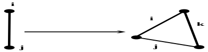

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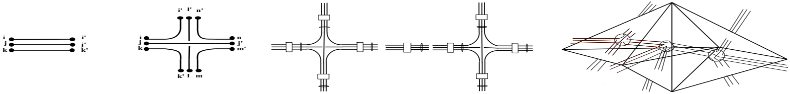

Tensor models

- Z defined as sum over all 3d simplicial complexes (manifolds and pseudo-manifolds)
  (pseudo-manifold = neighbourood of point not homeomorphic to a 3-Ball)
- why are they not good?
  - no topological expansion of amplitudes - no control over topology of diagrams
  - no way to separate manifolds from pseudo-manifolds
  - no direct/nice relation with 3d simplicial (classical or quantum) gravity - not enough structure/data in the amplitudes, and in boundary states
- in $d > 2$, gravity is -much- less trivial, both classically and quantum-mechanically
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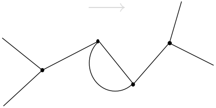
FROM POINT PARTICLES TO FIELDS, FROM MATRICES/TENSORS TO GFT

point particles $\longrightarrow$ fields

$$S(X) = \frac{1}{2}X^2 + \frac{\lambda}{3}X^3$$



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$\downarrow$ $\downarrow$

matrices $\longrightarrow$ Group Field Theory

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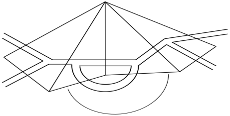

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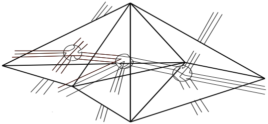

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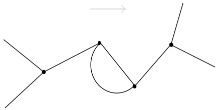

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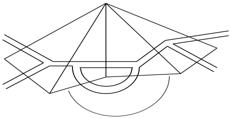

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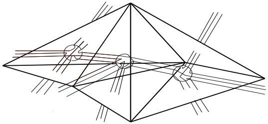

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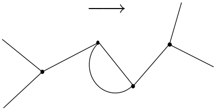

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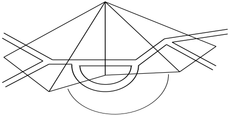

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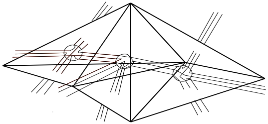

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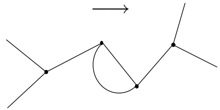

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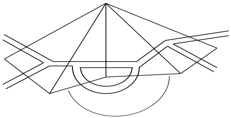

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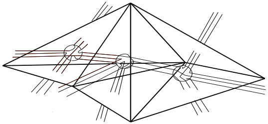

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....

The Group Field Theory formalism

general reviews:
Freidel, ’05, Oriti, ’06, ’07, ’10

work by:
Baratin, Ben Geloun, Bonzom, Boulatov, De Pietri, Fairbairn, Freidel, Girelli, Gurau, Livine, Louapre, Krajewski, Krasnov, Magnen, Noui, Ooguri, Oriti, Perez, Reisenberger, Rivasseau, Rovelli, Ryan, Smerlak, Tanasa, ...

GFTs can be defined, a priori, in any dimension and signature; here: focus on 3d Riemannian gravity $\rightarrow$ use $\mathrm{SU}(2)$ (local gauge group of gravity)

3D QUANTUM GRAVITY AS A GFT : KINEMATICS OF 2D QUANTUM SPACE

"tensor models plus pre-geometric data" guided by LQG, simplicial QG, NCG

- Triangle in $\mathbb{R}^3$; (2nd quantized) kinematics encoded in field φ (space of triangle geometries)
- triangle geometries parametrized by three $\mathfrak{su}(2)$ Lie algebra elements x_i attached to edges = discrete triad variables (discretization of triad fields along edges)

$$\varphi \; : \; (x_1, x_2, x_3) \in \mathfrak{su}(2)^3 \longrightarrow \varphi(x_1, x_2, x_3) \in \mathbb{R}$$

