Existential Import

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The present paper deals with the semantics of *all*, *some* and *not* in English and related languages. Since these semantics imply a logic, it also deals with the logic of quantification in natural language. To the extent that the linguistic findings are universally valid, the results are relevant to the study of human cognition in general.

Since standard quantification theory (SQT) replaced Aristotelian Predicate Calculus (APC), philosophers of language and semanticists have struggled with the fact that APC corresponds better with natural semantic intuitions than SQT. Pragmatics was called in to restore APC on non-logical, pragmatic grounds. The replacement of APC by SQT was motivated on the grounds that (a) APC suffers from improper existential import, and (b) SQT is a straightforward application of Boolean algebra, and as such preserves syllogistic reasoning on a mathematical basis.

Logically speaking, SQT differs from APC in that the Aristotelian subaltern entailments are abolished, which makes the Aristotelian Square collapse but saves the equivalences (conversions) between $\neg\forall\neg$ and $\exists$, and $\neg\exists\neg$ and $\forall$. In SQT, the quantifying predicate $\exists$ over pairs of sets $<X, Y>$ yields truth iff $X \cap Y \neq \emptyset$ and $\forall$ does so iff $X \subseteq Y$. The author recently found that if the condition $X \neq \emptyset$ is added to the condition for $\forall$ and the conversions are changed into one-way entailments (i.e. $\forall x(Fx,Gx) \vdash \neg\exists x(Fx,\neg Gx)$ and $\forall x(Fx,\neg Gx) \vdash \neg\exists x(Fx,Gx)$, but not vice versa), APC is restored without improper existential import (only the subcontraries are lost). The Boolean basis is unaffected since whenever $\forall$ yields truth, it is still so that $X \subseteq Y$. The resulting revised Aristotelian predicate calculus (RAPC), is represented in the Hexagon of fig. 1 (arrows stand for entailments, 'C' for contraries, 'CD' for contradictories). Note that the traditional (Boethian) letter types **A**,.**I**, **E** and **O** have been replaced with $\forall$, $\exists$, $\forall\neg$ and $\exists\neg$, respectively, since APC contains only the standard quantifiers $\forall$ and $\exists$, plus the standard negation $\neg$. (This answers the question, raised by Horn (1972, 1989:252-67) and Levinson (2000:69-71), of why the **O**-corner in APC is never lexicalised: there ís no **O**-corner to be lexicalized!)

The revision of APC is easily shown by means of a valuation space interpretation (VSI) (Van Fraassen 1971). Let the valuation space (VS) of a sentence *A*, /*A*/, be the set of situations in a universe **U** of possible situations in which *A* is true. To say that *A* is true now amounts to saying that the actual situation $s_a \in /A/$. Clearly, $/\neg A/ = \mathbf{U} - /A/$, $/A \wedge B/ = /A/ \cap /B/$, and $/A \vee B/ = /A/ \cup /B/$, and the whole of standard propositional calculus can be derived. APC can be rendered in VSI terms as in fig. 2, where each ring (circle) is marked for the VS of each Aristotelian sentence type. Given the fact that, as indicated in fig. 2, relations of entailment, contrariety, subcontrariety and contradiction can be read from the VSI diagram, the whole of APC is represented in fig. 2.

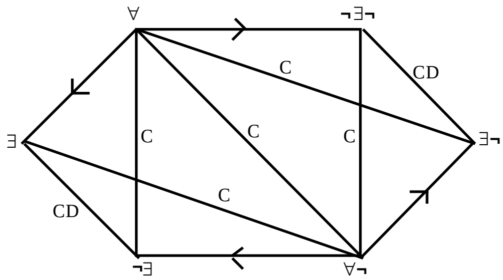

Figure 1

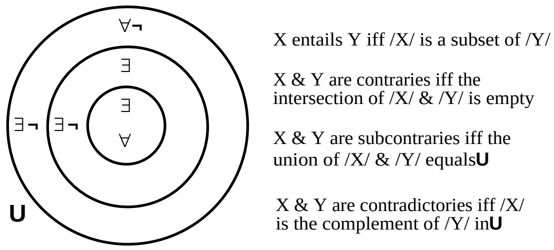

Figure 2

APC, however, fails to take into account the set of situations where the F-class is empty. Therefore, **U** in fig. 2 is incomplete and must be extended with a further ring containing those situations in which there is no representative of the F-predicate, as shown in fig. 3a.

