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KeV Sterile Neutrino Warm Dark Matter in Gauge Extensions of the Standard Model

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Abstract: A sterile (right-handed) neutrino with mass of keV scale is known to be a good Warm Dark Matter candidate. It is the main objective of this thesis to study how this possibility could be realized in the context of extensions of the Standard Model gauge group.

Having such a setup with an additional symmetry breaking scale between the electroweak and Planck scales, the naïve expectation leads to large thermal overproduction of sterile neutrinos. However, we show that sufficient entropy release due to out-of-equilibrium decay of other long-lived right-handed neutrinos is capable to dilute the density of the keV scale sterile neutrinos so that the observed Dark Matter abundance is achieved. The universal requirements that should be satisfied by such gauge extensions of the Standard Model, containing right-handed neutrinos, to be viable models of Warm Dark Matter, are presented.

Within this scenario, we discuss the possibility of the type I see-saw mechanism and provide a simple example in the context of a left-right symmetric model.

Keywords: keV scale sterile (right-handed) neutrinos, Warm Dark Matter, gauge extensions of the Standard Model, thermal production, out-of-equilibrium decay, see-saw mechanism, left-right symmetric model

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Sterile keV Neutrinos als Kandidaten für Warme Dunkle Materie in Erweiterungen der Standard Modell Eichgruppe

Kurzfassung: Es ist bekannt, dass ein steriles (rechts-händiges) Neutrino mit einer Masse im keV Bereich ein guter Kandidat für Warme Dunkle Materie ist. Das Hauptziel dieser Arbeit ist das Untersuchen einer solchen Realisierung im Rahmen von Theorien, die Erweiterungen der Standard Modell Eichgruppe darstellen.

Betrachtet man ein solches Modell mit einer zusätzlichen Symmetriebrechungsskala, die sich zwischen denen der elektroschwachen Wechselwirkung und der Planck Masse ansiedelt, so führt die naive Betrachtung zu einer großen thermalen Überproduktion von sterilen Neutrinos. Wir zeigen, dass durch ausreichende Entropiefreisetzung—verursacht durch ein anderes langlebiges rechts-händiges Neutrino, das zu einem Zeitpunkt zerfällt während dem es sich außerhalb des thermodynamischen Gleichgewichts befindet—die Möglichkeit besteht, die Teilchendichte der keV sterilen Neutrinos effektiv zu reduzieren und der beobachteten Menge von Dunkler Materie anzugleichen.

Die allgemeinen Erfordernisse und Bedingungen, die von solchen Eicherweiterungen erfüllt werden müssen um geeignete Theorien für Warme Dunkle Materie darzustellen, werden diskutiert. Desweiteren wird in einem solchen Szenarium die Möglichkeit eines Typ I See-saw Mechanismus untersucht und ein einfaches Beispiel im Rahmen eines links-rechts-symmetrischen Modells vorgestellt.

Stichwörter: Sterile (rechts-händige) Neutrinos, Warme Dunkle Materie, Eichgruppenerweiterungen des Standard Modells, thermale Produktion, Zerfall außerhalb des thermodynamischen Gleichgewichts, See-saw Mechanismus, links-rechts-symmetrisches Modell

To my parents
Elke and Hans-Peter Hettmansperger
with gratitude

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Chapter 1

Introduction

Dark Matter (DM) is one of the experimentally observed indications of physics beyond the Standard Model (SM). A wide variety of astrophysical and cosmological observations confirm that $\Omega_{\rm DM} \simeq 0.2$ part of the total energy density of the Universe is composed of some unknown form of non-baryonic matter, which interacts very weakly [1]. Evidence for DM come from the study of dynamics of stellar objects spanning the range from the dwarf galaxies to galaxy clusters [2], from the analysis of the Cosmic Microwave Background (CMB) [3], from observations of non-coinciding distributions of luminous and non-luminous matter in galaxy clusters [4], etc.

Most common particle physics explanations point towards Weakly Interacting Massive Particles (WIMPs), which are heavy, non-relativistic and weakly interacting thermal relics, leading to Cold Dark Matter (CDM).[1] The other extreme, referred to as Hot Dark Matter (HDM) has ultra-relativistic velocities. As a consequence of its large free streaming length, it prevents the formation of the observed small scale structures in the Universe and thus contradicts experiment. The intermediate situation, Warm Dark Matter (WDM), is, however, less explored. It may even provide a solution to some of the problems of the CDM simulations, reducing the number of Dwarf satellite galaxies, or smoothing the cusps in the DM halos [6].

Besides the identity of DM, another unsolved mystery in particle physics, presenting a further hint of physics beyond the SM, are the astonishing small masses of the usual left-handed neutrinos. Experiments over the past decade have shown that neutrinos oscillate from one flavour to another, which is only possible if they are massive particles [7]. Because the SM, predicting massless neutrinos, cannot explain such observations, there is no doubt that it must be extended. Perhaps the simplest and most aesthetic way to incorporate these phenomena—restoring at the same time the symmetry between the quark and lepton sectors—is to introduce right-handed neutrinos. These hypothetical neutral particles are, like all other right-handed chiral lepton fields, uncharged under weak interactions and thus represent total singlets of the SM symmetry group. Physicists characterise these neutrinos as *sterile* to emphasise that their only interaction with ordinary matter is gravitational or due to weak currents via very small mixing with ordinary SM neutrinos.

Given such weakly interacting and useful particles, which arise naturally in most extensions of the SM, we are motivated to ask whether there is a connection between the existence of DM and that of right-handed neutrinos:

Can sterile neutrinos represent Warm Dark Matter?

In fact it was pointed out in 1993 by Dodelson and Widrow [8] that sterile neutrinos can be either hot, warm, or cold dark matter candidates.

[1] Note that another common candidate for CDM is the axion, which is light, but due to a specific generation mechanism it has extremely small temperature (see Ref. [5]).

A simple model which predicts WDM as light sterile neutrinos [9][2] is the νMSM [12, 13], where only three singlet fermions with Majorana masses and Dirac mixing with ordinary (*active*) neutrinos are added to the SM. Then, the mass of one sterile neutrino can be chosen in the range of several keV, and with very small mixing with active neutrinos it will provide a particle with the lifetime exceeding the age of the Universe. This is the WDM candidate. The virtue and at the same time the problem of this model is that the sterile neutrino with such a small mixing (the *only* interaction of this particle is via the Yukawa couplings) never enters into thermal equilibrium, and can be produced only by some non-thermal mechanism. If this was not true, and the neutrino reached thermal equilibrium at some moment in the early Universe, then without any additional mechanism, the thermal relics with mass of over about 90 eV would overclose the Universe. Below or equal to this mass bound a HDM scenario would result, which is excluded by structure formation arguments as we mentioned above. Furthermore note that in order to calculate unambiguously the abundance of sterile neutrino DM in the νMSM, its initial amount is required, which consequently needs knowledge of the physics before the beginning of thermal evolution of the Universe (see Refs. [14, 15, 16]).

The possibility analysed in this work is opposite to the νMSM. We assume that there is some additional (gauge) scale between the electroweak and Planck scales, and that the sterile neutrinos are charged under these additional gauge transformations. Then, for general not too high intermediate scale, these particles have the chance to enter thermal equilibrium at some moment in the Universe's evolution. We find that in such a scenario, it is possible to reconcile the thermal overproduction of WDM with the observations. To do this, the abundance of the sterile neutrino should be diluted *after* it drops out of the thermal bath. This happens if some non-relativistic long-lived particle decays after the freeze-out of the DM sterile neutrino while being out of equilibrium and dominating the total energy density of the Universe. Thereby considerable entropy is released such that the amount of DM sterile neutrinos relative to the overall energy balance of the Universe is effectively reduced. The most natural candidate for this long-lived heavy particle is assumed to be another (heavier) sterile neutrino. Analysing this possibility, we are led to various constraints and bounds on the parameters in the model required by cosmological considerations and various experimental results. This set of requirements can serve as a basis for model builders whose intention is to implement WDM as right-handed neutrinos in, for example, a Grand Unified Theory.

The work starts by introducing the basic formalism of massive fermions—especially neutrinos—in gauge theories (see Chap. 2 and 3) where our main concern lies in the discussion of the see-saw mechanism (cf. Sec. 3.1 and Sec. 3.2) and the motivation of the introduction of right-handed neutrinos (see, e.g., Sec. 3.3). Section 3.4 gives an important parametrization of the Dirac Yukawa in the context of see-saw mechanisms, and Sec. 3.5 covers neutrino masses in a LR symmetric model. Then, before we dive into the main part, we present a review of fundamentals of cosmology in Chap. 4. In particular we will discuss the dynamics of important cosmological scenarios (Sec. 4.1 and 4.2); outline basic equilibrium thermodynamics (Sec. 4.3.1 and 4.3.2); and describe in Sec. 4.3.3 the production mechanism of thermal relics in the early Universe. We will conclude this chapter with Sec. 4.3.4, where our study will focus on the description of the entropy generating out-of-equilibrium decay.

[2]Note that WDM can also be many other particles, like a gravitino or even heavy particles, see, e.g., Refs. [10, 11].

In Chap. 5 we formulate the requirements on the properties of the DM sterile neutrino and the out-of-equilibrium decaying particle to make our model consistent with existing observations and bounds. This generic analysis, important for all possible models of this type, is made in the Sec. 5.1 and summarised in the end of it. In Sec. 5.2 and 5.3 we analyse the possibility to realise these constraints in the simplest models. Thereby we make use of the other (heavy) right-handed neutrinos to dilute the density of the DM sterile neutrino. Especially, in Sec. 5.2 we show that it is impossible for the same right-handed neutrinos to be involved in the dilution of the DM abundance and at the same time to give the masses to the active neutrinos via a type I see-saw like mechanism. Finally, in Sec. 5.3 a working example is provided, where the active neutrino masses are generated by a type II see-saw mechanism resulting from the scalar sector of a left-right (LR) symmetric model.

The appendices are devoted to the calculation of the total decay width of the sterile neutrinos in the models (App. A), to the derivation of the radiative decay width (App. B), and to the Higgs potential of the left-right symmetric model (App. C).

Chapter 2

Fermion Masses in Gauge Theories

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In particle physics the Lagrangian $\mathscr{L}$ plays the fundamental role. This quantity has to be a scalar of space-time symmetry (the Lorentz group) to be consistent with relativity. Furthermore, from gauge theory we know that $\mathscr{L}$ must be invariant under some internal symmetry transformations whose groups underlie the considered system [17, 18, 19].[1] This is guaranteed if every term in the Lagrangian represents a total symmetry group singlet.

For uncharged spin $\frac{1}{2}$ fermions, the space-time symmetry allows two types of mass terms in the Lagrangian: The *Dirac* and the *Majorana* mass terms.

2.1 Dirac Mass Term

A generic spinor ψ can always be decomposed into its chiral left- and right-handed components:

$$\psi = \psi_L + \psi_R \equiv \frac{1-\gamma^5}{2}\psi + \frac{1+\gamma^5}{2}\psi \; . \tag{2.1}$$

For this reason it is convenient to define the following chirality projection matrices

$$P_R \equiv \frac{1+\gamma^5}{2} \; , \tag{2.2}$$

$$P_L \equiv \frac{1-\gamma^5}{2} \; , \tag{2.3}$$

which satisfy $P_R + P_L = I$, $P_L^2 = P_L$, $P_R^2 = P_R$ and $P_L P_R = P_R P_L = 0$.

A free Dirac field with mass m is described by the Dirac Lagrangian [18]

$$\mathscr{L} = i\,\overline{\psi}\partial\!\!\!/\psi - m\overline{\psi}\psi \; , \tag{2.4}$$

where the first term on the right-hand side specifies the kinetic term, and the one proportional to m is referred to as the Dirac mass term. By using the decomposition (2.1) in chiral fields, the latter can be written as follows

$$\mathscr{L}_m^{\mathrm{D}} = -m\overline{\psi}\psi = -m\left(\overline{\psi_R}\psi_L + \overline{\psi_L}\psi_R\right) \; . \tag{2.5}$$

[1] To be accurate, we want to mention that $\mathscr{L}$ has to be invariant only up to a total divergence. The theorem by Emmy Noether then implies that in an anomaly free quantum theory the current is conserved (up to contact terms), see, e.g., Ref. [18].

This expression describes transitions between left- and right-handed fields and is a singlet of the Lorentz group $SL(2,\mathbb{C})$. Furthermore, because barred fields are in the conjugate representation, the bilinears in Eq. (2.5) would be invariant under gauge transformations if both, the right- and the left-handed chiral components would be in the same representation. In such a case the Dirac mass term is a total singlet.

2.2 Majorana Mass Term

The chiral fermion fields, described in the previous section, are the building blocks of the Standard Model (SM) [20, 21, 22] as well as of modern gauge theories. They are the smallest irreducible representations (irrep) of the Lorentz group, from which all other representations can be constructed. In particular, if parity is not a symmetry of the theory, an irrep can represent a massless fermion. Working in the chiral representation of the γ-matrices, this becomes manifest because the left- and right-handed parts of the Dirac equation decouple if the particle mass vanishes.

Now, one can ask if a 4-component spinor is required for the description of a massive fermion. The answer is *no*! E. Majorana discovered in 1937 [23] that there could be a connection between ψ_R and ψ_L [24]:

$$\psi_R = \psi_L^c \equiv (\psi_L)^c = i\gamma^2\psi_L^* = \mathcal{C}\overline{\psi_L}^T \,, \tag{2.6}$$

where $\mathcal{C} \equiv i\gamma^2\gamma^0$ defines the charge conjugation matrix. Fields satisfying this relation are called Majorana fields. Indeed, Eq. (2.6) makes sense since ψ_L^c is right-handed. By using the well-known commutation properties of the γ-matrices [18, 19], we get

$$P_L\psi_L^c = iP_L\gamma^2\psi_L^* = i\gamma^2 P_R\psi_L^* = 0 \,, \tag{2.7}$$

which confirms this statement.

Applying the Majorana condition (2.6) on the field ψ in Eq. (2.1),

$$\psi = \psi_L + \psi_R = \psi_L + \mathcal{C}\overline{\psi_L}^T = \psi_L + \psi_L^c \,, \tag{2.8}$$

we arrive at

$$\psi = \psi^c \,. \tag{2.9}$$

This relation implies that Majorana particles are their own antiparticles and thus must be neutral.

The possibility to construct a mass term out of only one chiral spinor component, which forms a singlet of $SL(2,\mathbb{C})$, leads to the Majorana mass term

$$\mathscr{L}_{m,L}^{\mathrm{M}} = -\frac{1}{2}M_L\overline{\psi_L^c}\psi_L + \mathrm{h.c.} \,. \tag{2.10}$$

Because of the anticommuting fermionic spinors (mathematically they are treated as Grassmann variables), and the antisymmetry property of the charge conjugation matrix $\mathcal{C}^T = \mathcal{C}^\dagger = \mathcal{C}^{-1} = -\mathcal{C}$ (see, e.g., Ref. [18]), the Majorana mass matrix M_L is symmetric, i.e. $M_L^T = M_L$.

What can we say about gauge invariance? A charge conjugate field is in the conjugate representation of the gauge group such that a barred charge conjugate field is in the same irrep as the field itself. Thus, a Majorana mass term is only possible if there is a gauge group singlet in the product of these spinors. For example, such a term is *not* invariant under the global $U(1)$ transformation (whereas the Dirac mass term is!)

$$\psi \to \mathrm{e}^{i\varphi}\psi \ , \tag{2.11}$$

leading to a total lepton number violation by two units.

Furthermore, in the SM of particle physics, where the lepton number conservation is an accidental symmetry, such terms are not allowed since this theory includes no fermions with zero hypercharge Y corresponding to the $U(1)_Y$ generator in the gauge group of electroweak interactions (see the following section).

2.3 Masses in the Standard Model

The Standard Model is a renormalizable [25] gauge theory [17] based on the symmetry group (see, e.g., Refs. [18, 19, 26])

$$G_{\mathrm{SM}} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \ , \tag{2.12}$$

where $SU(3)_C$ describes strong interactions i.e. quantum chromodynamics (QCD), and $SU(2)_L \times U(1)_Y$ the Glashow-Weinberg-Salam model of electroweak interactions.

The fermions in the SM appear in three families, each with the same quantum numbers under G_{SM}. Because the right- and left-handed chiral fields are treated differently under G_{SM}—left-handed fields are doublets whereas right-handed ones are singlets under $SU(2)_L$—the SM forbids bare mass terms for fermions. Therefore, all fermions of the Standard Model would be massless were it not for the mechanism of spontaneous symmetry breaking (SSB), also referred to as Higgs mechanism [27, 28, 29].

This process is based on the introduction of the famous but still hypothetical Higgs boson ϕ. Being a doublet of $SU(2)_L$, it can couple with fermion bilinears to form renormalizable terms of the following type:

$$\mathscr{L}_{\mathrm{Yuk}} = -y_{ab}\overline{L_L^a}\phi e_R^b + \mathrm{h.c.} \ . \tag{2.13}$$

L_L is the left-handed lepton doublet of $SU(2)_L$, e_R the right-handed lepton singlet of $SU(2)_L$ and y_{ab} is the coupling constant with generation indices. Such so-called Yukawa interactions represent total singlets of G_{SM}. However, because the ground state of the Higgs potential is *not* invariant under this symmetry group, the neutral component of the Higgs doublet acquires a non-zero vacuum expectation value v (VEV). Thus, $SU(2)_L \times U(1)_Y$ spontaneously breaks down to $U(1)_Q$, the gauge group of electromagnetic interactions, and the VEV v, substituted into Eq. (2.13), leads to Dirac masses for the charged fermions:

$$\mathscr{L}_m^{\mathrm{D}} = -v y_{ab}\overline{e_L^a}e_R^b + \mathrm{h.c.} \ . \tag{2.14}$$

However, we want to mention that in the SM, mass terms for neutrinos are not allowed. First of all, it contains no right-handed neutrino fields so that Yukawa terms such as in Eq. (2.13) can not be formed. Secondly, Majorana mass terms for the left-handed field ν_L are forbidden because on the one hand, such a term in bare form would violate electroweak gauge symmetry (see the discussion at the end of Sec. 2.2), and on the other hand, there is no way to produce it via SSB since there are no further scalar bosons in the

SM, which have appropriate quantum numbers, to form a renormalizable[2] gauge group singlet together with the bilinear $\overline{\nu_L^c}\nu_L$. Hence, to predict massive neutrinos, one has to go beyond the SM.

[2]From power counting of divergent diagrams, Lagrangian terms which contain products of fields with mass dimension larger than four are nonrenormalizable (see, for example, Refs. [18, 25]).

Chapter 3

Massive Neutrinos

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Nowadays we know that the SM cannot be a complete theory of everything. One of the shortcomings is that neutrinos oscillate and therefore must be massive particles. There exist several extensions of the SM which predict neutrino masses (see, e.g., Ref. [30]).

In this chapter we want to go beyond. We shall consider a general situation, where we assume that both, Dirac as well as Majorana mass terms for the neutrinos are present. This will lead to a special case—known as the see-saw scenario—which can explain the astonishing small active neutrino masses. Then we will show how such Dirac or Majorana mass terms can be realised in some specific extensions of the SM, discuss the see-saw mechanism in the context of effective field theories and conclude the chapter with a short review of a left-right symmetric model.

3.1 The Dirac-Majorana Mass Term

As mentioned in the end of the previous Sec. 2.3, the hypothetical right-handed neutrino is not present in the SM. Nevertheless, in order to have the possibility to form Dirac mass terms for the neutrinos, we will assume that it is there. Furthermore, we will allow for Majorana masses which are forbidden in the SM but could be present in some extended theories. The question as to what models and what scalar sectors could give rise to such terms will be postponed to Secs. 3.3 and 3.5.

Given the left- and right-handed neutrino fields, ν_L and N_R respectively,[1] one obtains the general mass Lagrangian by writing down all possible Dirac and Majorana mass terms. For one generation, one has

$$\mathscr{L}_m^{\mathrm{D+M}} = -m_D \overline{\nu_L} N_R - \frac{1}{2} M_L \overline{\nu_L^c} \nu_L - \frac{1}{2} M_R \overline{N_R^c} N_R + \text{h.c.} \,. \tag{3.1}$$

By defining the vector

$$\mathcal{N}_L \equiv \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix} \,, \tag{3.2}$$

the expression (3.1) can be written in a convenient matrix form

$$\mathscr{L}_m^{\mathrm{D+M}} = -\frac{1}{2} \overline{\mathcal{N}_L^c} \mathcal{M} \mathcal{N}_L + \text{h.c.} \,, \tag{3.3}$$

with the symmetric complex matrix

$$\mathcal{M} = \begin{pmatrix} M_L & m_D \\ m_D & M_R \end{pmatrix} \,. \tag{3.4}$$

Note that in this derivation we have considered a real M_R. This is always possible by an appropriately chosen phase of the field N_R. Then, once the phase of N_R is fixed, we can choose that of ν_L in such a way that m_D becomes real. However, if there is a phase left in M_L, it is no more possible to get rid of it because we can not change the phases of the chiral fields anymore. Thus, in general we can consider a complex $\mathcal{M}$ with real m_D and M_R but complex M_L.