- $\mathfrak{su}(2)$ is non-commutative space; φ should reflect this non-commutativity
- from LQG (simplicial BF): phase space for edge = $\mathcal{T}^*\mathrm{SU}(2) \simeq \mathfrak{su}(2) \times \mathrm{SU}(2)$
  - use non-commutative Fourier transform (Majid, Freidel, Livine, Mourad, Noui,...): $C(\mathrm{SU}(2)) \leftrightarrow C(\mathfrak{su}(2))$
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    $e_g(x) : \mathfrak{su}(2) \times \mathrm{SU}(2) \to \mathbb{C} : (x, g) \to e^{i\frac{1}{2}Tr(xg)}$ (fundamental representation)
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3D QUANTUM GRAVITY AS A GFT : KINEMATICS OF 2D QUANTUM SPACE

"tensor models plus pre-geometric data"guided by LQG, simplicial QG, NCG

- Triangle in $\mathbb{R}^3$; (2nd quantized) kinematics encoded in field φ (space of triangle geometries)
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- $\mathfrak{su}(2)$ is non-commutative space; φ should reflect this non-commutativity
- from LQG (simplicial BF): phase space for edge = $\mathcal{T}^*\mathrm{SU}(2) \simeq \mathfrak{su}(2) \times \mathrm{SU}(2)$
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    $e_g(x) : \mathfrak{su}(2) \times \mathrm{SU}(2) \to \mathbb{C} : (x, g) \to e^{i\frac{1}{2}Tr(xg)}$ (fundamental representation)
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$$\varphi(x_1, x_2, x_3) = \int [dg]^3 \, \varphi(g_1, g_2, g_3) \, e_{g_1}(x_1) e_{g_2}(x_2) e_{g_3}(x_3)$$

group elements = parallel transports of connection along links dual to the edges

- In order to define a geometric triangle, edge vectors have to 'close':

$$\varphi(x_1, x_2, x_3) = (C * \varphi)(x_1, x_2, x_3), \quad C(x_1, x_2, x_3) = \delta_0(x_1 + x_2 + x_3)$$

with delta functions:

$$\delta_x(y) := \int dg \, e_{g^{-1}}(x) e_g(y) \quad \text{s.t.} \quad \int d^3y \, (\delta_x * f)(y) = \int d^3y \, (f * \delta_x)(y) = f(x)$$

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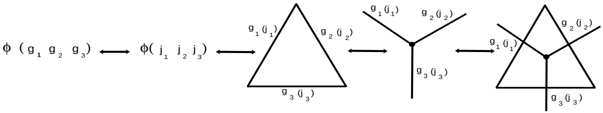
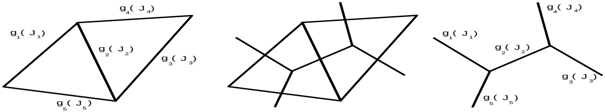
3D QUANTUM GRAVITY AS GFT: KINEMATICS OF 2D QUANTUM SPACE

- φ is building block of (quantum) 2d space



- fields can be convoluted (in group or Lie algebra picture) or traced (in representation picture) with respect to some common argument → gluing of multiple triangles along common edges → more complex simplicial structures, or, dually, more complicated graphs (many-GFT-particle states)



- generic observable/state/boundary configuration: $O(\varphi) = \sum_n O_n \left(\varphi^{*\, n}\right)$
- in representation space, generic (polynomial) state is labeled by spin networks (also kinematical quantum states in Loop Quantum Gravity approach)

