Gravitational Waves

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Abstract

In this year (1989), four groups around the world will propose to their funding bodies the construction of large-scale laser interferometric gravitational wave detectors. I shall review the design of these detectors, the problems of analysing their data, and the theory of the sources of the gravitational waves that they are designed to detect.

1 Introduction

At the present time we stand on the threshold of the development of gravitational wave astronomy. Four research groups will propose this year the construction of large-scale laser interferometric gravitational wave detectors that should attain sufficient sensitivity to detect the gravitational waves that are predicted to come from supernovaexplosions and the coalescence of compact-object binary systems at great distances. It is appropriate, therefore, to review the properties of these detectors and the predictions about gravitational wave sources. For further details, readers are referred to a number of reviews in the literature[1-5]. I will deal exclusively with ground-based detectors and the sources they can see. Searching for ultra-low-frequency sources with detectors in space is under active consideration (indeed, has already been done with tracking data from interplanetary spacecraft[6]). See the first two references above or the proceedings of a recent NASA-sponsored meeting[7] for details of this.

1.1 General relativity

Although gravitational waves are interesting to us because they are a consequence of general relativity, surprisingly little general relativity is needed to understand the discussion in this review. The real problems of gravitational wave detection are technical — in the construction of the apparatus — and astrophysical — in the prediction of likely sources of gravitational waves. Here is what we need:

- The polarisation pattern of gravitational waves, as illustrated in Figure 1. If we orient a bar detector across the centre of the polarisation circle, then it is clear that it will be stretched by the "+" polarisation and not affected by the "×" polarisation. Similarly, if we place the central mass of an interferometer at the centre of the circle, and the two end masses at 12 o'clock and 3 o'clock on the circle, the "+" polarisation will change the relative lengths of the two arms but the "×" polarisation will leave them equal. Since an interferometer senses the changes in the difference between the lengths of the two arms, it will respond to one polarisation but not the other. Both kinds of detectors are, therefore, linearly polarised detectors.

- The quadrupole formula. There are really two formulas, one that predicts the amplitude of the radiation from a nearly-Newtonian source and another that gives the radiation-reaction effects within it. When gravitational wave observations are made, their interpretation will require better calculations than are possible within the quadrupole approximation, but for the purpose of estimating the strength of gravitational waves and the properties of their sources, the errors (factors of 2 or so) of the approximation are only comparable to the other uncertainties in our astrophysical models.

2 Bar Detectors

Bar detectors are descendants from the first gravitational wave detector, devised by J. Weber in the early 1960's[9]. Interestingly, Weber also invented the laser interferometric detector, but chose to build a bar detector because the technology of the 1960's was not up to the job of interferometric detection. Today there are a large number of detectors, as given in Table 1.

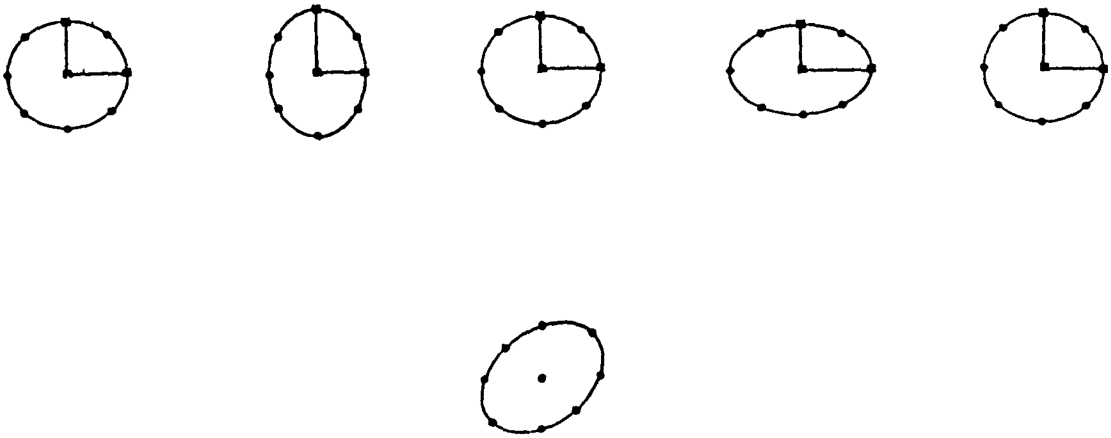


Figure 1: The polarisation diagram of gravitational waves[8]. The circle is a ring of free particles in empty space. When a normally incident gravitational wave arrives with the "+" polarisation (top), it distorts the circle into an oscillating ellipse. The orthogonal "×" polarisation (bottom) distorts the circle into an ellipse rotated by 45°.

Table 1: The active bar-detector groups

Institution	Room-temperature	Torsion	Cryogenic	Ultracryogenic
Stanford U.	-	-	√	√
Louisiana State U.	-	-	√	√
U. of Maryland	√	-	√	√
U. of Rome	√	-	√	√
U. W. Australia (Perth)	-	-	√	-
Moscow State U.	√	-	-	-
Tokyo U.	-	√	-	-
China (& Beijing)	√	-	-	-

The room-temperature bars typically have a sensitivity of $h \approx 10^{-16}$, where h is the amplitude of the wave, or alternatively the strain produced by the wave in the detector. The cryogenic bars at temperatures of 4.2 K reach to 10^{-18}. The ultra-cryogenic bars will operate at temperatures below 100 mK, and perhaps reach sensitivities approaching 10^{-20}.

The principal practical problem facing bars is the well-known *quantum limit*. For a typical modern bar detector, instrumented to measure the excitation of the fundamental mode of longitudinal vibration, the energy deposited in the bar (on classical arguments) by a burst with amplitude roughly 10^{-20} is equal to the energy of one phonon associated with the vibration of that mode. It was once thought that bars could not measure gravitational wave amplitudes below this, which was called the quantum limit.

Now it is understood that by carefully choosing the observable that one measures, one can measure arbitrarily small amplitudes with a bar[10]. This is done basically by choosing a conjugate pair of observables that have the crucial property that if one of them is measured to arbitrarily high accuracy, the resulting disturbance to the other (via the uncertainty principle) will not feed back through the dynamical evolution of the variables to make a large uncertainty in the measured variable a moment later. Such variables can be found for harmonic oscillators, so that it is in principle possible to measure very weak gravitational waves with ordinary bar detectors.

However, no one has yet found a practical way of implementing this for bars. The variable whose uncertainty is to be "squeezed" — we say that the bar is to be placed in a *squeezed state* — is difficult to measure. Squeezing has been demonstrated to reduce the thermal noise of a bar[11], but not yet to get below the quantum limit. Until this can be done, bars of the present design will not reach below about 10^{-20}. With new techniques[12], one might make a bar with a quantum limit of 10^{-22}. This is the same as the projected sensitivity of laser interferometers, albeit only within the narrow bandwidth of the resonant bar.

Although bars may have limitations in the long term, they remain our best — indeed essentially our only — method for detecting gravitational waves in the short term. Laser interferometers will not be ready to make observations at the 10^{-21} level for at least five years. Until then, it is very important that bar detectors should be run for as large a fraction of the time as possible. Cryogenic (4.2 K) bars in at least two places — Stanford and Rome — are now being run in observing mode 24 hours a day, and groups at Louisiana State University and the University of Western Australia (Perth) expect to

Table 2: The active laser interferometer collaborations

Institution	Prototype	Special Purpose	Plans for Large Project
U.S.A.: Caltech & M.I.T. (LIGO)	√	√	√
U. Glasgow & M.P.I. Quantenoptik, Garching	√	-	√
I.N.F.N. Pisa & C.N.R.S. Orsay/Paris (VIRGO)	-	√	√
U.W. Australia & A.N.U. (AIGO)	-	-	√
Tokyo	√	-	√?
U.S.S.R.	-	√	√?

join them soon.