From the expression in Eq. (3.3), it is clear that the chiral fields ν_L and N_R are in general not the mass eigenstates. In order to find the fields of massive neutrinos, one has to diagonalise the matrix in Eq. (3.4). This can be done by a unitary transformation W

$$\mathcal{N}_L = W n_L \,, \tag{3.5}$$

with the fields in mass basis

$$n_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix} \,. \tag{3.6}$$

Then, the mass matrix transforms as

$$W^T \mathcal{M} W = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \,. \tag{3.7}$$

To simplify the formulas, we consider real mass matrices. In such a case the eigenvalues of $\mathcal{M}$ are

$$m_{2,1} = \frac{1}{2} \left[M_L + M_R \pm \sqrt{(M_L - M_R)^2 + 4 m_D^2} \right] \,, \tag{3.8}$$

and the angle of the rotation which diagonalises $\mathcal{M}$ can be derived out of

$$\tan 2\vartheta = \frac{2 m_D}{M_R - M_L} \,. \tag{3.9}$$

[1] We take the name N_R instead of ν_R because in case of Majorana neutrinos, we have the relation $(\nu_L)^c = \nu_R$. However, the right-handed field describes a totally different particle. In the SM it is, for example, a total singlet of G_{SM} whereas the usual active neutrino ν_L is not (see the discussion at the end of Sec. 3.2.1). Thus, we should not identify it with $(\nu_L)^c$.

3.2 The See-Saw Mechanism

In this part we describe a scenario which is able to explain, why the neutrino masses are so small compared to that of charged leptons. We speak about the famous see-saw mechanism. This process can be realised in different ways in various extensions of the Standard Model (see, e.g., Ref. [30]). Here we want to describe the most common types.

3.2.1 Type I See-Saw

Assume that for some reason we have the scenario

$$m_D \ll M_R \, , \qquad M_L = 0 \, . \tag{3.10}$$

From Eq. (3.8), we obtain[2]

$$m_1 \simeq -\frac{m_D^2}{M_R} \, , \qquad m_2 \simeq M_R \, . \tag{3.11}$$

The Eqs. (3.11) show that ν_2 is as heavy as M_R and ν_1 is very light. This is because m_D is suppressed by the small factor m_D/M_R, i.e. the heavier the mass of the neutrino ν_2, the lighter the mass of ν_1. This is the reason why physicists call it "see-saw" mechanism.

In the considered "type I" scenario [31, 32, 33] the mixing angle (see Eq. (3.9)), which describes the transformation from flavour to mass basis, is very small:

$$\vartheta = \frac{m_D}{M_R} \ll 1 \, , \tag{3.12}$$

This implies that ν_1 is composed mainly of ν_L, whereas ν_2 is composed mainly of N_R. In particular, we have

$$W = \begin{pmatrix} i\cos\vartheta & \sin\vartheta \\ -i\sin\vartheta & \cos\vartheta \end{pmatrix} \, , \tag{3.13}$$

such that the mass eigenstates are given by

$$\nu_{1L} \simeq -i\,\nu_L + i\frac{m_D}{M_R}\,N_R^c \simeq -i\,\nu_L \, , \tag{3.14a}$$

$$\nu_{2L} \simeq \frac{m_D}{M_R}\,\nu_L + N_R^c \simeq N_R^c \, . \tag{3.14b}$$

At this point let us do a little excursion. We want to show, how important the see-saw mechanism is.

One of the unsolved questions of neutrino physics is the explanation of the tiny neutrino masses compared to that of charged leptons. This problem can be solved by the type I see-saw mechanism in a very beautiful way if we do a *minimal extension* of the SM—we add the right-handed neutrino. Note, that it is one of the peculiarities of the SM that it contains left- and right-handed chiral projections of all fundamental fermions except for neutrinos. However, in the epoch when the SM has been invented, neutrinos were thought to be massless, and therefore, right-handed neutrino fields have not been introduced. Like all the other right-handed fields of the SM, the N_R is assumed to be a $SU(2)_L$ singlet. The definition of electric charge, $Q = T_3 + Y/2$, then implies that this electrical neutral

[2] The minus sign in the formula for m_1 can be absorbed by a redefinition of the field ν_1 in mass basis (see Eq. (3.14)).

neutrino has in addition zero hypercharge ($Y = 0$). This means that the right-handed neutrino is a total singlet of the entire SM symmetry group $G_{\rm SM}$. In other words, this neutrino is uncharged under all SM interactions. Therefore, people call it *sterile*. On the contrary, the usual left-handed neutrinos, which participate in weak interactions, are usually called *active*.

Let us show how the scenario from above (see Eq. (3.10)) is realised in such a simple SM extension. The Majorana mass term for ν_L, which contains M_L, is not allowed in the SM because this field is contained in the $(\mathbf{1},\mathbf{2},\textbf{-1})$ representation[3] of $G_{\rm SM}$, and the Kronecker product

$$(\mathbf{1},\mathbf{2},\textbf{-1}) \otimes (\mathbf{1},\mathbf{2},\textbf{-1}) = (\mathbf{1},\mathbf{1},\textbf{-2}) \oplus (\mathbf{1},\mathbf{3},\textbf{-2}) \; , \tag{3.15}$$

contains no singlet. Thus, this term is forbidden by renormalizability and the symmetries so that we have $M_L = 0$. The Dirac mass term, which contains m_D, is produced by the Higgs mechanism in the same way like it was described in the case of charged fermions in Sec. 2.3. Therefore, m_D is like all other charged fermion masses proportional to the VEV of the Higgs boson. This is the reason why it cannot be much larger than the electroweak scale $\sim \mathcal{O}(10^2)$ GeV—one says it is protected by the symmetries of the SM. On the contrary, since the Majorana mass term of the right-handed sterile neutrino N_R is a total singlet of $G_{\rm SM}$, the mass M_R is not protected by the SM symmetries. Instead, it will be determined by some scale corresponding to new physics beyond the SM, where the chiral field N_R will belong to some non-trivial multiplet of the high-energy gauge group. The mass M_R is then protected by high energy symmetries, and its order of magnitude will be determined by their breaking scale, which may, for example, be at the grand unification scale $\sim \mathcal{O}(10^{14} \div 10^{16})$ GeV. Hence, the type I see-saw mechanism predicts a light active neutrino mass m_1 for neutrino ν_1 (cf. Eq. (3.11)), which is suppressed with respect to the electroweak or charged lepton mass scale ($\sim m_D$) by the very small ratio $m_D/M_R \sim 10^{-14} \div 10^{-12}$.

3.2.2 Type II See-Saw

The type II see-saw scenario [34, 35, 33] is realised when the left-handed Majorana mass M_L is small but non-zero, and if the relation

$$M_L \ll m_D \ll M_R \; , \tag{3.16}$$

holds. A non-zero M_L can, for example, be induced by extending the scalar sector of the SM by an Higgs triplet (see Eq. (3.15) and Sec. 3.3.2). There are models which predict a suppression of the mass M_L by a new scale of physics beyond the SM. A very important example will be given in Sec. 3.5, where we describe the left-right (LR) symmetric model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

Motivated by such theories, let us consider Eq. (3.16) with

$$M_L \equiv a \frac{m_D^2}{\Lambda} \; . \tag{3.17}$$

[3]The representation of a particle in the SM is a combination of an irrep of $SU(3)_C$, an irrep of $SU(2)_L$ and an irrep of $U(1)_Y$. The values of the dimensions of these irreps are usually collected in parentheses (cf. Eq. (3.15)).

Here a is a numerical coefficient and Λ denotes the high-energy scale. By applying Eq. (3.8), we obtain

$$m_1 \simeq M_L - \frac{m_D^2}{M_R} = a\frac{m_D^2}{\Lambda} - \frac{m_D^2}{M_R} , \qquad m_2 \simeq M_R . \tag{3.18}$$

Conventionally one speaks of a type II see-saw if $|M_L| \gg m_D^2/M_R$, whereas the type I see-saw refers to the case in which $|M_L| \ll m_D^2/M_R$.

3.2.3 See-Saw with three Generations

So far, the discussion was for simplicity and illustration focussed on the case of one generation. However, as we know from measurements of the Z boson decay width [36], there are three flavours of active neutrinos in nature. This motivates us to introduce three[4] right-handed neutrino fields, one for each generation. The general neutrino mass matrix which corresponds to the Dirac-Majorana mass term is now a 6×6 complex symmetric matrix:

$$\mathcal{M} = \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} , \tag{3.19}$$

where M_L and M_R are complex symmetric 3×3 blocks, and m_D is an arbitrary complex 3×3 matrix. The vector of left-handed fields $\mathcal{N}_L$, defined in Eq. (3.2), has the form

$$\mathcal{N}_L = \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix} , \tag{3.20}$$

with

$$\nu_L = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^T , \qquad N_R = (N_{eR}, N_{\mu R}, N_{\tau R})^T . \tag{3.21}$$

It is obvious that due to the matrix character of its entries, the diagonalization of the matrix in Eq. (3.19) is more complicated than in the case of one generation.

If all the eigenvalues of M_R are much larger than all the elements of m_D and M_L, $\mathcal{M}$ can be diagonalised by blocks following Ref. [39]. Up to corrections of the order of $M_R^{-1} m_D$, we have

$$W^T \mathcal{M} W \simeq \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix} , \tag{3.22}$$

with

$$W \simeq \begin{pmatrix} 1 - \frac{1}{2} m_D^* (M_R M_R^*)^{-1} m_D^T & (M_R^{-1} m_D^T)^\dagger \\ -M_R^{-1} m_D^T & 1 - \frac{1}{2} M_R^{-1} m_D^T m_D^* (M_R^*)^{-1} \end{pmatrix} \tag{3.23}$$

whereas the in general non-diagonal 3×3 matrices, m_ν and M_N, are given by

$$m_\nu = M_L - m_D M_R^{-1} m_D^T , \qquad M_N \simeq M_R . \tag{3.24}$$

These are the type II see-saw formulas in the case of three left- and three right-handed neutrinos.

[4] It is important to note, that the presence of sterile neutrinos is irrelevant for anomaly cancellations [37, 38].

3.3 Neutrino Mass in $SU(2)_L \times U(1)_Y$ Gauge Theories

As often mentioned before, to explain observed evidences for neutrino masses, one must go beyond the SM. However, this does not necessarily mean that one has to consider high-energy extensions of the gauge group $SU(2)_L \times U(1)_Y$. Even with the same gauge group as the SM, one can conjecture modifications in the scalar or fermionic sectors of the new model so that neutrino mass terms like in Eq. (3.1) are possible.

We already discussed such an extension in the fermionic sector (see Sec. 3.2.1): the adding of the right-handed neutrino to the matter content of the SM.

3.3.1 Introduction of right-handed Neutrinos

The right-handed neutrino N_R is assumed to be sterile, i.e. it is a total singlet of the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ such that its only interaction is gravitational. Nobody knows if it exists, but the introduction of this particle is a very aesthetic extension of the SM. Adding three of them—one for each generation—would remove the asymmetry between the lepton and quark sectors so that there is a right-handed chiral partner for every left-handed fundamental fermion field. Due to SSB the right-handed neutrinos provide the possibility of Dirac mass terms and in addition, Majorana masses for N_R are possible if lepton number conservation is violated. In such a case the scenario of the type I see-saw, discussed in Sec. 3.2.1, could be a possible explanation of the small active neutrino masses. At the same time, the heavy right-handed neutrinos would be non-observable if the mass scale of new physics beyond the SM would be at GUT scale. We want to mention that with right-handed neutrinos present, $U(1)_{B-L}$ is anomaly free. Thus, it would be possible to forbid Majorana mass terms by imposing a gauged $B - L$ symmetry so that models with massive Dirac neutrinos would result. However, such models are considered to be rather unnatural because very small Dirac Yukawas are required to predict the small active neutrino masses.

In the end of Sec. 3.2.1 we showed that a bare Majorana mass term for the left-handed neutrino field ν_L is forbidden by the SM symmetry group (cf. Eq. (3.15)). Does this mean that without introducing the right-handed neutrinos or extending the gauge group G_{SM}, neutrino masses are impossible? No! One can add new Higgs particles to the scalar sector of the model which can lead to Majorana masses for the left-handed neutrino. In the next section we shall present some well-studied examples.

3.3.2 Enlargement in the Scalar Sector

In the SM, the lepton fields and Higgs scalars have the following $SU(2)_L \times U(1)_Y$ transformation properties (see, e.g., Ref. [18]):

$$\begin{aligned} L_{ai,L} = \begin{pmatrix} \nu_a \\ l_a^- \end{pmatrix}_L &\sim \quad (\mathbf{2},\mathbf{-1})\ , \\ l_{a,R}^- &\sim \quad (\mathbf{1},\mathbf{-2})\ , \\ \phi_i = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} &\sim \quad (\mathbf{2},\mathbf{1})\ , \end{aligned} \tag{3.25}$$

where the first and second entries in the parentheses are the $SU(2)_L$ and $U(1)_Y$ quantum numbers, respectively. We adopt the convention of taking the hypercharges Y to be

$2(Q - I_3)$, where Q is the charge of the fermion and I_3 its third weak isospin component. a denotes the flavour and i the $SU(2)$ index. Let us consider the Kronecker products of the lepton representations:

$$\begin{aligned} \overline{L_{i,L}}\, l_R^- &\sim (\mathbf{2},\mathbf{1}) \otimes (\mathbf{1},\textbf{-2}) = (\mathbf{2},\textbf{-1})\ , \\ \overline{(L_{i,L})^c}\, L_{j,L} &\sim (\mathbf{2},\textbf{-1}) \otimes (\mathbf{2},\textbf{-1}) = (\mathbf{1},\textbf{-2}) \oplus (\mathbf{3},\textbf{-2})\ , \\ \overline{(l_R^-)^c}\, l_R^- &\sim (\mathbf{1},\textbf{-2}) \otimes (\mathbf{1},\textbf{-2}) = (\mathbf{1},\textbf{-4})\ . \end{aligned} \tag{3.26}$$

The first term in Eq. (3.26), which conserves lepton number, is well known. The coupling with the SM Higgs doublet (cf. Eq. (3.25)) leads to massive charged Dirac fermions due to SSB, like it was discussed in Sec. 2.3. Because the Higgs particle carries *no* lepton number, the mechanism of SSB will not spoil this global symmetry.

The other two bilinears in Eq. (3.26) violate lepton number by two units. Furthermore, there is no scalar particle in the SM which could join these expressions to a gauge invariant Yukawa coupling. Thus, to obtain mass terms out of these bilinears, we have to enlarge the scalar sector of the SM. The study of the Kronecker products in Eq. (3.26) gives us the following types of additional Higgs particles:

- triplet: $\Delta \sim (\mathbf{3},\mathbf{2})$,
- singlet with charge +1: $h^+ \sim (\mathbf{1},\mathbf{2})$,
- singlet with charge +2: $k^{++} \sim (\mathbf{1},\mathbf{4})$.

Note that although the last two terms in Eq. (3.26) have a net $B - L$ number one can restore this symmetry by assigning appropriate lepton numbers to the new Higgs particles. How is it then possible to obtain lepton number violating Majorana mass terms in such theories?

Adding a Higgs triplet Let us consider the case of the Higgs triplet. It joins the second term in Eq. (3.26) to form a gauge invariant Yukawa term as follows

$$\mathscr{L}_\Delta = -y_{ab}\overline{(L_{ai,L})^c}\varepsilon^{ik}\Delta^j_k\, L_{bj,L} + \text{h.c.}\ , \tag{3.27}$$

where $\varepsilon^{ik} = i(\sigma_2)_{ik}$ is the total antisymmetric 2-dimensional Levi-Civita tensor, which can be used to raise and lower $SU(2)$ indices.[5] Note that the term $\varepsilon^{ik}\Delta^j_k$ is symmetric under $i \leftrightarrow j$. Hence, it follows from Grassmann nature of the fermion fields and the antisymmetry property of the charge conjugation matrix $\mathcal{C} = i\gamma^2\gamma^0$ (see Sec. 2.2) that the Yukawa coupling y_{ab} is symmetric. The triplet scalar Δ in matrix form is given by

$$\begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \sim (\mathbf{3},\mathbf{2})\ , \tag{3.28}$$

where the charge assignments of the fields δ are easily determined by applying the Gell-Mann-Nishijima charge operator $Q = I_3 + 1/2\,Y$ on Δ:

$$Q\Delta = \frac{1}{2}[\sigma_3, \Delta] + \Delta\ . \tag{3.29}$$

[5] With σ_j, we denote the usual Pauli matrices. Furthermore, we use the convention that the fundamental representation $\mathbf{2}$ has a lower index, and its conjugate $\overline{\mathbf{2}}$ has an upper index.

Now let us assume that the parameters in the scalar potential of the theory are such that at the minimum the Higgs particle Δ acquires the VEV[6]

$$\langle\Delta\rangle = \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix} . \tag{3.30}$$

Then, after SSB, the expression (3.27) produces a Majorana mass term for the left-handed neutrinos

$$\mathscr{L}_m^{\mathrm{M}} = -\frac{1}{2} M_{ab}\, \overline{\nu_{a,L}^c} \nu_{b,L} + \text{h.c.} , \tag{3.31}$$

with the symmetric mass matrix

$$M_{ab} = 2\, v y_{ab} . \tag{3.32}$$

Thus, even if Δ carries lepton number 2 so that the expression in Eq. (3.27) is $B-L$ invariant, this symmetry will be broken as the Higgs particle develops a VEV. Note that by enforcing $U(1)_{B-L}$ to be a symmetry of the entire classical Lagrangian, this breakdown happens spontaneously implying the existence of a massless Nambu-Goldstone boson called Majoron. However, such interesting models, proposed by Gelmini and Roncadelli [40], are ruled out by now because the Z-boson decay into such massless scalars gives a significant contribution to its width and therefore contradicts measurement results. On the contrary, if we do not insist in having $B-L$ invariance, and write down the full gauge-invariant scalar potential involving the SM Higgs doublet ϕ and the triplet Δ, there will be a term

$$\mathscr{L} \supset A \phi_i^T \varepsilon^{ik} \Delta_k^j \phi_j + \text{h.c.} , \tag{3.33}$$

that breaks $B-L$ explicitly. Allowing for this term, there would be *no* spontaneous $B-L$ breakdown and the Majoron would be absent.

Adding Higgs singlets The Higgs singlet h^+ can have the following gauge invariant Yukawa term

$$\mathscr{L}_{h^+} = -y_{ab} \overline{(L_{ai,L})^c} \varepsilon^{ij} L_{bj,L} h^+ + \text{h.c.} . \tag{3.34}$$

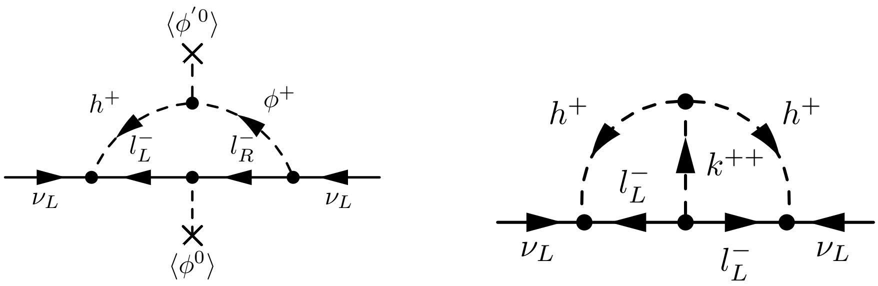
with an antisymmetric coupling $y_{ab} = -y_{ab}$. Here the scalar h cannot develop a VEV because otherwise the vacuum would be charged and electromagnetic gauge symmetry would be spontaneously broken. Thus, we need another source of $B-L$ violation in order to generate Majorana masses for ν_L. Anthony Zee found [41] that if one introduces besides the singlet h another scalar doublet ϕ' (in addition to the SM Higgs boson ϕ), it will be possible to form a lepton number violating scalar coupling of the form $\varepsilon\phi\phi' h^-$. In such a case Majorana mass terms are produced through loop diagrams such as Fig. 3.1(a).

