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REVIEW ARTICLE

Gravitational wave sources and their detectability†

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Abstract. This paper reviews sources of gravitational radiation that are likely to be detected by gravitational wave detectors now under development. We first develop back-of-the-envelope formulae that are useful for estimating the strength of gravitational waves and their *detectability*, i.e. their signal-to-noise ratio in either bar detectors or laser interferometric detectors. We show that, provided one can filter wide-band data optimally for a class of signals, the detectability of a signal in a given detector depends only on its distance, its total energy within the detector's bandwidth and (for wide-band detectors) on the dominant frequency of the signal. We then survey the most plausible sources: supernovae, coalescing compact-object binaries, neutron stars either spinning down or being driven by accretion, and the stochastic background. Several important points emerge from comparing detectability. For example, at a given distance, supernovae and coalescing binaries give comparable signals in a typical bar detector, but coalescing binaries are potentially much easier to detect than supernovae using interferometers. As another example, if a supernova is detected, then the subsequent spindown of any newly-formed neutron star may be just as easy to detect, again using an interferometer. The review concludes with an extensive discussion of coalescing binaries, their likely event rates, detection rates and astrophysical importance. They are one of astronomy's best 'standard candles'—observation of a coalescing binary by four interferometric detectors would determine its distance, independently of any knowledge of the masses of the stars in the system. With likely detection rates for coalescences of two-neutron-star systems of 1 to 10^4 per year, and with the strong possibility of detecting several two-black-hole coalescences per year at quasar distances, the prospects for interesting astrophysical information from gravitational wave detectors are very exciting.

1. Introduction

In the three decades since Joseph Weber first seriously proposed that gravitational waves from astronomical sources could be detected in laboratories, theorists would appear to have had plenty of time to decide what the most likely detectable sources of gravitational waves might be. However, after a flurry of activity to try—unsuccessfully—to explain Weber's initial observations [1], followed by the disappointment that subsequent bar experiments failed to confirm those observations, research into sources largely fell into the doldrums. The recent revival of the subject is due to a combination of factors: new discoveries and new understanding in astronomy regarding supernovae, evolution of binary stars and cosmology; the availability of computers with sufficient power to make realistic source calculations in general relativity practical; progress with cryogenic bar detectors to a sensitivity better than 10^{-18} that makes the detection of galactic supernovae a real possibility during the next decade; and the development

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of broad-bandwidth detectors (based on laser interferometry) with anticipated sensitivity approaching 10^{-22} that will be able to search for new classes of relatively narrow-band sources, such as pulsars, coalescing neutron-star binaries and Wagoner stars (all described in § 3 below).

Predictions about source strengths and detectability play two roles in gravitational wave research. First, because experiments are costly and somewhat risky, hard-pressed scientific funding agencies need as much reassurance as they can get that gravitational waves really are out there, at a detectable level. Second, the design of experiments and especially of data analysis strategies is influenced by the characteristics that gravitational waves are predicted to exhibit.

However, it must be borne in mind that the prediction of gravitational wave source strengths has a significant handicap: it relies almost entirely on information gathered from *electromagnetic* waves. Moreover, although gravitational wave observations will be made at relatively low frequencies (10^2–10^4 Hz), our predictions are based on the observations of electromagnetic waves at much higher frequency: 10^7 Hz for the longest radio waves, 10^{14} Hz for optical observations.

This difference in frequency is very significant, because the wavelength of a wave is strongly correlated with the size of the emitting region. Gravitational waves will be generated by the bulk motions of large masses confined to regions 10–100 km in size. Observed visible photons are typically generated by transitions inside atoms or by electron scattering of even shorter wavelength photons. The inference from such microscopic sources of what to expect of the bulk motions of huge masses is called modelling, and it is an enormous step. The success of modelling in many areas of astronomy, such as the spectral types of stars or accretion-disc models for x-ray binaries, should not blind us to the possibility that many models are necessarily oversimplified, or that the regions emitting observable electromagnetic radiation may not have much connection with the regions that may emit gravitational waves.

As an example, supernova models generally assume a spherically symmetric non-rotating collapse, and it may be argued that the observable expanding cloud of gas is so large that any initial rotation the progenitor star may have had will be insignificant, so that electromagnetic observations of supernovae may never tell us what role rotation plays in the initial gravitational collapse†. Nevertheless, rotation seems to be crucial for gravitational radiation: a spherical collapse generates none, while a modest amount of initial rotation may be enough to trigger non-axisymmetric instabilities that will produce copious amounts of gravitational radiation [4, 5]. My impression is that supernova theorists tend to be pessimistic about gravitational radiation from supernovae; the relative success of their models gives them confidence in the assumption of spherical symmetry. But such confidence may be misplaced, simply because any loss of symmetry in the gravitational-wave-generating core of a supernova may have little effect on the propagation of the shock through the stellar envelope, which produces all the observable electromagnetic radiation.

Another example is counting the number of binary systems composed of two neutron stars. A proportion of these will ultimately evolve, through the effects of gravitational radiation reaction, into the sources we call the coalescing binaries [6]‡.

† This may not be true of nearby supernovae. In fact SN1987A in the Large Magellanic Cloud has provided abundant electromagnetic evidence of rapid rotation [2]. The most dramatic evidence is the report of a 2 kHz pulsar [3].

‡ I know of two early discussions of coalescing neutron-star binaries as sources of detectable gravitational waves [7, 8]. These were remarkably prescient, having been written before the discovery of pulsars.

Only one such system is observed, the famous binary pulsar, PSR1913 + 16. The number of nearby binaries that are unobserved may be in the hundreds: not only does pulsar beaming make it likely that there are several nearby systems containing active pulsars that we do not see, but also the relatively short lifetime of a pulsar means that there are surely many more binary systems locally consisting of 'dead' neutron stars. Our estimates of the event rate for binary coalescences (§ 4 below) depend on electromagnetic observations of only one system, PSR1913 + 16, and the rate prediction is therefore very uncertain. If we could observe the dead neutron star binaries we would have a much better idea of what to expect in our detectors, but unfortunately these systems will probably not be discovered until space-based gravitational wave detectors operating at ultra-low frequencies are in place. (We might find them if we could receive the very-low-frequency radio waves emitted by their rotating magnetic dipole fields, but electromagnetic waves of this frequency do not propagate through the interstellar plasma.)