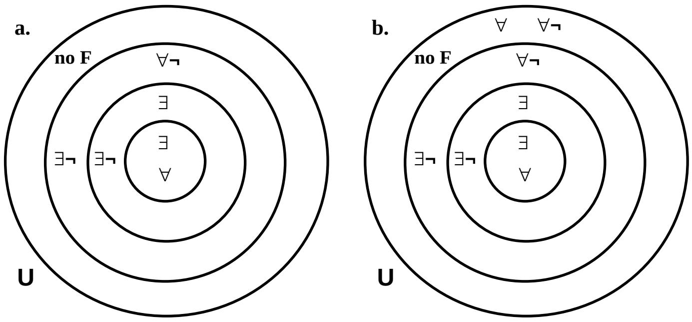

Figure 3

Now, however, the logical relations have changed. The subcontraries have gone and the conversions have been changed into one-way entailments as shown in the Hexagon of fig. 1. One notes that in fig. 3a, which represents RAPC, ∀, ∃, ∀¬ and ∃¬ are all false in cases where there is no F. SQT, on the contrary, declares both ∀ and ∀¬ true in such cases, as shown in fig. 3b. This, in fact, is the only difference between RAPC and SQT.

From the point of view of natural intuitions, the loss of the subcontraries does not appear serious, as it is easy to see the simultaneous falsity of “Some F is G” and “Some F is not-G” for cases where the F-class is non-instantiated. The replacement of the equivalences by one-way entailments from the universal quantifier ∀ likewise looks empirically promising. The inference from “All students did not pass the exam” to “No student passed the exam” seems correct, whereas the inference from “No proof of the man’s guilt has been found” to “All proofs of the man’s guilt have not been found” does not. Similarly, “All students passed” seems to license

"No student did not pass", but not vice versa, since many speakers will judge the former false but the latter true in cases where there were no students. If these judgements are correct, the Hexagon, i.e. RAPC, corresponds even better to natural intuitions than the original APC.

Like SQT, however, RAPC still fails to account for intensional (i.e. imagined) entities and intensional predicates (i.e. predicates that do not require real existence of the argument term referent for truth). For example, a sentence like "Some gods are worshipped in that temple" may well be true without it being necessary to conclude that there exist real gods. For that reason, the quantificational calculus must be modified and generalised to account for intensional phenomena as well.

Since natural language refers to and quantifies over intensional objects in precisely the same way as it does with regard to extensional (really existing) objects, there appears to be a prima facie requirement that single theories of reference and of quantification should account for both the extensional and the intensional cases. This makes it mandatory to accept an ontology containing incompletely defined *intensional objects*, as proposed by the Austrian philosopher Alexius Meinong (1853-1920). In an intensional theory of quantification, the universe of individuals **I** must contain all really existing as well as all imagined entities (objects). The quantifiers are still higher order predicates over pairs of sets (generalized quantifiers). However, the restrictor set (R-set) is no longer the standard extension of the predicate Fx, [[Fx]], comprising the set of entities that satisfy Fx, but the *intensional extension* {Fx} or the set of entities that satisfy Fx plus those that satisfy Π(Fx) (where Π is an intensional predicate/operator). One notes that {Fx} cannot be empty, since whenever Fx is mentioned it has automatically been imagined. This makes it possible to remove the condition $X \neq Ø$ from the satisfaction conditions of ∀.

At this point the minimal, presupposition-preserving negation "~" must be defined. We take it that the satisfaction conditions of a predicate *P* are divided into two subsets, the *preconditions* and the *update conditions* (cp. Seuren at al. 2001). The former define the *presuppositions* of the proposition *Pa*, i.e. the conditions of contextual coherence ('discourse anchoring') for *Pa*; the latter define the *semantic contribution* made by *Pa* to the discourse at hand. Together they form the truth conditions of *Pa*. Since an unanchored sentence lacks a truth value and does not express a proposition (as when I say to you now "The man was right after all", without any explanation as to the identity of the man or the issue at hand), the preconditions are truth-conditional, not just pragmatic, as is widely held in pragmatic circles. For good functional reasons of coherent discourse, the normal default negation in natural language toggles between satisfaction and non-satisfaction of the update conditions, leaving the preconditions unaffected. This negation is called the *minimal negation*, represented by "~". In VSI terms, we say that for each sentence *A* there is a *subuniverse* of possible situations $\mathbf{U}_A$, where the presuppositions of *A* are true (cp.