Another model is from Babu [42]. Instead of adding a second scalar doublet, he takes the doubly charged singlet k^{++} from above. Again it is possible to form a $B-L$ violating trilinear coupling of scalar particles, $k^{++}h^-h^-$, so that Majorana masses are produced by 2 loop diagrams as shown in Fig. 3.1(b). Note that such radiatively produced mass terms are small because of the factor $\frac{1}{8\pi^2}$ resulting from the loop integrals, and the proportionality to powers of small coupling constants. Hence, such mechanisms are besides the see-saw scenario another aesthetic explanation of the smallness of neutrino masses.

Finally, let us emphasise that the ρ parameter, defined by

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} , \tag{3.35}$$

[6]Note that because of conservation of charge, only the neutral component δ^0 can acquire a VEV.



(a) Onle-loop contribution to $\overline{\nu_L^c}\nu_L$ in the Zee model.

(b) Two-loop contribution to $\overline{\nu_L^c}\nu_L$ in the Babu model.

Figure 3.1: Processes generating the active neutrino mass radiatively.

is sensitive to such scalar sector modifications and could deviate from the SM value, $\rho = 1$, at tree level if the additional Higgs bosons acquire VEVs that contribute to electroweak symmetry breakdown. To show this explicitly, we give a more general expression. Using as usual the scalar kinetic terms with the VEVs inserted, for determining the gauge boson masses, we obtain in case of an arbitrary number of Higgs multiplets Φ_k, including the SM doublet, the following relation [43]:

$$\rho = \frac{\sum_k \left[I^k\left(I^k+1\right) - (I_3^k)^2\right] |v_k|^2 c_k}{2\sum_k (I_3^k)^2 |v_k|^2} \; . \tag{3.36}$$

Here I^k is the weak isospin of the Higgs multiplet Φ_k, and I_3^k is the third component of the weak isospin of the component of Φ_k, which has the VEV v_k. The quantity c_k equals $1/2\,(1)$ for real (complex) representations. Because experiments strongly restrict this parameter to $\rho = 1.0004^{+0.0008}_{-0.0004}$ [36], one has strong bounds on the VEVs of additional Higgs bosons.

3.4 Effective Majorana Mass

In some of the previous sections we demonstrated that without extending the SM it is impossible to have a Majorana mass term for the left-handed chiral neutrino field ν_L (see Sections 2.3, 3.2.1 and 3.3.2).

Then, at first sight, it seems to be curious why there exist a non-renormalizable expression, composed of SM fields, which respects G_{SM} at high energies and reduces to a Majorana mass term for ν_L at low energies. We speak about the lepton number violating Weinberg operator [44]

$$\mathscr{L}_5 \equiv -\kappa (L_L^T \varepsilon \phi) \mathcal{C}^\dagger (\phi^T \varepsilon L_L) + \text{h.c.} \, , \tag{3.37}$$

being the lowest dimensional unique term satisfying this property. Indeed due to electroweak symmetry breakdown, i.e.

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \xrightarrow{\text{SSB}} \quad \langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \; , \tag{3.38}$$

this term leads to

$$\mathscr{L}_m^M = -\frac{1}{2} m_{\nu_L} \overline{\nu_L^c} \nu_L + \text{h.c.} \, . \tag{3.39}$$

with the Majorana mass

$$m_{\nu_L} = 2\kappa v^2 \, . \tag{3.40}$$

Of course, the Lagrangian from above, Eq. (3.37), is non-renormalizable. Representing a dimension-5 operator,[7] this term would produce non-renormalizable divergences and, therefore, is not acceptable in the SM. Nevertheless, as we will demonstrate, it plays an important role in the framework of an effective field theory (EFT).

Over the past several decades it has become clear that the SM fails to explain a number of observed phenomena in particle physics, astrophysics and cosmology. Besides the neutrino oscillations (suggesting massive neutrinos) this model cannot explain the nature of Dark Matter, that of Dark Energy, the processes of baryon asymmetry and of inflation. These drawbacks indicate that the SM cannot be complete. It is rather the low energy manifestation of a deeper structure, some unknown ultimate theory (see, e.g., Ref. [45]).

The concept of EFT allows us to make predictions at low energies (or equally long distances) even if we don't know anything about the ultimate theory. More precisely it describes the low energy limit of a full theory by a series expansion of its Lagrangian [46]:

$$\mathscr{L} = \mathscr{L}^{(0)} + \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \ldots \, . \tag{3.41}$$

In our case $\mathscr{L}^{(0)} = \mathscr{L}_{\mathrm{SM}}$ corresponds to the renormalizable SM Lagrangian. The following $\mathscr{L}^{(i)}$'s are effective non-renormalizable dimension-$(4+i)$ terms, which are low energy residues of the high energy theory. The latter must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies, so that the operators $\mathscr{L}^1$, $\mathscr{L}^2$, ... are singlets of the gauge symmetry G_{SM}. However, since any Lagrangian term must have a total mass dimension of 4 the coupling constants of dimension-$(4+i)$ operators are proportional to Λ^{-i}, where Λ is a heavy mass, characteristic of the symmetry-breaking scale of the high energy "full" theory. This is the reason why the operators in Eq. (3.41) with $i > 0$ are highly suppressed compared to the SM Lagrangian such that the observability of the low-energy effects of the new physics beyond the SM is limited.

The unique candidate for $\mathscr{L}^{(1)}$, i.e. the effective low energy non-renormalizable operator with lowest dimensionality, compatible with the symmetries of the SM, is the dimension-5 operator $\mathscr{L}_5$ introduced in Eq. (3.37). In this formalism it is obvious, why the neutrino masses are small. The coupling constant of $\mathscr{L}_5$ is proportional to Λ^{-1} so that the mass term in Eq. (3.39) has the form

$$\kappa v^2 = a \frac{v^2}{\Lambda} \, , \tag{3.42}$$

with a, a dimensionless numerical coefficient. Note that this relation is of the same structure as that obtained with the see-saw mechanism described in Sec. 3.2.1 and 3.2.2. In the following we shall show that the see-saw mechanism can indeed be the source of the dimension-5 operator at low energies.

3.4.1 The See-Saw Mechanism and the Dimension-5 Operator

In this section we will reveal a connection between the see-saw mechanism and the effective dimension-5 operator $\mathscr{L}_5$.

[7] A Lagrangian term which contains a product of fields with energy or mass dimension d is called a dimension-d operator

The type I see-saw scenario is characterised by condition (3.10) and the presence of right-handed neutrinos N_R with mass M_R. To calculate the active neutrino masses, we will now present a new method proposed in Ref. [47]. Let us consider the Lagrangian

$$\mathscr{L}_m^{\text{D+M}} = -m_D(\overline{N_R}\nu_L + \overline{\nu_L}N_R) - \frac{1}{2}M_R(\overline{N_R^c}N_R + \overline{N_R}N_R^c)\ . \tag{3.43}$$

Above the scale of electroweak symmetry breaking, one can write this term in the form

$$\mathscr{L}_m^{\text{D+M}} = -y(\overline{N_R}\tilde{\phi}^\dagger L_L + \overline{L_L}\tilde{\phi}N_R) - \frac{1}{2}M_R(N_R^T\mathcal{C}N_R + N_R^\dagger\mathcal{C}^\dagger N_R^*)\ , \tag{3.44}$$

where $\tilde{\phi} = \varepsilon\phi^*$ is the charge conjugate SM Higgs field. Now comes the point. If the mass M_R is very heavy, the right-handed chiral field N_R can be integrated out. This is because at SM energies, it can be considered in the static limit in which the kinetic term in the equation of motion is neglected. Thus, we obtain with Eq. (3.44)

$$0 = -\partial_\mu\frac{\partial\mathscr{L}(x)}{\partial(\partial_\mu N_R(x))} + \frac{\partial\mathscr{L}(x)}{\partial N_R(x)} \simeq \frac{\partial\mathscr{L}_m^{\text{D+M}}}{\partial N_R(x)} = -M_R N_R^T\mathcal{C} - y\overline{L_L}\tilde{\phi}\ , \tag{3.45}$$

so that N_R in its static approximation is given by

$$N_R \simeq -\frac{y}{M_R}\tilde{\phi}^T\mathcal{C}\overline{L_L}^T\ . \tag{3.46}$$

Substituting this back into Eq. (3.44), we obtain

$$\mathscr{L}_m^{\text{D+M}} \simeq \frac{1}{2}\frac{y^2}{M_R}(L_L^T\varepsilon\phi)\mathcal{C}^\dagger(\phi^T\varepsilon L_L) + \text{h.c.}\ . \tag{3.47}$$

If we define the massive field ν_{1L} as in Eq. (3.14)

$$\nu_{1L} \simeq -i\nu_L\ , \tag{3.48}$$

the operator in Eq. (3.47) generates after SSB ($\phi \to \langle\phi\rangle$, cf. Eq. (3.38)) the Majorana mass term

$$\mathscr{L}_m^{\text{D+M}} \simeq -\frac{1}{2}\frac{y^2v^2}{M_R}\overline{\nu_{1L}^c}\nu_{1L} + \text{h.c.}\ . \tag{3.49}$$

which gives us because of

$$m_D = vy\ , \tag{3.50}$$

the same results as in the scenario of the type I see-saw (cf. Sec. 3.2.1).

The expression in Eq. (3.47) coincides with the dimension-5 operator in Eq. (3.37) if we identify

$$\Lambda = M_R, \qquad a = -\frac{y^2}{2}\ . \tag{3.51}$$

This procedure demonstrates that at low energies the type I see-saw mechanism effectively reduces to $\mathscr{L}_5$ so that Λ determines the scale of the right-handed neutrino mass M_R. However, we want to mention that there are other $\mathscr{L}_5$ contributions possible. The pure type II see-saw—induced by a very heavy Higgs triplet, which couples to the left-handed chiral neutrino fields, as described in Sec. 3.3.2—is, for example, another candidate. This can be illustrated impressively by Feynman diagrams and their corresponding amplitudes.

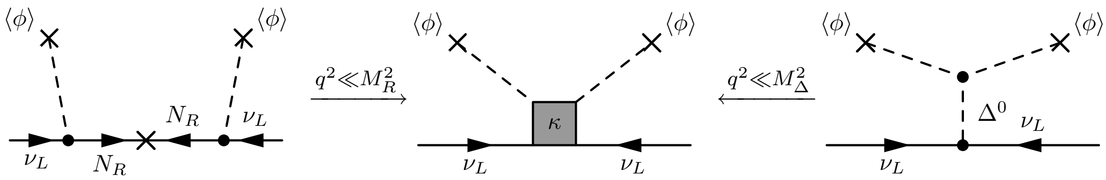


Figure 3.2: At energies far below the masses of the heavy intermediate particles, both the type I (left-hand side) and the type II (right-hand side) seesaw mechanisms effectively lead to the dimension-5 operator (centre).

In Fig. 3.2 we consider this procedure in the case of a type I and a pure type II see-saw. The intermediate particles in the diagram on the left are the right-handed neutrinos, whereas in the diagram on the right, we have the Higgs triplet propagating on the internal line. Because of their heavy mass, these particles cannot get on-shell at SM energies and thus are integrated away. In the language of Feynman diagrams this means that at low energies the propagator of these particles becomes effectively independent of the transferred four-momentum q^2 so that it contracts to an effective four particle vertex[8] where the 2 Higgs bosons of the SM meet the left-handed chiral neutrino fields. The resulting Feynman diagram corresponds to the effective dimension-5 operator, making it impossible to distinguish between these two scenarios at low energies.

To conclude Sec. 3.4, we present a very useful parametrization, which is based on the effective character of the see-saw mechanism.

3.4.2 The Casas-Ibarra Parametrization

Let us consider a Majorana mass matrix with the pattern

$$\mathcal{M} = \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} . \tag{3.52}$$

If the scenario of the type II see-saw is realized, we obtained out of this matrix the relation (cf. Sec. 3.2.3)

$$m_\nu = M_L - m_D M_R^{-1} m_D^T , \tag{3.53}$$

where m_ν denotes the active neutrino mass matrix. In the previous section we have seen that at low energies the heavy degrees of freedom are integrated away so that the number of high energy parameters, i.e. of those contained in M_L, m_D and M_R, exceeds the number of low energy parameters contained in m_ν. The idea is now to parametrize these unknown high energy quantities to obtain all possible Dirac mass matrices m_D which lead to the considered m_ν if M_R is known. Within the type I see-saw, such a parametrization of our ignorance was suggested by Casas and Ibarra [48]. We will give an extended version, following Ref. [49], which is also applicable to the scenario of the type II see-saw.

By defining the symmetric and in general complex matrix

$$X_\nu \equiv m_\nu - M_L , \tag{3.54}$$

we can rewrite Eq. (3.53) in the form

$$X_\nu = -m_D M_R^{-1} m_D^T , \tag{3.55}$$

[8] Note that this is well-known from Fermi's theory of weak interactions.

The matrices X_ν and M_R can be diagonalised by unitary transformations:

$$X_\nu = V_\nu^* X_\nu^{\rm diag} V_\nu^\dagger = \left[V_\nu^* (X_\nu^{\rm diag})^{\frac{1}{2}}\right] \left[V_\nu^* (X_\nu^{\rm diag})^{\frac{1}{2}}\right]^T , \tag{3.56a}$$

$$M_R = V_R^* M_R^{\rm diag} V_R^\dagger . \tag{3.56b}$$

Multiplying Eq. (3.55) by $\left[V_\nu^* (X_\nu^{\rm diag})^{\frac{1}{2}}\right]^{-1}$ from the left and by $\left\{\left[V_\nu^* (X_\nu^{\rm diag})^{\frac{1}{2}}\right]^T\right\}^{-1}$ from the right and using Eqs. (3.56), we find

$$I = R\, R^T , \tag{3.57}$$

with

$$R = \pm i \left(X_\nu^{\rm diag}\right)^{-\frac{1}{2}} V_\nu^T m_D V_R \left(M_R^{\rm diag}\right)^{-\frac{1}{2}} . \tag{3.58}$$

Eqs. (3.57) and (3.58) mean that the type II see-saw relation requires R to be a complex orthogonal matrix, but otherwise does not constrain it. In this way we obtain for the Dirac mass term in the basis where M_R is diagonal

$$m_D = \pm i\, V_\nu^* \sqrt{X_\nu^{\rm diag}}\, R \sqrt{M_R^{\rm diag}} V_R^\dagger . \tag{3.59}$$

R is an arbitrary complex orthogonal matrix and can be parametrized as

$$R = \pm\, R_{12}\, R_{13}\, R_{23} , \tag{3.60}$$

where R_{ij} is the matrix of rotation by a complex angle ω_{ij} in the ij-plane.

The formula for the type I see-saw can easily derived out of (3.59). Because of $M_L = 0$, X_ν simply corresponds to the active neutrino mass matrix m_ν and the transformation matrix V_ν to the PMNS matrix U. We arrive at

$$m_D = \pm i\, U^* \sqrt{m_\nu^{\rm diag}}\, R \sqrt{M_R^{\rm diag}} V_R^\dagger . \tag{3.61}$$

This is the so called Casas-Ibarra parametrization of the Dirac mass matrix.

3.5 Neutrino Mass in a left-right symmetric Model

Section 3.3 was devoted to the explanation of neutrino masses in models based on the SM electroweak symmetry $SU(2)_L \times U(1)_Y$. Here the aim is to give masses to the neutrinos in the context of a model with a bigger gauge group. Our choice is to focus on the left-right (LR) symmetric model, which seems to be one of the most straightforward extensions of the SM. As the name says, in this model, the left- and right-handed chiralities of fermions are treated identically at high energies above all symmetry breaking scales. This implies that in the symmetric phase the weak interactions must conserve parity, i.e. the Lagrangian of this model involves both $V - A$ as well as $V + A$ charged-currents. The dominance of $V - A$ at low energies is then caused by the fact that there is a non-symmetric vacuum under space reflections. Moreover, in this model right-handed neutrinos are naturally included, because the LR symmetric treatment of weak interactions at high energies has the consequence that every left-handed fundamental fermion must have a right-handed partner. The minimal gauge group that implements such a symmetry is

$$G_{\rm LR} \equiv SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} , \tag{3.62}$$

where $SU(3)_C$ is the usual QCD and $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ the high energy electroweak symmetry with gauged[9] $U(1)_{B-L}$. In the symmetric phase, besides $B-L$, the baryon and lepton numbers are conserved separately, since there are no couplings between leptons and quarks. Moreover, the term $B-L$ appears in the electric charge formula, which in the LR symmetric model has the attractive form [50, 51]

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2} \; . \tag{3.63}$$

I_{3L} and I_{3R} are the third components of the weak isospins of $SU(2)_L$ and $SU(2)_R$, respectively. People think that $B-L$ makes more physical sense than the hypercharge Y contained in the Gell-Mann Nishijima formula of the SM.

Equation (3.63) has important consequences. For instance, at an energy scale beyond the electroweak one ($\sim \mathcal{O}(100-200)$ GeV), $\Delta I_{3L} = 0$ holds and since charge is conserved, one obtains from Eq. (3.63)

$$\Delta I_{3R} = -\frac{1}{2}\Delta(B-L) \; . \tag{3.64}$$

This relation connects the breakdown of parity with that of local $B-L$ symmetry. If SSB of $SU(2)_R$ is chosen so as to give $I_{3R} = 1$, we have in the case of neutrinos ($\Delta B = 0$) a lepton number violation of $\Delta L = 2$. This leads to Majorana mass terms and neutrinoless double beta decay. Indeed, we will see in what follows that the neutrino masses are implemented by a type II see-saw mechanism and that there is a connection to the symmetry breaking scale of the gauge group $SU(2)_R$.

Such left-right symmetric models were first proposed around 1973-1974 by Pati and Salam [52] but also Rabindra N. Mohapatra and Goran Senjanovic are very active in this field [53, 54, 55].

Below, we will give a short review where our main attention will focus on the realisation of massive neutrinos. The general Lagrangian of such a model consists of three parts:

$$\mathscr{L}_{\rm LR} = \mathscr{L}_{\rm kin} + \mathscr{L}_{\rm Y} - \mathcal{V} \; . \tag{3.65}$$

$\mathscr{L}_{\rm kin}$ contains the kinetic terms dictated by the gauge group and the assignment of fermions and Higgs bosons to this model, $\mathcal{V}$ is the Higgs potential, and $\mathscr{L}_{\rm Y}$ the part which contains the Yukawa couplings. We start with specifying $\mathscr{L}_{\rm Y}$.

3.5.1 Yukawa Couplings

Quarks and Leptons As mentioned before, the fermions in LR symmetric models are treated symmetrically. They include the right-handed neutrino N_R and are assigned to the following irreducible representations [56]

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (\mathbf{2}, \mathbf{1}, \mathbf{1/3}) \; , \qquad q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, \mathbf{1/3}) \; , \tag{3.66a}$$

$$\Psi_L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} \sim (\mathbf{2}, \mathbf{1}, \textbf{-1}) \; , \qquad \Psi_R = \begin{pmatrix} N_R \\ l_R \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, \textbf{-1}) \; , \tag{3.66b}$$

where the parentheses following the matrix expressions indicate the $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ quantum numbers.

[9] As already mentioned in Sec. 3.3.1, the group $U(1)_{B-L}$ is free from anomaly because of the presence of right-handed neutrinos.

Higgs particles What kind of scalar fields do we need to form gauge invariant Yukawa terms leading to Dirac or Majorana mass terms after SSB? To produce a Dirac mass term, the bidoublet

$$\phi \equiv \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \sim (\mathbf{2}, \mathbf{2}^*, \mathbf{0}) \ , \tag{3.67}$$

is the obvious choice because the Kronecker product of the bilinears $\overline{q_L} q_R$ and $\overline{\Psi_L}\Psi_R$ is $(\mathbf{2}^*, \mathbf{2}, \mathbf{0})$. Thus we have the Yukawa Lagrangian

$$\begin{aligned} \mathscr{L}_m^{\mathrm{D}} = & f_{ij} \overline{\Psi_L}^i \phi \Psi_R^j + g_{ij} \overline{\Psi_L}^i \tilde{\phi} \Psi_R^j + \text{h.c.} \\ & + \mathcal{F}_{ij} \overline{q_L}^i \phi q_R^j + \mathcal{G}_{ij} \overline{q_L}^i \tilde{\phi} q_R^j + \text{h.c.} \ , \end{aligned} \tag{3.68}$$

where we included the terms with the charge conjugate bidoublet defined by $\tilde{\phi} \equiv \sigma_2 \phi^* \sigma_2 \sim (\mathbf{2}, \mathbf{2}^*, \mathbf{0})$, which transforms in the same way as ϕ. The coefficients f, g, $\mathcal{F}$ and $\mathcal{G}$ are the Yukawa couplings.

