LOGIC, LANGUAGE AND THOUGHT

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1. Introduction

The point of this presentation is to undermine a few generally accepted ideas about logic by showing that modern standard logic, in the form in which it has crystallised since roughly 1900, is far from the one and only possible immutable logic it has been taken to be. On the contrary, a simple inspection of the foundations of logic quickly reveals ways of producing a mass of logically sound alternatives that deviate in often interesting ways from standard logic. We will demonstrate this for predicate calculus. Propositional calculus will be left alone, even though there, too, one sees interesting and fruitful ways of varying upon the standard theme.[1]

The interest of this new approach lies in the fact that modern standard logic may be of great use to mathematics but corresponds rather badly with intuitive judgements of truth and falsity produced by ordinary speakers. By contrast, traditional predicate calculus, which we shall call 'Aristotelian' even though it came to us in a form streamlined by the philosopher Boethius, scores significantly better in this respect, but it has the serious drawback of suffering from one fatal logical defect and a few

[1]See, for example, Seuren *et al.* (2001) for a three-valued propositional calculus serving the purposes of presupposition theory and discourse-dependency.

1º Encontro Nacional de Filosofia Analítica, coord. de H. Jales Ribeiro (Coimbra: Faculdade de Letras, 2003), pp.259 - 276.

minor ones. For that reason it was unceremoniously dumped around the year 1900 and replaced with modern standard predicate calculus.

This placed philosophers of language and semanticists of natural language in an uncomfortable dilemma. In the beginning they were browbeaten by logicians like Bertrand Russell, who called natural language ambiguous, vague and generally unreliable for logical or scientific purposes. Soon, however, it became clear that natural language is of significant interest to logic and deserves further study from this point of view. Yet even though after 1970, under the influence of the American logician Richard Montague, research to that effect became fashionable, and to some extent also fruitful, in the context of so-called *formal* or *model-theoretic semantics*, the discrepancies with natural intuitions remained. And as this fact was increasingly recognised, a new discipline, called *pragmatics,* appeared some thirty or so years ago, where attempts are made to bridge the gap between formal logical analysis and natural intuitions by an appeal to general principles of rational communicative behaviour of people engaging in linguistic interaction.

These attempts, however, have met with less than complete success. Pragmatics keeps being plagued by a lack of precision, which makes it hard to falsify, and thus scientifically weak. One of the things pragmaticists try to achieve is to restore tradi-tional Aristotelian predicate calculus, not, however, as a sound system of logic but as a communicatively functional manifestation of modern standard logic. One may have different views about that. But when it turns out, as it now does, to be possible to produce a gamut of logically sound alternative logics, including varieties that incorporate the logical properties of Aristotelian predicate calculus, then clearly an entirely new perspective arises with regard to the logical properties of natural language. It is shown in this paper how Aristotelian predicate calculus, or, if that is preferred, a close relative of it, can be restored to its rightful place as the logic of language and thus, in important respects, also as the logic of thinking, no longer as a pragmatic ersatz but as a well-defined logico-semantic system that needs no outside support. Not only will the uncomfortable dilemma that semanticists and philosophers of language had to face thus be removed, we also get a number of interesting new insights into the nature and the foundations of logic into the bargain.

2. Some basic notions

Let us first review some indispensable basic notions. First, what is logic? The least controversial modern answer to this question is to say that *logic is the formal calculus of semantic entailments.* But what is a formal calculus, and what is a semantic entailment? Let us consider semantic entailments first. When we say that a sentence *A semantically entails* a sentence *B*, we mean that in all cases where *A* is true, *B* must of necessity also be true in virtue of the meanings of *A* and *B.* Thus, whenever it is true to say *John has been murdered*, it is also true to say that *John is dead*, since it is in the meaning of *have been murdered* that whoever has been murdered is dead. In fact, if, as speakers of English, we know the meanings of English sentences and words, we also know what is semantically entailed by the sentences of English. Normally, speakers of a language make themselves aware of the semantic entailments of the sentences they use by way of mere *intuition* or *introspection*: one consults one's own inner awareness of what has been said and decides that such and such a sentence follows or does not follow. This is, for example, what one will do in the case of *John has been murdered* entailing *John is dead.*

