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Superfluid spherical Couette flow

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Abstract. We solve numerically the two-fluid, Hall–Vinen–Bekarevich–Khalatnikov equations for a He-II-like superfluid contained in a differentially rotating, spherical shell, generalizing previous simulations of viscous spherical Couette flow (SCF) and superfluid Taylor–Couette flow. The system tends towards a stationary but unsteady state, where the torque oscillates persistently, with amplitude and period determined by dimensionless gap width δ and rotational shear $\Delta\Omega$. In axisymmetric superfluid SCF, the number of meridional circulation cells multiplies as the Reynolds number Re increases. In nonaxisymmetric superfluid SCF, three-dimensional vortex structures are classified according to topological invariants. We find that the mutual friction is "patchy"; that is, it takes different forms in different parts of the vessel, a surprising new result.

1. Introduction

Spherical Couette flow (SCF), is observed when a viscous fluid fills a differentially rotating, spherical shell [1]. However, the problem of superfluid SCF, for example in He II, has not yet been explored numerically or experimentally [2]. Even in cylindrical (Taylor–Couette) geometry, only a limited amount of information exists regarding state transitions in the superfluid problem, for the special cases of very small gap widths ($\delta \sim 0.02$) and small Reynolds numbers ($Re \lesssim 380$) [3].

In this paper, we employ a numerical solver recently developed to solve the two-fluid Hall–Vinen–Bekarevich–Khalatnikov (HVBK) equations for a rotating superfluid [4] to study the *unsteady* behaviour of SCF in viscous (Navier–Stokes) fluids and superfluids, in two and three dimensions. We study the effect of the normal fluid/superfluid dynamics on the time-dependence of the macroscopic hydrodynamics. Differential rotation drives a meridional counterflow which can excite microscopic turbulence in the superfluid. We study the coupling between the macroscopic flow and microscopic superfluid turbulence, which has an important effect on the form and strength of the mutual friction between the normal and superfluid components and hence on the torque on the container.

2. HVBK theory

The motion of a rotating superfluid is described by the HVBK equations, a generalization of the two-fluid Landau-Tisza theory for He II that includes the physics of quantized vortices [5–7]. Quantized vortex lines mediate an interaction between the normal fluid and the superfluid component known as mutual friction. For a rectilinear vortex array the mutual friction is anisotropic, with $\mathbf{F} \propto \hat{\boldsymbol{\omega}}_s \times \boldsymbol{\omega}_s \times \mathbf{v}_{ns}$, where $\mathbf{v}_s$ the velocity of the superfluid, $\mathbf{v}_n$ the velocity of the normal fluid, $\mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$, and $\boldsymbol{\omega}_s = \nabla \times \mathbf{v}_s$ [6]. If the counterflow speed v_{ns} exceeds a threshold, growing Kelvin waves are excited along the vortex lines (the Donnelly-Glaberson instability, DGI) and the rectilinear array is disrupted to form a self-sustaining, reconnecting, "turbulent" vortex tangle [8]. In this case the mutual friction per unit mass is isotropic, and takes the Gorter-Mellink (GM) form, with $\mathbf{F} \propto v_{ns}^2 \mathbf{v}_{ns}$ [6–8].

We solve the HVBK equations using a pseudospectral collocation method for the spatial discretization and a time-split algorithm to step forward in time. A detailed description of the method can be found in [7, 9].

3. Unsteady, axisymmetric flow states

We investigate the *unsteady* behaviour of superfluid SCF by performing a set of axisymmetric ($N_\phi = 4$) and nonaxisymmetric ($N_\phi = 12$) numerical experiments with rotational shear in the range $0.1 \leq \Delta\Omega \leq 0.3$, in medium and large gaps ($0.2 \leq \delta \leq 0.5$), with HV and GM mutual friction. Figure 1 depicts the meridional streamlines of the normal (left) and superfluid (right) components in superfluid SCF, for $Re = 10^4$, $\delta = 0.5$, and $\Delta\Omega = 0.3$, with HV mutual friction. In the equator we observe large circulation cells adjacent to the inner boundary, each one containing twin cores circulating in the same sense. The flow in each hemisphere is symmetric about the equatorial plane. This flow pattern is characteristic of moderately high Reynolds numbers ($Re \gtrsim 10^4$). The HV mutual friction couples normal and superfluid components strongly, so that their meridional streamlines are similar. At lower Reynolds numbers ($Re \lesssim 10^3$), the streamlines of the two components differ markedly. The normal component behaves like a viscous, Navier–Stokes fluid at low Re, with a small number ($\lesssim 3$) of large circulation cells on each side of the equatorial plane. The superfluid is influenced less by the normal fluid, due to the stiffness provided by the vortex tension force [2]. When GM mutual friction operates, the normal and superfluid components behave similarly, both at low and high Reynolds numbers, but the superfluid displays a richer variety of circulation cells, while the normal component behaves like an uncoupled Navier–Stokes fluid.