3 Interferometric Detectors

3.1 General remarks

Although Weber was the first to consider laser interferometric detectors, the first to build one was Weber's former student, R.L. Forward, who was then working for Hughes Aircraft Company[13]. The development of this approach began in earnest in the mid-1970's, after the limitations of bars caused some bar groups to change direction. Today there are a number of groups working in this area, as summarised in Table 2.

The prototype interferometers have progressed to the point where their sensitivity approaches that of the best cryogenic bars. They could be developed a bit further, but for reasons that we will see below any large increase in sensitivity requires a significant increase in arm length. The interferometers also have quantum limits on their sensitivity, but the limits occur at much smaller values of h. The primary quantum limit is the shot-noise or photon-counting limit, determined by the available laser power. We will study this in some detail below. Just as for bars, it is possible in principle to circumvent this limit by squeezing, in this case by the use of *squeezed light*[14]. Unlike the case of bars, squeezing to below the quantum limit has in fact been demonstrated, reducing shot noise by factors of up to 3 so far[15]. It is likely that squeezing will be implemented in laser interferometers in the future[16].

Although the existing interferometers are usually called prototypes, they have done some observing runs. Most notable is a 100-hour coincidence experiment between Glasgow and the Max Planck instrument early in 1989. The data are still being analysed, but the run did demonstrate that interferometers are not as temperamental as some had feared: both instruments achieved duty cycles better than 90%.

3.2 Proposed gravitational wave detectors

Four groups are submitting proposals for full-scale laser interferometric gravitational wave detectors in 1989. The designs all have a number of features in common. The arms of the interferometers are in the range 3–4 km. The interferometers have to sit in a good vacuum, and have to have good isolation from seismic noise. Seismic noise is likely to be relatively straightforward to eliminate above about 100 Hz, but it may need special techniques for lower frequencies. Typically, experimenters expect to develop the instruments to their full sensitivity in various stages spread over a period of 10 years or more. The first stage would reach a sensitivity level of 10^{-21} over a 1 kHz bandwidth. This would be followed by progress to 10^{-22} at kHz frequencies. In some proposals, a second detector optimised for low-frequency observing down to 100 Hz would be built as well, and there would be a final extension of sensitivity down to lower frequencies.

Briefly, the proposals are:

- *The British-German collaboration*[34]. The proposal will be for an installation with three-kilometre arms, either at right angles to each other or at 60°, as in Figure 2*a*. When the arms of an interferometer make an angle θ with one another, its sensitivity is reduced relative to one with the same arms at a right angle by a factor of $\sin\theta$. The only reason for making one with arms at 60° to one another is that it provides the possibility of a significant enhancement at some time in the future, by completing the equilateral triangle and installing three independent differently polarised detectors. Such an arrangement would give good polarisation information and fairly uniform sky coverage. As Figure 2*b* shows, an enhancement is also possible for the right-angle detector, but this has a poorer antenna pattern.

  The British-German detector[34] would have two separate interferometers in a single vacuum system. One of the interferometers would be optimised for high-frequency gravitational radiation, which means 1 kHz and above. The second would be optimised for the range 100–1000 Hz. The site of this detector has not yet been decided.

- *The Italian-French collaboration (VIRGO).* This proposal is for a single detector with three-kilometre arms at a right angle. It has as a principal goal the attainment of good sensitivity at very low frequencies, perhaps down to 10 hz. It would be built on a site near Pisa.

- *The US detectors (LIGO).* At the present time it appears that the American proposal will be for two detectors, each with 4 km arms at right angles, one located on the East Coast and one on the West Coast of the USA. Inside each housing are likely to be several interferometers (perhaps 3), each of which has its optics in a separate vacuum system from the others so that working on one does not disturb the others. Two of the interferometers would be optimised for high and low frequencies, as in the British-German detector. Any other interferometer would be for special purposes, perhaps for the use of other groups that wish to do such research.

- *The Australian proposal (AIGO).* This would be built on a site near Perth, which would permit the construction of arms at least 3 km long. This proposal envisions a collaboration with the British-German group, and its design is based on the design put forward by them. A collaboration with Japan is also possible.

3.3 How an interferometer works

All laser interferometric gravitational wave detectors are essentially developments of the familiar Michelson interferometer that was used in the famous Michelson-Morley experiment. Light from a laser is divided by a beamsplitter into separate beams travelling into the two arms. The light is reflected from mirrors at the ends of the arms and passes through the beamsplitter on its return. Each beam is divided by the beamsplitter, and the result is two coherent superpositions of the two return beams, travelling respectively in the directions indicated in Figure 3*a* by the letters A and B. The light going in direction A reaches a photodetector, whose output is the signal from the detector. The light in direction B has no use in this simple configuration, and is lost. It turns out, very importantly, that one gets the best signal-to-noise

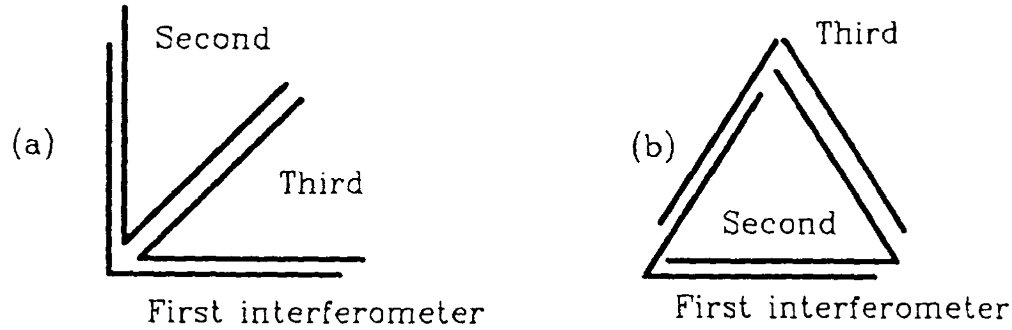


Figure 2: Possible configurations of the British-German detector.[34]

(a) The conventional 90° arrangement, with a possible later extension that would permit three differently polarised detectors on the same site. (b) A 60° configuration, with its equilateral extension accomplishing the same thing with a more uniform sky coverage.

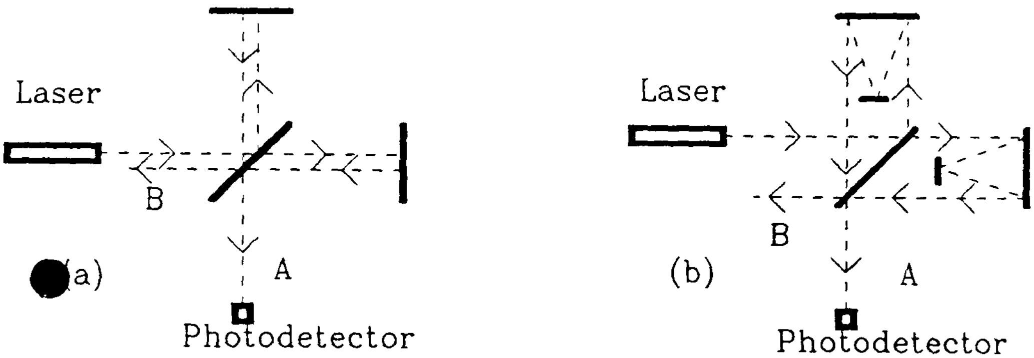
ratio by arranging for complete destructive interference in direction A when there is no gravitational wave. It is best to have a null signal from the photodetector for a null observation: the absence of a gravitational wave. When a wave comes in, then it will produce a nonzero intensity on the photodetector.

