

On secular effects in the binary pulsar

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Summary. We show that a recent suggestion by Lapiedra, Portilla & Sanz, that casts doubt on present interpretation of the binary pulsar observations, is incorrect. In particular, their suggestion that, if the binary system is moving relative to us then there will be a long-period modulation of its observed period, violates special relativity near the observer. In order to explain how the error in their conclusion arises, we give a detailed discussion of the correspondence between observable quantities and various Newtonian and post-Newtonian constructs.

1 Introduction

In a recent paper, Lapiedra *et al.* (1983, hereafter referred to as LPS) examine certain post-post-Newtonian (p^2N) terms in the equation of motion for a nearly Newtonian binary star, and suggest that if the total momentum of the binary is non-zero, then certain periodic terms of very long period 'are observationally significant, i.e. in practice they will appear as if they were secular effects'. If true, this would mean that the observed secular decrease in the orbital period P of the binary pulsar system (Taylor & Weisberg 1982; Boriakoff *et al.* 1982) might in fact be a mixture of these p^2N effects with the effects of gravitational radiation reaction, the proportion of this mixture being dependent upon the binary's (unknown) velocity relative to us. Since the observed dP/dt agrees well with the predictions of Einstein's equations regarding gravitational radiation reaction (Futamase 1983), this assertion by LPS would be significant. The purpose of this note is to point out that this assertion is untrue. Quite independently of the details of the post-Newtonian calculation, it is easy to show that an effect which is absent in the centre-of-momentum frame (as LPS assert) and yet causes real confusion in other frames violates special relativity in the neighbourhood of the observer, where the complicated effects of general relativity may be ignored.

2 Observations

The LPS term is supposed to cause periodic changes in P on time-scales long compared to the present span of observations (about 10 years). If we wait long enough to average out the

long-period terms, LPS expect us to see only the radiation-reaction terms. But on the present short time-scale it would be impossible to separate one effect from the other observationally.

Consider observations made in the centre-of-momentum frame, where LPS claim their new effect vanishes. The observer is far away and simply records a series of pulses from the pulsar. By fitting the changes in the intervals between pulses to a model of a nearly-Newtonian binary system, the observer determines all the published parameters of the system. If this observer concludes that the decrease of the period is as predicted by general relativity, then presumably LPS would agree with this conclusion. Now, the transformation to another observer for whom the system has non-vanishing momentum is accomplished simply by a Lorentz transformation in the neighbourhood of the original observer. The only difference in the second observer's data is that the intervals between pulses now have an overall constant doppler shift. There can be no extra long-period modulation of the pulse intervals of the type suggested by LPS.

3 Discussion

Where, then, do LPS go astray? While the above argument, based on special relativity alone, is easy to understand, the following discussion will inevitably be more technical. The main problem with the LPS argument is that they do not calculate the rate of change of the actual period which the observers measure. Instead, they calculate the rate of change of a fictitious Newtonian period P_N , which differs from the true period by terms of the order of LPS's 'new' effect. This is a rather subtle point, which has caused confusion a number of times in the history of radiation-reaction calculations, so I will discuss it in some detail. The point of view which I shall take was to my knowledge first proposed by Futamase (1983) and then elaborated by Schutz (1984).

Let us suppose for the moment that the post-Newtonian equations up to p^2N order are an exact description of the binary's motion, so we neglect radiation reaction. These equations are conservative, and the radial motion of their reduced two-body problem is periodic with period P . This period is a function of the initial data set $I = \{x_1, x_2, v_1, v_2\}$ of the two stars. It is nearly equal to the Newtonian period $P_N(I)$, which is the period the binary would have if the same initial data I were evolved according to Newton's equations. One may therefore speak of post- and post-post-Newtonian corrections to the period $P_1(I)$ and $P_2(I)$, such that:

$$P(I) = P_N(I) + P_1(I) + P_2(I). \quad (1)$$

This splitting up of P into different pieces is, however, no more than a theoretician's convenience. An observer who measures the period from variations in the intervals between different pulses measures P itself, not its various pieces. When using the full equations of motion through p^2N order, P is exactly constant, whereas P_N , P_1 , and P_2 are not: only their sum is constant. For example, the p^2N corrections to the equations of motion cause $P_N(I)$ to change as the data I evolve, but these changes are compensated by changes in $P_2(I)$. Such changes have no observable consequences, since the intervals between pulses arriving at the observer are regulated only by P .

Confusion can arise when we realise that the process of fitting the data to a model of the binary in order to measure P actually uses a Newtonian model with some post-Newtonian corrections. This is purely a question of the accuracy of the observations, and it would be incorrect to suppose that the period quoted by observers is the functional $P_N(I)$ in equation (1). The data contain only P : P_N is not observable. But the observations have limited accuracy, partly because of instrumental effects but mainly because of the inherent irregularity of the

pulses themselves. For data spanning a few months, observational errors are large enough to obscure the difference between P and P_N at that time. But, again supposing the p^2N equations to be exact, at a later time a similar observation will now measure the same value for P , and certainly not the value that the functional P_N would now have if it had been evolved by the p^2N equations from the original observation.

What LPS calculate is the second effect, the rate of change of $P_N(I)$ as I evolves according to the p^2N equations of motion. If there is a long-period variation in $P_N(I)$ caused by p^2N terms, this will necessarily be compensated by variations in $P_2(I)$, which LPS do not calculate. (Actually, LPS calculate only changes in the energy, not the period. I have phrased this argument in terms of the period because that is the observable, but similar considerations apply to the energy. In particular, our final conclusion will be the same because the Newtonian period functional P_N is a function only of the Newtonian energy functional E_N).

Now we can relax the assumption that the p^2N equations are exact. The next order beyond p^2N (called $p^{2.5}N$) involves the radiation-reaction terms. Taking the full equations through $p^{2.5}N$, it is easy to see that as the data I evolve, the changes in P come only from changes in I produced by $p^{2.5}N$ terms, since all lower-order terms leave P constant. The lowest-order change in P is therefore the same as the change in P_N caused by only the $p^{2.5}N$ terms, and corresponding changes in P_1 and P_2 will be of higher order. The conclusion we reach is that the full change in the observable P through $p^{2.5}N$ order is correctly calculated as the change in P_N using *only* the $p^{2.5}N$ corrections to the equations of motion, but it would be incorrect to include also the changes in P_N produced by p^2N terms. It is these terms which give rise to the LPS long-period effect.

There is therefore no reason to think that the current interpretation of the binary pulsar observations is incorrect.

References

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