Aristotle, however, discovered the principle of logic. He found that the meanings of some words can be described in such a way that they allow for a formal calculus that derives entailments, that is, by the application of strictly formal rules based on the semantic description of the words in question. This is the case, for example, with the words *all*, *some* (that is, *at least one*) and *not*, which together define what is now known as *predicate calculus.* Semantic entailments that can be derived by way of calculus, i.e. not intuitively but formally, are called *logical entailments*, as distinct from merely semantic entailments. For *A logically entails B* we write: $A \vdash B$.

The words *all*, *some* and *not* are the *logical constants* of the language of predicate calculus. By this we mean the following. Every logical calculus must be expressed in a language, which may be a regimented form of a natural language or a formal language designed strictly for the purpose at hand. In any such language, a few *logical constants* are singled out, which enable one to *compute* entailments instead of deriving them intuitively. In the simplest form of predicate calculus the logical con-stants are *all*, *some* and *not.* The remaining elements that may occur in the sentences at issue are represented as variables (usually *F*, *G*, etc.), which take lexical predicates as values. (In propositional calculus, the standard logical constants are, again, *not*, and also *and*, *or* and *if-then*, and the variables stand for propositions. In modal logic, the modal predicates are added as logical constants.)

Aristotle, or better the Aristotelian tradition, distinguished four sentence types:

type **A** : All F is G
type **I** : Some F is G
type **E** : All F is not-G (or: No F is G)
type **O** : Some F is not-G (or: Not all F is G)

The names **A**, **I**, **E** and **O** were introduced by the early Christian philosopher Boethius (± 500 AD), who named the types **A** and **I** after the first two vowels of the Latin word *affirmo* ('I affirm'), and the types **E** and **O** after the vowels of the Latin word *nego* ('I deny'). Given Boethius's influence on medieval logic, his notation became standardly accepted during the Middle Ages and later.

Boethius cast the system of Aristotelian Predicate Calculus (APC) into the simple and perspi-cuous shape of the famous *Square of Oppositions*, which defines the logical relations of the four sentence types with respect to each other, as shown in fig. 1.

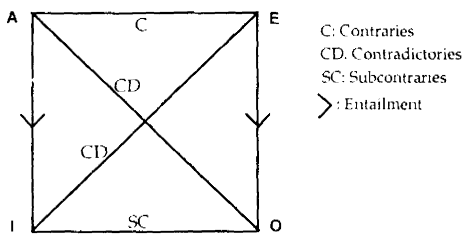

Figure 1 APC as summarised by Boethius in the *Square of Oppositions*

We see that (type) **A** entails (type) **I** (**A** |– **I**), and **E** entails **O** (**E** |– **O**). These are the *sub-altern entailments*. Moreover, **A** and **E** are

Contraries: they cannot both be true at the same time, but they can be false at the same time. Therefore, **A** |– **not-E** (and **E** |– **not-A**). Then there are the *Contradictories*, which cannot be either simultaneously true or simultaneously false: **I** and **E**, and **A** and **O** are pairs of Contradictories: **I** |– **not-E** and **not-I** |– **E**; **A** |– **not-O** and **not-A** |– **O**. Contradictories may be defined in terms of *Conversions* or *Equivalences*. Equivalence is logical entailment in both directions. **I** and **not-E** are equivalent, or formally: **I** ≡ **not-E** (and thus: **not-I** ≡ **E**), and **A** ≡ **not-O** (and thus: **not-A** ≡ **O**). Finally, there are the *Subcontraries* **I** and **O**. These can be true at the same time, but not false: **not-I** |– **O** (and thus **not-O** |– **I**).

Two successive negations cancel each other out: *not-not-A* is logically equivalent with *A* for any sentence *A*, and likewise *not-not-P* is logically equivalent with *P* for any predicate *P*. Furthermore, the sentence types **I** and **O** are considered to have *existential import.* That is, they license the conclusion that, whenever they are true, there exists at least one member of the *F*-class: there is at least one *F*.