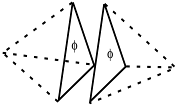
3D QG AS GFT: CLASSICAL DYNAMICS OF QUANTUM SPACE

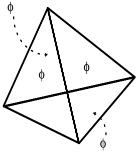
- Define classical action for $\varphi_{123} = \varphi(x_1, x_2, x_3)$
  - interaction term: four geometric triangles glued pairwise along common edges to form tetrahedron
  - kinetic term: gluing of tetrahedra along common triangles, by edge identification
  - no gravity, no continuum, no GR input

$$S = \frac{1}{2} \int [dx]^3 \, \varphi_{123} * \varphi_{123} - \frac{\lambda}{4!} \int [dx]^6 \, \varphi_{123} * \varphi_{345} * \varphi_{526} * \varphi_{641}$$

where $\phi_i * \phi_i := (\phi * \phi_-)(x_i)$, with $\phi_-(x) = \phi(-x)$

- propagator and a vertex:





$$\int_{SU(2)} dh_t \prod_{i=1}^{3} (\delta_{-x_i} * e_{h_t})(y_i), \quad \int_{SU(2)} \prod_t dh_t \prod_{i=1}^{6} (\delta_{-x_i} * e_{h_{tt'}})(y_i)$$

3D QG as GFT: classical dynamics of quantum space

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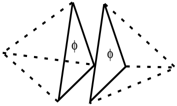

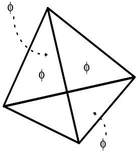

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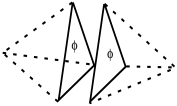

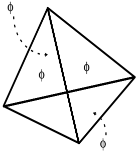

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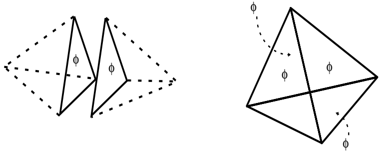

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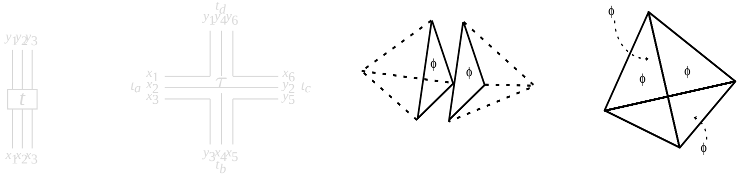

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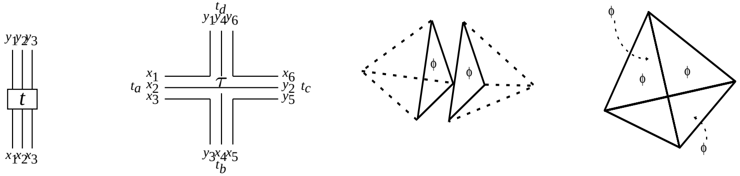

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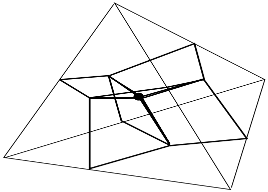
3D QG AS GFT: CLASSICAL DYNAMICS OF QUANTUM 2D SPACE

- geometrical meaning:



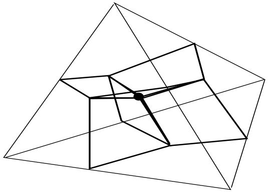
  - pair of variables in two fields (x_e, y_e) associated to the same edge e = edges vectors seen from the frames associated to the two triangles t, t' sharing it
  - vertex functions: the two variables are identified, up to parallel transport, and up to a sign for two opposite edge orientations

- in group picture (Boulatov, '92):

$$\mathcal{K}(g_e, \tilde{g}_e) = \int dh_t \prod_{e=1}^{3} \delta(g_e h_t \tilde{g}_e^{-1}) \qquad \mathcal{V}(g_{tt'}) = \prod_{t\tau=1}^{4} \int dh_{t\tau} \prod_{t \neq t'} \delta(g_{tt'} h_{t\tau} h_{t'\tau}^{-1} \tilde{g}_{tt'}^{-1})$$

- geometric meaning: flatness of each wedge (portion of face inside tetrahedron): piecewise-flat context, trivial matching at boundary