It will turn out that the sensitivity of the laser interferometer over a wide range of frequencies will be limited by the available power of light in the arms. This limit is called the shot-noise or photon-counting limit, and arises essentially because of the discrete nature of photons. It is an important limit and one that is easy to understand, so I shall give a simple heuristic derivation.

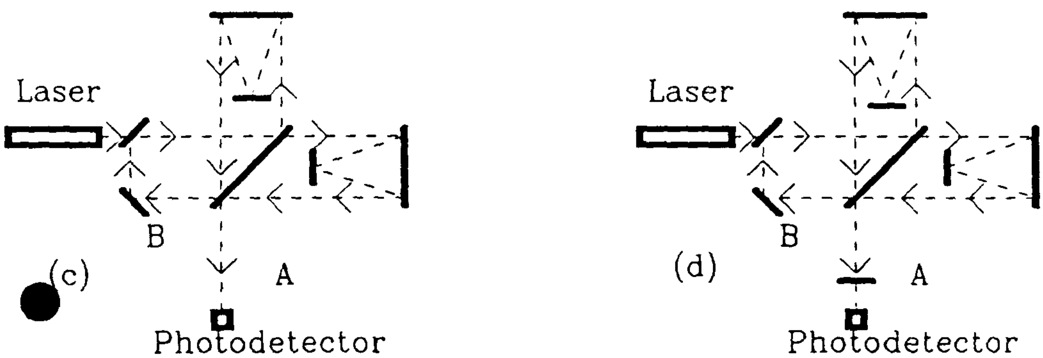
3.3.1 Simple Michelson interferometer

The idea of the detector is to measure small shifts in the interference fringe at the photodetector. This means measuring small lengths. With a single photon one would not expect to be able to measure a length smaller than,say, half its wavelength λ, so the random error would be about $\delta\ell = \lambda/2$. Using N photons to make the same measurement, one would hope to reduce this by the usual statistical factor of $\sqrt{N}$: $\delta\ell = \lambda/2\sqrt{N}$. To use an interferometer of arm-length ℓ to reach a sensitivity limit h, we note that (for the optimum orientation of the wave) the arms change by an amount $h\ell/2$ in opposite directions, and a photon's path changes in length by twice the lengthening of

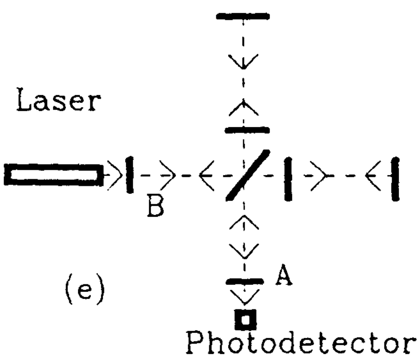
Figure 3: Basic design of a laser interferometer for gravitational wave detection.



(a) The simple one-reflection Michelson interferometer. (b) A multi-reflection delay-line Michelson.



(c) A delay-line interferometer with recycling: the light that formed from constructive interference is returned to the interferometer. (d) Dual recycling in a delay-line: the gravitational wave signal is recycled back through the interferometer.



(e) The Fabry-Perot alternative to the delay line, with recycling and dual recycling mirrors.

an arm. These changes add to give a net fringe shift of $2h\ell$. By setting this equal to the error $\lambda/2\sqrt{N}$, we find that we require

$$N = \left(\frac{\lambda}{4h\ell}\right)^2$$

photons to make such a measurement. Given that each photon has an energy[1] $2\pi\hbar c/\lambda$, and that we want to perform a measurement in a finite time τ, we require a power

$$P_{\text{Michelson}} = \frac{\pi\lambda\hbar c}{8h^2\ell^2\tau}. \tag{1}$$

In order to get broadband frequency coverage at least to 1 kHz we need to take $\tau = 10^{-3}$ s. To reach a sensitivity $h = 10^{-22}$ in a 3 km detector with green laser light ($\lambda = 5 \times 10^{-7}$ m) in this simple Michelson interferometer, we require a power

$$P \approx 70 \text{ MW}.$$

No laser is capable of continuous power outputs of this size, and no mirrors could handle it for long.

3.3.2 Delay-line interferometer

Fortunately, there are simple ways to get the same sensitivity with lower laser power, and the development of these ideas has been crucial for our current optimism about reaching 10^{-22}. The first idea is simply to bounce the light up and down the arms many times, as in Figure 3*b*. This makes a Michelson *delay-line* interferometer. The point is that if the arm length changes by an amount $\delta\ell$, then after N_{dl} passes up and down the arm, the light emerges having travelled a distance $N_{\text{dl}}\delta\ell$ extra. (The simple Michelson interferometer has $N_{\text{dl}} = 2$.) This reduces the power requirement by $(N_{\text{dl}}/2)^2$ compared to a simple Michelson. One cannot keep bouncing the light up and down forever, however; in fact, if one keeps it in the arms longer than half of a period of the gravitational wavethat one wants to detect, then the wave's effect on the arm will change sign and begin to wipe out the signal. So the optimum number of bounces is

$$N_{\text{dl}} = \frac{c}{2f_{\text{gw}}\ell}.$$

[1] Unfortunately, we are stuck with using h for both the wave amplitude and Planck's constant. To avoid confusion, I will always use the reduced Planck constant $\hbar$.

For 1 kHz waves and a 3 km interferometer, this is 50 bounces, and the saving in power is a factor of 625:

$$P_{\text{delayline}} = \frac{2\pi\lambda\hbar f_{\text{gw}}^2}{h^2 c\tau} \approx 110\ \text{kW}. \qquad (2)$$

This is better, but still unrealistically large.

3.3.3 Recycling

The next key idea is called *recycling* and was devised independently in 1982 by Drever[17] and R. Schilling (unpublished). It simply rests on the observation that the power in the beam B in Figure 3*b* is normally equal to the laser power input, and it is all wasted. By simply adding two mirrors as in Figure 3*c*, one can *recycle* the light back through the system. Instead of wasting light, the power in the arms will build up until the losses at the mirrors (through scattering and absorption) equals the input power from the laser. Since good mirrors are available now with reflectivity R within 5×10^{-5} of unity, the power in the arms will be 2×10^4 times the input laser power.

To calculate the gain in sensitivity over the delay-line, note that when the light emerges from the delay line it has already suffered N_{dl} reflections, each of which has removed light by scattering. So the light returns to the recycling mirrors not $(1-R)^{-1}$ times, as it would in the simple Michelson arrangement, but only $[(1-R)N_{\text{dl}}]^{-1}$ times before being removed from the beam by mirror imperfections. This is the factor by which the power is enhanced, and leads to the laser power requirement of

$$P_{\text{recycling}} = \frac{\pi\lambda\hbar f_{\text{gw}}(1-R)}{h^2\tau\ell} \approx 280\ \text{W}. \qquad (3)$$

If one does the calculations of τ and of the measurement error more carefully, one finds that the power required is actually about 50 W. This is not expected to be very difficult to achieve, even with the high frequency stability that is also required of the laser. Current plans envision using Nd:YAG lasers.