3. The Boethian representation of APC improved

In a moment we will start shooting at APC. But before we do so, we must first revise the Boethian representation of APC, with its simple use of the vowels of *affirmo* and *nego* for the sentence types. One problem with the Boethian Square is that the role of (internal or external) negation is not spelled out explicitly, even though nega-tion plays a systematic structural role in the entailment schemata. Let us use the signs '¬' for external negation and '*' for internal (clausal predicate) nega-tion. This reduc-es the number of quantifiers to just *all* and *some*, as in standard modern predicate cal-culus. The corners of the Square are now named **A**, **I**, **A*** (or ¬**I**), and **I*** (or ¬**A**).

Moreover, the Conversions are not expressed explicitly in the Boethian Square. If we do express them explicitly, we get a somewhat different configuration, which we will call the *Improved Square.* It is shown in fig. 2, where the pairs of heavy horizontal lines stand for logical equivalence. It must be noted that the actual *logic* has not changed: we are still dealing with APC. Only the representation has been improved.

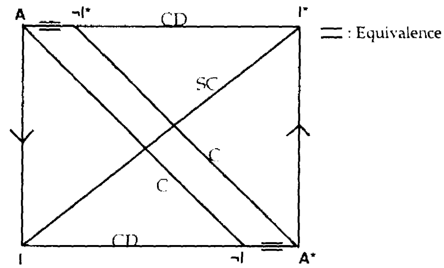

Figure 2 The Improved Square as representation of APC

What we have here is, in fact, a combination of two logically analogous triangles, each with the corners **A**, **I** and ¬**I**, that is, sentence types characterized by the quantifiers *all*, *some* and *no*. The only difference between the two triangles is that the one on the left works with a positive predicate, i.e. *G*, whereas the one on the right works with the negative counterpart, i.e. *not-G*. But since the specific choice of predicate has no influence on the logical properties of the calculus, this difference must be logically irrelevant, which it turns out to be: the two triangles display exactly the same logical entailment schemata. Moreover, the triangle on the right hand side stands, so to speak, upside down with respect to the triangle on the left hand side, which enables us to express the equivalences explicitly: **A** ≡ ¬**I*** and ¬**I** ≡ **A***.[2]

4. Critique of APC

Now we can start shooting at APC proper. Around 1900 it became clear that APC collapses in cases where the *F*-class is empty. Consider the sentence:

[2]The Improved Square also solves a (quasi-)problem raised by Horn (1972, 1989:252-67) and Levinson (2000:69-71), both practitioners of pragmatics. These authors wonder why, in the Boethian Square, the **A**-,**I**- and **E**-corners are lexicalized, in virtually all languages of the world, as one single word (in English, for example, as *all*, *some* and *no*), whereas the **O**-corner systematically requires more than one word as its lexical expression. They seek the answer in pragmatic principles. Although their observation appears to be correct, the Improved Square shows that the problem is imaginary: it is merely an artifact of the deficient way Boethius formalized APC. The Boethian **O**-corner has lost its place to the three quantifier expressions *all*, *some* and *no*.

(1) All gnomes live in Norway.

This is an impeccable sentence, with the predicate *gnome* in the *F*-position, and the predicate *live in Norway* in the *G*-position. Both predicates are good Eng-lish, but it so happens that the former is uninstantiated: there are no gnomes in this world, the predicate *gnome* is empty. The question is now: is sentence (1) true or false in this world? According to APC, but also according to modern standard logic, it has to be one of the two. A third possibility is not available ('tertium non datur').

Now suppose we take the sentence to be true. Then it follows, by the subaltern entailment schema, that some gnomes live in Norway (type **I**). But that again entails, by existential import, that there are gnomes, which is not the case. Therefore, senten-ce (5) cannot be true. But it cannot be false either. For if it were, it would follow, by the equivalence $\neg\mathbf{A} \equiv \mathbf{I}^*$, that some gnomes do not live in Norway (type **I***), which again has existential import. Therefore, sentence (1) can be neither true nor false, which goes against the very charter of both traditional and modern logic. In fact, the whole of APC turns out to have existential import, as it requires the non-emptiness of the *F*-class. Only non-empty predicates are allowed in the *F*-position.