With this fundamental reservation in mind, let us begin our review of what we can predict about gravitational wave sources. I will begin in § 2 with some simple formulae that are useful in estimating source strengths and, more importantly, their *detectability* in detectors of various kinds. Section 3 is a round-up of the principal likely sources, with the waves classified as is customary into bursts, continuous waves and stochastic waves. Burst sources include the supernova/gravitational collapse and the coalescing binary. Sources of continuous waves include neutron-star spindowns, instabilities in neutron stars driven by accretion, and pulsars. The stochastic background could come from many classes of source, including cosmic strings. In § 4 I return to the coalescing binaries, which may in a decade from now be the most important gravitational wave source. I discuss estimates of the event rate and of the rate at which a network of detectors would observe such events. Then I suggest the sort of astrophysical information that we might expect to learn from observations of coalescing binaries, emphasising especially that coalescing binaries are one of astronomy's best 'standard candles', or estimators of absolute distance. The final section is a brief conclusion.

Further details on much that is discussed here can be found in the comprehensive review by Thorne [9], in two recent conference proceedings [10] and in two forthcoming books [11].

2. Back-of-the-envelope estimates

Given the uncertainties in many of the astrophysical variables that go into source calculations, it is useful to have rough, easy-to-use approximations for the principal things one wants to calculate. I will concentrate on two important variables: source strengths in the quadrupole approximation and basic estimates of the detectability of different sources.

2.1. Source strengths

Most estimations of the intensity of radiation use the so-called 'quadrupole formula', which is the low-velocity, weak-field approximation. This approximation has a long history, but recent investigations have shown that it is a consistent asymptotic approximation in a considerable range of situations [12–18]. Even when the source is fairly relativistic, I will use the approximation to give a rough idea of how such radiation may come off.

The quadrupole formula for the transverse traceless part of the radiation field from a nearly Newtonian source is [19, 20]

$$h_{ij}^{\mathrm{TT}} = \frac{4}{r}\,\{\text{trace-free part of}\}\int_{\text{source}} T_{ij}\,\mathrm{d}^3x \tag{2.1}$$

where the stress tensor T_{ij} includes the Newtonian gravitational stresses as well as the usual fluid stress:

$$T_{ij} = \rho v_i v_j + p\delta_{ij} + \frac{1}{4\pi}\,(\Phi_{,i}\Phi_{,j} - \tfrac{1}{2}\delta_{ij}\Phi_{,k}\Phi^{,k}). \tag{2.2}$$

Here ρ and p are the fluid density and pressure, v_i is the velocity of the fluid and Φ is the Newtonian gravitational potential. I shall use units in which $c = G = 1$. Taking the trace-free part in (2.1) eliminates the δ_{ij} terms in (2.2) and uses only the non-isotropic part of the remainder. Let us consider a source in which (i) gravity plays an important role in the dynamics and (ii) the motions are highly non-spherical. Then the gravitational stresses in (2.2) will be of the same order as the kinetic stresses, and these are bounded by twice the kinetic energy density. A typical component of h_{ij}^{TT} in a highly non-spherical situation might then be roughly approximated by

$$|h| \sim 2T_{\text{non-spherical}}/r \tag{2.3}$$

where $T_{\text{non-spherical}}$ is the kinetic energy associated with the non-spherical motions.

This result has the interesting interpretation that gravitational radiation is basically the 'Newtonian' field of the non-spherical kinetic energy distribution, except that unlike a Newtonian field it propagates at the speed of light.

An upper limit on h comes from the dynamics of self-gravitating systems [21]. If the motions in the system are generated by gravitational effects, as in gravitational collapse, then

$$T \sim |W| \sim \tfrac{1}{2}M|\Phi_{\text{int}}|$$

where W is the gravitational potential energy of the system and Φ_{int} is a typical value of the gravitational potential *inside* the source. Since this bounds the non-spherical part of T, we have

$$|h| \leqslant M|\Phi_{\text{int}}|/r \sim \Phi_{\mathrm{N}}\Phi_{\text{int}} \tag{2.4}$$

where Φ_{N} is the Newtonian potential $-M/r$ of the whole source at the observer's distance r. When the system is highly non-spherical, such as a binary star system, then this formula gives a good approximation. When the system is mostly spherical, such as in collapse with little rotation, it may give a bound a factor of 10^3 or more too high (cf § 3.2.3).

2.2. *Detectability*

In this section I will give useful formulae for estimating whether one source might be harder or easier to detect than another, using a given detector. Different considerations apply depending upon whether the detector has a smaller bandwidth than the signal or a larger one.

2.2.1. Narrow-band detectors. Bar detectors typically have relatively narrow bandwidths, say 10 Hz at 1 kHz. Suppose the detector has a central frequency f_0 and a bandwidth Δf_0. If the signal has a broader bandwidth than this, we may approximate its energy distribution per unit frequency E_f as relatively constant over Δf_0. Then the amplitude signal-to-noise ratio in the detector will be proportional to the square root of the ratio of the excitation energy of the detector to the noise energy in the detector system:

$$S/N \propto (E_f(f_0)\Delta f_0/kT_{\rm eff})^{1/2} \qquad \text{(narrow-band detector)} \tag{2.5}$$

where $T_{\rm eff}$ is the effective temperature of the detector and k is Boltzmann's constant. The factors left out of (2.5) include the detector's cross section and the distance to the source. Equation (2.5) is not written in the usual way that bar-detection physicists express sensitivity; in particular, the effective temperature of the detection is a complex function of many parameters, including the signal itself. Full discussions of bar-detector sensitivity may be found in the literature [22].

2.2.2. Wide-band detectors. The laser interferometric gravitational-wave observatories (LIGO) currently being planned by a number of groups are designed to have bandwidths larger than those of most expected sources. In this case it is appropriate to characterise the noise by $S_h(f)$, the spectral density of amplitude noise in the detector. (More precisely, 'amplitude' means the value of h inferred from the detector's output.) The amount of noise present in an observation depends on the range of frequencies over which S_h contributes. If S_h is a slowly varying function of f, and if the signal has frequency f, bandwidth Δf, and amplitude h, then if we perform a Fourier transform of the output of the detector, the signal will only compete with the noise in the signal bandwidth:

$$S/N \simeq h/(S_h(f)\Delta f)^{1/2} \qquad \text{(wide-band detector).} \tag{2.6}$$

For a burst of radiation with $\Delta f \sim f$, we have

$$S/N \simeq h/(S_h(f)f)^{1/2} \qquad \text{(wide-band detector, observing bursts).} \tag{2.7}$$

Now, the energy in the signal is related to its amplitude h and frequency f by [19, 20]

$$E \propto f^2 h^2 \tau \tag{2.8}$$

where τ is the duration of the signal. We may use this to eliminate h in favour of E in (2.6). Again for a burst of radiation, where $\tau \sim f^{-1}$, we have

$$S/N \propto (1/f)(E/S_h(f))^{1/2} \qquad \text{(wide-band detector, observing bursts).} \tag{2.9}$$

If the detector is a LIGO, the form of $S_h(f)$ depends on how the detector is configured [9]. A 'simple' detector with fixed characteristics, observing over a wide range of frequencies at the photon-counting noise limit, has white noise, with S_h independent of f:

$$S/N \propto E^{1/2}/f \qquad \text{(simple LIGO, not optimised, observing bursts).} \tag{2.10a}$$