3D QG AS GFT: CLASSICAL DYNAMICS OF QUANTUM 2D SPACE



- geometrical meaning:
  - pair of variables in two fields (x_e, y_e) associated to the same edge $e =$ edges vectors seen from the frames associated to the two triangles t, t' sharing it
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3D QG AS GFT: CLASSICAL DYNAMICS OF QUANTUM 2D SPACE

- in representation space:

$$S(\varphi) = \frac{1}{2}\sum_{\{j\},\{m\}} \varphi^{j_1 j_2 j_3}_{m_1 m_2 m_3}\,\varphi^{j_3 j_2 j_1}_{m_3 m_2 m_1} - $$
$$- \frac{\lambda}{4!}\sum \varphi^{j_1 j_2 j_3}_{m_1 m_2 m_3}\,\varphi^{j_3 j_4 j_5}_{m_3 m_4 m_5}\,\varphi^{j_5 j_2 j_6}_{m_5 m_2 m_6}\,\varphi^{j_6 j_4 j_1}_{m_6 m_4 m_1}\left\{\begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array}\right\}$$

- from which:

$$\mathcal{K} = \mathcal{K}^{-1} = \delta_{j_1\tilde{j}_1}\delta_{m_1\tilde{m}_1}\delta_{j_2\tilde{j}_2}\delta_{m_2\tilde{m}_2}\delta_{j_3\tilde{j}_3}\delta_{m_3\tilde{m}_3}$$
$$\mathcal{V} = \delta_{j_1\tilde{j}_1}\delta_{m_1\tilde{m}_1}\delta_{j_2\tilde{j}_2}\delta_{m_2\tilde{m}_2}\delta_{j_3\tilde{j}_3}\delta_{m_3\tilde{m}_3}\delta_{j_4\tilde{j}_4}\delta_{m_4\tilde{m}_4}\delta_{j_5\tilde{j}_5}\delta_{m_5\tilde{m}_5}\delta_{j_6\tilde{j}_6}\delta_{m_6\tilde{m}_6}\left\{\begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array}\right\}$$

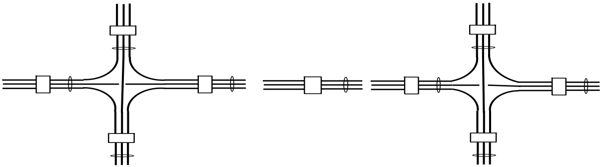
- geometry rather obscure - however, dynamics directly in terms of quantum numbers labelling quantum states of the theory

3d QG as GFT: microscopic quantum dynamics

- the quantum theory is defined by the partition function, in Feynman expansion:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} Z(\Gamma)$$

- building blocks of FD are:
  - lines of propagation, with 3 labelled strands (dual to triangles),
  - vertices of interaction (made of 4×3 labelled strands re-routed following the combinatorics of a tetrahedron)
- this produces: 2-cells, identified by strands of propagation passing through several vertices, and then closing (for closed FD), dual to edges; 'bubbles'= 3-cells bounded by the above 2-cells, dual to vertices of simplicial complex



- Feynman graphs Γ are fat graphs/cellular complexes topologically dual to 3d triangulated (pseudo-)manifolds of ALL topologies

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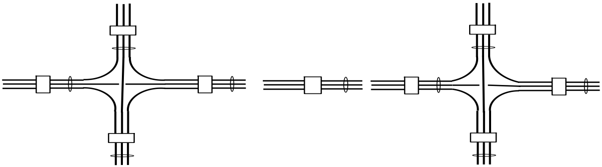

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3d QG as GFT: GFT Feynman amplitudes

Feynman amplitudes $Z(\Gamma)$ obtained by convoluting vertices with propagators

They can be expressed, equivalently, in Lie algebra, group or representation picture

In the Lie algebra (non-commutative) representation we obtain (A. Baratin, DO, '10):

$$Z(\Gamma) = \int \prod_L dh_L \prod_f dx_f \, e^{i \sum_f \mathrm{Tr}\,(x_f H_f)}$$