Until the early 1980's, there was little point in doing recycling, because mirror quality did not give much gain. The advent of superior mirror coating techniques, originally developed for military use in laser-gyroscope-based navigational equipment, has been crucial to our confidence that 10^{-22} is a realistic goal for the large detectors.

3.3.4 Dual recycling and narrow-band observing

A further development of the optical design has recently made a big impact on the proposals submitted this year. This is the technique called *dual recycling*, devised by Meers[19]. Since there is normally no light going in the direction A in Figure 3c, any light that does go there carries the gravitational wave signal. Meers suggests placing a further mirror in the path of this light, to recycle the signal itself. Depending on the reflectivity and position of this mirror, one can transform the interferometer into a narrow-band detector with any bandwidth down to about 2 Hz at any frequency in the observing range. One can also keep the bandwidth broad but compensate for defects elsewhere, such as too short a storage time in the arms for the signal being searched for. There have been other proposals for narrow-banding these detectors, namely the resonant recycling design of Drever[17], and the detuned recycling idea of Vinet and collaborators[19], but dual recycling offers some significant advantages. Figure 3d shows a simplified version of the dual recycling configuration proposed for the British-German detector.[34]

By making a detector narrow-band, one concentrates all the photons in the narrow bandwidth. This improves the sensitivity by the square root of the ratio of the ordinary bandwidth to the narrow bandwidth, a factor of up to $\sqrt{1000/2} = 22$. But this is still the sensitivity for measurements taking one millisecond. If we observe a pulsar, for example, it may be practical to take data for periods of up to, say, $\tau = 10^7$ s (four months). This provides a further enhancement by the ratio $\sqrt{10^7/10^{-3}} = 10^5$. One can therefore use laser interferometers to look for sources of continuous radiation with amplitudes below 10^{-28}.

3.3.5 Fabry-Perot interferometers

I have concentrated on the Michelson-type interferometer and its more or less direct descendants, because it is somewhat easier to see how they work. But there is an alternative design, developed in Glasgow and now adopted for most of the proposed interferometers, which is based upon Fabry-Perot cavities instead of multiple-reflection delay-lines. The cavities are resonant with the laser light when no gravitational wave is present, and the effect of the gravitational wave is to destroy the resonance, preventing light from entering the cavities and increasing the intensity at the photodetector. Fabry-Perot interferometers perform essentially identically to delay-line ones, and one can implement recycling and dual recycling in them. A fully recycling Fabry-Perot interferometer is shown in Figure 3e. The reasons why groups would choose to build a Fabry-perot instead of a delay line have to do with cost and ease of implementation. I will not go into them here.

3.4 Other noise sources

Although shot noise is the limit that we would like our detectors to reach, other types of noise might get in the way. Some of the noise sources that have been studied and whose effect is detailed in the proposals submitted by the various groups are:

- *Seismic noise.* Seismic and other ground-motion noise has a spectrum that peaks at low frequencies. By passive isolation — consisting of good foundations, stacks with alternating layers of rubber and metal, and a two-stage pendulum suspension — good isolation is expected above about 100 Hz. Similar techniques work well on the prototypes and on bar detectors. The VIRGO collaboration are developing special suspension/isolation systems that may allow us to reach down even to 10 Hz. The noise reaching the detector falls off steeply with frequency, as shown in the sensitivity plots given in Figures 4 and 5 below.

- *Mirror vibrations.* The mirrors are at room temperature, so they vibrate thermally. The more reflections one uses in an interferometer, the more mirror noise there will be. This affects the design of the interferometer, as we discuss below. If the mirrors are made more massive, the amplitude of vibration will be reduced (because the energy of the mode remains at $kT/2$). The effect of this noise is alleviated by making the mirrors resonant at a frequency outside the range of interest of the observations, and using a material with a high Q, so that the resonant response of the mirrors in the observation range is small. The effect of this noise for realistic values of Q is shown in Figures 4 and 5.

- *Suspension thermal noise.* The mirrors and other optical components are suspended from their seismic isolation mounts by thin wires. There is therefore a pendulum mode of vibration of about 1 Hz that will also have $kT/2$ energy at room temperature. A high Q in the pendulum reduces this noise in the same way as for mirror vibrations. It affects mainly the low-frequency sensitivity of the detectors, and its effect is shown in Figures 4 and 5.

- *Laser noise.* Lasers can introduce many kinds of noise: frequency fluctuations, amplitude fluctuations, changes in the direction of the beam, and so on. These can all be controlled, sometimes as a result of some very clever optical feedback systems.

- *Vacuum pressure fluctuations.* Small fluctuations in the residual air in the arms causes changes in the index of refraction that can mimic a gravitational wave signal. All the proposals expect to achieve sufficiently high vacuum so that this is negligible. Fabry-perot systems may require pressures below 10^{-8} torr.

- *Miscellaneous noise sources.* Various other kinds of noise have to be controlled: light scattering from mirrors and the walls of the vacuum tube; "violin modes" of the suspension wires, acoustic noise from the environment, possible electromagnetic interference, and so on.

Another possible limitation on detector performance, although not really a noise source, is mirror heating. Although the losses per reflection from mirrors are small, the light intensity builds up until all the laser power is being dissipated in the mirrors. The resultant heating can change the shape and index of refraction of the mirrors, which affects the quality of the interference pattern they produce at A and B in Figure 3. The present proposals use sufficiently small laser power to avoid this, but if it is desired to go to higher sensitivity in the future by using higher laser power, this could be a limiting factor. How easy it will be to cure will depend on developments in materials science.

3.5 Why 3 kilometres?

Most of the expense of a laser interferometer is in the cost of the long vacuum pipes and the vacuum pumps. Why, then, do the groups all propose long detectors? The simple answer is that longer arms have a larger response to a given gravitational wave, but as we have seen in Section (3.3) this really only applies to the simple Michelson interferometer. With multiple reflections, light can be stored in much shorter arms and still achieve the same response if the limit is the shot noise. The problem with doing this in an arm of, say, 100 m, is that the large number of reflections required for a 1 kHz signal (1500), or the even larger number required for a 100 hz signal (15,000), bring their own problems. Each reflection adds thermal noise from the mirror vibrations and the suspension. Each reflection also subtracts light intensity, making recycling less and less effective. We can see the effects of this by comparing the way length affects the power required to reach a certain sensitivity in Equations (1) to 3. All the research groups agree that it would be essentially impossible to reach 10^{-22} over distances less than 1 km, and that by going to 3 km one allows some margin of safety for aspects of the system that may not perform optimally.

3.6 Squeezed light

As I remarked in Section (3.1) above, the photon shot noise limit is not a fundamental limit; it is a quantum limit that can be beaten by squeezing techniques[14] that have already been demonstrated in the lab[16]. There are technical limitations, particularly with regard to mirror losses, but it has been shown that squeezing is compatible with both ordinary (broadband) recycling and dual recycling. It is not impossible that the presently proposed detectors could be pushed to 3×10^{-23} with no increase in laser power. This factor of 3 translates into a factor of 27 in the volume of space that can be reached, and therefore into a factor of 27 in the event rate for detected gravitational waves.

4 Sources of Gravitational Waves

As we will see below, the maximum amplitude of gravitational waves expected from sources in our Galaxy is 10^{-18}, and that only once every decade or so. The maximum amplitude expected from extragalactic sources that might occur more than once per year is 10^{-21}, which could come from supernovae in the Virgo Cluster, where there are several thousand galaxies. The proposed designs are aimed at providing amplitude sensitivity ten times better than this.