If we can optimise the storage time of light in the arms of a simple LIGO for any desired frequency f, then the attainable S_h varies as f^2, and

$$S/N \propto E^{1/2}/f^2 \qquad \text{(simple LIGO, optimised, observing bursts).} \tag{2.10b}$$

If the LIGO is configured in a recycling mode [23], again optimised for signals of frequency f, then $S_h \propto f$ and

$$S/N \propto E^{1/2}/f^{3/2} \qquad \text{(recycling LIGO, optimised, observing bursts).} \tag{2.10c}$$

If the signal has more structure than a simple burst, so that it lasts for n cycles near frequency f, then *if the waveform is known in advance* the best signal-to-noise ratio will be obtained by matched filtering of the data [24, 25]. A rough estimate of what this will be is provided by setting the effective bandwidth of the signal Δf equal to the inverse of the *duration* $\tau = n/f$:

$$S/N \simeq h\sqrt{n}/(S_h(f)f)^{1/2} \qquad \text{(wide-band detector, filtered).} \qquad (2.11)$$

Comparing this with (2.6), we see that *matched filtering improves the signal-to-noise ratio by a factor of* $n^{1/2}$ *over a burst of the same amplitude.* This may not be as remarkable as it first looks, because a signal that lasts n times as long as a burst and has the same frequency and amplitude carries n times as much energy. Using (2.8) with $\tau = n/f$ to estimate the energy in the signal, we find

$$S/N \propto (1/f)(E/S_h(f))^{1/2} \qquad \text{(wide-band detector, filtered).} \qquad (2.12)$$

This is *identical* to its counterpart for bursts, (2.9): *two signals from the same distance with the same energy and frequency have the same signal-to-noise ratio provided that they are extracted from the noise (if necessary) by matched filtering.* If a signal lasts longer than the observing time (such as the signal from a pulsar), then the relevant energy is that which is emitted during the observation period.

The only other source characteristic which affects the signal-to-noise ratio is its distance r: S/N is proportional to r^{-1}. The ratio E/r^2 is called the *fluence* of the source, so we see that: for any given detector (and provided one can apply matched filtering to signals of narrow bandwidth), the signal-to-noise ratio depends only on the source's characteristic frequency and its *fluence* within the detector's bandwidth and observing time. The signal-to-noise formulae are summarised in table 1. I will use them in the next section to assess the detectability of various sources. The proportionality signs leave out properties of the detectors that determine their absolute sensitivity. See Thorne [9] for the full formulae.

Table 1. Summary of dependence of signal-to-noise ratio S/N on the characteristics of the source (energy and frequency) for various detectors. For wide-band detectors, $S_h(f)$ is the spectral density of noise.

Type of detector	$S_h(f) \propto$	$S/N \propto$
Narrow bandwidth Δf_0 about f_0	—	$(E_f(f_0)\Delta f_0)^{1/2}$
Wide bandwidth	—	$E^{1/2}S_h(f)^{-1/2}f^{-1}$
Simple LIGO, not optimised	f^0	$E^{1/2}f^{-1}$
Simple LIGO, optimised	f^2	$E^{1/2}f^{-2}$
Recycling LIGO, optimised	f^1	$E^{1/2}f^{-3/2}$

3. Round-up of sources

The conventional division of sources is into three classes: burst, continuous waves and the stochastic background. This division reflects the need for different kinds of data analysis for the different classes. A 'burst' is any event for which one can neglect the Doppler shift produced by the rotation of the Earth [26]. The maximum duration of such a 'burst' is a time τ such that the frequency resolution of an observation lasting a time τ, $2/\tau$, is equal to the net shift due to Earth's having rotated for a time τ,

$f\Omega_\oplus^2 R_\oplus \tau$, where f is the maximum frequency we look at, $\Omega_\oplus$ is the Earth's angular velocity, and $R_\oplus$ is the Earth's radius (we take $c=1$). This gives $\tau = \Omega_\oplus^{-1}(\frac{1}{2}fR_\oplus)^{-1/2} \simeq$ 70 min for $f = 1$ kHz. We will therefore adopt a working limit of ~30 min on any signal that qualifies to be a burst. Any coherent wave train longer than this is called a continuous wave. Any confusion-limited wave train, where the coherence length of an individual wave is longer than the time before another (uncorrelated) wave arrives, contributes to the stochastic background.

3.1. Bursts

3.1.1. Gravitational collapse. Historically, this has been the class of sources which has driven the development of gravitational wave detectors. It is still the most likely source to be observed by bar detectors, although LIGO observations may be dominated by coalescing binaries (below). We can use our upper limit, (2.4), to bound the value of h we might expect.

Suppose collapse forms a 1 $M_\odot$ *neutron star*, with all the rest of the mass expelled. Taking an upper limit on $|\Phi_{\rm int}| \sim 0.2$, we find

$$h \leqslant 2\times 10^{-21}(15\ {\rm Mpc}/r) \qquad f \sim 1\ {\rm kHz} \tag{3.1}$$

where r is the distance to the source. (The Virgo cluster is roughly 15 Mpc away.) Unfortunately, this strength is not likely to be reached except when rotation dominates the collapse and rotational instabilities produce non-axisymmetries [2-5], and even in this case only the 'effective' h, $h\sqrt{n}$, will be likely to approach this level. The total radiated energy, even over several cycles, is not likely to exceed half the binding energy,

$$E \leqslant 0.05\ M_\odot. \tag{3.2}$$

If instead a *black hole* of $M \sim 10\ M_\odot$ forms, efficiencies can in principle approach 100%. More plausibly, if we estimate 10% efficiency we find

$$h \sim 1.5\times 10^{-20}(15\ {\rm Mpc}/r) \qquad f \sim 1\ {\rm kHz} \qquad \Delta E \sim 1\ M_\odot. \tag{3.3}$$

These figures are very much upper limits, for it is in principle possible for gravitational collapse to be perfectly spherical, and so to produce no gravitational radiation at all. In practice, one can expect a range of initial conditions, and therefore a range of gravitational wave amplitudes. Numerical calculations can give us an idea of what this range may be. Present computers are able to calculate the emission from rotating axisymmetric collapse [27]. But the most powerful sources are expected to be those in which rotational instabilities produce non-axisymmetries, and the rate of occurrence of such systems in nature is hard to estimate from present observations. Three-dimensional collapse calculations now under development on the fastest supercomputers will give us a better idea of the range of initial conditions that produce big bursts [28] although they will need to be rather accurate before they can produce realistic results [4]. Another uncertainty is the rate at which collapses occur in any galaxy. The observed supernova rate (still uncertain) is a lower limit, but Blair [29], Bahcall and Piran [30] and Schramm [31] present evidence that the rate of collapse may be as large as one collapse every one to five years. The 'excess' collapses may be electromagnetically quiet, or they may produce supernovae that are not seen in external galaxies: perhaps they are hidden in dense clouds, or perhaps they are underluminous (like SN1987A). If collapses occur in our Galaxy at the rate of, say, one per three years, then a gravitational wave detector sensitive at the 10^{-19} level stands an excellent chance of observing a burst, particularly since neutrino detectors are capable of signalling the occurrence of the event [32, 33]. Moreover, a detector sensitive at the 10^{-22} level could see up to one event per day from the Virgo cluster.