H_f = total holonomy around boundary of face $f \in \Gamma$, dual to edge of triangulation Δ

This is simplicial path integral of 1st order 3d gravity

continuum theory: $S(e,\omega) = \int tr\,(e \wedge F(\omega))$

for open FD, one gets 3d gravity with boundary terms (fixed boundary triad)

Explicit link with simplicial gravity path integrals
(solution to first problem of tensor models)

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volume of space of flat (discrete) connections (consistent with continuum picture)

In terms of group representations:

$$Z(\Gamma) = \left(\prod_f \sum_{j_f} \right) \prod_f (2j_f + 1) \prod_v \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\}$$

Ponzano-Regge spin foam (state sum) model

spin foam models are sum over histories of spin networks in Loop Quantum Gravity

exact duality spin foam model ↔ simplicial gravity path integral

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- GFTs are (combinatorially) non-local field theories on groups (Lie algebras), interpreted as 2nd quantized theories (generalization of matrix models)
  - of simplicial geometry and
  - of canonical LQG (QFT of spin networks)
- field ϕ represents "2nd quantized simplex"or "2nd quantized spin net vertex"
- arguments of field have interpretation of pre-geometric data
- microscopic dynamics dictated by discrete (minimal) geometric considerations
- both geometry and topology are dynamical
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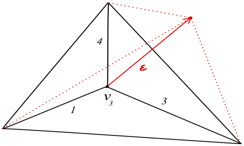
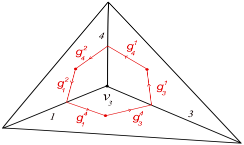
- diffeomorphisms in GFT
- GFT perturbative renormalization

DIFFEOMORPHISM SYMMETRY IN 3D (DISCRETE) GRAVITY

In discrete gravity diffeos are not defined; however, one can identify a discrete analogue of them and a corresponding symmetry of discrete action (at least in 3d with $\Lambda = 0$) (Rocek-Williams '84)(Freidel-Louapre '02)(Dittrich-Bahr '09):

- discrete translation symmetry of triad variables (in 1st order theory):
  $B_e \longrightarrow B_e + \phi_{v1}(g_L) - \phi_{v2}(g_L) \qquad \phi_v \in \mathfrak{su}(2)$
  becomes discrete diffeo transformations of edge lengths in Regge calculus (2nd order theory)
- corresponds to vertex translations in $\mathbb{R}^3$ embedding





- in canonical gravity, it implies *flatness constraint* on boundary connection:
  $H_l\, \Psi(\{g_L\}) = \Psi(\{g_L\}) \quad \forall \quad closed \quad loop\ l$

To identify diffeomorphism symmetry, need to work in (non-commutative) triad representation of GFT action - (necessary to) use "colored model"

DIFFEOMORPHISM SYMMETRY IN BOULATOV MODEL (A. BARATIN, F. GIRELLI, DO, '10)

label vertices in tetrahedron by $i = 1, 2, 3, 4$ - edges are labeled as $e = (ij)$ - color triangles of tetrahedron by their 3 vertices - define 4 fields: ϕ_{ijk} (coloring needed for field transformation)

$$S(\{\phi_{ijk}\}) = \sum_{(ijk)} \int [dx_{ij}](\phi_{ijk} * \phi_{ijk})(x_{ij}, x_{jk}, x_{ki}) +$$

$$+ \quad \frac{\lambda}{4!} \int \phi_{123}(x_{12}, x_{23}, x_{31}) * \phi_{234}(x_{32}, x_{34}, x_{41}) * \phi_{124}(x_{21}, x_{24}, x_{14}) * \phi_{134}(x_{13}, x_{43}, x_{43})$$