Seuren et al. 2001). If *A* has no presuppositions (i.e. the predicate of *A* has no preconditions), $\mathbf{U}_A = \mathbf{U}$. We now say that for all sentences *A*, $/A/ \subseteq \mathbf{U}_A$ and $/{\sim}A/ \subseteq \mathbf{U}_A$, and $/{\sim}A/ = \mathbf{U}_A - /A/$. There is also a *radical negation* $\simeq$, such that $/{\simeq}A/ = \mathbf{U} - \mathbf{U}_A$. This, however, is left out of consideration here. But note that $/{\sim}A/ \cup /{\simeq}A/ = /\neg A/$, as shown in fig. 4.

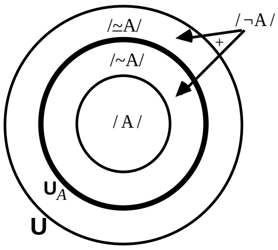

Figure 4

We now define [<Fx>], the *presuppositional extension* of the predicate *Fx*, as the set of entities that satisfy the preconditions of *Fx*, i.e. for which either *Fx* or *~Fx* yields truth. Clearly, if *Fx* has no preconditions, [<Fx>] = **I** and {Fx} = [[Fx]]. We define the universal quantifier ∀ as taking the precondition that {Fx} ∩ [<Gx>] ≠ Ø, and the update condition that {Fx} ∩ [<Gx>] ⊆ [[Gx]]. In other words, “All F is G” is true iff for all *e* ∈ {Fx} ∩ [<Gx>] (i.e. *e* ∈ {Fx} qualifies for the predicate Gx), *e* ∈ [[Gx]]. For example, “All Englishmen are rich” is true iff all members of {Englishman(x)} that qualify for the predicate “rich” are indeed rich. Since “rich” has a precondition of existence for its subject term, the class of imaginary Englishmen is automatically excluded from consideration. On the other hand, “All unicorns are imaginary” is true in this world, since the predicate “imaginary” has no existential precondition, so that [<imaginary(x)>] = **I**. Since {unicorn(x)} ∩ **I** = {unicorn(x)}, it is sufficient that {unicorn(x)} ⊆ [[imaginary(x)]], which is the case. Existential import is thus taken away from the existential quantifier and placed in the G-predicate. “Some Englishmen are rich” now entails the existence of Englishmen, but “Some gods are worshipped” does not entail the existence of gods, since “rich” is extensional but “be worshipped” intensional with regard to the subject term.

Whether the precondition {Fx} ∩ [<Gx>] ≠ Ø specified for ∀ should also be specified for (the negation of) ∃, depends on natural intuitions. If a sentence like “No unicorn likes hay” is deemed true in this world, which is far from unlikely, then the answer is No, otherwise Yes. Pending the availability of reliable data, we leave both options open. In fact, French speakers report that “Aucun unicorne n’aime le foin” is more likely to be taken to be false, whereas “Il n’y a pas d’unicorne qui aime le foin” is clearly true. There may thus be different varieties of the (negated) existential quantifier.

This account restores the equivalence of “not-all F is G” and “some F is not G”, which was lost in RAPC. In RAPC, $\neg\forall(Fx,Gx) \not\equiv \exists(Fx,\neg Gx)$, since when [[Fx]] = Ø, $\neg\forall(Fx,Gx)$ is true while $\exists(Fx,\neg Gx)$ is false. In the intensional calculus, however, when the matrix predicate Gx is extensional, $\sim\forall(Fx,Gx)$ is false in cases where [[Fx]] = Ø, owing to existential presupposition failure, while $\exists(Fx,\sim Gx)$ is likewise false. When, on the other hand, Gx is intensional, it is immaterial whether or not [[Fx]] = Ø, since with intensional matrix predicates the conditions for both ∀ and ∃ apply to {Fx}, which is automatically nonempty (see above). Fig. 5 shows this more clearly: the shaded area represents /~∀(Fx,Gx)/, which coincides with /∃(Fx,~Gx)/. If the G-predicate has no preconditions, **U**$_\forall$ = **U**, which leaves the equivalence intact. This again improves the empirical status of the calculus.