The torque exerted by the normal fluid component on the inner and outer spheres, is plotted versus time in Figures 2a and 2b. It oscillates, with peak-to-peak amplitude $\sim 10^{-3}$ for $t \leq 30$ and $\sim 10^{-5}$ for $t \geq 30$. These oscillations, with period $\approx 2\pi/\Omega$, persist as long as the differential rotation is maintained, up to $t = 214$ in our longest simulation. They are observed at all the Reynolds numbers considered in this paper ($1 \times 10^2 \leq Re \leq 3 \times 10^4$). The oscillation amplitude is greater for HV friction; oscillations are still observed for GM friction, but with peak-to-peak amplitude $\sim 10^{-6}$. Superfluid SCF is intrinsically unsteady and quasiperiodic.

In axisymmetric superfluid SCF, the torque oscillates persistently during steady differential rotation (after initial transients die away), with typical period $\sim \Omega^{-1}$ and fractional amplitude $\sim 10^{-2}$. The amplitude of the oscillations increases with Re. If the outer sphere is impulsively set into corotation with the inner sphere after a period of differential rotation, the relaxation time scale is set mainly by the angular velocity change $\Delta\Omega$, while the long-term evolution of the torque is controlled by δ. The viscous torque exerted by a superfluid with GM mutual friction is approximately three times smaller when compared to the torque when HV friction is acting.

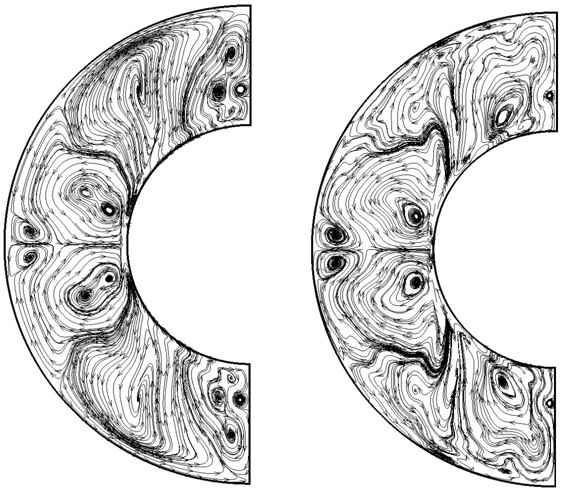

Figure 1. Snapshots at $t = 214$ of meridional streamlines for the normal (left) and superfluid (right) components in superfluid SCF, with $Re = 10^4$, $\delta = 0.5$, and $\Delta\Omega = 0.3$.

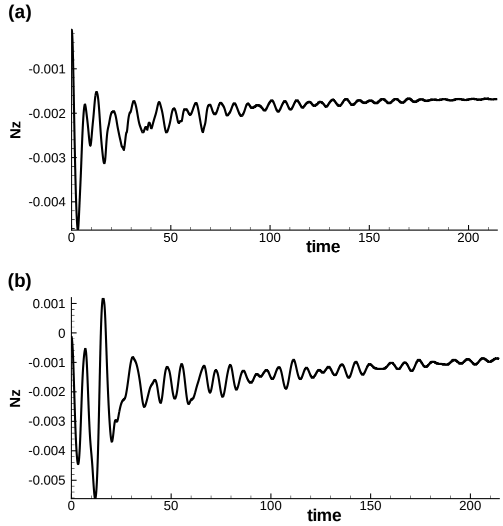

Figure 2. Viscous torque exerted on the (a) inner and (b) outer spheres as a function of time in superfluid SCF, with HV mutual friction, $\delta = 0.5$, $\Delta\Omega = 0.3$, and $Re = 10^4$.