- transformation of GFT field (for $\epsilon_v \in \mathfrak{su}(2)$) (translation of triangle vertices):

$$\left(T_{\{\epsilon_v\}} \triangleright \phi_{123}\right)(x_{12}, x_{23}, x_{31}) = `\phi(x_{12} - \epsilon_1 + \epsilon_2, x_{23} - \epsilon_2 + \epsilon_3, x_{31} - \epsilon_3 + \epsilon_1)'$$

$$\left(T_{\{\epsilon_v\}} \triangleright \phi_{123}\right)(g_{12}, g_{23}, g_{31}) = e^{i\mathrm{Tr}(\epsilon_1(g_{31}g_{12}^{-1}))}\, e^{i\mathrm{Tr}(\epsilon_2(g_{12}g_{23}^{-1}))}\, e^{i\mathrm{Tr}(\epsilon_3(g_{23}g_{31}^{-1}))}\phi(g_{12}, g_{23}, g_{31})$$

- can show that action $S(\{\phi_{ijk}\})$ is *invariant* (care with ordering, $*$-products,...) invariance implies *flat connection* on boundary of tetrahedron (GFT vertex)
- nice match of simplicial gravity and canonical LQG results in single formalism
- this is a *quantum group* symmetry
- from QFT point of view, it is a *global* symmetry

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$$S(\{\phi_{ijk}\}) = \sum_{(ijk)} \int [dx_{ij}](\phi_{ijk} * \phi_{ijk})(x_{ij}, x_{jk}, x_{ki}) +$$

$$+ \frac{\lambda}{4!} \int \phi_{123}(x_{12}, x_{23}, x_{31}) * \phi_{234}(x_{32}, x_{34}, x_{41}) * \phi_{124}(x_{21}, x_{24}, x_{14}) * \phi_{134}(x_{13}, x_{43}, x_{43})$$

- transformation of GFT field (for $\epsilon_v \in \mathfrak{su}(2)$) (translation of triangle vertices):

$$\left(T_{\{\epsilon_v\}} \triangleright \phi_{123}\right)(x_{12}, x_{23}, x_{31}) = `\phi(x_{12} - \epsilon_1 + \epsilon_2, x_{23} - \epsilon_2 + \epsilon_3, x_{31} - \epsilon_3 + \epsilon_1)'$$

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- can show that action $S(\{\phi_{ijk}\})$ is *invariant* (care with ordering, $*$-products,...) invariance implies *flat connection* on boundary of tetrahedron (GFT vertex)
- nice match of simplicial gravity and canonical LQG results in single formalism
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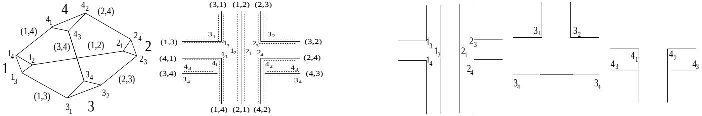
- see intertwiner of single copy of $DSU(2)$ translation at each vertex of Δ



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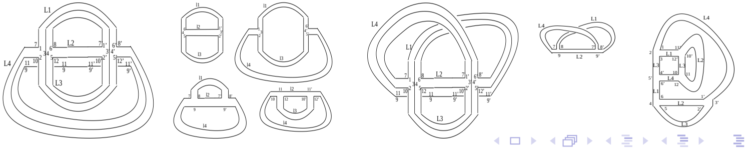

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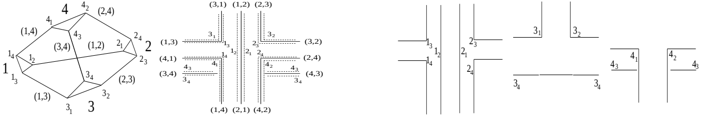

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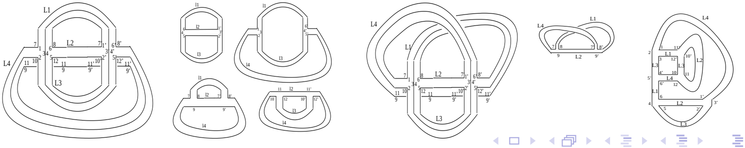