This is fatal for any logic. For if a logic is, as we have agreed it is, a method for the computation of entailments merely on grounds of the semantic definition of the logical constants, then it must be irrelevant whether or not there exist, right now, any gnomes or any one hundred year old Scots, or any black swans.

5. Modern predicate calculus (MPC)

Modern predicate calculus has tackled and solved this problem in a radical way. The makers of modern logic simply applied classical set theory, according to which the null set, usually written '$\emptyset$', is a subset of any set: for all sets X, $\emptyset \subseteq X$. A sentence of the **A**-type is accordingly said to be true in precisely those cases where the *F*-class is a subset of the *G*-class, whether the former is empty or non-empty. A sentence of type **I** is said to be true in precisely those cases where the *F*-class and the *G*-class intersect non-emptily: there is at least one F and at least one G.[3]

As a result of this, however, virtually the whole of the Square, whether Boethian or Improved, has to go. The Subalterns are no longer valid. Nor are the Contraries, since **A** and **A*** both count as true when the *F*-class is empty. The same fate befalls the Subcontraries, since both **I**-type and **I***-type sentences count as false when the *F*-class is empty. Only the Conversions remain intact. But that is hardly sufficient to speak of a Square of logical relations. In other words, APC has had to give way to a new predicate calculus which, compared to APC, is badly impoverished. Yet that new calculus is our MPC, which was introduced around 1900 and has since been looked up to with the greatest reverence and respect. This respect is not unjustified, as MPC is eminently suitable for the formulation of mathematical statements and proofs, which makes MPC an optimal tool in all applications of mathematics in the sciences and in technology. But for natural language MPC is an unmitigated disaster.

Consider, for example, the sentences (2a, b). In the MPC book they should both be true in this world, where no real gnomes exist:

(2) a. Some visitors talked with all gnomes.
    b. Some visitors talked with no gnome.

In fact, MPC makes both sentences equivalent, given the absence of gnomes, to the statement that there was at least one visitor. Yet ordinary people will consider (2a) false. And (2b) may have to count as true, but only in an insipid way. Pragmatic principles make both sentences equally inappropriate in the given context, but fail to explain why the one is felt to be false while the other is taken to be true. More examples of this nature are easily thought up. In practically all cases the conclusion is that APC fits natural intuitions much better than MPC, even though the latter appears to reign supreme in the world of logic.

[3]While discussing MPC we disregard the formal *language* of MPC, which derives from Peano via Russell and Whiteheadís *Principia Mathematica*. Since the language of MPC allows for the possibility of recursive embedding of quantified propositional functions, MPC's logical power is vastly superior to that of APC in its language of the Improved Square as shown in fig. 2. How-ever, APC is easily reformulated in the language of MPC, and vice versa, since in the formal language of MPC the recursive embeddings are themselves always formulated in terms of the Boethian sentence types. For simplicity's sake we restrict ourselves here to representations in the language of the Improved Square. The analysis is in no way affected by this restriction.

6. A new point of view

But we will not give up. After all, since we have decided that logic, any logic, is a formal system for the computation of entailments on grounds of the meanings of the logical constants involved, it should be possible to vary the semantic descriptions of the logical constants concerned while keeping the logic sound. And if we do that, we may as well keep the presumed logical constants of natural language in mind. We may then think up formal logical languages intended to reflect, by way of hypothesis, what is going on in language, because that is what we are finally interested in.

The strategy we will follow is based on the notion of *valuation space* introduced in Van Fraassen (1971). The valuation space */A/* of a sentence *A* is the set of possible situations in which *A* is true. *A* divides the universe **U** of all possible situations up into two subsets, those in which *A* is true, or */A/*, and those in which *A* is false, or */¬A/*, where */¬A/* is the complement of */A/* in **U**.

A valuation space model is the optimal graphic representation of APC, MPC and all possible variants. Fig. 3 shows a valuation space model for APC.