Although gravitational waves are difficult to detect, they carry enormous amounts of energy. The energy flux of a gravitational wave of frequency f and amplitude h is given within the Isaacson approximation[8,20] by

$$\mathcal{F}_{\rm gw} = 3.2 \times 10^{-3} \left(\frac{f}{1\ \rm kHz}\right)^2 \left(\frac{h}{10^{-22}}\right)^2\ \rm W\ m^{-2}. \tag{4}$$

In astronomers' language, a 1 kHz wave with amplitude 10^{-22} is as bright as a star of apparent magnitude -13, which is 10^5 times brighter than the brightest star in the night sky. By integrating Eq. (4) over a sphere of radius r for a time τ we find the relation between the amplitude h of an isotropic wave and the energy E it carries during a time τ:

$$h = 1.6 \times 10^{-21} \left(\frac{E}{10^{-2}\ M_{\odot}c^2}\right)^{1/2} \left(\frac{f}{1\ \mathrm{kHz}}\right)^{-1} \left(\frac{\tau}{1\,\mathrm{ms}}\right) \left(\frac{r}{15\ \mathrm{Mpc}}\right)^{-1} . \quad (5)$$

For convenience we normalise total energies to $10^{-2}\,M_{\odot}c^2$ (probably a fairly large burst for a conventional supernova) and distances to the distance of the Virgo Cluster.

4.1 The worldwide gravitational wave network

Gravitational wave detectors cannot operate alone; detections must be confirmed by coincidences between two separated detectors. But two detectors alone cannot supply enough information to reconstruct the gravitational wave itself, *i.e.* to infer its amplitude, polarisation, and direction of travel. Solving this "inverse problem" is crucial to getting scientific information from the detectors, and it requires at least three detectors around the world. In order to allow for non-optimal orientation, operational down-time, special-purpose uses such as narrow-banding, and detector development, the goal of the worldwide detector community is to have detectors at four separated sites as a sensible minimum, and five as a highly desirable next step.

Since detector sensitivity is limited by internal and local noise sources, presumably uncorrelated between separated sites, coincidence experiments have a lower false-alarm rate (noise-generated coincidence rate) at a given threshold than individual detectors do. For a given false-alarm rate (we adopt once per year for our estimates in this review), the more detectors a network has, the lower will be the threshold it can operate at. Thus, each additional detector improves the performance of all the others. These considerations plus the formidable complexity of laser interferometric detectors account for the highly cooperative spirit that exists among the various gravitational wave groups today.

The following list of sources of gravitational waves is by no means exhaustive, but it contains those that are the most likely to be detected, based on our present understanding of them. The uncertainties are to a large extent the justification for building detectors; if we knew all about the sources we would not want to build detectors to study them!

4.2 Supernovae

Supernovae, or more generally gravitational collapses, have been the primary goal of gravitational wave detector development. Supernovae occur in supergiant stars whose cores are supported by electron degeneracy pressure because they have exhausted their nuclear fuel. When the mass of the core builds up to the maximum that can be supported, it collapses to neutron-star densities. If the collapse halts there, a rebound will occur that drives off the outer envelope of the star. This causes the optical display that we see as a supernova.

If the collapse is spherical, there will be no radiation. The main way to achieve nonsphericity is through rotation. If the core has sufficient angular momentum — it does not need much — and if this is conserved during the collapse, then rotational instabilities will deform the core into a tumbling cigar-shaped object that could even fission into two lumps. The tumbling and orbital motion should be strong radiators. The energy available is large, plausibly up to $0.1 M_\odot$ in extreme cases. A more moderate guess is that nonaxisymmetric collapse will give perhaps $0.01 M_\odot$.

We know little about the precise waveform to expect, but on general grounds one expects that the burst may last about a millisecond and have very little structure. If the burst has $0.01 M_\odot$ energy and comes from Virgo, it would have an amplitude greater than 10^{-21}. A network of detectors would be able to identify such a burst reliably as far away as 40–50 Mpc. This is three times the distance to Virgo and is a volume of space in which there are perhaps a thousand supernovae per year. If only a small fraction are very nonaxisymmetric, there could still be a good event rate in our detectors.

The amount of rotation in a typical collapse is hard to predict, but the case for occasional events in which rotation dominates would be strong if the report of a pulsar in the remnant of SN1987a, with a pulse period of nearly 2 kHz, is confirmed[21]. Studies of neutron star models show that a star rotating that fast must either still be subject to a slowly-growing gravitational wave instability, or else it must be very close to such an instability[22]. It would be very hard to form such a star without its having undergone nonaxisymmetric deformation, with the emission of considerable gravitational wave energy either as a burst or over a relatively few cycles of rotation. The data in which the pulsar was discovered are extremely clean, and can be fit with a simple model: remove the Doppler shifts produced by the Earth's motion and fit the residual frequency shifts with a Keplerian orbit. The fit is so good[23] that it is very hard to reject this data, despite the fact that it is hard (but not impossible[24]) to explain theoretically and the fact that the pulsar has not yet been seen again. My own personal view is that the pulsar is (or was) real, and that therefore this supernova was certainly rotation-dominated. The case for others in other galaxies is therefore strong.

4.3 Coalescing binaries

Since most stars begin as members of binary star systems, it is likely that a substantial fraction remain binary after the individual stars have completed their evolution and become either white dwarfs, neutron stars, or black holes. A small number of these will have been brought so close together during earlier phases of binary evolution that their orbital lifetime against the loss of energy to gravitational radiation is less than the age of the universe. Such systems will coalesce in an astronomically short time. Systems containing white dwarfs are not of interest for ground- based detectors, since they coalesce before their orbital radiation reaches a frequency accessible from the ground. (They would be visible from space.) But those composed of highly compact objects — neutron stars and/or black holes — can produce observable radiation.

4.3.1 How we detect coalescing binaries— matched filtering

Coalescing binaries give off a great deal of energy before they coalesce. As the radiation from a system consisting of two $1.4\,M_{\odot}$ neutron stars changes from 100 to 200 Hz, the waves carry away some $5 \times 10^{-3}\,M_{\odot}c^2$ in energy, comparable to a decent supernova burst. Because this energy is spread out over many cycles, the waves' amplitude is smaller than one would expect from a supernova. Nevertheless, by the data analysis technique called *matched filtering* it is possible to attain a much higher signal-to-noise ratio than for a comparable supernova.

Matched filtering is simply a linear pattern-matching technique designed to find signals of known shape in noisy data. As long as the noise is random with respect to the signal shape, it can make a significant improvement in signal-to-noise ratio. For signals with n cycles in their waveform, matched filtering can improve the signal-to-noise ratio by a factor of roughly $\sqrt{n}$. (This is why, in the sensitivity diagram, Figure 4, the vertical scale $h_{\rm eff}$ is approximately $h\sqrt{n}$. This allows one to compare in one graph sources on which various amounts of filtering can be performed.)