What about Majorana mass terms? The Kronecker products of the bilinears $\overline{\Psi_L^c}\Psi_L$ and $\overline{\Psi_R^c}\Psi_R$ contain $(\mathbf{3}, \mathbf{0}, \mathbf{-2})$ and $(\mathbf{0}, \mathbf{3}, \mathbf{-2})$, respectively, so that the scalar triplets

$$\Delta_L \equiv \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix} \sim (\mathbf{3}, \mathbf{0}, \mathbf{2}) \ , \tag{3.69}$$

$$\Delta_R \equiv \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix} \sim (\mathbf{0}, \mathbf{3}, \mathbf{2}) \ , \tag{3.70}$$

can join these bilinears to form gauge invariant Yukawa terms

$$\mathscr{L}_m^{\mathrm{M}} = (h_L)_{ij} \overline{\Psi_L^c}^i \varepsilon \Delta_L \Psi_L^j + (h_R)_{ij} \overline{\Psi_R^c}^i \varepsilon \Delta_R \Psi_R^j + \text{h.c.} \ , \tag{3.71}$$

with $h_L = h_L^T$ and $h_R = h_R^T$.[10] The entire Yukawa Lagrangian which leads, as we will see, to a type II see-saw mechanism is thus given by

$$\mathscr{L}_{\mathrm{Y}} = \mathscr{L}_m^{\mathrm{D}} + \mathscr{L}_m^{\mathrm{M}} \ . \tag{3.72}$$

Discrete Left-Right Symmetry In addition to the LR gauge symmetry, models of this kind are usually assumed to possess a discrete LR symmetry, which is broken at a scale that may or may not coincide with the $SU(2)_R$ breaking scale (see the following discussion). It can be introduced in different ways. One possibility is the parity symmetry defined by [56]

$$\Psi_L^i,\ q_L^i \leftrightarrow \Psi_R^i,\ q_R^i \ , \quad \phi \leftrightarrow \phi^\dagger \ , \quad \Delta_L \leftrightarrow \Delta_R \ . \tag{3.73}$$

This leads to some restrictions on the parameters contained in the Lagrangian. The most obvious consequence is that the gauge couplings of the left- and the right-handed $SU(2)$ groups are equal, i.e. $g_L = g_R \equiv g$. Considering Eq. (3.68) and (3.71), we furthermore have

$$h_L = h_R \equiv h_M \ , \tag{3.74}$$

and hermiticity of the Dirac-type Yukawa couplings: f, g, $\mathcal{F}$ and $\mathcal{G}$.

[10] Note that we have already discussed such a coupling in Sec. 3.3.2.

3.5.2 The Pattern of Symmetry Breaking and the Higgs Potential

Here we shall show that with the two scalar triplets and the bidoublet defined in the previous section it is possible to break the gauge group of the LR symmetric model spontaneously down to that of the SM. There are in fact two scales of symmetry breakdown, in contrast with just one in the SM [57]. In the first stage, the neutral component δ_R^0 of Δ_R acquires a VEV v_R. This breaks the gauge symmetry G_{LR} down to the electroweak symmetry[11] $SU(2)_L \times U(1)_Y$, where Y is the usual SM hypercharge determined by

$$\frac{Y}{2} = I_{3R} + \frac{B-L}{2} \,. \tag{3.75}$$

In the second stage the VEVs κ and κ' of the bidoublet and v_L of the scalar triplet Δ_L break the electroweak gauge group down to $U(1)_Q$. Explicitly, we have

$$\langle\phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}, \qquad \langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix} \,. \tag{3.76}$$

We shall assume for simplicity in what follows that all the vacuum expectation values are real.[12] Furthermore note that we are not considering the case where the breaking scale of the discrete parity symmetry is separated from that of $SU(2)_R \times U(1)_{B-L}$. In our case it breaks at the first stage when δ_R^0 acquires a VEV.[13]

There exist an important relation between the VEVs in Eq. (3.76) resulting from the minimisation conditions of the Higgs potential.

Minimising the Higgs Potential The most general renormalizable expression for the Higgs potential $\mathcal{V}$ satisfying the gauge symmetry G_{LR} and the discrete parity transformation defined in Eq. (3.73), which one can write down, is given in App. C. The minimisation procedure, as described in more detail in Ref. [56], is as follows. We insert the VEVs of the Higgs particles (cf. Eq. (3.76)) into Eq. (C.2) and obtain

$$\begin{aligned}\mathcal{V}(\kappa,\kappa',v_L,v_R) = &-\mu_1^2\left(\kappa^2+\kappa'^2\right) - 4\mu_2^2\kappa\kappa' - \mu_3^2\left(v_L^2+v_R^2\right) \\ &+ \lambda_1\left(\kappa^2+\kappa'^2\right)^2 + (8\lambda_2+4\lambda_3)\,\kappa^2\kappa'^2 + 4\lambda_4\kappa\kappa'\left(\kappa^2+\kappa'^2\right) \\ &+ \rho_1\left(v_L^4+v_R^4\right) + \rho_3 v_L^2 v_R^2 \\ &+ \left[\alpha_1\left(\kappa^2+\kappa'^2\right) + 2\left(\alpha_2+\alpha_2^*\right)\kappa\kappa' + \alpha_3\kappa'^2\right]\left(v_L^2+v_R^2\right) \\ &+ 2\left[\beta_1\kappa\kappa' + \beta_2\kappa^2 + \beta_3\kappa'^2\right] v_L v_R \,. \end{aligned} \tag{3.77}$$

Then, using the extremising conditions $\partial\mathcal{V}/\partial v_L = \partial\mathcal{V}/\partial v_R = 0$, the following relation between the VEVs result:

$$0 = v_R\frac{\partial\mathcal{V}}{\partial v_L} - v_L\frac{\partial\mathcal{V}}{\partial v_R} = -2\left(v_L^2 - v_R^2\right)\left[\beta_2\kappa^2 + \beta_1\kappa\kappa' + \beta_3\kappa'^2 + (\rho_3 - 2\rho_1)v_L v_R\right] \,. \tag{3.78}$$

The two possible solutions to this equation are

[11] We are not interested in strong interactions and therefore ignore the QCD gauge group $SU(3)_C$ in our formula.

[12] This can always made true for a proper range of free parameters of the potential $\mathcal{V}$ (cf. App. C).

[13] The other case, however, can be achieved by introducing a parity odd neutral scalar field with a non-zero VEV (see, e.g., Ref. [57]).

- $v_L \neq v_R$,
- $v_L^2 = v_R^2$.

We will focus on the first possibility because, under a choice of parameters of the Lagrangian, this solution will give a minimum of $\mathcal{V}$ [56]. By defining the parameter

$$\gamma \equiv \frac{\beta_2 \kappa^2 + \beta_1 \kappa \kappa' + \beta_3 \kappa'^2}{(2\rho_1 - \rho_3)\kappa_+^2} \quad \text{with} \quad \kappa_+^2 \equiv \kappa^2 + \kappa'^2 \, , \tag{3.79}$$

we get out of Eq. (3.78) the VEV see-saw constraint [56]:

$$v_L = \gamma \frac{\kappa_+^2}{v_R} \, . \tag{3.80}$$

This expression relates the widely varying VEV scales. In fact, because κ_+ contains the VEVs which break the SM symmetry group down to $U(1)_Q$, one expects that it is of electroweak scale ($\sim \mathcal{O}(200)$ GeV). Furthermore, the parameters of the Higgs potential, β_i and ρ_i, are naturally[14] assumed to be of order unity so that by definition $\gamma \sim \mathcal{O}(1)$. The VEV v_R breaks parity and thus should be of high scale beyond the electroweak one. As a consequence, Eq. (3.80) enforces $v_L \ll \kappa_+$.

This VEV see-saw will play an important role in the following section, where the type II see-saw, generating the neutrino masses in the LR symmetric model is described.

3.5.3 Neutrino Masses

In the presented LR symmetric model, the type II see-saw is realised in a fairly natural way. To find the mass terms for the neutrinos that arise due to SSB, we need to consider the Yukawa terms that couple the lepton fields Ψ to the Higgs particles $\Delta_{L,R}$ and ϕ. We already know these expressions from Sec. 3.5.1. Let us see what happens when the gauge symmetry is broken the two steps (cf. Sec. 3.5.2) down to $U(1)_Q$. For convenience, we will work in what follows with one lepton generation. To identify the mass contributions, we insert the VEVs for the Higgs fields (cf. Eq. (3.76)). At the first stage the neutral component of Δ_R acquires a VEV v_R so that the relevant term $h_M \overline{\Psi_R^c} \varepsilon \Delta_R \Psi_R + \text{h.c.}$ in Eq. (3.71) reduces to (cf. procedure for the Higgs triplet in Sec. 3.3.2)

$$\mathscr{L}_m^{\text{M},R} = h_M v_R \overline{N_R^c} N_R + \text{h.c.} \, . \tag{3.81}$$

At the second stage the VEVs $\langle \phi \rangle$ and $\langle \Delta_L \rangle$ break the SM gauge group down to the electromagnetic one so that the residual expression $h_M \overline{\Psi_L^c} \varepsilon \Delta_L \Psi_L + \text{h.c.}$ in Eq. (3.71) passes with $\langle \Delta_L \rangle$ into a Majorana mass term for the left-handed chiral neutrino field

$$\mathscr{L}_m^{\text{M},L} = h_M v_L \overline{\nu_L^c} \nu_L + \text{h.c.} \, . \tag{3.82}$$

Due to $\phi \xrightarrow{\text{SSB}} \langle \phi \rangle$, also Dirac mass terms result below the electroweak scale. The first line in Eq. (3.68) leads to the charged lepton mass term $\mathscr{L}_m^{\text{D},l}$ and the neutrino mass term $\mathscr{L}_m^{\text{D}}$,

$$\mathscr{L}_m^{\text{D},l} = (f\kappa' + g\kappa)\overline{l_L} l_R + \text{h.c.} \, , \qquad \mathscr{L}_m^{\text{D}} = (f\kappa + g\kappa')\overline{\nu_L} N_R + \text{h.c.} \, , \tag{3.83}$$

[14] Smaller values are possible but require fine-tuning and much bigger values would violate unitarity and lead to a non-perturbative theory [56].

whereas the second line produces Dirac masses for the quarks. To summarize, we have the following neutrino mass terms after two stages of SSB:

$$\begin{aligned} -\mathscr{L}_m^{\mathrm{D+M}} &= \mathscr{L}_m^{\mathrm{M},R} + \mathscr{L}_m^{\mathrm{M},L} + \mathscr{L}_m^{\mathrm{D}} \\ &= \frac{1}{2} M_R \overline{N_R^c} N_R + \frac{1}{2} M_L \overline{\nu_L^c} \nu_L + m_D \overline{\nu_L} N_R + \text{h.c.}\,. \end{aligned} \tag{3.84}$$

with

$$M_{L,R} \equiv 2\, h_M v_{L,R}\,, \quad \text{and} \quad m_D \equiv h_D \kappa_+ \equiv f\kappa + g\kappa'\,. \tag{3.85}$$

This Lagrangian, already covered in Sec. 3.1, can be written in the convenient matrix form

$$\mathscr{L}_m^{\mathrm{D+M}} = -\frac{1}{2} \overline{\mathcal{N}_L^c} \mathcal{M} \mathcal{N}_L + \text{h.c.}\,, \tag{3.86}$$

with $\mathcal{N}_L$ defined in Eq. (3.2), and

$$\mathcal{M} = \begin{pmatrix} M_L & m_D \\ m_D & M_R \end{pmatrix} = \begin{pmatrix} 2h_M v_L & h_D \kappa_+ \\ h_D \kappa_+ & 2h_M v_R \end{pmatrix}\,, \tag{3.87}$$

From previous discussions, we know that $v_L \ll \kappa_+ \ll v_R$, which implies that the type II see-saw scenario, i.e. Eq. (3.16), is realised. Thus, from Eq. (3.18), we get the masses

$$\begin{aligned} m_1 &= M_L - \frac{m_D}{M_R} = 2h_M v_L - \frac{(h_D \kappa_+)^2}{2h_M v_R}\,, \\ m_2 &= M_R = 2h_M v_R\,, \end{aligned} \tag{3.88}$$

where m_1 corresponds to the light active neutrino $\nu_{1L} \simeq -i\nu_L$ and m_2 to the heavy right-handed neutrino $\nu_{2L}^c \simeq N_R$.

Note that this model is an example where M_L is suppressed by a new scale $\Lambda \sim v_R$ beyond the SM (see discussion in Sec. 3.2.2)—the scale of parity breakdown. In fact, because of the VEV see-saw relation (3.80), we have

$$m_1 = 2h_M \gamma \frac{\kappa_+^2}{v_R} - \frac{(h_D \kappa_+)^2}{2h_M v_R} = \left(2h_M \gamma - \frac{h_D^2}{2h_M}\right) \frac{\kappa_+^2}{v_R} \propto \frac{1}{v_R}\,, \tag{3.89}$$

creating the pleasant situation that if $v_R \to \infty$ the left-handed neutrino mass vanishes. As a consequence, a massless neutrino and a maximally parity-violating weak Lagrangian seem to go hand in hand in the LR symmetric model [58].

In the next section we outline the derivation of the boson masses. We will discover that the mass of a new right-handed $SU(2)_R$ gauge boson, denoted by W_R goes with v_R, so that its $(V + A)$ charged-current interaction disappears in the limit $v_R \to \infty$. This confirms the latter statement.

3.5.4 Boson Masses

The gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ has 3 generators of each $SU(2)$ and one generator of $U(1)$, so that there must be in total 7 gauge bosons in the LR symmetric theory. As in the SM, these bosons get their masses through the Higgs mechanism from the kinetic terms of the scalars. Remember that our scalar sector consists of one bidoublet and one set of left-right symmetric lepton-number carrying triplets. The relevant kinetic part of the Lagrangian is given by

$$\mathscr{L}_{\text{kin}} \supset \mathscr{L}_{\text{kin}}^{\Delta_{R,L},\phi} = \text{Tr}(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L) + \text{Tr}(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R) + \text{Tr}(D_\mu \phi)^\dagger (D^\mu \phi) \; , \quad (3.90)$$

with the covariant derivatives

$$D_\mu \Delta_{L,R} = \partial_\mu \Delta_{L,R} + \frac{1}{2} ig \left[\vec{W}_{L,R} \cdot \vec{\sigma}, \Delta_{L,R} \right] + \frac{1}{2} ig' B \Delta_{L,R} \; , \quad (3.91)$$

$$D_\mu \phi = \partial_\mu \phi + \frac{1}{2} ig \left(\vec{W}_L \cdot \vec{\sigma} \phi - \phi \vec{\sigma} \cdot \vec{W}_R \right) \; . \quad (3.92)$$

The charged gauge bosons are $W^1_{L,R}$ and $W^2_{L,R}$, and the three uncharged are $W^3_{L,R}$ and B.

Expanding the scalar fields about the vacuum (see Eq. (3.76)), one obtains out of Eq. (3.90) the mass terms

$$-\mathscr{L} \supset \begin{pmatrix} W_L^+, & W_R^+ \end{pmatrix} \mathbf{M}_c^2 \begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix} + \frac{1}{2} \begin{pmatrix} W_L^3, & W_R^3 & B \end{pmatrix} \mathbf{M}_n^2 \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix} \; , \quad (3.93)$$

where $W^\pm$ are defined by $W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp W^2)$ and

$$\mathbf{M}_c^2 = \frac{g^2}{2} \begin{pmatrix} (2v_L^2 + \kappa_+^2) & -2\kappa\kappa' \\ -2\kappa\kappa' & (2v_R^2 + \kappa_+^2) \end{pmatrix} \; , \quad (3.94)$$

$$\mathbf{M}_n^2 = \begin{pmatrix} \frac{g^2}{2}(\kappa_+^2 + 4v_L^2) & -\frac{g^2}{2}\kappa_+^2 & 2gg'v_L^2 \\ -\frac{g^2}{2}\kappa_+^2 & \frac{g^2}{2}(\kappa_+^2 + 4v_R^2) & 2gg'v_R^2 \\ 2gg'v_L^2 & 2gg'v_R^2 & g'^2(v_R^2 + v_L^2) \end{pmatrix} \; . \quad (3.95)$$

Taking into account the phenomenological requirement $v_L \ll \kappa_+ \ll v_R$, the eigenvalues of matrix (3.94) lead to the following charged gauge boson masses

$$m_{W_1}^2 \simeq \frac{1}{2} g^2 \kappa_+^2, \quad \text{and} \quad m_{W_2}^2 \simeq \frac{1}{2} g \kappa_+^2 + g^2 v_R^2 \; . \quad (3.96)$$

The corresponding physical eigenstates (i.e. the mass eigenstates)

$$\begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix} = \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix} \; , \quad (3.97)$$

are mixtures of the flavour states W_L and W_R, with mixing angle ζ:

$$\tan 2\zeta = -\frac{2\kappa\kappa'}{v_R^2 - v_L^2} \; . \quad (3.98)$$

From the experimental values [59] of $g \simeq 0.65$ (at the Z pole) and $m_{W_1} = m_W \simeq 80.4$ GeV, we find with Eq. (3.96)

$$\kappa_+ \simeq 174 \text{ GeV} \; , \quad (3.99)$$

which confirms our previous assumption: $\kappa_+ \sim \mathcal{O}(100-200)$ GeV. Furthermore note that in our framework $\zeta \ll 1$ so that $W_1 \simeq W_L$, and $W_2 \simeq W_R$.

To obtain the neutral eigenstates of the mass matrix (3.95), it is convenient to work in the basis ($g\sin\theta_W \equiv e$) [57]:

$$A = \sin\theta_W(W_L^3 + W_R^3) + \sqrt{\cos 2\theta_W}B \, , \tag{3.100}$$

$$Z_L = \cos\theta_W W_L^3 - \sin\theta_W \tan\theta_W W_R^3 - \tan\theta_W\sqrt{\cos 2\theta_W}B \, , \tag{3.101}$$

$$Z_R = \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W}W_R^3 - \tan\theta_W B \, . \tag{3.102}$$

Here A is the massless photon which corresponds to the unbroken $U(1)_Q$. The two massive neutral gauge bosons denoted by Z_1 and Z_2 are mixtures of the fields Z_L and Z_R:

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} Z_L \\ Z_R \end{pmatrix} \, , \tag{3.103}$$

where

$$\tan 2\xi \simeq \frac{(\cos 2\theta_W)^{3/2}}{2\cos^3\theta_W}\frac{\kappa_+^2}{v_R^2} \, . \tag{3.104}$$

$$M_{Z_L}^2 \simeq \frac{g^2}{2\cos^2\theta_W}(\kappa^2 + \kappa'^2 + 4v_L^2) \, , \tag{3.105}$$

$$M_{Z_R}^2 \simeq 2(g^2 + g'^2)v_R^2 \, . \tag{3.106}$$

These equations are valid if $v_L \ll \kappa_+ \ll v_R$. In this limit ξ is small, and we have $Z_1 \simeq Z_L$ as well as $Z_2 \simeq Z_R$.

Note that if $v_R \to \infty$, only the SM gauge bosons (W_L and Z_L) are detectable. This is also the case in the scalar sector. Without fine-tuned parameters in the Higgs potential all massive Higgs bosons besides the one which corresponds to the SM doublet will have a mass of order v_R [56].

Let us conclude this section with specifying the charged-current Lagrangian in the LR symmetric model. As in the SM the coupling of the gauge bosons to quarks and leptons can be obtained out of the fermion kinetic energy Lagrangian. Prior to symmetry breakdown, we have the following charged-current interaction term

$$-\mathscr{L}_{cc} = \frac{g}{\sqrt{2}}\sum_a \left[W_L^\mu\left(\overline{d_{aL}}\gamma_\mu u_{aL} + \overline{l_{aL}}\gamma_\mu \nu_{aL}\right) + W_R^\mu\left(\overline{d_{aR}}\gamma_\mu u_{aR} + \overline{l_{aR}}\gamma_\mu N_{aR}\right)\right] + \text{h.c.} \, . \tag{3.107}$$

Because $M_{W_R} \gg M_{W_L}$, all low-energy weak processes appear the same as in the $SU(2)_L \times U(1)_Y$ model, with small corrections proportional to $M_{W_L}^2/M_{W_R}^2$, undetectable in experiments performed to date. These contributions of the right-handed boson W_R vanish if $M_{W_R} \simeq M_2 \propto v_R \to \infty$ such that weak interactions become pure $(V - A)$ type and, in addition, zero active neutrino masses result ($m_1 \propto \mathcal{O}(1/M_{W_R})$, cf. Eq. (3.89)).