3.1.2. Coalescing binaries. Consider two 1.4 $M_\odot$ neutron stars in a circular orbit, which decays due to the emission of gravitational radiation. When the gravitational wave frequency f is 100 Hz (this is twice the orbital frequency), the stars are separated by 160 km and $|\Phi_{\rm int}| \sim 10^{-2}$. Our upper limit (2.4) is a good estimate here, since all the mass is moving non-spherically. This gives

$$h \simeq 5\times 10^{-23}(100\ \mathrm{Mpc}/r) \qquad f = 100\ \mathrm{Hz}. \tag{3.4}$$

The energy radiated as f increases to 200 Hz is

$$E \simeq 6\times 10^{-3}\, M_\odot \qquad \text{in } n \equiv (f^2/\dot f) \simeq 500 \text{ cycles}. \tag{3.5}$$

Alternatively, if a binary system consists of a 10 $M_\odot$ black hole and a 1.4 $M_\odot$ neutron star, then

$$h \simeq 3\times 10^{-22}(100\ \mathrm{Mpc}/r) \qquad f = 100\ \mathrm{Hz} \tag{3.6}$$

$$E \sim 3\times 10^{-2}\, M_\odot \qquad \text{in } n \simeq 130 \text{ cycles}. \tag{3.7}$$

These values for h are the maximum one can expect: geometrical factors (the relative orientations of the orbital plane of the binary and the antenna pattern of the detector) will reduce this. In § 4 I will incorporate these factors. Also in § 4 I will argue that the detection rate for coalescence of black-hole–neutron-star systems may not be very different from that for neutron-star–neutron-star binaries.

3.1.3. Detectability: comparing collapses with coalescences. Suppose a collapse releasing a *moderate* 0.01 $M_\odot$ of energy and a binary coalescence of two 1.4 $M_\odot$ stars occur at the same distance. Which is easier to detect?

In a *narrow-band* detector, observing at 800 Hz, the comparison hinges on the energy per unit frequency. For a collapse, with a bandwidth of ~1 kHz, this is roughly $10^{-5}\, M_\odot\, \mathrm{Hz}^{-1}$. For the coalescence, the orbital energy radiated as the gravitational wave frequency increases by 1 Hz at 800 Hz is $3\times 10^{-5}\, M_\odot$. The result is

$$\frac{(S/N)_{\rm coalescence}}{(S/N)_{\rm collapse}} = \left(\frac{\mathrm{d}E/\mathrm{d}f_{\rm coalescence}}{\mathrm{d}E/\mathrm{d}t_{\rm collapse}}\right)^{1/2} \sim 2. \tag{3.8}$$

A narrow-band detector at 800 Hz can pick up collapses and coalescences about equally easily.

For a *wide-band detector*, the behaviour of the S/N depends on the frequency behaviour of $S_h(f)$, because a wide-band detector will observe collapses at about 1 kHz but it can also be optimised to look for coalescences at lower frequencies, such as 100 Hz. If we observe each source with an *optimised recycling* LIGO, for example, then we find from table 1:

$$\frac{(S/N)_{\rm coalescence}}{(S/N)_{\rm collapse}} = \left(\frac{E_{\rm coalescence}}{E_{\rm collapse}}\right)^{1/2}\left(\frac{f_{\rm collapse}}{f_{\rm coalescence}}\right)^{3/2} \sim 25 \tag{3.9}$$

where we have taken $f_{\rm coalescence} = 100$ Hz and $E_{\rm coalescence}$ to be the energy radiated as the waves move from 100 Hz to 200 Hz. This ratio explains the current interest in coalescing binaries: they can be seen some 25 times further away than moderate collapses, or in a volume some 10^4 times as large. I will discuss what is known about the event rate per unit volume in § 4.1 below; the analysis there of the many uncertainties suggests that a network of five recycling LIGO should see between 1.5 and 1.5×10^4 events per year. In this case, the event rate uncertainty is entirely caused by poor information about the number of events; the amplitude one can expect from each event is not greatly uncertain.

Notice that the radiation we are discussing comes from the orbital motion of the two stars, not from the later coalescence event as the two stars collide. Any radiation—gravitational or visible—that comes from the collision will be an added bonus that increases the detectability of the event [34]. In fact, gamma-ray observations in the near future may well be able to constrain the event rate [34].

In this section, I have compared collapses with coalescences in given detectors. Dewey [35] has compared the detectability of coalescences in bars with their detectability in LIGO.

3.2. Continuous waves

By our definition, continuous waves last longer than about 30 min, so that any attempt to find them in noisy data must compensate for Doppler shifts produced by the Earth's motion. This compensation depends on the assumed direction of the source, so that there will be different filters for different directions. The computational difficulties this raises if the observing period is as long as a few months are so formidable that it is likely that, at least initially, searches at maximum sensitivity will not be possible except for candidates whose positions and/or frequencies are already known [26, 36]. Cross correlation techniques among three LIGO seem promising for discovering bursts [26, 37] but not for continuous waves.

3.2.1. Instability-driven spindown. If a neutron star is formed in a gravitational collapse, it may start out rotating very rapidly. If its period is less than about 1 ms, then it may be subject to a class of gravitational radiation driven instabilities. Their effect will be to radiate angular momentum away until the star spins down to a marginally stable condition.