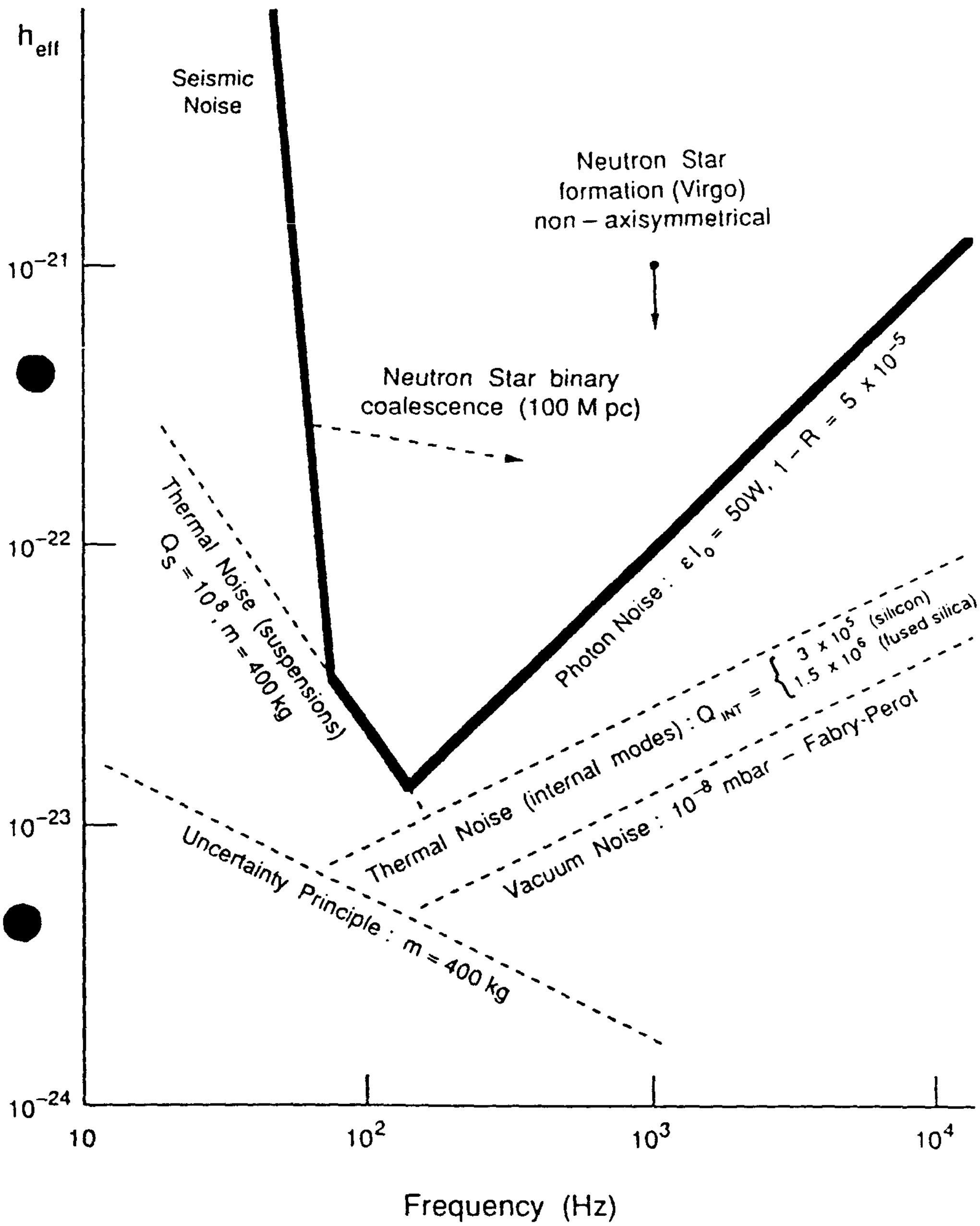


Figure 4: The sensitivity h_{eff} to short bursts of radiation of a 10^{-22} detector optimised for observing at frequency f. A number of interesting possible sources are shown. The effects that give rise to the sensitivity limits are indicated. (For an explanation of h_{eff}, see Section (4.3.1).) Taken from Ref. [34].

Filtering works in a simple manner. Given an output stream of data $s(t)$ (mostly noise) and an expected signal $h(t)$, one constructs the correlation

$$c(\tau) = \int_{-\infty}^{\infty} h(t)s(t+\tau)dt. \tag{6}$$

The time-delay τ is variable, and the correlation will peak at a value of τ that represents the best match to any signal buried in the noisy data $s(t)$.

In practice, the correlations are performed by employing fast Fourier transforms. Note that the technique is only as good as the quality of the predicted waveform, so theoretical studies can prove very important in helping to extract signals from the noise. Moreover, in practice, there will be a large number of different waveforms, perhaps up to 1000, that will require filters. The technique is standard in signal analysis of all kinds, but for gravitational wave detection it presents its own special problems.

4.3.2 The waves expected from coalescing binaries

It is interesting to look at the amplitude of the signals expected from coalescing binaries. The maximum amplitude, taken over all orientations of the binary and the detector, is

$$h_{max} = 3.6 \times 10^{-23} \left(\frac{M_T}{2.8M_\odot}\right)^{2/3} \left(\frac{\mu}{0.7M_\odot}\right) \left(\frac{f}{100\mathrm{Hz}}\right)^{2/3} \left(\frac{100\mathrm{Mpc}}{r}\right), \tag{7}$$

where M_T is the total mass of the binary and μ its reduced mass, and where r is its distance and f the frequency at which it is radiating (twice its instantaneous orbital frequency).

Although the amplitude looks small, matched filtering can bring up the signal-to-noise ratio in this case by as much as a factor of 25. This means that coalescing binaries can be seen as far away as 650 Mpc, much further away than supernovae, provided detectors can operate down to 100 Hz. If seismic noise can be eliminated down to 40 Hz, this range is roughly doubled to 1.3 Gpc.

Notice that the masses of the stars influence the amplitude of radiation only through the combination

$$\mathcal{M} = \mu^{3/5} M_T^{2/5}, \tag{8}$$

which we call the *mass parameter* of the binary system. Remarkably, the mass parameter also determines how fast the frequency of the signal increases as the orbit decays:

$$\frac{f}{\dot{f}} = 7.8 \left(\frac{f}{100\ \mathrm{Hz}}\right)^{8/3} \left(\frac{\mathcal{M}}{M_\odot}\right)^{5/6} \mathrm{s}. \tag{9}$$

Equations (7) and (9) contain an important relation: the masses of the stars enter only via the mass parameter $\mathcal{M}$. By measuring the signal amplitudes and the rate of change of the frequency, the two equations can be combined to yield a single unknown, the distance to the source. In fact there is not enough information from a single detector to determine the maximum signal amplitude used in Eq. (7), but a network of three detectors could do so. In this way, *coalescing binaries become distance indicators*, and this has important implications for astronomy, to which we return later.

Not all binaries will consist of neutron stars. The statistics of X-ray binaries suggest that perhaps one percent will contain a black hole of roughly 14 $M_\odot$; it is possible that a further one percent of these might consist of two such black holes. The signal-to-noise ratio of the two-black hole system would be some 6.3 times larger than that of two neutron stars at the same distance, so the range for detecting binary black holes would be more than 4 Gpc, approaching a cosmological redshift of 0.5.

The maximum range of the detector is not the whole story, since orientation-dependent effects will reduce the probability that any given coalescence will be seen. Tinto[25] has made a detailed study of these effects in a network of 4 or 5 detectors, and has concluded that it would register about 50% of all coalescences that take place within half of its maximum range, and it will see some 5–10% of all those within its maximum range. These numbers would not change greatly if the worldwide network had only 3 or 4 detectors.