Chapter 4

Cosmology and Dark Matter

Contents

4.1 Fundamentals of Standard Cosmology

4.1.1 Homogenity and Isotropy—The Robertson-Walker Metric

Our Universe appears to be homogeneous and isotropic around us on scales of more than 100 megaparsecs, so that on this scale the density of galaxies is very smooth, and all directions from us appear to be equivalent. From these observations, one is led to the cornerstone of cosmology—the "Cosmological Principle", which states that the Universe looks the same from all positions in space, i.e. there is no privileged point in the Universe playing a particular role. One of the best indications which supports this principle comes from the astonishing uniform temperature corresponding to the perfect black-body spectrum of the cosmic microwave background (CMB) radiation arriving us from different parts of the sky [60, 61].

We may therefore approximate the Universe as a spatially homogeneous and isotropic three-dimensional space, which may expand or contract as a function of time. The metric, which describes a Universe of this type, is necessarily of the Robertson-Walker (RW) form with a space-time interval given by [62]

$$ds^2 = dt^2 - R^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right\} . \tag{4.1}$$

The variables (t, r, θ, ϕ) are comoving coordinates, which are carried along with the expansion or contraction of the Universe. The coordinate t in Eq. (4.1) is the time measured by an observer at rest in the comoving frame, i.e. $(r, \theta, \phi) = \text{const}$. The cosmic scale factor is denoted by $R(t)$ and the constant k describes the curvature. There are three possibilities—negative, zero or positive k—which correspond to the three possible spatial geometries—hyperbolic, flat and spherical, respectively.

A particular useful quantity, which can be obtained from the cosmic scale factor, is the Hubble parameter

$$H \equiv \frac{\dot{R}(t)}{R(t)} \,. \tag{4.2}$$

It relates the velocity v with which the most distant galaxies are receding from us to their distance d from us via

$$v = Hd \,. \tag{4.3}$$

This law was discovered by Edwin Hubble in 1929 (cf. Ref. [63]) and has been verified to high accuracy by modern experimental methods. The present value of H is measured to be positive so that the Universe must be in an expanding phase. Usually it is written as

$$H_0 = 100\, h \, \frac{\text{km Mpc}}{\text{s}} \,, \tag{4.4}$$

with the normalized dimensionless Hubble constant h, which is, for example, known with about 10% precision from the Hubble Space Telescope Key Project [64]:

$$h = 0.72 \pm 0.08 \,. \tag{4.5}$$

This value is consistent with results from independent methods such as the global fitting of CMB data by the Wilkinson Microwave Anisotropy Probe (WMAP) collaboration (see, e.g., Ref. [3]).

4.1.2 The Einstein Equations

The RW metric is a purely kinematic consequence of requiring an isotropic and homogeneous Universe. In this section we shall study the dynamics in the form of differential equations governing the evolution of the scale factor $R(t)$. These equations follow from the fundamental equations of general relativity—the Einstein field equations (see, e.g., Ref. [62])

$$\mathcal{G}_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R} g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} \,, \tag{4.6}$$

where $\mathcal{G}_{\mu\nu}$ is the Einstein tensor, G Newton's gravitational constant, $T_{\mu\nu}$ the stress-energy tensor and Λ is the cosmological constant. To determine the Ricci tensor[1]

$$\mathcal{R}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\nu}\Gamma^{\sigma}_{\lambda\sigma} - \Gamma^{\sigma}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} = \mathcal{R}_{\nu\mu} \,, \tag{4.7}$$

and the Ricci scalar

$$\mathcal{R} \equiv g^{\mu\nu}\mathcal{R}_{\mu\nu} \,, \tag{4.8}$$

we have to use the components of the RW metric $g_{\mu\nu}$ and the Christoffel symbols $\Gamma^{\lambda}_{\mu\nu}$.

[1] with "," we denote the partial derivative with respect to the following component, and with ";" we will denote the covariant derivative with respect to the following component.

Comparing Eq. (4.1) with $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ and identifying $(x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)$, we obtain

$$g_{00} = 1 \,, \qquad g_{11} = -R^2/(1-kr^2) \,,$$
$$g_{22} = -R^2 r^2 \,, \qquad g_{33} = -R^2 r^2 \sin^2\theta \,. \tag{4.9}$$

Furthermore, out of $g_{\alpha\nu}g^{\nu\beta} = \delta^\beta_\alpha$, it follows $g^{\mu\nu} = 1/g_{\mu\nu}$. The non-zero Christoffel symbols are then related through $\Gamma^\alpha_{\beta\gamma} \equiv 1/2\, g^{\alpha\varepsilon}(g_{\varepsilon\beta,\gamma} + g_{\alpha\gamma,\beta} - g_{\beta\gamma,\varepsilon}) = \Gamma^\alpha_{\gamma\beta}$. Substituting the derived expressions back into that of the Ricci tensor, Eq. (4.7), leads to $\mathcal{R}_{\mu\nu} = 0$ if $\mu \neq \nu$, and

$$\mathcal{R}_{00} = -3\frac{\ddot{R}}{R} \,, \tag{4.10}$$

$$\mathcal{R}_{11} = \frac{R\ddot{R} - 2\dot{R}^2 + 2k}{1-kr^2} \,, \tag{4.11}$$

$$\mathcal{R}_{22} = r^2(R\ddot{R} + 2\dot{R}^2 + 2k) \,, \tag{4.12}$$

$$\mathcal{R}_{33} = r^2 \sin^2\theta (R\dot{R} + 2\dot{R}^2 + 2k) \,. \tag{4.13}$$

Thus, the Ricci scalar in the RW metric is

$$\mathcal{R} = -6\left(\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2}\right) \,. \tag{4.14}$$

4.1.3 Stress-Energy Tensor and the Fluid Equation

The stress-energy (or energy-momentum) tensor $T_{\mu\nu}$ in the Einstein equations (4.6), describes the density and flows of the 4-momentum $(E, -p_1, -p_2, -p_3)$. To be consistent with the symmetry of the RW metric, this tensor must be diagonal, and by isotropy the spatial components must be equal. The simplest realisation of such a tensor is that of a perfect fluid, i.e. a fluid that has no heat conduction or viscosity. It is fully parametrized by its mass density ρ, the 4-velocity u^μ and the pressure p (see, e.g., Ref. [65]):

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - p g^{\mu\nu} \,. \tag{4.15}$$

For a comoving observer, the 4-velocity is given by $u = (1, 0, 0, 0)$, so that the stress-energy tensor reduces with Eq. (4.9) to

$$T^{00} = \rho \,, \qquad T^{11} = p\frac{(1-kr^2)}{R^2} \,,$$
$$T^{22} = \frac{p}{(rR)^2} \,, \qquad T^{33} = \frac{p}{(rR\sin\theta)^2} \,. \tag{4.16}$$

The conservation of energy and momentum in a general metric is simply incorporated by

$$T^{\mu\nu}{}_{;\nu} = 0 \,. \tag{4.17}$$

In special relativity, for example, the flat space-time is described by the Minkowski metric which leads together with Eq. (4.17) to the well-known law

$$\frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0 \,. \tag{4.18}$$

Equation (4.17) can be used to determine how the components of the energy-momentum tensor evolve with time. The covariant derivative of the tensor $T^{\mu\nu}$ is

$$T^{\mu\nu}{}_{;\nu} = T^{\mu\nu}{}_{,\nu} + \Gamma^{\mu}_{\nu\sigma} T^{\sigma\nu} + \Gamma^{\nu}_{\nu\sigma} T^{\mu\sigma} = 0 \, . \tag{4.19}$$

Inserting the Christoffel symbols and using the expressions (4.16), the only non-trivial expression we obtain is that with $\mu = 0$:

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0 \, . \tag{4.20}$$

This relation, called fluid equation, allows to determine the energy (mass) density evolution of some material. As we see, there are contributions from two terms. The first one in the bracket describes the dilution in case of increasing volume, whereas the second one corresponds to the loss of energy due to work done by the pressure of some material during the expansion of the Universe.

4.1.4 The Friedmann Equation

What can we learn from the Einstein equations? The $0-0$ component of Eq. (4.6) gives precisely the famous Friedmann equation, which governs the time evolution of the scale factor $R(t)$:

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \, . \tag{4.21}$$

The $i-i$ component gives

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = -8\pi G p + \Lambda \, . \tag{4.22}$$

Subtracting the Friedmann equation from it leads to the acceleration equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \, . \tag{4.23}$$

Out of the first term on the right-hand side, we can learn that if material has any pressure, this will increase the gravitational force, and thus decelerate the expansion. Furthermore, we want to mention that the fluid equation derived previously (cf. Eq. (4.20)) is a consequence of Eq. (4.21) and (4.22). It can be obtained by solving for $\dot{\rho}$ in Eq. (4.21) and eliminating $\ddot{R}$ with the help of Eq. (4.22). This is because general relativity automatically encodes the energy conservation, namely $T^{\mu\nu}{}_{;\nu} = 0$ (cf. Eq. (4.17)), since the Einstein tensor in Eq. (4.6) satisfies the Bianchi identities [65]:

$$\mathcal{G}^{\mu\nu}{}_{;\nu} = 0 \, . \tag{4.24}$$

4.2 The History of the Universe

4.2.1 Cosmological Scenarios

In order to solve Eqs. (4.20) and (4.21), we need to classify the various types of matter that could give a possible contribution to the dynamics of the Universe according to their pressure-to-density ratio (also called equation of state). There are three classes distinguished in cosmology:

MATTER

It refers to any type of material in the Universe which is pressureless and has non-relativistic velocity, i.e.

$$v_M \ll c\,, \quad p_M = 0\,. \tag{4.25}$$

By applying the fluid equation (4.20), we derive

$$\dot{\rho}_M = -3\frac{\dot{R}}{R}\rho_M \quad \Rightarrow \quad \rho_M \propto \frac{1}{R^3}\,. \tag{4.26}$$

Examples are stars, planets, clouds of gas or galaxies seen as a whole.

RADIATION

It refers to ultra-relativistic particles. This includes photons, and other light particles like for example neutrinos travelling nearly at the speed of light. A calculation based on statistical mechanics gives the interesting result that the pressure of a relativistic gas is related to its energy density. We have the characteristics

$$v_R \simeq c\,, \quad p_R = \frac{\rho}{3}\,. \tag{4.27}$$

For this type of particles the evolution of the energy density is (use Eq. (4.20))

$$\dot{\rho}_R = -3\frac{\dot{R}}{R}\left(1+\frac{1}{3}\right)\rho_R = -4\frac{\dot{R}}{R}\rho_R \quad \Rightarrow \quad \rho_R \propto \frac{1}{R^4}\,. \tag{4.28}$$

COSMOLOGICAL CONSTANT Λ

This quantity appears in the last term of the Einstein equations (cf. Eq. (4.6)) and corresponds to the observed huge "*Dark Energy*" fraction in the Universe. Physically it is equivalent to vacuum energy in quantum field theories even though its nature is still a mystery to physicists. Proposals are based on a constant energy density filling space homogeneously, or quantum fields, such as quintessence or moduli, being dynamic quantities [66].

Describing Λ as if it were a fluid with constant density ρ_Λ and pressure p_Λ, we obtain

$$\rho_\Lambda = \text{const.} \quad \Rightarrow \quad p_\Lambda = -\rho_\Lambda\,. \tag{4.29}$$

To study the dynamics of the Universe in the presence of these types of matter, it is convenient to split the density $\rho = \rho_R + \rho_M$ into its radiation and matter part and write the Friedmann equation in the form

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho_R + \frac{8\pi G}{3}\rho_M - \frac{k}{R^2} + \frac{\Lambda}{3}\,, \tag{4.30}$$

Note that we chose the right-hand side in the order: *radiation*, *matter*, *spatial curvature*, *cosmological constant*. This is because of its scaling with $R(t)$. In fact, the terms evolve with respect to the scale factor as R^{-4}, R^{-3}, R^{-2} and R^0. If R keeps growing, and if we assume that all terms on the right-hand side are present, there is a chance that they all dominate the Universe expansion one after each other. In the following we study these four possible stages. For each scenario we provide a solution of the Friedmann equation, using Eqs. (4.26), (4.28) and (4.29)

I **Domination of *radiation*:**

$$\rho_R \propto R^{-4} \quad \Rightarrow \quad \left(\frac{\dot{R}}{R}\right)^2 \propto R^{-4}; \quad R(t) \propto t^{1/2}; \quad H(t) = \frac{1}{2t} . \tag{4.31}$$

In this scenario, the Universe expands forever, but the rate of expansion, described by $H(t)$, decreases with time.

II **Domination of *matter*:**

$$\rho_M \propto R^{-3} \quad \Rightarrow \quad \left(\frac{\dot{R}}{R}\right)^2 \propto R^{-3}; \quad R(t) \propto t^{2/3}; \quad H(t) = \frac{2}{3t} . \tag{4.32}$$

Here the expansion is faster than in the radiation dominated scenario, because the pressure is missing, which would supply an extra deceleration (see Eq. (4.23)).

III **Domination of *curvature*:** In the case of negative curvature ($k < 0$), we obtain

$$k < 0 \quad \Rightarrow \quad \left(\frac{\dot{R}}{R}\right)^2 \propto R^{-2}; \quad R(t) \propto t; \quad H(t) = \frac{1}{t} . \tag{4.33}$$

Such an open Universe, dominated by its curvature expands linearly.

In the case of positive curvature ($k > 0$), and for absent cosmological constant Λ, the right-hand side of the Friedmann equation goes to zero so that the expansion of such a closed Universe stops. Afterwards, H decreases and the Universe collapses.

IV **Domination of the *cosmological constant*:**

$$\Lambda = \text{const.} \quad \Rightarrow \quad \left(\frac{\dot{R}}{R}\right)^2 \propto R^0; \quad R(t) \propto \exp\left(\sqrt{\Lambda/3}\, t\right); \quad H = \sqrt{\Lambda/3} . \tag{4.34}$$

If the cosmological term never decays into matter or radiation, the Universe will end up in an exponentially accelerated expansion.

4.2.2 The Matter Composition—Evolution of the Universe

To determine the past and the future evolution of our Universe, we have to measure besides the present Hubble parameter the current density of radiation, matter and Λ. For this purpose let us consider another convenient form of the Friedmann equation. If we evaluate Eq. (4.30) today and denote the values corresponding to this moment with a subscript "0", we arrive at

$$1 = \frac{8\pi G}{3H_0}\rho_{R0} + \frac{8\pi G}{3H_0}\rho_{M0} - \frac{k}{R^2 H_0} + \frac{\Lambda}{3H_0} . \tag{4.35}$$

By using the definitions

$$\Omega_R \equiv \frac{8\pi G}{3H_0}\rho_{R0}, \qquad \Omega_k \equiv \frac{k}{R^2 H_0},$$
$$\Omega_M \equiv \frac{8\pi G}{3H_0}\rho_{M0}, \qquad \Omega_\Lambda \equiv \frac{\Lambda}{3H_0} , \tag{4.36}$$

Description	Symbol	Value
Hubble constant	h	0.705 ± 0.013
Total density	Ω_0	$1.0050^{+0.0060}_{-0.0061}$
Dark Energy density	Ω_Λ	0.726 ± 0.015
Matter density	$\Omega_M h^2$	$0.1358^{+0.0037}_{-0.0036}$
Baryon density	$\Omega_b h^2$	$0.02267^{+0.00058}_{-0.00059}$
Dark Matter density	$\Omega_{\rm DM} h^2$	0.1131 ± 0.0034

Table 4.1: Cosmological parameters of combined fit WMAP+SN+BAO (with 68% CL uncertainties) [3, 67].

it follows

$$1 = \Omega_R + \Omega_M - \Omega_k + \Omega_\Lambda \ . \tag{4.37}$$

We deduce that the Universe is flat provided that

$$\Omega_0 \equiv \Omega_R + \Omega_M + \Omega_\Lambda = 1 \ . \tag{4.38}$$

In such a case the sum of the densities of radiation, matter and Λ is equal at *any* time to the critical density

$$\rho_c(t) \equiv \frac{3H(t)}{8\pi G} \ . \tag{4.39}$$

Note that Ω_x with $x \in \{R, M, \Lambda\}$ is usually defined by the ratio of the present (or observed) density ρ_{x0} to the critical density ρ_{c0} of the Friedmann Universe, where $\rho_{\Lambda 0} = \rho_\Lambda = \Lambda/(8\pi G)$. These density parameters represent, together with the Hubble constant $H(t)$, the four *cosmological parameters*, which entirely describe the evolution of the Universe.

One of the main tasks of cosmology was to measure these quantities. Table 4.1 summarizes the recent experimental results obtained by using WMAP data combined with measurements of Type Ia supernovae (SN) and Baryon Acoustic Oscillations (BAO) in the galaxy distribution (see Ref. [3, 67] and references therein). They astonishingly satisfy relation (4.38) leading to a zero or negligible small curvature parameter k and thus to a flat Universe. Furthermore, the data shows, that our present Universe is composed out of two components—nearly 30% matter and 70% Dark Energy. Therefore, according to the previous discussion, we can expect that its evolution will end up in an accelerated expansion driven by the non-zero cosmological constant. In fact, one can easily show (see, e.g., Ref. [65]) that if our approach is correct we would be at the very beginning of the Dark Energy dominated epoch, leaving behind a matter dominated and eventually an early radiation dominated era such that the expansion history of the Universe would follow:

$$\textit{Radiation dom.} \dashrightarrow \textit{Matter dom.} \dashrightarrow \textit{Dark Energy dom. (today and future)} \ . \tag{4.40}$$

To conclude, note that the results in Tab. 4.1 reveal a clear discrepancy between the two density parameters Ω_M and Ω_b, being evidence that most of the matter in our Universe is non-baryonic. The huge fraction given by $\Omega_M - \Omega_b$ must be some kind of so-far unknown non-luminous and non-baryonic material, referred to as Dark Matter (DM). In chapter

5 we will propose the sterile neutrino as a viable DM candidate. To be able to analyse its cosmological properties, especially its production mechanism in the early Universe, we will outline in what follows some important facts of thermodynamics.

4.3 Thermodynamics in the early Universe

If we follow the sequence in Eq. (4.40) backwards, we will cross a point where the scale factor $R(t)$ becomes so small that due to the large density and rapid interactions the particles of matter and radiation are in local thermal equilibrium. As we will see, this can be maintained as long as the interaction rate Γ dominates H, the expansion rate of the Universe. Roughly speaking, a particle species in the early Universe has to interact sufficiently or it decouples from the thermal bath.

In this section we quickly review some basic equilibrium thermodynamics, describe, using the Boltzmann equation, a departure from thermal equilibrium—the freeze-out from thermal bath, and finally present a further non-equilibrium process, which can generate entropy during the expansion of the Universe.

4.3.1 Equilibrium Thermodynamics

Let us consider a particle species χ with number density n_χ, energy density ρ_χ and pressure p_χ. We are interested in the early hot Universe where these particles are assumed to be in thermal equilibrium. Statistical mechanics [68] then predicts

$$n_\chi = \frac{g_\chi}{(2\pi)^3} \int f_\chi(\vec{p}) \mathrm{d}^3 p \,, \tag{4.41a}$$

$$\rho_\chi = \frac{g_\chi}{(2\pi)^3} \int E_\chi f_\chi(\vec{p}) \mathrm{d}^3 p \,, \tag{4.41b}$$

$$p_\chi = \frac{g_\chi}{(2\pi)^3} \int \frac{p^2}{3E_\chi(\vec{p})} f_\chi(\vec{p}) \mathrm{d}^3 p \,. \tag{4.41c}$$

The quantity g_χ is the number of internal degrees of freedom (spin states of particle χ), and $E_\chi(\vec{p}) = (p^2 + m_\chi^2)^{1/2}$ with $p = |\vec{p}|$ is the energy. The statistical distribution function $f_\chi(\vec{p})$ is given by the familiar Fermi-Dirac or Bose-Einstein distribution,

$$f_\chi(\vec{p}) = \frac{1}{e^{(E_\chi - \mu_\chi)/T_\chi} \pm 1} \,, \tag{4.42}$$

where $+$ accounts for fermions and $-$ for bosons. This equilibrium distribution has two parameters, the temperature T_χ, and the chemical potential μ_χ.