4. Unsteady, nonaxisymmetric flow states

Using the discriminant criterion D_A for identifying vortical structures in the flow [7, 10], we study a system that exhibits nonaxisymmetric flow: a differentially rotating shell in which the rotation axes of the inner and outer spheres are mutually inclined by an angle $\theta_0 = 3$ °, with $\mathbf{v}_s$ weakly coupled to $\mathbf{v}_n$ via GM mutual friction [7]. We show the topology of the superfluid in Figure 3. We present isosurfaces of $D_A = 10^{-4}$ (Figures 3a–d) and $D_A = -10^{-4}$ (Figures 3e–h) for $\mathbf{v}_s$ in superfluid SCF with $Re = 10^3$. Throughout most of the volume, the flow is focal in nature. Strain-dominated regions, shown in orange, also exist, but are less widespread. They have a threaded structure (Figures 3e–h). The normal fluid dynamics (not shown), on the other hand, is almost completely dominated by vorticity, with strain-dominated regions only detected in small regions close to the poles.

Nonaxisymmetric superfluid SCF induced by tilting the rotation axis of the inner and outer spheres (for angles $\theta_0 \leq 3$ °), is focal throughout most of the volume of the shell, with strain-dominated regions confined inside narrow toroidal threads. Vorticity isosurfaces have a characteristic wedge shape that drifts along the equator. Persistent torque oscillations are also observed in all three dimensional flows considered, with period $\sim 6\Omega^{-1}$.

5. Applications

The results on superfluid dynamics summarised in this paper are also relevant to laboratory experiments by Tsakadze and Tsakadze [11], the only systematic experimental study of spherical Couette flow in He II undertaken to date, which studied the deceleration of axisymmetric vessels made of glass and plastic and filled with He II, after an impulsive acceleration. The Tsakadze experiments — and by extension, the numerical results in this paper — are of general interest in understanding the physics of superfluid turbulence in rotating systems [7, 8]. One interesting effect is that the meridional circulation can generate *patchy* mutual friction: the DGI is excited in parts of the superfluid (e.g. near the walls, on the rotation axis, and at the equator) but not elsewhere [8] (see for example Fig. 9 of [8]). We are currently performing numerical simulations of the Tsakadze experiments using a patchy friction force. The results of this investigation will

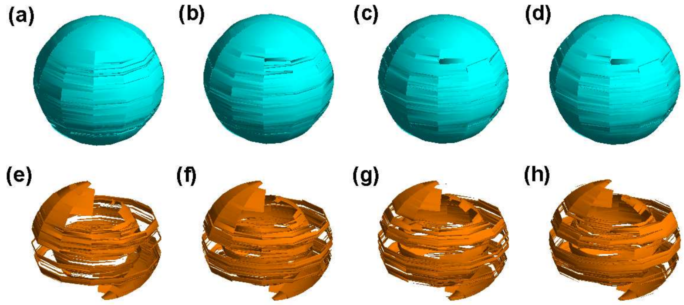

Figure 3. Nonaxisymmetric superfluid SCF when the inner and outer spheres are mutually inclined by $\theta_0 = 3$, with $Re = 10^3$, and $\delta = 0.3$: instantaneous flow topology of the superfluid component. Isosurfaces in light blue for $D_A = 10^{-4}$ at (a) $t = 20$, (b) $t = 30$, (c) $t = 40$, and (d) $t = 50$; and in orange for $D_A = -10^{-4}$ at (e) $t = 20$, (f) $t = 30$, (g) $t = 40$, and (h) $t = 50$.

be presented in a forthcoming paper.

Patchy friction in superfluids may be also important in the dynamics of glitches in pulsars [4]. Glitches are characterized by a sudden increase in the angular velocity of the pulsar, followed by a period of exponential relaxation [12]. The long relaxation time after a glitch, and the temperatures in a neutron star imply that the interior of the star is a superfluid [6, 12]. We find that, if the meridional circulation is fast enough, a vortex tangle is alternatively created and destroyed in the outer core of the star (and indeed any spherical container) [8]. Before a glitch, differential rotation in the outer core drives a nonzero, poloidal counterflow which excites the DGI, and the vortices evolve into an isotropic tangle. Right after a glitch, the differential rotation ceases, so does the poloidal counterflow, the vortex tangle decays, a rectilinear vortex array forms, and the mutual friction changes to HV form, suddenly locking the normal and superfluid components and leading to a spin-up of the crust. The very high Reynold numbers found in neutron star interiors ($Re \gtrsim 10^{11}$) make a numerical study of this problem very challenging [8]. However, experiments of the Tsakadze type have provided promising results on the relaxation of rotating superfluids. New experiments of this type can test which aspects of the turbulent-laminar transition are caused by the normal and superfluid components respectively.

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