An important question that we touched on above is the quality of the predicted waveform that is used to construct a filter for these signals. It assumes a point-mass Newtonian binary with a circular orbit. The circular assumption is a good one for binaries this close, since eccentricity decreases faster than orbital radius under the action of gravitational radiation reaction. The influence of tidal, mass-exchange, and post-Newtonian effects have been considered in detail by Krolak[26], who found that they begin to be important above 500 Hz or so. The estimates we have used here for signal-to-noise ratios are dominated by the power in the signal between 100 and 200 Hz, so they will be insensitive to any such corrections. On the other hand, if such effects can be modelled reliably then they can be used to improve the signal-to-noise ratios we have quoted here, and to extract further astrophysical information from the observations.

4.3.3 How many there might be

There is one well-known binary coalescence *precursor* system in our Galaxy: the famous Binary Pulsar, PSR 1913+16, which has a remaining lifetime of about 10^8 years before it will coalesce. Importantly, a second such system, PSR 2127+11C, has very recently been discovered in the globular cluster M15[27].

Its pulse and orbital characteristics are remarkably similar to those of the classic binary, PSR 1913+16. The masses of the component stars will not be known until the periastron shift is measured, but if we assume a pulsar mass of $1.4\,M_\odot$, then the minimum companion mass allowed by the mass function is $0.94\,M_\odot$. This implies a *maximum* remaining lifetime of 10^9 years. If we take the likely companion mass to be $1.4\,M_\odot$, then the lifetime of PSR 2127+11C is half that of PSR 1913+16.

Furthermore, it is possible that PSR 0021-72A, in the globular cluster 47 Tuc, is also a precursor with a short lifetime[28]. Unfortunately, this pulsar is at the limit of detectability, and exhibits a number of unexplained features, so it is too soon to draw conclusions about it.

The event rate out to any distance is very uncertain, however, since our estimates rely on the precursor systems we can see. Based on PSR 1913+16, Clark, *et al*[29], concluded that the most likely value for the rate is probably 3 events per year out to a distance of 100 Mpc, which would give an event rate in a network of some 40–80 events per year. I have elsewhere estimated[30] that this rate may be uncertain by a factor of 100 either way, allowing a detection rate that could range from one every two years up to several thousand per year. The more recently discovered precursors mentioned above raise this rate somewhat and reduce the uncertainties in it. Further searches now going on will reduce the uncertainties even more.

4.4 Pulsars and other continuous-wave sources

There are many possible long-lived or continuous sources of gravitational radiation in the frequency range accessible to our proposed detector. These include pulsars with "lumps" in their crust; unstable pulsars spinning down after having been formed with too large an angular velocity; and unstable accreting neutron stars where the instability is being driven by the accretion of angular momentum ("Wagoner stars"[31]).

The sensitivity of a detector to such long wavetrains increases as the square root of the time of observation. If the dominant source of noise were photon shot noise, the sensitivity would be extremely good. However, it appears that thermal mirror vibrations will dominate or at best equal photon noise in this case, unless materials with extremely high Q can be found. As Figure (5) shows, thermal noise may dominate the photon noise by a factor of perhaps 5.

As I mentioned above, the sensitivity achievable on a continuous source increases with the square root of the observation time τ. One might contemplate narrow-banding a detector for a period of up to a few months in order to make an important observation; a significantly longer observation might not be desirable, given the importance of searching for bursts and for continuous sources at other frequencies.

There may be pulsars in the solar neighbourhood that are not visible electromagnetically (because they are beamed elsewhere or because they are old and radio-quiet), but which could still be radiating gravitational waves. But the problem of conducting an all-sky search for such signals is formidable: the Earth's motion produces Doppler effects that need to be removed from any observations lasting longer than about 30 minutes, and these corrections are different for each different location on the sky. The longer an observation lasts, the better will be its directional resolution, and therefore the greater will be the number of possible locations that have to be looked at. For example, an observation lasting 10^7 s would be able to define the location of a pulsar to an accuracy of 0.2 arcsec[32], better than optical observations can do using ground-based telescopes.

If we wanted to search the whole sky for such pulsars, simply performing the data analysis on 4 months' worth of data over a 1 kHz bandwidth would be beyond the capacity of present and foreseeable computers[32], because one would have to search separately all 0.2-arcsec-square boxes on the sky: some 10^{13} in all. Given the data set of some 10^{10} points, it is not hard to see why this is impossible at present. Instead, the detectors will perform searches of more limited sensitivity, such as all-sky searches limited to narrow frequency bands and/or short observation times.

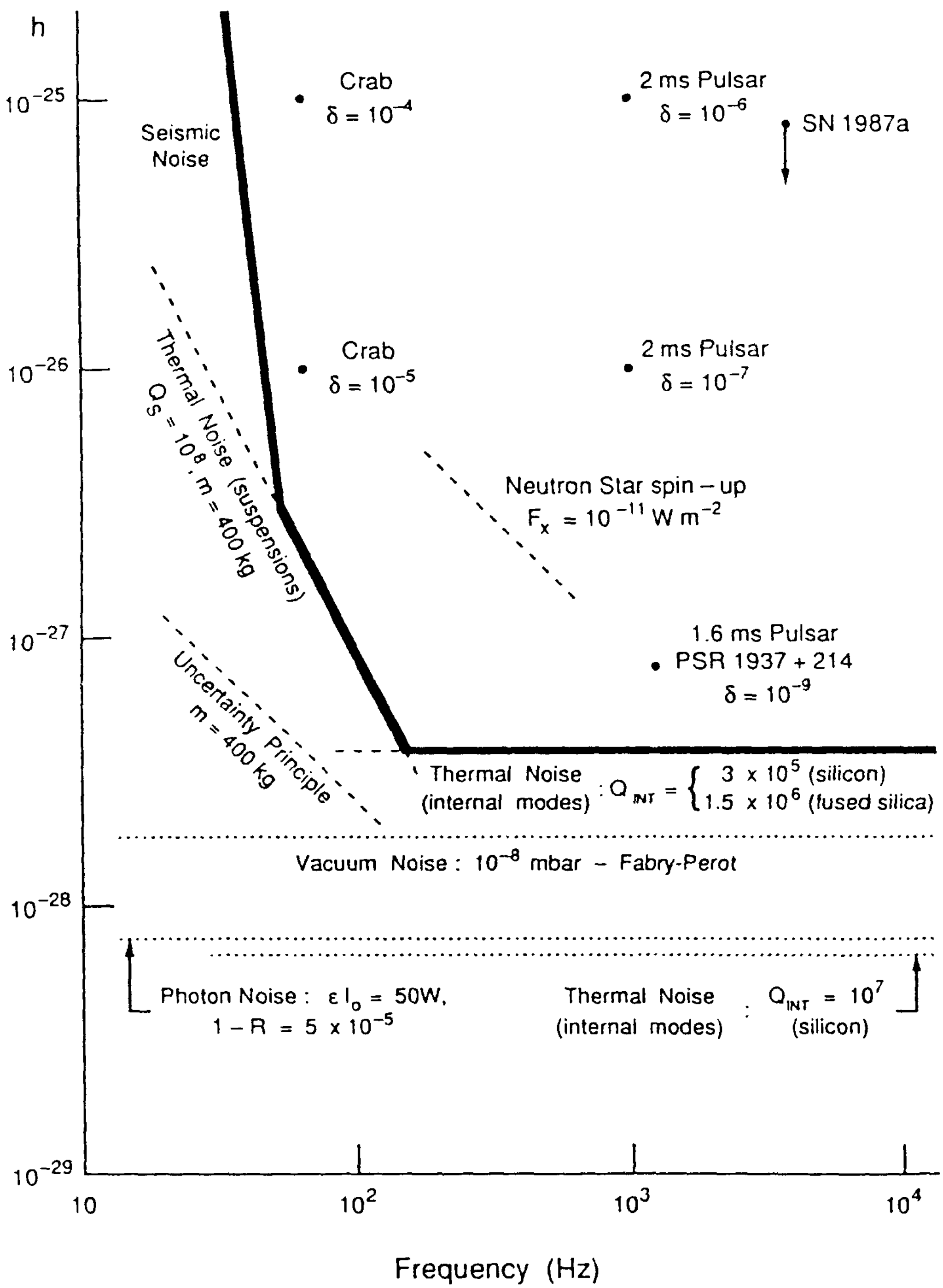


Figure 5: Sensitivity of a 10^{-22} detector to continuous wavetrains, assuming an integration time of 10^7 s. Depending on the Q of the mirror material that can be achieved, either mirror noise will dominate or it will be comparable to shot noise. Taken from Ref. [34]