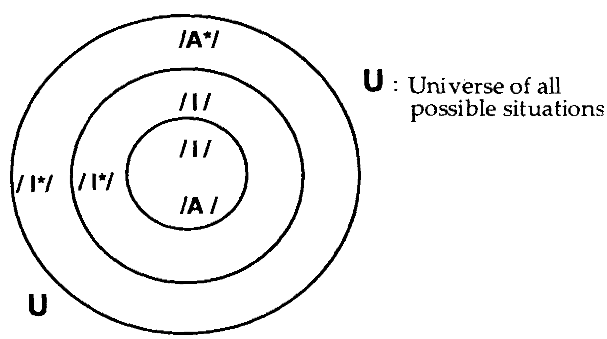

Figure 3 Valuation space model for APC

In fig. 3 the innermost circle is the space for all situations in which the types **A** and **I** are true. The middle ring contains exactly those situations where the types **I** and **I*** are true, and the outer ring those where the types **A*** and **I*** are true.

Such diagrams enable one to read entailment schemata directly from them, since for each pair of sentences *X* and *Y* it is the case that:

X |– Y iff	/X/ is a subset of /Y/
CD(X,Y) iff	/X/ is the complement of /Y/
C(X,Y) iff	/X/ and /Y/ have no element in common
SC(X,Y) iff	/X/ and /Y/ together form **U**
≡ (X,Y) iff	/X/ and /Y/ are identical.

The entire logical system of APC can now be read from fig. 3. The Subalterns are valid because /**A**/ is a subset of /**I**/ and /**A***/ is a subset of /**I***/. The Contradictories are valid because /**I**/ is the complement of /**A***/ and /**A**/ of /**I***/. This means that the Equivalences (Conversions) are valid: /**A**/ = /¬**I***/ and /**I**/ = /¬**A***/. The Contraries are valid because /**A**/ and /**A***/ have no element in common. The Subcontraries, finally, are valid because /**I**/ and /**I***/ together form **U**.

The problem is, however, that the space **U** of fig. 3 does not contain all possible situations with respect to the sentence types involved, because the situations where the *F*-class is uninstantiated are not represented. But with a valuation space model this is quickly remedied: we simply add a further ring to fig. 3 for those situations where the *F*-class is empty. The result is fig. 4, which shows a valuation space modelling for what we call the *revised Aristotelian predicate calculus* or RAPC.

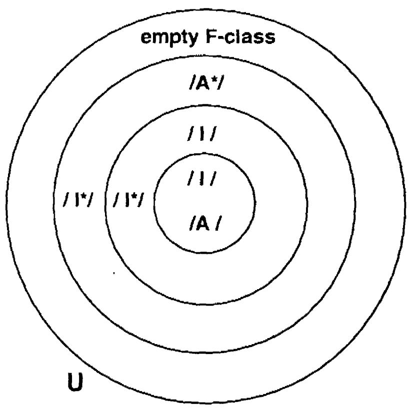

Figure 4 Valuation space model for RAPC

Now **U** contains all possible situations with respect to the four sentence types and their negations. But the addition of the outer ring in fig. 4 has changed the entailment schemata. The Subalterns have remained intact, and also the Contraries, whose number has even increased. But the Subcontraries have disappeared, and so have the Equivalences, which have been replaced with one-way entailments: **A** $\vdash$ ¬**I*** en **A*** $\vdash$ ¬**I**, but not vice versa. Therefore, the Improved Square of fig. 2 must give way to fig. 5, the *Hexagon of Logical Relations* in RAPC.