These instabilities were first discovered by Chandrasekhar [38] and later shown by Friedman and Schutz [39] to be generic in rotating stars. This 'CFS' class of instabilities has been reviewed by Friedman and by Schutz [5]. It is characterised by unstable modes with given axial eigenvalues m (eigenfunction proportional to $e^{im\phi}$). For any given m, there is a point of instability in a sequence of stars of increasing angular momentum J, such that for larger J, all stars have at least one unstable mode of that eigenvalue m. The unusual feature of this instability is that it tends to set in *earlier* along the sequence for larger m. In the absence of viscosity, for any $J\neq0$, there is some $m_G(J)$ such that all $m\geqslant m_G(J)$ have unstable modes; $m_G(J)$ increases as J decreases. In other words, every rotating, perfect-fluid star has infinitely many unstable modes! However, the growth time of such modes increases exponentially with m [40], and equally importantly viscosity ν in the star can remove the instability for all $m>m_v(\nu)$ [41]. As ν increases, $m_v(\nu)$ decreases. The result is that realistic stars rotating less rapidly than about 1 kHz may have no instabilities, but as the rotation period decreases, the instabilities for $m=5$ (possibly), 4 (likely), 3 and 2 appear in succession. So far, calculations of the onset and growth rates of these instabilites have been approximate [41], but the situation should improve soon, with mode calculations now underway [43]. However, there will remain a number of uncertainties when making predictions about any particular star: the equation of state and viscosity of neutron stars are still subject to much debate; and a star's mass and temperature (which determines its viscosity [44]) both affect the modes' growth rates and frequencies, and the critical angular momentum for which it would be marginally stable. Observations of the minimum period of rotation of neutron stars could shed light on these questions, but indications [45] that this period is 1.6 ms have been called into question by the apparent observation of a pulsar with a period of 0.5 ms in the remnant of SN1987A [3]. Further observations will be needed to settle this.

We can make crude estimates of the radiation we might expect from instability-driven spindown. The $m=2$ and $m=3$ modes have very short timescales, roughly 1 s and 100 s, respectively, so if a burst is detected this radiation may follow on immediately. The $m=4$ and $m=5$ timescales might be of order several hours and a few weeks respectively, and it is important to know whether detectors stand any chance of seeing this radiation. Assuming that the instability in one mode causes the star's rotation period to increase by 0.1 ms at about 1 ms, then a 1.4 $M_\odot$ star will radiate $\sim 3\times 10^{-3}\, M_\odot$ during the spindown timescale τ. The frequency of the radiation will gradually change from its initial value (anything up to 5 kHz, but more likely around 300 Hz or so) down to zero. Using the numbers we can estimate

$$E \sim 3\times 10^{-3}\, M_\odot \text{ at } 0<f<5\text{ kHz}$$

$$h \sim 6\times 10^{-26}\left(\frac{E}{3\times 10^{-3}\, M_\odot}\right)^{1/2}\left(\frac{f}{300\text{ Hz}}\right)^{-1}\left(\frac{\tau}{1\text{ mo}}\right)^{-1/2}\left(\frac{r}{15\text{ Mpc}}\right)^{-1}. \tag{3.10}$$

Let us now compare the detectability of the initial burst and the subsequent spindown in a broadband detector that can follow the frequency over the whole spindown time τ:

$$\frac{(S/N)_{\text{burst}}}{(S/N)_{\text{spindown}}} \sim \left(\frac{E_{\text{burst}}}{E_{\text{spindown}}}\right)^{1/2}\left(\frac{f_{\text{spindown}}}{f_{\text{burst}}}\right)^{p}$$

where p is 1, 1.5, or 2 for LIGO, depending on their configuration. Since $f_{\text{burst}} \sim 3 f_{\text{spindown}}$ and $E_{\text{burst}} \sim 3\, E_{\text{spindown}}$ (again taking $E=0.01\, M_\odot$ for a moderate burst), we find

$$\frac{(S/N)_{\text{burst}}}{(S/N)_{\text{spindown}}} \sim 0.3 \text{ to } 1. \tag{3.11}$$

Therefore, if a burst is detected it seems well worthwhile to look for instability-driven spindown radiation. It would confirm the presence of a neutron star.

3.2.2. Accreting neutron stars (Wagoner mechanism). If a slowly rotating neutron star accretes mass from a binary companion, it can be spun up until it reaches an instability of the CFS class whose growth time is shorter than the accretion timescale. When this happens, Wagoner [46] pointed out, the instability will grow until the accreted angular momentum is balanced by the angular momentum carried away by gravitational waves. This would produce radiation at a single frequency, steady on the accretion timescale. Since there is accretion, the system should also be an x-ray source, and moreover the energy radiated in gravitational waves should be *proportional* to the energy emitted in x-rays. This leads to the formula [9]

$$h \simeq 2\times 10^{-28}\left(\frac{300\text{ Hz}}{f}\right)\left(\frac{F_x}{10^{-10}\text{ erg cm}^{-2}\text{ s}^{-1}}\right)^{1/2} \tag{3.12}$$

where f is the frequency of the radiating mode (very uncertain) and F_x is the x-ray flux. Notice that the distance to the source does not enter (3.12), because the x-ray flux F_x scales with distance in the same way as the gravitational wave energy flux. The mass, temperature, viscosity and equation of state of the star determine whether the instability exists at all, and if it does then they and the accretion rate determine f.

Several galactic binary x-ray sources have fluxes well above $10^{-10}\,\mathrm{erg\,cm^{-2}\,s^{-1}}$. However, to pick the signal out of the noise with a resonantly recycling LIGO, one must have a good idea of the frequency to 'tune' the detector to†. A satellite has been proposed [49] that would have a large enough collecting area to allow it to make searches for millisecond modulations of the x-ray flux at the level of a few per cent. If such observations (or others) give a candidate frequency, then proposed LIGO would have more than enough sensitivity to reach the level given in (3.12) [9].

3.2.3. Lumpy pulsars. An axisymmetric rotating star gives off no gravitational radiation, so any radiation emitted by pulsars depends on their degree of non-axisymmetry. There must be some non-axisymmetry due to whatever mechanism produces the beams of electromagnetic radiation, but this may be quite small. However, larger 'lumps' or 'mountain ranges' may become frozen into the crust of a neutron star as it cools down. If a lump has a mass m and the star rotates with period P, then the lump's kinetic energy is

$$T_{\text{non-spherical}} \simeq 2.5\times10^{-5}(m/1M_\odot)(30\ \mathrm{ms}/P)^2. \tag{3.13}$$

Putting this into (2.3) gives

$$h \simeq 3\times10^{-25}\left(\frac{m}{10^{-4}\,M_\odot}\right)\left(\frac{3\ \mathrm{kpc}}{r}\right)\left(\frac{30\ \mathrm{ms}}{P}\right)^2. \tag{3.14}$$

Notice that using (2.4) would give a considerable overestimate, because the lump is not at a virial velocity (unless the star is rotating close to its break-up angular velocity), and only a small fraction of the star's mass is in the lump. A direct upper limit of about 10^{-21} on the Crab pulsar's radiation has been set by the Tokyo group's specially constructed bar detector [50]; this is still above an indirect limit of about 10^{-24} (i.e. $m \sim 10^{-3}\,M_\odot$) determined by attributing all of the Crab's slowdown to the loss of energy to gravitational radiation, a situation which is surely an overestimate. Current LIGO designs anticipate being able to reach $m \sim 10^{-5}$ to $10^{-6}\,M_\odot$ in the Crab and possibly in the Vela pulsar, provided adequate seismic isolation can be achieved down to about 10 Hz. This is a design goal of the systems being developed for the Italian-French VIRGO detector.