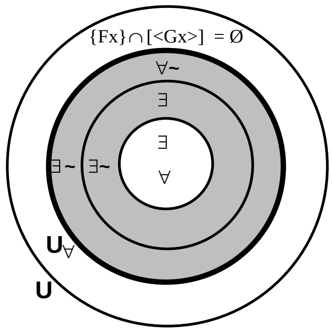

Figure 5

On the other hand, however, there is a problem (signalled by a number of authors, e.g. Zalta 1988, Castañeda in Haller 1985/6:58, Lejewski in Haller 1985/6:232), with the truth conditions of simple sentences of the form “Holmes is an Englishman”. We want to say that “Some Englishmen are imaginary” is true, since {Englishman(x)} ∩ [[imaginary(x)]] ≠ Ø. If we are then asked to mention an instance of an imaginary Englishman, we want to be able to produce, for example, Sherlock Holmes as imagined by Conan Doyle. Yet under the terms specified so far “Holmes is an Englishman” is (radically) false owing to presupposition failure, since to be an Englishman one first has to exist, which Holmes does not do. We need, therefore, a second interpretation under which “Holmes is an Englishman” is true, which makes this sentence ambiguous. Our solution consists, in principle, in adding the hedge “who/which qualifies for the (main) predicate Gx” not only to all quantified terms but also to instantiations adduced in a chain of argument. Not only would “Some Englishmen are imaginary” then be read as “Some Englishmen who qualify for the predicate “imaginary” are imaginary”, but “Holmes is an Englishman” would then likewise be read as “Holmes is an Englishman who qualifies for the predicate “imaginary”, or “Holmes is a case in point”. However, the mechanism for the distribution of such hedges has not been developed yet (see Castañeda in Haller 1985/6:58 for a similar view, but without formal elaboration).

One possible implication for the study of human cognition should be mentioned here. It appears that human cognition does not naturally develop the concept of null set until a very high degree of mathematical abstraction is achieved. The question is whether a satisfactory logic and semantics of quantification can be developed without the help of Ø to account for the lower levels of abstraction where natural language operates. As has been shown above, intensional predicate calculus eliminates Ø as an option for the R-set (F-predicate). It remains to eliminate Ø for the matrix set (G-predicate): a sentence like "Some logicians are 450 years old", where the predicate "450 years old" is uninstantiated, must be processable and result in the value False. This may be achieved by treating the quantifiers as binary predicates over pairs of R-sets and predicate intensions (satisfaction conditions of predicates). "All F is G" now means: "all members of {Fx} that qualify for Gx satisfy the conditions of Gx". One notes that this would be a return to the Aristotelian notion of a proposition as the mental assignment of a property to an entity or set of entities. It would also place the medieval theory of distributive supposition in a new formal light.

There is, furthermore, the peculiar fact that, at a non-reflective default level of cognitive operation, *some* is naturally interpreted as "partial", and thus equivalent with "partial not". The corresponding extensional calculus is shown in fig. 6 (with "**P**" for 'partial' 'and "=" for 'equivalent'):

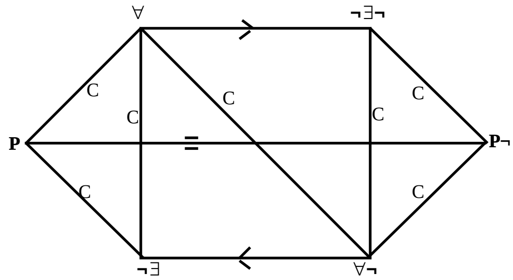

Figure 6

It may be assumed that at this default level of abstraction the notion 'subset' (denoted by "some F") is defaultwise interpreted as 'proper subset'. The combination of the 'no-null-set' hypothesis with the 'proper subset' hypothesis opens an interesting perspective on further research into the logical properties of cognition.

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