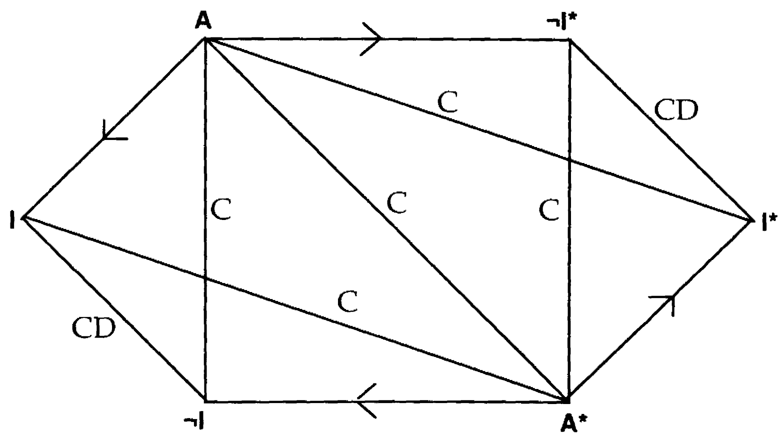

Figure 5 The Hexagon of Logical Relations in RAPC

What fig. 4 does but fig. 5 does not show is the fact that in RAPC, when the *F*-class is empty, all four sentence types **A**, **I**, **A*** and **I*** count as false. The difference with MPC is that MPC counts the types **A** and **A*** as true in such cases. This again means that the entailment schemata of MPC differ significantly from those in RAPC, as one can see from fig. 6. Perhaps surprisingly, the only difference between the valuation space models for MPC and for RAPC consists in the fact that, in MPC, **A** and **A*** count as true with an empty *F*-class, whereas in RAPC they count as false.

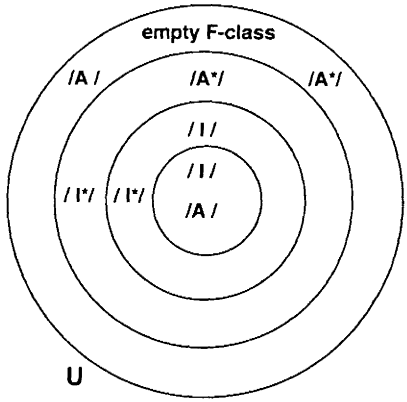

Figure 6 Valuation space model for MPC

What is striking about RAPC is its Aristotelian character. RAPC is also much richer than MPC, which has kept only the Equivalences (Conversions). RAPC, by contrast, has kept the Subalterns and has increased the number of Contraries, while it has eliminated the Subcontraries and replaced the Equivalences with one-way entailments. These changes are likely to be productive for a logico-semantic description of natural language. The loss of the Subcontraries probably is an advantage, since a sentence like (3), which instantiates the simultaneous falsity of **I** and **I***, appears appropriate in a situation without any gnomes:

(3) I didn't see any gnome there, nor was there any gnome I did not see.

In this respect both MPC and RAPC seem to be superior to APC. But for the Equi-valences the situation is different. It does not seem to be absurd to regard a sentence like (4a) as true while taking (4b) to be false, although MPC deems them both true:

(4) a. There was no gnome present.
b. All gnomes were not present (absent).

It seems reasonable, moreover, to suppose that for natural intuition (4b) entails (4a), but not vice versa, which would favour RAPC over both APC and MPC. On the other hand, it looks as if sentences like (5a) and (5b), which are likewise both considered true in MPC, count as false for natural intuition:

(5) a. No gnome said anything.
  b. All gnomes said nothing.

One also feels, however. that (5a) entails (5b) and (5b) entails (5a), which makes the two equivalent. If that is correct, MPC (or APC if that were sound) should be favoured over RAPC.

If experimental evidence were to corroborate these provisional intuitions, we would have several systems co-existing side by side. Perhaps we should think of two distinct existential quantifiers in natural language, with partly different realisations in surface structure. Negation of the one would then yield truth in case of an empty *F*-class, whereas negation of the other would yield falsity. In view of what is proposed in the following section this may well be an interesting thought.

7. More possibilities

But we can go further than this. Since Strawson (1950) it has been widely accepted that the normal, unmarked negation in natural language negates only part of the semantic content of the sentence while leaving the remainder unaffected. The unaffected remainder is what we call the *presuppositions* of the sentence; the negated part is the *assertive content.* A sentence like (6a), for example, *presupposes* that John wants to go by bike and *asserts* that John is the only one who wants that. But (6b), though the negation of (6a), still presupposes that John wants to go by bike, although now the assertion that he is the only one has been denied:

(6) a. Only John wants to go by bike.
  b. Not only John wants to go by bike.