3.3. Stochastic background

There are a wide variety of possible sources of a stochastic background: binaries, various phase transitions in the early universe [51], primordial nucleosynthesis, formation and binary coalescence of black holes at the time of galaxy formation, and cosmic strings. Sazhin [51] has reviewed predictions about the background from phase transitions; these and the binary backgrounds may be out of the frequency range of

† LIGO may be tuned by a variety of techniques. 'Resonant recycling' was introduced by Drever [23]. 'Detuned recycling' is a more recent invention of Vinet *et al* [47]. An especially promising technique for adjusting the bandwidth and overcoming storage time limitations is 'dual recycling', devised by Meers [48].

ground-based detectors. An interesting source of background that could be detectable in bars and LIGO is the decay of cosmic strings. Strings might provide seeds for galaxy formation; if they do, their decay should produce a background with about 10^{-7} of the closure energy density. See Hogan and Rees [52], or for more detail, Vachaspati and Vilenkin or Brandenburger *et al* [53]. Radio observations of the 'millisecond pulsar' PSR1937+21 are providing an increasingly strong constraint on this figure at low frequencies, of the order of 10^{-7} Hz, due to this pulsar's extraordinary stability [54].

It is possible, however, that LIGO could reach to 10^{-9} of closure density at a frequency of ~100 Hz [9]. The stochastic background can be observed by cross-correlating the outputs of two antennae; if the resulting noise is higher than one expects on the basis of the Gaussian noise statistics of either detector, then the excess noise may be due to the background radiation. Ideally, the two detectors should be closer than half a wavelength of the radiation, which at 100 Hz means a separation not larger than 1500 km. Detectors separated by more than that begin to lose coherence in their responses to the random waves, so that the background does not produce correlations between antennae. Michelson [55] has discussed this in detail.

4. Coalescing binaries in detail

Because coalescing binaries are so much more easily detected by LIGO than bursts are (3.9), it is important to look carefully at what we can predict about the event rate of coalescences, at how much information a network of detectors could extract from a signal, and at what the astrophysical importance of such observations may be.

4.1. Event rate

At first I will consider only binaries consisting of two neutron stars. A *precursor* system is a binary system which contains two neutron stars and is in an orbit whose gravitational radiation decay time is less than a Hubble time. Our estimates of the event rate hinge on the fact that we have observed *one* precursor system: the famous 'binary pulsar' PSR1913+16, which will coalesce in 1.4×10^8 yr.

A naive estimate of the coalescence rate makes two assumptions: (i) that there is a steady state, with the coalescence rate equal to the precursor birthrate, and (ii) that PSR1913+16 (and any other precursors) are typical pulsars that just happen to be in binary systems. Assumption (ii) implies that, since we see one precursor out of about 400 known pulsars, the precursor birthrate is 1/400th of the pulsar birthrate. Assumption (i) then implies that the present coalescence rate is also 1/400th of the pulsar birthrate. If we adopt a canonical pulsar birth rate of 1 per 40 years in our Galaxy [56], we conclude that there should be one coalescence per 16 000 years in our Galaxy. There are about 10^5 galaxies out to a distance of 100 Mpc, of which roughly half are spirals in which pulsars are still forming. The result is that we expect three coalescences per year out to 100 Mpc. This agrees with the estimate of Clark *et al* [57], who used a similar method based on the observed supernova rate rather than the pulsar birthrate.

Assumptions (i) and (ii) can both be challenged, however, and there are other misgivings one can have about this 'naive' method. I shall list various possible corrections to the naive result.

1. Assumption (ii) is wrong: PSR1913+16 is not typical. Pulsars in binary systems (of which more than a dozen are known) tend to have the shortest periods and longest spindown times of all pulsars. They may be formed from 'old' neutron stars that are spun up by accretion in the binary system [58]. Judging from their spindown times, their lifetimes may be 10^2 to 10^3 times longer than typical pulsars, which means their birthrate should be 10^{-2} to 10^{-3} of our naive estimate. This multiplies the coalescence rate by 10^{-2} to 10^{-3}.

2. Small number statistics. Since we have only one observed precursor, our inferences could easily be in error by a large factor. Statistics of pulsars in binaries, however, suggest that PSR1913+16 is not unusual: of the known pulsars in binaries, at least two (PSR0655+64 and PSR1831−00, both of which probably have white-dwarf companions) may have gravitational wave decay times of the order of a Hubble time. It seems reasonable, therefore, to allow for the true rate to differ from the observed rate of occurrence at the 90% confidence limits of a Poisson distribution: a factor of 0.1 to 3 times the naive rate.

3. Selection effects in pulsar observations. It is clear that pulsars are harder to observe if they are in binaries than if they have no companion, because the discovery of a pulsar requires that the pulse rate be a constant in the solar system's barycentric frame for as long an integration time as is needed to make it stand up above the noise. The orbital Doppler shift of a pulsar's signal can smear it out and make it unobservable. Compensating this is the fact that radio astronomers have searched very hard in the last 10 years for short-period pulsars, which tends to favour pulsars in binaries. At present there are no available quantitative estimates of these effects, but it is very likely that the difficulty of detecting pulsars in binaries will be the dominant effect on estimates of coalescence rates, since it gets stronger for short orbital periods, precisely the systems among which precursors will be found. I will multiply the naive rate by a factor of 3–10 for selection effects, recognising that this is to some extent an arbitrary choice.

4. Assumption (i) is wrong: there is no steady state. We are dealing with systems which decay on a Hubble time scale, so there is no reason to expect that some sort of equilibrium has been reached. If early epochs of star formation in galaxies (Population II) produced binaries and massive stars with a frequency comparable to that in contemporary Population I star formation, then a better estimate of the coalescence rate today is to multiply the current precursor formation rate by the ratio of the historical mean star formation rate to the current star formation rate in any volume of space. In our Galaxy this ratio may be about 5. If we take a large volume of space, we must include the elliptical galaxies that we omitted from the naive calculation, which raises this factor to 10: coalescences may be occurring in ellipticals as often as in spirals, despite the fact that they are not forming stars now. This estimate of the factor due to early star formation assumes a uniform distribution of orbital decay times among pulsar-containing binaries. In fact, it might be thought more likely that the distribution would be skewed towards longer decays: a larger probability of finding a decay time between 1 and 2 Hubble times than between 0 and 1. If this is the case, it *raises* the present coalescence rate, since our one observed precursor has a relatively short decay period. (The observed binaries are consistent with a level distribution or one that increases modestly with decay time.) Other factors we have omitted are the possibility of Population III binaries and the likelihood that (due to metallicity effects) early generations of stars would have had a greater proportion of massive stars than recent generations. Ignoring these effects, which would push the coalescence rate up, I shall adopt a correction factor of 10 to the naive rate out to 100 Mpc.