We evaluate the integrals in Eqs. (4.41) in the following useful limits:

Relativistic Limit For $T_\chi \gg m_\chi$ and $m_\chi \gg \mu_\chi$, we have

$$f_\chi(\vec{p}) \simeq \frac{1}{e^{p/T_\chi} \pm 1} \,, \tag{4.43}$$

so that

$$n_\chi \simeq \begin{cases} \frac{3}{4} g_\chi \frac{\zeta(3)}{\pi^2} T_\chi^3 & (\chi = \text{fermion}) , \\ g_\chi \frac{\zeta(3)}{\pi^2} T_\chi^3 & (\chi = \text{boson}) , \end{cases} \tag{4.44a}$$

$$\rho_\chi \simeq \begin{cases} \frac{7}{8} g_\chi \frac{\pi^2}{30} T_\chi^4 & (\chi = \text{fermion}) , \\ g_\chi \frac{\pi^2}{30} T_\chi^4 & (\chi = \text{boson}) , \end{cases} \tag{4.44b}$$

$$p_\chi \simeq \frac{\rho_\chi}{3} , \tag{4.44c}$$

where $\zeta(3) \simeq 1.202$ is the Riemann zeta function of 3. The average energy is given by

$$\langle E_\chi \rangle \equiv \frac{\rho_\chi}{n_\chi} \simeq \begin{cases} \frac{7\pi^4}{180\zeta(3)} T_\chi & (\chi = \text{fermion}) , \\ \frac{\pi^4}{30\zeta(3)} T_\chi & (\chi = \text{boson}) . \end{cases} \tag{4.45}$$

Note that we already encountered Eq. (4.44c) in the definition of radiation in Sec. 4.2.1.

Non-Relativistic Limit For $m_\chi \gg T_\chi$ and $m_\chi \gg \mu_\chi$ the Bose-Einstein as well as the Fermi-Dirac function can be approximated by a Boltzmann distribution function. We have

$$f_\chi(\vec{p}) \simeq e^{(\mu_\chi - m_\chi)/T_\chi} e^{-p^2/(2m_\chi T_\chi)} , \tag{4.46}$$

for both bosons and fermions. The equations (4.41) lead to

$$n_\chi = g_\chi \left(\frac{m_\chi T_\chi}{2\pi} \right)^{3/2} e^{\frac{\mu_\chi - m_\chi}{T_\chi}} , \tag{4.47a}$$

$$\rho_\chi = m_\chi n_\chi , \tag{4.47b}$$

$$p_\chi \simeq n_\chi T_\chi , \tag{4.47c}$$

valid for non-relativistic bosons and fermions. In this case the average energy is given by

$$\langle E_\chi \rangle \simeq m_\chi + \frac{3}{2} T_\chi . \tag{4.48}$$

Out of Eqs. (4.47), we see that p is negligible small compared to the energy density ρ_χ so that the definition of matter in Sec. 4.2.1 is well-defined.

Since the early Universe was radiation dominated, it is convenient to express its total energy density in terms of the photon temperature T (cf. Eq. (4.44b)):

$$\rho = \frac{\pi^2}{30} g_* T^4 , \tag{4.49}$$

The parameter g_* is then given by (use Eq. (4.41b))

$$g_* = \frac{15}{\pi^4} \sum_\chi g_\chi \left(\frac{T_\chi}{T} \right)^4 \int_{x_\chi}^\infty \frac{(u^2 - x_\chi^2)^{1/2} u^2 \mathrm{d}u}{\exp(u - y_\chi) \pm 1} . \tag{4.50}$$

with $x_\chi \equiv m_\chi / T_\chi$ and $y_\chi \equiv \mu_\chi / T_\chi$. Because of the exponential suppression factor in Eq. (4.47b), the energy density contribution of non-relativistic particles is small compared to that of radiation. In the early radiation dominated Universe it is therefore a good approximation to consider only the relativistic degrees of freedom with $m_\chi \ll T$ so that Eq. (4.50) simplifies to (cf. Eq. (4.44b))

$$g_* \simeq \sum_{\chi=\text{bosons}} g_\chi \left(\frac{T_\chi}{T}\right)^4 + \frac{7}{8} \sum_{\chi=\text{fermions}} g_\chi \left(\frac{T_\chi}{T}\right)^4 , \tag{4.51}$$

Then, using the Friedmann equation and Eq. (4.49), we can write the Hubble parameter in the form

$$H = \left(\frac{8\pi^3 G}{90}\right)^{1/2} \sqrt{g_*}\, T^2 = \frac{2\pi^{3/2}}{3\sqrt{5} m_{\text{Pl}}} \sqrt{g_*}\, T^2 , \tag{4.52}$$

with the Planck mass $m_{\text{Pl}} \equiv G^{-1/2}$.

4.3.2 Entropy

Assuming a sufficiently slow Universe expansion, so that the interaction rates of particles dominated the Hubble expansion rate H (in particular in the early epoch), thermal equilibrium should have been maintained in any local comoving volume element $\mathrm{d}V$. Because, in such a case, there is no net inflow or outflow of energy, the second law of thermodynamics implies the conservation of entropy and thus an adiabatic evolution of the Universe:

$$T\mathrm{d}S = \delta Q = 0 \quad \Rightarrow \quad \mathrm{d}S = 0 , \tag{4.53}$$

In order to derive the energy density and pressure dependence of the entropy S, we make use of the first law of thermodynamics (energy conservation):

$$\delta Q = T\mathrm{d}S = \mathrm{d}(\rho V) + p\mathrm{d}V = \mathrm{d}[(\rho + p)V] - V\mathrm{d}p . \tag{4.54}$$

The quantities ρ and p are the equilibrium energy density and pressure calculated in the previous section. Applying the integrability condition [68]

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T} , \tag{4.55}$$

on Eq. (4.54), we obtain after a trivial integration

$$T\frac{\mathrm{d}p}{\mathrm{d}T} = \rho + p \quad \Leftrightarrow \quad \mathrm{d}p = \frac{\rho + p}{T}\mathrm{d}T . \tag{4.56}$$

With this relation, Eq (4.54) transforms into

$$\mathrm{d}S = \mathrm{d}\left[\frac{(\rho + p)V}{T}\right] , \tag{4.57}$$

so that the constant entropy per comoving volume $V = R^3$ is given by

$$S = \frac{\rho + p}{T} R^3 . \tag{4.58}$$

Because the entropy density $s \equiv S/R^3$ is dominated by the contribution of the relativistic particles, it is also convenient to express this quantity in terms of the photon temperature T. Using Eq. (4.58) together with Eqs. (4.44), we get

$$s = \frac{2\pi^2}{45} g_{*S} T^3 , \tag{4.59}$$

where

$$g_{*S} \simeq \sum_{\chi=\text{bosons}} g_\chi \left(\frac{T_\chi}{T}\right)^3 + \frac{7}{8} \sum_{\chi=\text{fermions}} g_\chi \left(\frac{T_\chi}{T}\right)^3 . \tag{4.60}$$

Here, like in Eq. (4.51), the sum goes over all relativistic species χ.

Note that s is proportional to the number density (4.44a) of relativistic particles, and that g_* equals g_{*S} if $T_\chi = T$. Furthermore, the conservation of S implies that $g_{*S}T^3R^3$ remains constant as the Universe expands, such that

$$T \propto g_{*S}^{-1/3} R^{-1} . \tag{4.61}$$

4.3.3 Freeze-Out—Decoupling from Thermal Bath

Consider a stable particle χ which interacts with some other particle Ψ through some process $\chi\bar{\chi} \leftrightarrow \Psi\bar{\Psi}$. In the early Universe, when the temperature was much higher than m_χ, the creation and annihilation processes of $\chi\bar{\chi}$ were equally efficient. Hence, we expect the abundance of χ to be comparable with that of other species in the hot plasma. However, if we assume these particles to remain in thermal equilibrium indefinitely, this would change dramatically as the Universe expands and its temperature drops below m_χ. The reason is that for $T < m_\chi$ only a small fraction of $\Psi\bar{\Psi}$ pairs has sufficient kinetic energy to create χ particles so that the processes $\Psi\bar{\Psi} \to \chi\bar{\chi}$ become inefficient, whereas the annihilations $\chi\bar{\chi} \to \Psi\bar{\Psi}$ continue unhindered. In fact, from Sec. 4.3.1, we know that in thermal equilibrium the density of non-relativistic particles is Boltzmann suppressed,

$$n_{\chi,\text{EQ}} = g_\chi \left(\frac{m_\chi T_\chi}{2\pi}\right)^{3/2} e^{\frac{\mu_\chi - m_\chi}{T_\chi}} . \tag{4.62}$$

leading quickly to cosmologically irrelevant values as the Universe cools down. However, there are a number of notable departures from thermal equilibrium, which circumvent this situation. For example, one speculates that baryons (we) are present nowadays because of a baryon to antibaryon number asymmetry in the very early Universe [62]. Due to this asymmetry not every baryon found an antibaryon to annihilate with so that a small amount survived until today. Further known departures are the decoupling of the CMB [62], the primordial nucleosynthesis [69] and also on the more speculative side the process of inflation [70, 62].

In this section we will focus on a simple mechanism called freeze-out or decoupling from thermal bath which is capable to explain the survival of a sizeable relic density of weakly interacting massive particles (WIMPs). Because there is strong evidence that DM must be of this kind, this mechanism could explain why this type of matter is representing $\Omega_{\text{DM}} \simeq 0.2$ part of the total energy density of the present Universe. It takes into account that the self-annihilation of the χ's, dominating their creation process at low temperatures, can be contained by the competing effect of the Hubble expansion. This is because the interaction rate Γ varies as the number density times an thermally averaged annihilation cross section, which both are quantities, decreasing with temperature T. Then eventually at some moment of the Universe's expansion, T will be sufficiently low and the χ's are that much diluted that they cease to interact with each other and survive to the present day. This moment of freeze-out is characterised by $\Gamma \simeq H$.

Quantitatively, one describes these competing effects of self-annihilation and Hubble expansion by the Boltzmann equation, given in the form (see, e.g., Ref. [62, 71]):

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}}|v|\rangle \left(n_\chi^2 - n_{\chi,\mathrm{EQ}}^2\right) \equiv \mathfrak{C} \, . \tag{4.63}$$

The quantity n_χ is the number density of WIMPs and $\langle\sigma_{\chi\bar{\chi}}|v|\rangle$ is the thermally averaged $\chi\bar{\chi}$ annihilation cross section multiplied by the relative velocity between the particles. The significance of the terms is manifest: The second term on the left-hand side of this relation describes the dilution of the particle density due to the expansion of the Universe, whereas the collision term $\mathfrak{C}$ on the right-hand side accounts for annihilation and creation processes that change the number of χ's present. Note that in the absence of these interactions, we have the solution $n_\chi \propto R^{-3}$.

It is useful to scale out the effect of the Hubble expansion by considering the number of particles in a comoving volume. Therefore, we define the variable

$$Y_\chi \equiv \frac{n_\chi}{s} \, , \tag{4.64}$$

with s, the entropy density of the Universe. Using Eq. (4.53)

$$\begin{aligned} sR^3 = S = \text{const.} \quad &\Leftrightarrow \quad \dot{s}R^3 + 3R^2\dot{R}s = 0 \\ &\Leftrightarrow \quad \dot{s} = -3Hs \, , \end{aligned} \tag{4.65}$$

we obtain

$$s\dot{Y}_\chi = \dot{n}_\chi - \frac{n_\chi}{s}\dot{s} = \dot{n}_\chi - 3Hn_\chi \, . \tag{4.66}$$

Thus, the Boltzmann equation can be written as

$$s\dot{Y}_\chi = \mathfrak{C} \, . \tag{4.67}$$

Furthermore, we want to change from time to temperature dependence and introduce

$$x \equiv \frac{m}{T} \quad \Rightarrow \quad x\mathrm{d}x = -\frac{m^2}{T^3}\mathrm{d}T \, , \tag{4.68}$$

where m is any convenient mass scale (usually the mass of the particle of interest). During the radiation dominated epoch the temperature T is related to the time t through (cf. Eq. (4.31) & (4.52))

$$t = \frac{1}{2H} = \frac{3\sqrt{5}m_{\mathrm{Pl}}}{4\pi^{3/2}T^2}g_*^{-1/2} = 0.301g_*^{-1/2}\frac{m_{\mathrm{Pl}}}{T^2} \, , \tag{4.69}$$

so that the total differential of t leads together with Eq. (4.68) to

$$\mathrm{d}t = 2t\frac{\mathrm{d}x}{x} = \frac{1}{H}\frac{\mathrm{d}x}{x} \, . \tag{4.70}$$

By using this relation in Eq. (4.67), we can write the Boltzmann equation in the form $(H(m) \equiv x^2 H)$

$$\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = \frac{\mathfrak{C}}{sxH(m)} = -\frac{x\langle\sigma_{\chi\bar{\chi}}|v|\rangle s}{H(m)}\left(Y_\chi^2 - Y_{\chi,\mathrm{EQ}}^2\right) \, , \tag{4.71}$$

or equivalently

$$\frac{x}{Y_{\chi,\mathrm{EQ}}}\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\Gamma_\chi}{H}\left[\left(\frac{Y_\chi}{Y_{\chi,\mathrm{EQ}}}\right)^2 - 1\right] , \tag{4.72}$$

with the interaction rate

$$\Gamma_\chi \equiv n_{\chi,\mathrm{EQ}}\,\langle\sigma_{\chi\bar{\chi}}|v|\rangle \; . \tag{4.73}$$

The equilibrium values $Y_{\chi,\mathrm{EQ}}$ in the relativistic and non-relativistic regimes are (use Eqs. (4.44a), (4.47a) together with Eq. (4.59))

$$Y_{\chi,\mathrm{EQ}}(x) = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{1/2}\frac{g_\chi}{g_{*S}}x^{3/2}\mathrm{e}^{-x} \qquad (\text{non-relativistic}: \quad x \gg 3) , \tag{4.74a}$$

$$Y_{\chi,\mathrm{EQ}}(x) = \frac{45\zeta(3)}{2\pi^4}\frac{g_{\chi,\mathrm{eff}}}{g_{*S}} \qquad (\text{relativistic}: \quad x \ll 3) , \tag{4.74b}$$

where $g_{\chi,\mathrm{eff}}$ equals g_χ for bosons, and $(3/4 g_\chi)$ for fermions.

The Boltzmann equation in the form of Eq. (4.72) is very informative. It shows that the relative change of χ's per comoving volume is controlled by the effectiveness of annihilations, the ratio Γ_χ/H, times a measure of the deviation from thermal equilibrium. Indeed, after decoupling where the annihilation rate drops below the expansion rate ($\Gamma_\chi/H \lesssim 1$), the relative change in particles is

$$-\frac{\Delta Y_\chi}{Y_\chi} \sim -\frac{\frac{\mathrm{d}Y_\chi}{\mathrm{d}x}x}{Y_{\chi,\mathrm{EQ}}} \sim \frac{\Gamma_\chi}{H} \lesssim 1 , \tag{4.75}$$

so that annihilations become ineffective, and the number of χ's in a comoving volume "freezes in".

In what follows we want to give approximate solutions of the Boltzmann equation for particles that are either relativistic or non-relativistic when they freeze-out (decouple) from thermal bath. For this purpose let us assume that $\Gamma_\chi \simeq H$ roughly occurs for $x = x_f \equiv m_\chi/T_f$ where T_f is called freeze-out temperature.

Hot Relics: $x_f \lesssim 3$ In this case the freeze-out happens when the species is still relativistic. We know from Eq. (4.74b) that in this regime $Y_{\chi,\mathrm{EQ}}(x)$ has only a weak temperature dependence through g_{*S}. Thus, its asymptotic value $Y_\infty \equiv Y(x \to \infty)$ is *insensitive* to the precise value of x_f and can simply be approximated by the equilibrium value at freeze-out:

$$Y_\infty = Y_{\chi,\mathrm{EQ}}(x_f) = \frac{45\zeta(3)}{2\pi^4}\frac{g_{\chi,\mathrm{eff}}}{g_{*S}(x_f)} \; . \tag{4.76}$$

Assuming that the expansion of the Universe remains adiabatic, the present number density of such hot relics is just given by (s_0 is the present entropy density)

$$n_{\chi 0} = Y_\infty s_0 \; . \tag{4.77}$$

On the contrary, if the entropy should increase, say by a factor $\mathfrak{S}$, the present number density would be reduced exactly by $\mathfrak{S}$. We will discuss a possible mechanism of entropy production in Sec. 4.3.4.

By using $\rho_{\chi 0} = n_{\chi 0} m_\chi$ together with s_0 and ρ_{c0} from Ref. [36], we derive the present density parameter of the hot relic χ

$$\Omega_{\chi 0}h^2 = 7.61 \times 10^{-2}\left(\frac{m_\chi}{1\ \mathrm{eV}}\right)\frac{g_{\chi,\mathrm{eff}}}{g_{*S}(x_f)} \; . \tag{4.78}$$

Through $\Omega_{\chi 0} \lesssim \Omega_0 \simeq 1$ (cf. Sec. 4.2.2), this formula gives an upper bound to its mass

$$m_\chi \lesssim 13.1 \, \frac{g_{*S}(x_f)}{g_{\chi,\text{eff}}} \text{ eV} . \tag{4.79}$$

As an example, let us calculate the cosmological bound on the mass of the usual light SM neutrinos ($\chi = \nu$). To determine $g_{*S}(x_f)$, we have to calculate the freeze-out temperature T_f. For this purpose we consider the ratio of interaction rate to expansion rate, which is given by

$$\frac{\Gamma_\nu}{H} \simeq \frac{G_\text{F}^2 T^5}{T^2/m_\text{Pl}} \simeq \left(\frac{T}{1 \text{ MeV}}\right)^3 . \tag{4.80}$$

Here we used Eq. (4.52) and the fact that the cross section of weakly interacting neutrinos is of the form $\sigma_{\nu\bar{\nu}} \simeq G_\text{F}^2 E^2 \simeq G_\text{F}^2 T^2$. The number density of hot relativistic relics goes with T^3 (cf. Eq. (4.44a)) so that $\Gamma_\nu = n\sigma_{\nu\bar{\nu}}|v| \simeq G_\text{F}^2 T^5$. Thus, at temperatures below 1 MeV the neutrinos are decoupled from thermal bath.

It is now easy to calculate g_{*S}. For $T \simeq 1$ MeV, the thermalized relativistic particles are the photons, electrons, positrons, and neutrinos, all with the same temperature, yielding (cf. Eq. (4.60))

$$g_{*S}(T \sim 1 \text{ MeV}) = 2 + 2\frac{7}{8}2 + 6\frac{7}{8} = 10.75 . \tag{4.81}$$

Thus, with $g_{\nu,\text{eff}} = 2 \times (3/4)$ we obtain out of Eq. (4.78) and Eq. (4.79)

$$\Omega_\nu h^2 = \frac{m_\nu}{94 \text{ eV}} , \tag{4.82}$$

such that

$$m_\nu \lesssim 94 \text{ eV} . \tag{4.83}$$

This implies that the SM neutrinos must be lighter than 94 eV, otherwise they would overclose the Universe.

Cold Relics: $x_f \gtrsim 3$ This case is more subtle since cold relics decouple when they are non-relativistic. By considering the equilibrium value $Y_{\chi,\text{EQ}}(x)$ in this regime, we identify a strong exponential dependence on x, so that the precise details of the freeze-out process are important.

Because this case will be of no further interest in this work, we will only sketch the derivation of the result and refer to [5] for more details. One starts with the Boltzmann equation in the form of Eq. (4.71) and parametrizes the annihilation cross section as

$$\langle \sigma_{\chi\bar{\chi}} |v| \rangle \equiv \sigma_\star (T/m)^n = \sigma_\star x^{-n} , \tag{4.84}$$

where $n = 0$ accounts for s-wave annihilation, $n = 1$ for p-wave annihilation, etc. Then the Boltzmann equation becomes

$$\frac{\text{d}Y_\chi}{\text{d}x} = -\lambda x^{-n-2}(Y_\chi^2 - Y_{\chi,\text{EQ}}^2) , \tag{4.85}$$

with

$$Y_{\chi,\text{EQ}} = 0.145(g/g_{*S}) x^{3/2} e^{-x} , \tag{4.86}$$

$$\lambda \equiv \left. \frac{x \langle \sigma_{\chi\bar{\chi}} |v| \rangle s}{H(m)} \right|_{x=1} = 0.264 (g_{*S}/g_*^{1/2}) m_\text{Pl} \, m \, \sigma_\star . \tag{4.87}$$

As shown in [5], Eq. (4.85) can be solved approximately to a very good accuracy. The results are

$$Y_\infty = \frac{3.79\,(n+1)\left(g_*^{1/2}/g_{*S}\right)x_f^{n+1}}{m\, m_{\rm Pl}\,\sigma_0} = \frac{3.79\,(n+1)\left(g_*^{1/2}/g_{*S}\right)x_f}{m\, m_{\rm Pl}\,\langle\sigma_{\chi\bar\chi}|v|\rangle}\ , \tag{4.88}$$

$$n_{\chi 0} = s_0 Y_\infty\ , \tag{4.89}$$

$$\Omega_{\chi 0}h^2 = 1.07\times 10^9 \frac{(n+1)x_f^{n+1}}{\left(g_{*S}/g_*^{1/2}\right)m_{\rm Pl}\sigma_\star}\ {\rm GeV}^{-1}\ . \tag{4.90}$$

From Eq. (4.88), we see that the smaller the annihilation cross section $\langle\sigma_{\chi\bar\chi}|v|\rangle$, the larger the abundance of cold relics. Thus, WIMPs, which are viable candidates for DM, are generally expected to be a by-product of our Universe's hot youth.