In other words, the negation word *not* denies only the *assertion* of (6a) but leaves the *presupposition* intact. If this is so, the standard logical negation does not represent normal, unmarked negation in natural language. While the standard logical negation corresponds with the valuation space diagram of fig. 7a, natural language negation is represented more adequately as in fig. 7b, with '~' for presupposition-preserving natural language *not*. The heavy circle in fig. 7b contains the situations where the presuppositions of a sentence *A* are true. The middle ring is the so-called *inner complement* of /*A*/, i.e. the set of situations where the presuppositions of *A* are true but not the assertive content of *A*. The outer ring is the *outer complement* of /*A*/, i.e. the set of situations where the presuppositions of *A* are false.

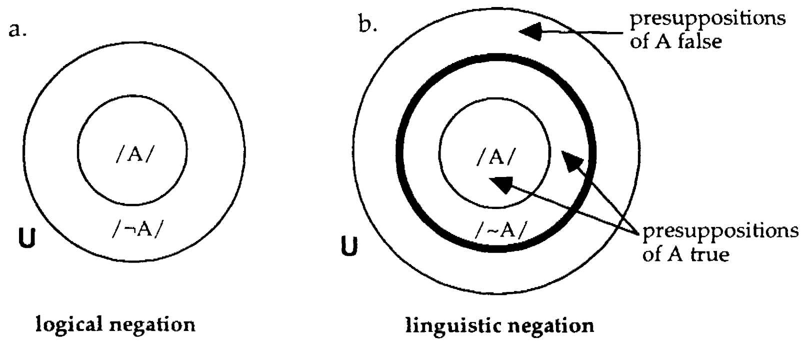

Figure 7 Valuation space diagrams for logical and linguistic negation

Suppose we accept this analysis (for extensive argumentation see Seuren et al. 2001). Then we can associate with the universal and the existential quantifier in natural language the presupposition that the *F*-class is non-empty. If we do that, the outer ring of the valuation space model for RAPC in fig. 7a becomes the outer complement of the entire logical calculus, while the inner complements of the various sentence types are restricted to the heavy circle, as shown in fig. 8.

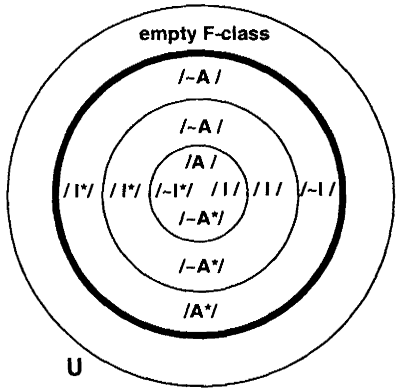

Figure 8 Valuation space model for (R)APC with presupposition-preserving negation and non-empty *F*-class presupposition for **A**- and **I**-type sentences

This restores the original calculus of APC except that the negation is no longer the standard bivalent negation but the new trivalent presupposition-preserving negation '~', as is shown in fig. 9. (One wonders whether Aristotle might not have been influenced unwittingly by his natural intuitions about natural language negation.)

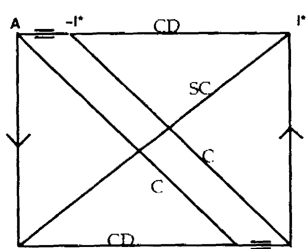

Figure 9 Improved Square with presupposition-preserving negation and non-empty *F*-class presupposition for **A**- and **I**-type sentences

We may even think up further variations on the same logical theme. We may, for example, restrict the presupposition of the non-empty *F*-class to the universal quantifier and leave the existential quantifier unaffected. This leads to a mixed system, whose valuation space model is given as fig. 10.