5. The pulsar formation rate may be larger. Our naive estimate was based on a pulsar formation rate in our galaxy of one per 40 years. Blair [29] has argued that the real gravitational collapse rate may be substantially higher, even up to one per year and other considerations suggest a rate between 3 and 15 per year is likely [30–32]. I will adopt a correction range of 1–10 for this effect.

The *result* of all these factors is that the coalescence rate is likely to lie in the range 10^{-2} to 10^{2} per year out to 100 Mpc. Only gravitational wave observations are likely to tell us where in this range the true rate lies. We will see below that even if the rate is at the lower bound of 10^{-2} per year out to 100 Mpc, a network of four LIGO detectors could pick up a few events per year.

Until now I have concentrated on double neutron-star precursor systems. But a certain fraction of close binaries will form neutron-star/black-hole systems. How large a fraction could this be? A suggestion comes from binary x-ray systems. Presumably all precursors pass through such a stage before the second star evolves to a neutron star or black hole. Observations suggest that perhaps 1% of all binary x-ray systems contain a massive black hole as the compact object. It is plausible, therefore, that 1% of binary coalescences in any given volume of space will be of a neutron star and a black hole. The energy emitted by a binary consisting of a 1.4 $M_\odot$ neutron star and a 14 $M_\odot$ black hole as its gravitational wave frequency goes from 100 to 200 Hz is a factor 5.8 times larger than for the two-neutron-star system, so the filtered signal-to-noise ratio will be 2.4 times larger. Such systems are detectable 2.4 times further away, which is to say in a volume 14 times larger. The detected event rate could, therefore be 14% of the two-neutron-star rate. A similar calculation for the coalescence of a binary consisting of two 14 $M_\odot$ black holes (occurring in any given volume 10^{-4} times as often as a binary neutron star coalescence) predicts a detected event rate roughly 3% of the two-neutron-star rate.

4.2. Observations with a network of LIGO

To convert source strength and event rate calculations into predictions about observations, one must know two more things: the basic sensitivity of a network of LIGO detectors, and the threshold on S/N one must set in a given network to be reasonably sure that an observed event is real and not a 'false alarm' due to detector noise.

A single LIGO with 4 km arms, 100 W effective laser power, and mirrors with losses no worse than 5×10^{-5} per reflection, operated in recycling mode optimised for signals in the 100–200 Hz bandwidth, can reach a photon-counting noise limit of

$$S_h(f)\sim(8.2\times10^{-25})^2\ \mathrm{Hz}^{-1} \tag{4.1}$$

at 100 Hz [9]. We can estimate the S/N obtainable for a coalescing binary by using (2.11). If we take $f=100$ Hz, $n=\frac{1}{2}f\tau\sim250$ and $h=3.6\times10^{-23}$ for an optimally oriented source at 100 Mpc (this is a more accurate value of h than our estimate in (3.4) [7–9]) then we find an optimum S/N of about 68. This is an overestimate, because it ignores the fact that the frequency changes and the S/N degrades during the observation. Doing the same calculation at 200 Hz (where S_f is up by a factor of 2.5, h is up by $2^{2/3}$, and n is down by $2^{-5/3}$) gives a lower bound on S/N of about 28. A full calculation [59] gives

$$(S/N)_{\max}=46 \tag{4.2}$$

for two 1.4 $M_\odot$ neutron stars at 100 Mpc, where 'max' means a maximum over all relative orientations of the antenna and source.

Put another way, such a LIGO could register a coalescing neutron-star binary at the 1σ noise level at a maximum distance of 4.6 Gpc. But this gives an unrealistic range for the detector, because a 1σ event will be masked by noise. We must next ask what the threshold of detectability is: how high must the threshold be set so that detector noise generates only one false alarm per year, say? For a single detector, assuming Gaussian noise, a detector time resolution of 10^{-3} s (i.e. assuming 3×10^{10} independent measurements per year), and allowing for the fact that the data will have to be filtered for signals from systems with a variety of possible stellar masses (we cannot be sure that every neutron star will have a mass of 1.4 $M_\odot$), we find [60] that the threshold should be set at about $7.4\,\sigma$. So a single detector could identify the 'standard' coalescence event as far away as 620 Mpc.

Of course, events seen in only one detector will not necessarily be believed, because there is always a background of rare, unmodelled noise that could be responsible for isolated events. Coincidence observations between two or more detectors are necessary. Two detectors would suffice for the identification of an event, and because their noise backgrounds are uncorrelated, the threshold can be reduced to 5.4σ. But two detectors cannot give us enough information to determine the intrinsic amplitude, polarisation and direction of an incoming wave: three detectors is the minimum number required for a complete reconstruction of the wave [26, 37], and four are desirable. Since five detectors worldwide seems a practical possibility, I shall calculate here their observing range and consider in the next section what astrophysics such a network can do.

For five detectors, the threshold for one five-way false alarm per year in observations of coalescing binaries can be set at $3.5\,\sigma$. The maximum range, if the detectors are operating at what one might call their realistically achievable design goals, as assumed for (4.1), is therefore 1.3 Gpc. (This corresponds to a cosmological redshift in the range 0.2 to 0.4, depending on the Hubble constant.) But most events in this volume will not be optimally oriented, and so will fall below the threshold. From detailed studies of the 'antenna pattern' of realistic networks one finds that about 7% of the events in this volume will be detected. Moreover, 50% of all the events nearer than about 0.4 of the maximum range will be detected by all detectors. For our case, this '50%' distance is about 500 Mpc.

We can now estimate the rate of detected events. If the rate of coalescences out to 100 Mpc is 10^{-2} to 10^2 per year, then the rate of detection of events from within 500 Mpc is between 0.6 and 6×10^3 per year. The total rate for detected events out to the maximum range is between 1.5 and 1.5×10^4 per year. Provided that the instruments reach their design goals, coalescing binaries seem a near certainty for detection.

Binaries containing black holes also have interesting detection rates. We estimated earlier that neutron-star/black-hole binaries would be seen at 14% of the two-neutron-star rate, while for two-black-hole systems the figure is 3%. These translate into detection rates between 0.2 and 2000 per year for one-black-hole systems and between 4.5×10^{-2} and 450 per year for two-black-hole binaries. Perhaps just as interesting, these systems can be seen much further away. For example, two thirds of the binary black-hole coalescences will occur between 4 and 9 Gpc, i.e. at redshifts of order 1.

At these distances, cosmological redshift effects become important. These effects make coalescing binaries *easier* to detect, because the cosmological expansion shifts down into our 100–200 Hz observing window the larger amplitude high-frequency radiation emitted by the system at a later stage in its coalescence. The effect [62] is to enhance the S/N by a factor of $(1+z)^{5/6}$ over that for the same system at the same *luminosity distance* [20] in a static Euclidean space. This may make black-hole systems observable at redshifts approaching 4, i.e. as far away as the most distant quasars.