4.3.4 Out-Of-Equilibrium Decay—Entropy Production

Let us consider an unstable particle ψ which is decoupled from thermal bath (i.e. $Y_\chi \gg Y_{\chi,{\rm EQ}}$) and sufficiently long lived so that it becomes non-relativistic before decay. Then, because the matter density grows relative to that of radiation as R (cf. Sec. 4.2.1), there is the chance that ψ decays while dominating the energy density of the Universe. We will show that under these circumstances such a mechanism of out-of-equilibrium decay can release a considerable amount of entropy during the expansion of our Universe [72].

For a rough estimate, assume that the decoupled species ψ has a pre-decay abundance of $Y_i = n_\psi/s$. The decay should occur at a time $t \sim H^{-1} \sim \tau = \Gamma^{-1}$, when the temperature of the Universe is $T = T_D$ and the energy density is dominated by $\rho \sim \rho_\psi = Y_i s m_\psi \sim Y_i g_* T_D^3 m_\psi$. Then, just before τ, the Friedmann equation gives:

$$H_D^2 \equiv H^2(t=\tau) \sim \rho/m_{\rm Pl}^2 \sim Y_i g_* T_D^3 m_\psi / m_{\rm Pl}^2 \sim \tau^{-2}\ . \tag{4.91}$$

Suppose that ψ decays into relativistic particles that rapidly thermalize, yielding a post-decay energy density of (cf. Eq. (4.44b))

$$\rho_R \sim g_* T_r^4\ . \tag{4.92}$$

Because of energy conservation this must correspond to $\rho_\psi = H_D^2 m_{\rm Pl}^2$ so that the ratio of the final to initial entropy per comoving volume is

$$\frac{S_f}{S_i} \sim \frac{T_r^3}{T_D^3} \sim g_*^{1/4}\frac{Y_i m_\psi}{(m_{\rm Pl}\Gamma)^{1/2}}\ . \tag{4.93}$$

Furthermore, it appears that due to this entropy release the Universe has been heated up:

$$\frac{T_f}{T_i} = \frac{T_r}{T_D} = \left(\frac{S_{\rm after}}{S_{\rm before}}\right)^{1/3} = g_*^{1/12}\frac{(Y_i m_\psi)^{1/3}}{(m_{\rm Pl}\Gamma)^{1/6}} \tag{4.94}$$

$$\Rightarrow \quad T_r = g_*^{-1/4}\sqrt{m_{\rm Pl}/\tau}\ . \tag{4.95}$$

As we will now see, the more careful analysis will confirm, that the estimates (4.93) and (4.95) are quite accurate. However, it will turn out that the temperature of the Universe never increases. What happens instead is that it just falls more slowly than it would in the absence of entropy release.

First let us take into account the fact that the decays are not simultaneous, but instead follow the usual exponential law, $\mathrm{d}(R^3 n_\psi)/\mathrm{d}t = -\tau^{-1}(R^3 n_\psi)$, which gives

$$\dot{n}_\psi + 3Hn_\psi = -\tau^{-1} n_\psi \ . \tag{4.96}$$

In the non-relativistic regime, we have $\rho_\psi = mn_\psi$ (cf. Eq. (4.47b)), and

$$\dot{\rho}_\psi + 3H\rho_\psi = -\tau^{-1}\rho_\psi \ . \tag{4.97}$$

This equation is integrated trivially:

$$\rho_\psi(R) = \rho_\psi(R_i)\left(\frac{R_i}{R}\right)^3 \exp(-t/\tau) \ , \tag{4.98}$$

where the subscript i denotes the value of the quantity at the initial epoch ($t = t_i$). The energy released from such ψ decays converts into relativistic particles which then rapidly thermalize. To determine the energy density ρ_R in these relativistic particles, we make use of its relation to S (compare Eq. (4.49) and (4.59)):

$$\begin{aligned}\rho_R &= \frac{3}{4}Ts \\ &= \frac{3}{4}\left(\frac{45}{2\pi^2 g_*}\right)^{1/3} R^{-4}S^{4/3} \ .\end{aligned} \tag{4.99}$$

The evolution of entropy per comoving volume (S in formula (4.99)) is derived from the second law of thermodynamics, $\mathrm{d}S = \delta Q/T$. With δQ, the heat added per comoving volume (see Eq. (4.97))

$$\delta Q = -\mathrm{d}(R^3\rho_\psi) = \tau^{-1}R^3\rho_\psi \mathrm{d}t \ , \tag{4.100}$$

and

$$S = (2\pi^2/45)g_* T^3 R^3 \ , \tag{4.101}$$

we obtain

$$S^{1/3}\dot{S} = \frac{\tau^{-1}R^3\rho_\psi}{T} = \left(\frac{2\pi^2}{45}g_*\right)^{1/3}\frac{R^4\rho_\psi}{\tau} \ . \tag{4.102}$$

A simple (but formal) integration gives, using Eq. (4.98)

$$S^{4/3} = S_i^{4/3} + \frac{4}{3}\rho_\psi(R_i)R_i^4\tau^{-1}\int_{t_i}^{t}\left(\frac{2\pi^2}{45}g_*\right)^{1/3}\frac{R(t')}{R_i}\exp(-t'/\tau)\mathrm{d}t' \ . \tag{4.103}$$

Before we provide a solution to this equation let us analyse the illustrative limit, where g_* is assumed to be constant (see Ref. [62]). In this case the first law of thermodynamics (energy conservation) leads to the following energy balance:

$$\mathrm{d}(R^3\rho_R) = -p_R\mathrm{d}(R^3) - \mathrm{d}(R^3\rho_\psi) \ , \tag{4.104}$$

Using $p_R = \rho_R/3$, valid for constant g_*, we can write

$$\mathrm{d}(R^3\rho_R) = -\frac{\rho_R}{3}\mathrm{d}(R^3) + \tau^{-1}R^3\rho_\psi\mathrm{d}t \ , \tag{4.105}$$

$$\dot{\rho}_R + 4H\rho_R = \tau^{-1}\rho_\psi \ . \tag{4.106}$$

The significance of Eq. (4.106) is manifest: it is exactly the fluid equation (see Eq. (4.20)) applied to radiation (cf. Eq. (4.28)), with $\tau^{-1}\rho_\psi$ as a source term which accounts for energy transfer from ρ_ψ to ρ_R. Note that in the limit $g_* = \text{const.}$ Eq. (4.106) and (4.102) are equivalent. For non-constant g_*, however, the former is not valid and we have to solve Eq. (4.102).

The Friedmann equation gives the evolution of the cosmic scale factor $R(t)$

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3m_{\rm Pl}^2}(\rho_\psi + \rho_R) , \tag{4.107}$$

and forms together with Eqs. (4.98), (4.106) (or (4.102)) and (4.107) a closed set of differential equations. Let us discuss the qualitative behaviour of their solutions [62].

Assume that the non-relativistic particles ψ start to dominate the energy density of the Universe at some moment $t = t_\psi$. To produce a significant amount of entropy, we necessarily need $t_\psi < \tau$. Then, in between of these values, t_ψ and τ, the Universe is matter dominated and $R(t) \propto t^{2/3}$ (see Eq. (4.32)). Furthermore, we have approximately $\rho_\Psi \simeq \rho_\psi(R_i)(R_i^3/R^3) \propto t^{-2}$ (cf. Eq. (4.98)) so that Eq. (4.106) can simply be solved:

$$\rho_R = \rho_R(R_i)\left(\frac{R_i}{R}\right)^4 + \frac{3}{5}\rho_\psi(R_i)\frac{t_i^2}{t\tau} , \tag{4.108}$$

with $\rho(R_i)$, the energy density at some initial moment $t_i < \tau$. The first term solves the homogeneous fluid equation and represents the original radiation. The second term, i.e. the particulate solution to Eq. (4.106), gives the energy density component produced by ψ decays. For convenience let us take $t_i \simeq t_\psi$ so that $\rho_R(R_i) \simeq \rho_\psi(R_i)$. Then, by comparison of the two terms in Eq. (4.108), we can determine the moment when the ψ-produced component starts to be the dominant component: $t_\bullet \simeq t_\psi(5\tau/3t_\psi)^{3/5}$. This means that the behaviour of ρ_R is the following: Until the time $t_\bullet$ the radiation behaves as usual, $\rho_R \propto R^{-4}$, whereas afterwards, during the time interval $t_\bullet \lesssim t \lesssim \tau$, we have $\rho_R \propto t^{-1} \propto R^{-3/2}$. For $t \gtrsim \tau$ the source term ρ_ψ/τ goes to zero due to the decays of ψ's, and ρ_R once again falls as $\propto R^{-4}$. Thus, we have found that the energy density in radiation and, hence, the temperature of the Universe ($T \propto \rho_R^{1/4}$) always *decrease*, even though at a much slower rate during the epoch of ψ decays. This is due to the facts that, on the one hand, we have no sudden and simultaneous decay at $t = \tau$, which was spuriously assumed in the rough estimates above, and, on the other hand, the Universe expands and red-shifts the decay-produced energy density in radiation.

Notice that during the period $t_\bullet \lesssim t \lesssim \tau$, where the ψ-produced radiation component is dominant, the entropy is growing. Out of Eq. (4.99), we obtain

$$S \propto R^3 \rho_R^{3/4} \propto R^{15/8} \propto t^{5/4} . \tag{4.109}$$

To determine the exact amount of entropy, produced as a result of such out-of-equilibrium decays, we will solve Eq. (4.103). Using $Y = n/s = nR^3/S$ together with $\rho_\psi = m_\psi n_\psi$ and Eq. (4.101), it follows from Eq. (4.103) that the ratio of the final ($t \gg \tau$) to initial ($t \ll \tau$) entropy per comoving volume can be written as [72]

$$\frac{S_f}{S_i} = \left[1 + \frac{4}{3}\left(\frac{45}{2\pi^2 g_*(T_i)}\right)^{1/3} \frac{m_\chi Y_i}{T_i} I\right]^{3/4} , \tag{4.110}$$

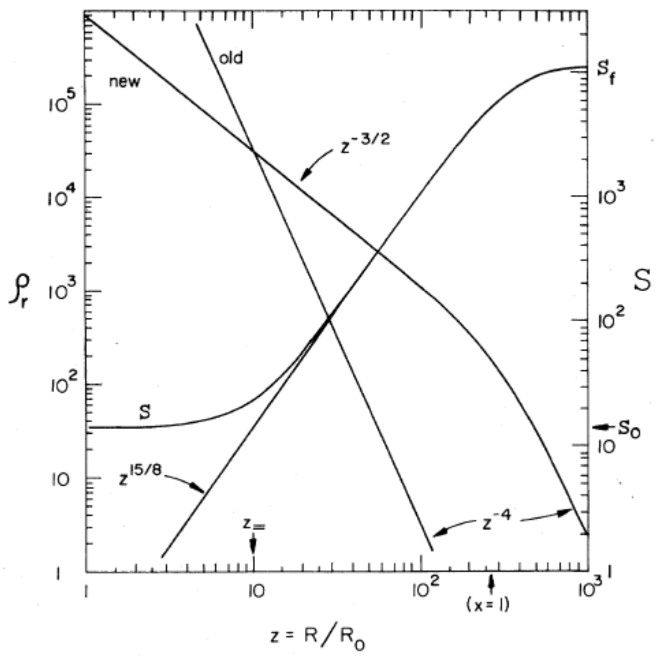


Figure 4.1: The evolution of ρ_R and S as a function of $z = R/R_0$ in the scenario where the ψ particles dominate the energy density of the Universe during the decay period. The "old"-line shows the energy density $\rho_R \propto R^{-4}$ in the original radiation component, whereas the "new"-line gives the energy density component of the decay products of ψ. From $z = z_=$ (or equally $t = t_\bullet$), when the decay products start to dominate radiation, until $t \simeq \tau$ denoted by $x \equiv t/\tau \simeq 1$, the density goes like $\rho_R \propto R^{-3/2}$, implying $T \propto R^{-3/8}$. For $t \gg \tau$, we have the usual behaviour $\rho_R \propto R^{-4}$. The entropy S is in good approximation constant before $t_\bullet$, then, during $t_\bullet \lesssim t \lesssim \tau$ increases proportional to $R^{15/8}$ and afterwards adopts the constant value S_f. Figure taken from Ref. [72].

with $(\tau^{-1} = \Gamma)$

$$I = \Gamma \int_0^\infty \left(\frac{2\pi^2 g_*}{45}\right)^{1/3} \frac{R(t')}{R_i} \mathrm{e}^{-t'/\tau} \mathrm{d}t' \; . \tag{4.111}$$

The entropy production is significant if the energy density of χ particles and their decay products dominate the total energy density during the decay epoch. In this case one can show that

$$I = 1.09 \left(\frac{8\pi\rho_\psi(R_i)}{3m_{\mathrm{Pl}}^2}\right)^{1/3} \Gamma^{-2/3} \left(\frac{2\pi^2 \bar{g}_*}{45}\right)^{1/3} , \tag{4.112}$$

where the number 1.09 is the result of a numerical integration [72] and $\bar{g}_*^{1/3}$ is the weighted average of $g_*^{1/3}$ in the interval near $t = \tau$. By substituting this back into Eq. (4.110), we find

$$\frac{S_f}{S_i} = \left(1 + 2.95 \left(\frac{2\pi^2 \bar{g}_*}{45}\right)^{1/3} \frac{(m_\chi Y_i)^{4/3}}{(m_{\mathrm{Pl}}\Gamma)^{2/3}}\right)^{3/4} . \tag{4.113}$$

Comparing with our rough estimate (Eq. (4.93)), we see that there is only a small difference by a numerical factor of order unity.

Finally let us calculate the "reheating" temperature which is defined as the temperature of the Universe at the end of the decay epoch where radiation dominates. Out of the Friedmann equation, it follows directly

$$H^2(t=\tau) \simeq \left(\frac{1}{2\tau}\right)^2 \simeq \frac{8\pi}{3m_{\rm Pl}^2}\frac{\pi^2 \bar{g}_*}{30}T_r^4\,, \tag{4.114}$$

$$T_r \simeq \frac{1}{2}\left(\frac{2\pi^2\bar{g}_*}{45}\right)^{-1/4}\sqrt{\Gamma m_{\rm Pl}}\,. \tag{4.115}$$

which again differs from Eq. (4.95) only by a numerical factor of order unity .

4.4 Dark Matter in the Universe

As mentioned in the introduction, a wide variety of cosmological and astrophysical observations provide evidence for the existence of Dark Matter. While we know that it interacts gravitationally but must be non-luminous and non-baryonic, the nature of DM is still a mystery to physicists and cosmologists. It is probably made of new kind of particles, but even though there are many ideas about what those could be, we have not yet succeeded in actually detecting and identifying a DM candidate in laboratory.

Evidence emerge at different scales. Without attempting to be complete, let us cite a few among them. At the galactic and sub-galactic scales hints come from weak gravitational lensing of distant galaxies with foreground structure [73], the incompatibility of the radial velocity-distribution of a galaxy with its mass distribution [74], from strong gravitational lensing of galaxies [75] etc. At the scale of galaxy clusters the famous "Bullet Cluster" with non-coinciding distributions of luminous and non-luminous matter represents evidence for DM and at the same time illustrates in a very beautiful way that DM must be some form of very weakly interacting material (thus, non-baryonic) [4]. Furthermore, measurements of the mass-to-light ratio of galaxy clusters via the application of the virial theorem, the method of weak gravitational lensing or the analysis of the X-ray profile lead to values which exceed that of the solar neighbourhood. This was first observed in 1933 by Fritz Zwicky who studied the velocity dispersion in the Coma Cluster and predicted for the first time in history that a large amount of non-luminous matter is required to be present [76]. On the cosmological scale the WMAP analysis of the small temperature-anisotropies of the Cosmic Microwave Background Radiation (CMBR), discovered in 1965 by Penzias and Wilson [60, 61], provided the possibility to determine with very good accuracy the total amount of DM in the present Universe. A combined WMAP+SN+BAO fit gives the following results (see Sec. 4.2.2 and Tab. 4.1):

$$\Omega_b h^2 = 0.02267^{+0.00058}_{-0.00059}, \qquad \Omega_M h^2 = 0.1358^{+0.0037}_{-0.0036}\,, \tag{4.116}$$

so that the DM fraction is

$$\Omega_{\rm DM}h^2 = \Omega_M h^2 - \Omega_b h^2 = 0.1131 \pm 0.0034\,. \tag{4.117}$$

We would like to point out that the value of $\Omega_b h^2$ is in good agreement with the predictions from Big Bang Nucleosynthesis (BBN) (see, e.g., Ref. [77]):

$$0.018 < \Omega_b h^2 < 0.023\,. \tag{4.118}$$

This is really striking, since these two measurements are done with completely different techniques: BBN uses nuclear physics and the CMB theory follows out of relativistic fluid mechanics. Taken together, all these observations lead to the conclusion that approximately 80% of matter in the Universe is composed out of DM.

Cosmologists consider several different types of this unidentified weakly interacting material, which they classify according to how fast the individual DM particles travel around us. Heavy and non-relativistic candidates are called Cold Dark Matter (CDM) while very light particles moving with ultra-relativistic velocities are referred to as Hot Dark Matter (HDM). Concerning structure formation in the Universe, these DM species behave totally different. Due to their large free streaming length, rapidly moving HDM particles prevent clumps of matter from accumulating such that formation of small scale structures is suppressed. Thus, in a HDM-dominated Universe, structures form in a top-down fashion: first, the large scale structures are born which afterwards fragment and form the small scale inhomogeneities like galaxy clusters or galaxies. In the case of slowly moving CDM, however, the particles have a negligible free streaming length leading to a bottom-up formation of structures: small scale structures of galactic size form first and then merge to larger structures like superclusters. A comparison of observations with structure formation simulations favours domination of CDM in the Universe. We already know a possible production mechanism for these heavy but weakly interacting particles (WIMPs). They satisfy the requirements of *cold* relics discussed in Sec. 4.3.3 and can possess a large relic abundance due to their small cross section which enters inversely in formula (4.88). Nevertheless, CDM has some problems. In particular, it implies the cuspy halo problem predicting rotation curves which are peaked too strongly. Furthermore, out of CDM simulations it follows that galaxies such as the Milky Way should have a large number of Dwarf satellites, which are, however, not observed. It is the intermediate situation between cold and hot—the Warm Dark Matter (WDM), which is able to ameliorate these problems. Contrary to CDM it causes structure formation to occur bottom-up from above its finite but not too large free-streaming scale, and top-down from below this scale such that densities of halo cores are lowered, large halos have fewer subhalos, and density fluctuations at the satellite scale or smaller are suppressed [78, 6].

In the following chapter we suggest thermally produced light sterile neutrinos as WDM candidates and show which properties these particles must possess for not to be in conflict with experimental bounds or cosmological predictions.

Chapter 5

KeV Sterile Neutrino Warm Dark Matter

Contents

Dark matter has been recognized as an essential part of matter for over 70 years now, and many suggestions have been made, what it could be. Most of these ideas have focused on CDM, particles that are predicted in extensions of the SM, such as supersymmetry. Since these concepts recently led to problems of structure formation simulations (see previous discussion), we want to consider another, less explored but maybe more viable one, that of WDM.

Natural candidates commonly considered for WDM are light sterile (right-handed) neutrinos. These hypothetical massive fermions, which arise in most extensions of the SM, are very weakly interacting singlets of the electroweak gauge group $SU(2)_L \times U(1)_Y$, and, in fact, their very useful properties (cf. Chap. 3) as well as the provided possibility to solve the mystery of DM are quite good reasons for these particles to exist.