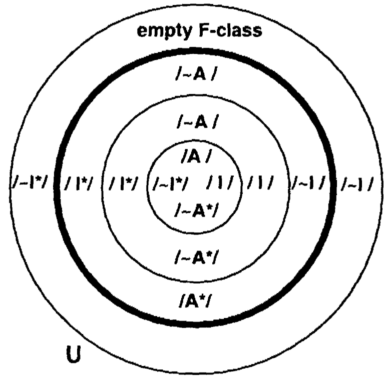

Figure 10 Valuation space model for RAPC with presupposition-preserving negation and non-empty *F*-class presupposition only for **A**-type sentences

Fig. 10 returns the Hexagon of Logical Relations of fig. 5, but now, as shown in fig. 11, with the presupposition-preserving negation '~' instead of '¬'.

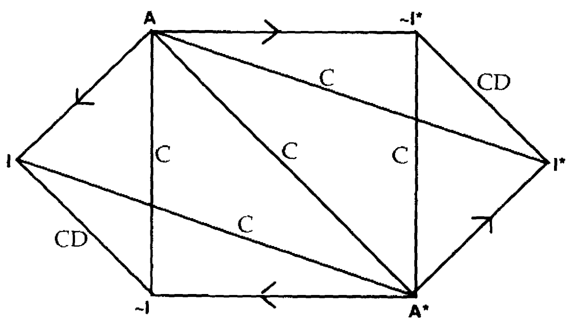

Figure 11 Hexagon with presupposition-preserving negation and non-empty *F*-class presupposition only for **A**-type sentences

Special care, however, must be taken, in this mixed system, with regard to entailment relations between **A**-type and **I**-type sentences. The definitions of entailment, contradictoriness, contrariety, subcontrariety and equivalence given above still stand, but they no longer simply translate into entailment schemata. Entailments from **~I**- or **~I***-type to **A**- or **A***-type sentences, with or without the negation ~, are, on the whole, no longer valid. To see which are and which are not valid, one only has to consult fig. 10, which allows one to read all entailment relations directly. Interestingly, though **I** and **A*** in fig. 11 are still Contraries, they are now also Contradictories within the inner complement for the **A**-type sentences.

The systems of fig. 8 and fig. 10 may be valid at the same time, if we accept two different existential quantifiers, one with and one without the non-empty *F*-class presupposition, as was suggested above, at the end of the previous section. In other words: options galore. But the question is: which of these options, or which com-bination of options, provides the most adequate account of the natural intuitions of native speakers in terms of a sound logic. The dilemma that beset philosophers of language and semanticists a century ago when modern logic was introduced may, after all, turn out less serious than has been thought.

8. Why?

In the given context one is naturally inclined to pose the question of the functionality of the specific variety or varieties of logic presumed to be present in language and thinking. Why do language and thinking deviate from the minimalist logic system that has been found adequate for mathematical purposes? This question, no matter how justified, cannot be answered at this stage. This is because the prior question as to the *actual nature* of the logic of language and thinking has not been answered yet.

The first large problem still awaiting a solution is posed by quantification involving intensional predicates and the related issue of quantification over virtual objects. This problem area is of considerable magnitude and a satisfactory account will no doubt necessitate drastic measures with regard to the existing machinery. Sentence (7), for example, shows that when an intensional predicate like *be worshipped* is used in the *G*-position, an **I**-type sentence can still be true even though the *F*-class is uninstantiated:

(7) A Greek god was worshipped there in the old days.

Such cases seem to warrant the conclusion that in predicate calculus as it occurs in language and thinking the existential quantifier does not have existential import (only ontological import), existential import being derived from the fact that extensional predicates require real existence for the reference objects of their argument terms. Such matters, however, are far from trivial.

It seems, moreover, that any investigation into questions of the functionality of the logical system or systems postulated for language and cognition had better be carried out in the context of certain hypotheses concerning the general nature of cognition. At this moment we have two specific hypotheses in mind. First we would consider the so-called *null set hypothesis*, which says that the concept 'null set' does not occur in natural human thinking. If this hypothesis is correct, the logic of language must be represented without the help of the symbol Ø – a task at which we are working. The second hypothesis is the *proper subset hypothesis*, which says that in natural human thinking the notion 'subset' is always interpreted as 'proper subset', just as a subcontinent is necessarily a part of a larger area called 'continent', or a subcontract is part of a large contract. These, however, are perspectives for further research about which we cannot report at this moment.

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