4.3. Doing astrophysics with coalescing binary observations

If a network is able to determine the direction, polarisation and intrinsic amplitude h of a gravitational wave event, then a remarkable result follows: it is possible to deduce the distance to the system, without knowing the masses of the stars! *Coalescing binaries are one of astronomy's 'standard candles'.* Here is how this is done. The amplitude h of the wave is given by [7-9]

$$h = (\text{orientation-dependent factors})\mu M^{2/3} f^{2/3} r^{-1} \tag{4.3}$$

where μ is the reduced mass of the binary, M its total mass and r its distance. The time scale for the frequency to change is

$$\tau = f/\dot{f} = k\mu M^{2/3} f^{11/3} \tag{4.4}$$

where k is a constant. Since τ can be measured from observation of the wave train, the product $h\tau$ is measurable:

$$h\tau = k' f^{-2} r^{-1} \tag{4.5}$$

where k is another constant that depends on the binary's angular position and orientation. The masses μ and M have dropped out! A three-detector measurement of the wave [26] determines its position and its polarisation and this fixes the binary's orientation. Since f is known, (4.5) may be solved for r. (In the cosmological context, this distance is the luminosity distance.) Notice that only the 'mass parameter' $\mu M^{2/3}$ can be determined from these observations, not the individual masses of the stars.

This distance measurement depends on the accuracy of our model for the binary system: we have represented it as two point masses, in a circular orbit determined by Newtonian gravity, which is losing energy at a rate given by the quadrupole formula for gravitational waves. One can imagine many possible corrections: eccentricity, tidal effects, magnetic interactions, post-Newtonian corrections. Krolak [59] has examined all of these and shown that only the post-Newtonian effects may be significant in the anticipated observations. If an event has S/N greater than 20 to 30, it may exhibit measurable post-Newtonian effects. Since these depend on the stellar masses in a different combination from the Newtonian ones, it will be possible in such cases to measure the individual masses of the stars.

Observations of coalescing binaries can feed information back into astrophysics in a variety of ways.

1. Measuring the Hubble constant. If a coalescence event is accompanied by an optical outburst that allows the galaxy containing the event to be identified, then the combination of an optical redshift measurement and the distance measurement from the gravitational wave observation (4.5) will determine the Hubble constant with unprecedented accuracy. However, even if optical identifications of coalescence events prove impossible, a statistical method based on galaxy clustering will give the Hubble constant to better than 10% accuracy after about 10 events from within 100 Mpc have been observed [63]†. Another method will be described in 4 below.

† It is shown in [26] that the determination of angular positions by a network of detectors will be much more accurate than was assumed in [63], thereby improving the speed with which H_0 can be measured.

2. Neutron star physics. At some point, as the stars in the binary get closer and closer, mass transfer between them will begin and the waveform will change drastically from our simple model. What happens from then on depends on the masses of the stars, and the nuclear matter equation of state [6]. Having identified a coalescing binary event, it may be possible to follow the subsequent evolution by tracking the gravitational waves. There may be a period of orbital stabilisation, during which mass is stripped from one of the stars, until it reaches the minimum mass of a neutron star, becomes unbound and explodes [34]. Alternatively, mass transfer may make the orbit evolve more quickly towards coalescence. After mass transfer one star may have too much mass, and subsequently collapse to a black hole. Gravitational wave observations, coupled with numerical simulations of binary coalescence, may provide strong constraints on neutron star physics.

3. Mass limits on neutron stars. Every observation gives a value of the mass parameter $\mu M^{2/3}$. After hundreds of events are observed, any upper bound on neutron star masses should show clearly in the distribution of this parameter. Post-Newtonian measurements on a few strong sources will provide individual masses all by themselves. Black-hole masses will be similarly constrained should binaries containing black holes be observed.

4. Cosmological distribution of stars out to ~ 1 Gpc. Observations give the distance and angular position of the binaries. Any large-scale clustering or holes in the distribution of stars should be apparent in the neutron-star coalescence events. It will be interesting to see whether this distribution follows the observed distribution of bright galaxies. If it does, then by matching the redshift of significant features in the galaxy distribution with the gravitational wave distance to corresponding features of the coalescing binary distribution, one can make an independent determination of the Hubble constant. Binary black holes, if they have a reasonable event rate, provide similar cosmological information, but over a much larger volume. It is possible that a maximum distance to black-hole binaries will become evident: this would be a clear indication that we have observed directly the onset of star formation.

5. Tests of general relativity. Any gravitational wave observations will provide tests of general relativity's predictions that waves propagate at the speed of light and have two transverse polarisations [9]. Observations of binary black-hole coalescences will provide another stringent test. Such coalescences will have been simulated numerically with good accuracy by the time detectors are operating. Since there are few parameters involved, observations will test relativity in the sort of strong-field, non-linear regime which is hard to access any other way.

If future technological developments allow the LIGO detectors' sensitivity to be improved by a further factor of 4 or 5, then even more interesting observations become possible [62]. Two such are 6 and 7 below.

6. Measurement of the deceleration parameter. If observations of distant ($z > 1$) coalescences are possible, their distribution of the mass parameter $\mu M^{2/3}$ should be the same as the 'local' one, except for redshift effects: the measured mass of a star in the binary is its true mass times $1+z$. by comparing these distributions for various values of the binary's distance (luminosity distance in this context), one might be able to see systematic effects due to q_0.

7. Gravitational lensing. Coalescing binaries at quasar distances should be lensed as frequently as QSO. The signature would be the observation of two identical waveforms from nearby points on the sky, but at different times. (Lensing introduces time differences of months or years for propagation on different paths.) Because the signals

are polarised and the time delay will automatically be measured, these observations provide different information about the lenses than optical observations typically do. Note, as well, that because of the 24 h, all-sky coverage afforded by gravitational wave detectors, lensing surveys will be more complete than has been possible up to now using optical means.

5. Conclusion

The supernova explosion in the Large Magellanic Cloud has highlighted the importance of pursuing the development of bar and interferometric detectors. With bar detectors likely soon to be linked into a worldwide observing network [64], and with the prospect that LIGO will be operating at very interesting sensitivity levels within ten years, we stand on the threshold of gravitational wave astronomy. General relativity, whose theoretical development in the last two decades has been nourished by many astronomical discoveries (neutron stars, black holes in x-ray binaries, massive black holes in galactic nuclei, the binary pulsar, and so on), will soon be responsible for astronomical observations of its own, observations that will further enrich our understanding of the Universe. Some of these observations—and their astronomical return—are more or less predictable, and we have reviewed them here. But surely the most exciting prospect of all is the expectation that gravitational waves will reveal completely unanticipated phenomena. This will be the real payoff for the enormous efforts now being made by the world's gravitational wave detector groups.

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