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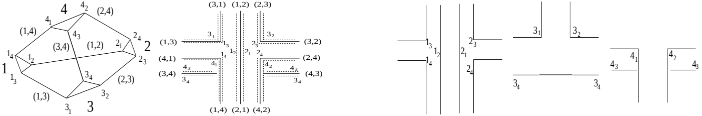

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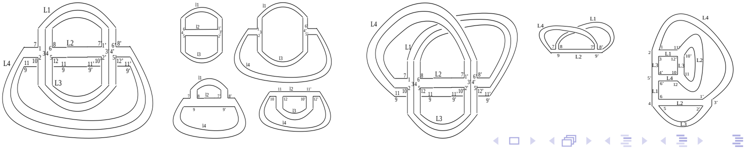

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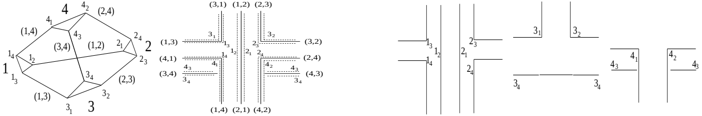

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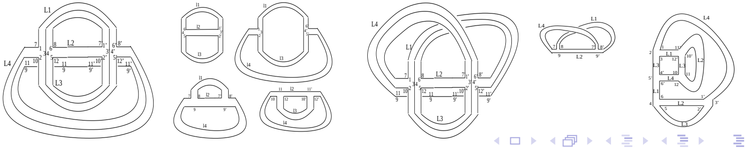

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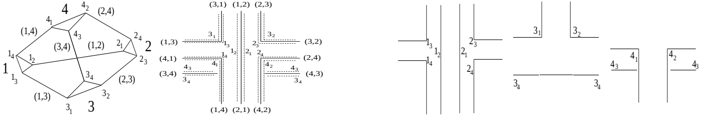

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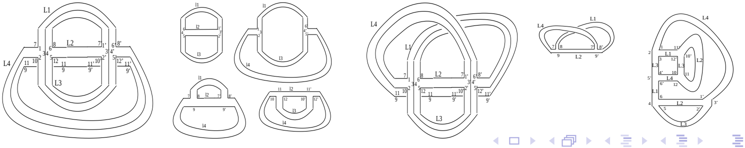

Perturbative GFT renormalization - the 3d case

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Question: can you control the perturbative GFT sum over Feynman diagrams (including sum over topologies)?

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$$Z(\Gamma) = \prod_{L\in\Gamma}\int dh_L \prod_f \delta(\prod_{L\in\partial f} h_L)$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorics, all topologies and pseudo-manifolds $\longrightarrow$ difficult to isolate divergences, unclear which FDs need renormalization
- results:
  - identification of ‘Type 1’graphs, generalization of 2d planar graphs, allowing for contraction procedure, later proved to be -manifolds- of -trivial topology-
  - exact power counting of divergences for this class of FD
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- Freidel-Louapre modification adding (different gluing of four triangles):

$$+\frac{\lambda\,\delta}{4!}\prod_{i=1}^{6}\int dg_i\,[\,\phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_4,g_2,g_6)\phi(g_6,g_5,g_1)]\,.$$

  - general perturbative bounds: $Z_\Gamma \leq K^n \Lambda^{6+3n}$, with n vertices
  - perturbative sum for partition function and free energy are Borel summable
- colored model (color each triangle in tetrahedron) (same amplitudes)

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- suggestions from condensed matter and analogue gravity systems (superfluid Helium-3, BEC) (Jacobson, Hu, Volovik, Laughlin, Visser, Unruh, Schuetzhold, Liberati, Sindoni, etc)
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- hypothesis: continuum is coherent, equilibrium many-particles physics for GFT quanta at low temperature (hydrodynamic approx): "quantum spacetime fluid"?
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