4.5 Unpredicted sources

There are many other possible sources of gravitational radiation, especially including several possible sources of a stochastic gravitational wave background. But it is hard to believe that we will not also find things we simply did not predict. As with the opening of any other window in astronomy, one can be confident that there will be unexpected sources of gravitational waves at some level. If they are strong enough to stand out above the broadband noise, then they will be readily detected and studied. If they are weaker but have some structure, such as the coalescing binary signal, then they may still be detectable using cross-correlation between detectors.

Cross-correlation is not as sensitive as filtering, but it might be the principal way of finding new things. For example, suppose two 10^{-22} recycling detectors optimised for broadband bursts observe a weak signal lasting 1 s with a typical frequency of 200 Hz, not very different from the parameters of a coalescing binary. Then a source radiating the same energy as a coalescing binary could be seen in the Virgo cluster. Whether such sources exist and are frequent enough to give a reasonable event rate is a question that will only be answered by observation. Certainly such correlations should be done after supernova events, for example, in order to look for neutron star spindown or any unpredicted aftermath radiation.

5 Impact on Astrophysics

Discussions of possible gravitational wave sources tend to concentrate on the likelihood of sources being there, as if the measure of the success of a gravitational wave detector is whether it detects a gravitational wave. But the long-term purpose of building these instruments is, in the end, the scientific information they can bring in. So in this section I will describe where in physics and astrophysics the data from detectors could have its greatest impact.

We will begin our list with three new tests of relativistic gravity that observations of gravitational waves can perform.

- *Test of gravitational wave polarisation.* Simply seeing gravitational waves would, of course, be a milestone for relativistic gravity. But many theories predict gravitational waves, and a network of detectors can distinguish among them. With four or more detectors, one has redundant information in the observations with which to reconstruct the amplitude, polarisation, and direction of the wave. If these data are self-consistent, then general relativity provides a good model of the wave, particularly of its polarisation properties. If they are not self-consistent, then a different theory of gravity may be necessary.

- *Speed of propagation of gravitational waves.* If a supernova at 15 Mpc were seen optically and detected by the gravitational wave network, there should be less than a day's delay between the gravitational wave and the optical detections, provided the gravitational wave travels at the same speed as the light from the supernova. Over a travel time of some 45 million years, the coincident arrival of the waves within a day would establish that their speeds were equal to within one part in 10^{10}.

- *Test of strong-field gravity.* A further test can be made if black hole coalescing binaries are detected. Computer simulation should soon be accurate enough to make detailed predictions of the dynamics of the merger of the holes, and of the radiation they emit, with only a few parameters (such as the masses, spins, total angular momentum, and impact parameter of the collision). Given a reasonable signal-to-noise ratio, matching the observations to the predictions could provide a stringent test of strong-field gravity.

Now we turn to the astrophysical "return" on the enormous investment these detectors would require.

- *Morphology of the supernova core.* Observations of bursts from gravitational collapses tell us a number of things about supernovae themselves. We could learn how many collapses do not produce visible supernovae; how often rotation plays an important role in the collapse; whether the collapse has formed a neutron star or a black hole; and what the mass and angular momentum of the compact object are.

- *Neutron star equation of state.* This is one of the most important areas where gravitational wave astronomy can provide information that is crucial to nuclear physics: the interactions of neutrons in these conditions are poorly understood and inaccessible to laboratory experiments. Supernova gravitational wave observations constrain the equation of state by telling us what the timescale of collapse and rebound is, what the mass and angular velocity of any neutron star formed in the collapse might be, and what the upper mass limit of neutron stars is. Coalescing binaries similarly offer information on neutron star masses (through the mass parameter $\mathcal{M}$) and on mass exchange once the initial point-mass approximation breaks down. Observations of pulsars radiating from frozen-in mass deformations constrains the solid crust equations of state.

- *Compact-object statistics.* It is very hard to devise unbiased indicators of the numbers and distribution of pulsars, old neutron stars, and black holes. Observations of gravitational waves from supernovae and coalescing binaries can give a new measure of the mass functions of these populations and of their formation rate. Searches for unknown pulsars, if successful, could give a relatively unbiased indication of their distribution in the solar neighbourhood.

- *Hubble's constant.* If the event rate of coalescing binaries is sufficient to give a few per year from within 100 Mpc, then the fact (noted in Section (4.3.2) above) that coalescing binaries are reliable distance indicators allows one to measure Hubble's constant to within a few percent in a year or two of observations[33]. This in turn will determine the age of the universe and the distance scale to external galaxies.

- *Cosmological mass distribution.* Given a reasonable event rate, coalescing binaries are good tracers of the stellar distribution out to 500 Mpc or (for black holes) a few Gpc. Their distribution would indicate structure out to 500 Mpc on length scales of 10–100 Mpc or so, scales on which we have little information at present. This would provide a stringent test of the homogeneity and isotropy of the universe.

- *The early universe.* By confirming or ruling out a stochastic background of gravitational radiation as predicted by cosmic string theory, gravitational wave observations can be crucial to the cosmic string theory of galaxy formation. If other backgrounds are detected, they will have to be explained by some physics in the early universe. If the explanation has to do with phase transitions, for example, then this would have implications for particle physics; if an early generation of very massive objects is the cause, then this has implications for galaxy formation as well.

- *Follow-up observations.* Once gravitational wave sources have been identified by the detectors, astronomers will want to look at them with optical, radio, X-ray, and other telescopes. These follow-up observations will be a further source of important information. They require the gravitational wave network to provide a position for the source, which it can do to within a degree or so for burst sources at threshold, and with more accuracy for stronger sources. In addition, the network ought to be able to identify a supernova event within an hour of its arrival in the detectors, enough time to notify optical astronomers who would like to catch its first optical brightening, which would occur any time from a couple of hours to a day after the gravitational wave signal.

We appear, therefore, to be on the threshold of gravitational wave astronomy. If we are lucky enough to have another nearby supernova explosion within the next few years, then bar detectors will either see it or set an upper limit, and we will have our first information about gravitational wave strengths from such events. If funding bodies provide the resources to build interferometers over the next few years, then one can be very confident that they will be making observations within five years or so, and that they will ultimately reach the sensitivity required to detect waves from very distant events. Provided Nature is not too stingy with such sources, such a network will provide a great deal of astronomical information. My betting is that a network of interferometers will register more than 100 events per year once they reach their design sensitivity. When that happens, general relativity will have become a full partner with the rest of physics in not only explaining but also exploring the universe.

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