Probably one of the simplest models which realizes sterile neutrino WDM is the νMSM [12, 13]. It is essentially the SM with no further scale between the electroweak and Planck scales, but with three sterile neutrinos, having Majorana masses and Dirac mixing with ordinary (active) neutrinos, added. Then, for a specific choice of parameters, this simple SM extension is very predictive. Besides having a WDM candidate, represented by the lightest added singlet fermion, it can explain the tiny masses of active neutrinos via the see-saw mechanism, and at the same time, the apparent matter-antimatter asymmetry in the Universe [13]. However, due to their weak interactions (they can only interact via the small Dirac mixing with active neutrinos), the sterile neutrinos *never* enter into thermal equilibrium (i.e. $\Gamma \ll H$), such that thermal production mechanisms (cf. Sec. 4.3.3), are not viable in this model. Instead, it is a non-thermal process [8] which produces the WDM in the νMSM, and it was argued in [14, 15, 16] that in order to calculate unambiguously its

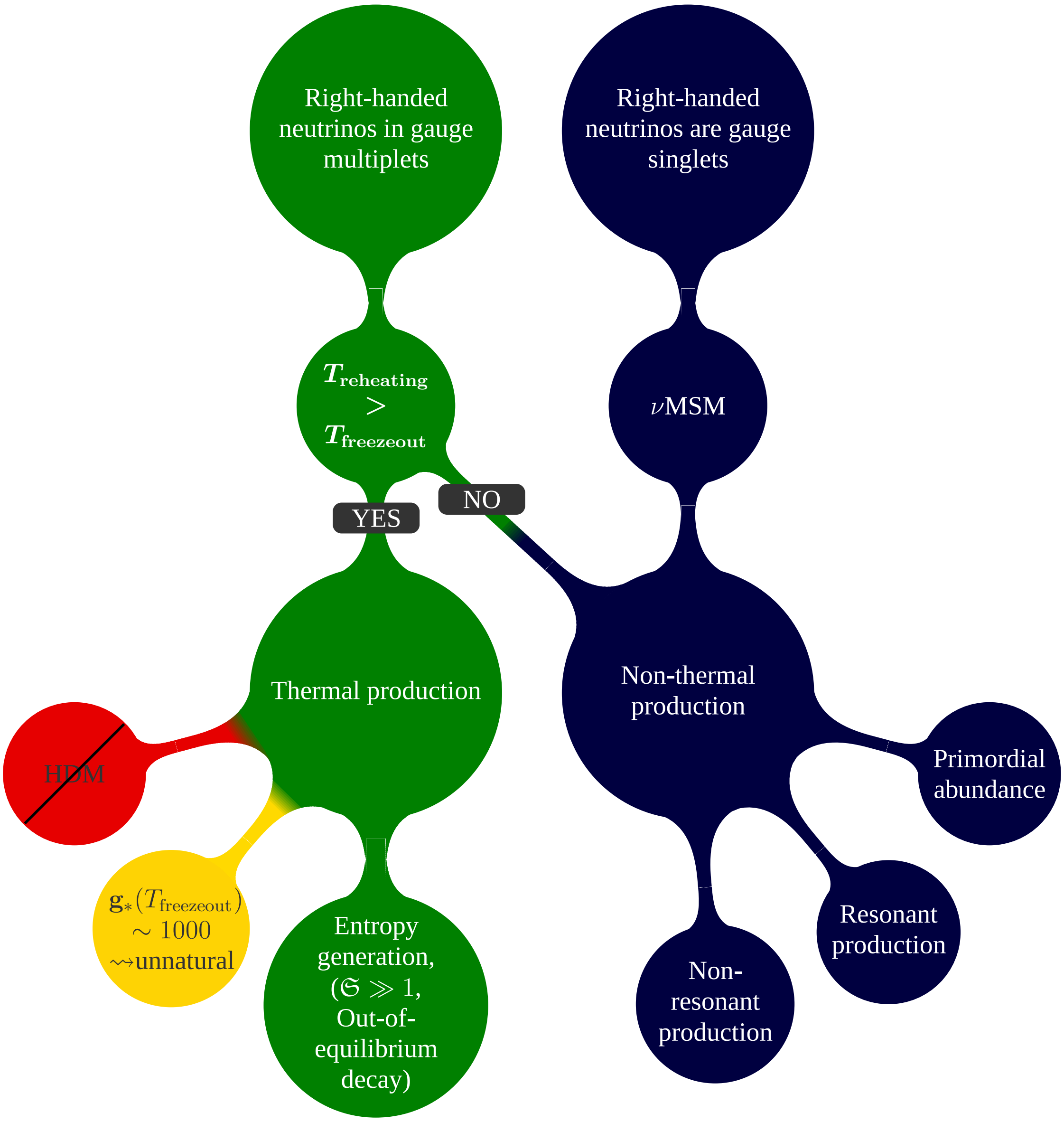


Figure 5.1: Schematic illustration of the direction we pursue. The tree should be read from top to bottom. Starting on the green left-hand side and taking the *YES* path corresponds to our analysis. The *NO* branch, which points to the blue right-hand side, would lead to a situation similar to the νMSM, which we don't further examine.

present abundance, the initial WDM amount is required. This, however, needs knowledge of physics before the beginning of the thermal evolution of the Universe.

Here we will analyse a possibility opposite to the νMSM. We postulate an additional (gauge) scale between the electroweak and Planck scales, and that the sterile neutrinos are charged under the associated gauge transformations. Then, for general not too high intermediate scale, it will be possible for these particles to reach thermal equilibrium at some moment in the early Universe. Assuming a relativistic decoupling (the hot relic case in Sec. 4.3.3), the thermal production mechanism predicts a viable DM abundance for sterile neutrinos with masses of about ~ 90 eV. This, however, would correspond to HDM and is forbidden by structure formation arguments like the Lyman-α bound, which excludes masses below ~ 1 keV.

Thus, in order to reconcile the thermal overproduction of a keV scale DM sterile neutrino with the observations, its abundance has to be diluted *after* it drops out of the thermal bath. A suitable mechanism, which can handle this is already known—the entropy production due to a long-lived particle which decays while dominating the total energy density of the Universe and being out of equilibrium (cf. Sec. 4.3.4). We will show that if sufficient large entropy is released *after* the decoupling of our DM sterile neutrino, its abundance can be sufficiently diluted to be in accordance with experimental results. The natural candidate for this long-lived particle is another (heavy) sterile neutrino.

For completeness, we will also mention other ways to avoid the WDM overproduction in the analysed class of models. One possibility is realised if all the new gauge interactions are at the GUT scale, while the reheating after inflation leads to temperatures below this scale. This situation is similar to the νMSM, because in this case the sterile neutrinos do not reach thermal equilibrium. Another possibility requires large (of the order of thousand) number of degrees of freedom at the moment of the sterile neutrino freeze-out, and does not seem natural.

The chart, Fig. 5.1, gives an overview of these possibilities and illustrates in which direction our discussion goes.

5.1 Cosmological Requirements and Constraints from Experiments

In this section, we introduce the generic framework we will work with, and discuss the various constraints and bounds resulting from cosmological considerations and various experimental results. Note that these constraints are rather general and apply to most variations of the specified model.

5.1.1 Assumptions and Definitions

In the following we will assume the existence of three right-handed (sterile) neutrinos, N_I ($I = 1, 2, 3$). As mentioned in Sec. 3.3.1 and 3.2.1 these sterile neutrinos are not charged under the SM gauge group, however, could be charged under the gauge transformations of an extended model (ultimately, emerging in the breaking chain of some GUT model). Though for most of the statements in this work the precise details of this gauge interaction are not important, we will use a specific LR symmetric extension of the SM, and stick to it to obtain definite numbers. This specific model (for a review see, e.g., Ref. [56] or Sec. 3.5) with the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ appears as a subgroup of many GUT theories.

In this framework the neutrinos interact with the charged gauge bosons through the interaction Lagrangian (cf. Eq. (3.107))

$$-\mathscr{L}_{cc}^{\nu N} = \frac{g}{\sqrt{2}} \sum_a \left(W_L^\mu \overline{l_{aL}} \gamma_\mu \nu_{aL} + W_R^\mu \overline{l_{aR}} \gamma_\mu N_{aR} \right) + \text{h.c.} \,, \tag{5.1}$$

where W_L is the SM W-boson, W_R is the corresponding right-handed boson from $SU(2)_R$, and l_a are the charged SM leptons. The neutrino masses arise due to spontaneous symmetry breakdown as discussed in detail in Sec. 3.5.2 and 3.5.3. Up to Sec. 5.3, however, we will not be interested in these details, and thus just write the general mass matrix as (cf. Eq. (3.19))

$$\mathscr{L}_m^{\text{D+M}} = -\frac{1}{2} \left(\overline{\tilde{\nu}_{aL}^c}, \overline{\tilde{N}_{aR}} \right) \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \tilde{\nu}_{aL} \\ \tilde{N}_{aR}^c \end{pmatrix} + \text{h.c.} \,, \tag{5.2}$$

where the tilde over the neutrino fields indicate that they are written in *flavour* basis. We assume that the block matrices in Eq. (5.2) obey in some sense the relations $M_L < m_D < M_R$ such that we can use the see-saw type formulas from Sec. 3.2.3. The transformation into to the mass basis is then given by (use Eq. (3.23) and neglect terms of order M_R^{-2})

$$\begin{pmatrix} \tilde{\nu}_{aL} \\ \tilde{N}_{aR}^c \end{pmatrix} \simeq \begin{pmatrix} 1 & (M_R^{-1} m_D^T)^\dagger \\ -M_R^{-1} m_D^T & 1 \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & V_R \end{pmatrix} \begin{pmatrix} \nu_{iL} \\ N_{IR}^c \end{pmatrix} \,, \tag{5.3}$$

where U is the standard PMNS matrix and the unitary matrix V_R describes the mixing in the right-handed sector:

$$M_L - m_D M_R^{-1} m_D^T = U^* \cdot \text{diag}(m_1, m_2, m_3) \cdot U^\dagger \,, \tag{5.4}$$

$$M_R = V_R^* \cdot \text{diag}(M_1, M_2, M_3) \cdot V_R^\dagger \,. \tag{5.5}$$

with m_i being the active neutrino masses and M_I the sterile neutrino masses. Note that if $M_L = 0$, then Eq. (5.4) is the usual see-saw formula.

For the analysis of the sterile neutrino decay, when the oscillations of the active neutrinos are not important, while the masses of the charged leptons are, it is helpful to make the described rotation only partially—without the PMNS rotation by the matrix U. Then we get the mixing angles between the mass states of the sterile neutrinos and SM flavours

$$\theta_{aI} \equiv \frac{(m_D V_R)_{aI}}{M_I} \,, \tag{5.6}$$

and also

$$\theta_I^2 \equiv \sum_{a=e,\mu,\tau} |\theta_{aI}|^2 \,. \tag{5.7}$$

As we will see, these squared mixing angles describe the overall strength of interaction (decay) of sterile neutrinos with the SM particles.

Before moving on to the analysis of the cosmological properties of sterile neutrinos, let us note an additional possible complication. Specifically, as discussed in Sec. 3.5.4, the W_L and W_R bosons in Eq. (5.1) do not have to coincide with the mass eigenstates, W_1

and W_2 (with masses $M_W \equiv m_{W_1}$ and $M \equiv m_{W_2}$), respectively, but be slightly mixed (cf. Eqs. (3.97) and (3.98)):

$$\begin{aligned} W_L &= \cos\zeta\, W_1 - \sin\zeta\, W_2 \;, \\ W_R &= \sin\zeta\, W_1 + \cos\zeta\, W_2 \;. \end{aligned} \tag{5.8}$$

Normally this can be neglected, but it may give significant contribution to the radiative decay of the DM sterile neutrinos, analysed in Sec. 5.1.6.

5.1.2 Temperature of Freeze-Out

Let us now calculate the moment of decoupling of the neutrinos N_1 in the early Universe. We will denote values corresponding to this moment by the subscript "f". As far as the sterile neutrino is relatively light and the freeze-out happens while it is still relativistic, the calculation is analogous to those for the usual active neutrinos (see the "Hot Relic" case in Sec. 4.3.3). The only difference is that the annihilation cross section is suppressed by the larger mass M of the right-handed gauge boson $W_R \simeq W_2$, compared to the SM W boson mass M_W,

$$\sigma_{N_1 N_1} \approx \sigma_{\nu\bar{\nu}} \left(\frac{M_W}{M}\right)^4 \sim G_{\rm F}^2 E^2 \left(\frac{M_W}{M}\right)^4 \;. \tag{5.9}$$

Here, $\sigma_{\nu\bar{\nu}}$ is the SM neutrino annihilation cross section, $G_{\rm F} = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi constant and E is the energy of the colliding sterile neutrinos. These are light and ultra-relativistic, so that according to Sec. 4.3.1, $v \simeq c = 1 \;\Rightarrow\; n \sim T^3$, and the energy in Eq. (5.9) scales as $E \sim T$. Thus, the interaction rate (per neutrino) is

$$\Gamma_{N_1} = n\, \sigma_{N_1 N_1} |v| \sim G_{\rm F}^2 T^5 \left(\frac{M_W}{M}\right)^4 \;. \tag{5.10}$$

Requiring the equality of the width Γ_{N_1} and the Hubble parameter (4.52)

$$H = \frac{2\pi^{3/2}}{3\sqrt{5} m_{\rm Pl}} \sqrt{g_*}\, T^2 \;, \tag{5.11}$$

we obtain, with the Planck mass $m_{\rm Pl} = \sqrt{G} = 1.22 \times 10^{19}$ GeV, the freeze-out temperature

$$T_{\rm f} \sim g_{*\rm f}^{1/6} \left(\frac{M}{M_W}\right)^{4/3} (1 \div 2) \text{ MeV} \;. \tag{5.12}$$

The quantity $g_{*\rm f}$ counts the effective number of relativistic degrees of freedom (d.o.f.) immediately after freeze-out. If freeze-out happens below 100 MeV, we have at least $g_{*\rm f} = 10.75$, (cf. Eq. (4.81)).

We recognize that for the not very large scale M, the sterile neutrino N_1 decouples at rather low temperature. Thus, it normally is in thermal equilibrium at the early stages of the Universe evolution, making it a thermal relic. This will be the possibility which we peruse in the current study. In this case calculation of the present day density of the sterile neutrinos is insensitive to the history of the Universe before $T_{\rm f}$.

Note, however, that if the reheating temperature after inflation is lower than Eq. (5.12), the neutrinos never enter the thermal equilibrium. In this case additional assumptions about the initial abundance of the sterile neutrinos are necessary to predict their current density, and the generation mechanism is very different form the case analysed here (see, e.g., Refs. [79, 80, 8, 81]). Such a scenario can, for example, be realised for a very low reheating temperature (see, e.g., Ref. [82]), or naturally if the right-handed scale is the GUT scale, $M \sim M_{\rm GUT}$, leading to $T_{\rm f} \sim M_{\rm GUT}$, and the reheating after inflation reached slightly lower temperatures. Another way to implement this situation is perused in the νMSM [12, 13], where *no new physics* is present up to Planck scale, leading to N_1 never entering the thermal bath.

5.1.3 Abundance of Sterile Neutrino Dark Matter

The number to entropy density ratio of our relic sterile neutrino N_1 (two fermionic degrees of freedom) after freeze-out is given by (cf. Eq. (4.44a) and Eq. (4.59) or use Eq. (4.76))

$$Y_{\rm f} \equiv \left.\frac{n_{N_1}}{s}\right|_{\rm f} \equiv \frac{n_{N_1}(t_f)}{s(t_f)} = \frac{1}{g_{*{\rm f}}}\frac{135\zeta(3)}{4\pi^4} \,. \tag{5.13}$$

While the Universe expands slowly with all the processes approaching thermal equilibrium ($\Rightarrow S$ constant), both the number density and entropy density scale inversely proportional to the volume of the Universe, and their ratio remains constant. If, however, non-equilibrium processes happen during the expansion, like, for example, an intermediate matter dominated stage caused by out-of-equilibrium decay of a heavy species, additional entropy release is possible, which we will take into account by the factor $\mathfrak{S}$

$$\mathfrak{S} \equiv \frac{S_0}{S_{\rm f}} = \frac{s(t_0)}{s(t_f)}\left(\frac{R(t_0)}{R(t_f)}\right)^3 \,. \tag{5.14}$$

This leads to the relation

$$\frac{n_{N_1}(t_0)}{n_{N_1}(t_{\rm f})} = \left(\frac{R(t_{\rm f})}{R(t_0)}\right)^3 = \frac{s(t_0)}{s(t_{\rm f})}\frac{1}{\mathfrak{S}} \,. \tag{5.15}$$

Let us calculate the contribution of N_1 to the present energy density. Rescaling the number to entropy density ratio at present moment by this factor, as compared to the freeze-out moment, we get for the sterile neutrino contribution to the energy density of the Universe Ω_{N_1}

$$\begin{aligned}\frac{\Omega_{N_1}}{\Omega_{\rm DM}} &= \left(\left.\frac{n_{N_1}}{s}\right|_{\rm f}\right)\frac{1}{\mathfrak{S}}M_1\frac{s_0}{\Omega_{\rm DM}\rho_c} \\ &\simeq \frac{1}{\mathfrak{S}}\left(\frac{10.75}{g_{*{\rm f}}}\right)\left(\frac{M_1}{1\ {\rm keV}}\right)\times 100 \,,\end{aligned} \tag{5.16}$$

where $\Omega_{\rm DM} = 0.113h^{-2}$ is the DM density, $s_0 = 2889.2\ {\rm cm}^{-3}$ the present day entropy density and $\rho_c = 1.05368 \times 10^{-5}h^2$ GeV cm^{-3} is the critical density of the Universe (see Tab. 4.1 and Ref. [36]). The observational requirement is $\Omega_{N_1}/\Omega_{\rm DM} \leq 1$ with equality being the nicest choice (all DM is made out of N_1), and inequality opting for multispecies DM.

Let us analyse Eq. (5.16) further. Without entropy release ($\mathfrak{S} = 1$) the Universe is overclosed, unless the neutrino is very light (see also the weaker bound in Eq. (4.79)), which corresponds to the Hot Dark Matter case, excluded by the structure formation in the Universe. Models with the number of degrees of freedom at freeze-out $g_{*\mathrm{f}}$ of order 1000 seem rather unnatural and will not be considered. The only opportunity is thus the entropy release after the freeze-out of N_1,

$$\mathfrak{S} \simeq 100 \left(\frac{10.75}{g_{*\mathrm{f}}}\right)\left(\frac{M_1}{1\ \mathrm{keV}}\right) . \tag{5.17}$$

Having this entropy release will lead to the observed DM abundance today. In the following, we will analyse possibilities of generation of this large amount in the model.[1]

5.1.4 Mass Bounds

The mass of the DM particle N_1 can not be too light, or the observed structure in the Universe would have been erased by a too hot DM. The simplest and most robust bound can be obtained from the phase space density arguments. The phase space density of a collision-less DM can only become smaller during the evolution of the Universe, as an effect of coarse-graining. Comparison of the primordial phase space density, which is calculated using the initial DM particle distribution function and the maximal modern one, derived from the observation of the Dwarf spheroidal galaxies [10, 83] gives the lower bound

$$M_1 > 1 \div 2\ \mathrm{keV} . \tag{5.18}$$

Another important bound is the Ly-α bound [84, 85]. This bound constrains the velocity distribution of the DM particles, from the effect of their free-streaming on the formation of the structure on the scales, probed by the Ly-α forest. It should be noted, that to convert this constraint into a bound for the mass of the DM particle, one needs to take into account the initial velocity distribution of the particles. In our case it takes the form of a usual thermal distribution, but with the temperature ($T \propto R^{-1}$) lowered by the dilution factor $\mathfrak{S}^{-1/3}$. This corresponds to the *thermal relic* case in Ref. [84], and not to the case of the non-resonantly produced sterile neutrinos, denoted m_{NRP} in Ref. [84]. Thus, the result of Ref. [84] should be rescaled as

$$M_1 > \frac{T}{T_\nu} m_{\mathrm{NRP}} , \tag{5.19}$$

where T is the present temperature of the DM neutrino diluted with the entropy factor (5.17), T_ν is the temperature of the usual relic neutrinos, and $m_{\mathrm{NRP}} = 8$ keV [84]. The ratio of the temperatures is obtained from the requirement of the observed Ω_{DM}. With Eq. (5.17) we have:

$$\frac{T}{T_\nu} = \mathfrak{S}^{-1/3} \simeq \left(\frac{0.01\ \mathrm{keV}}{M_1}\right)^{1/3} , \tag{5.20}$$

Equivalently, we can use Eq. (4.44a) and calculate $\Omega_{\mathrm{DM}}\rho_{c0} \equiv \Omega_{N_1}\rho_{c0} = M_1 n_{N_1} \propto T^3$. Dividing by the current active neutrino temperature $T_{\nu 0} = (4/11)^{1/3} T_{\gamma 0}$ (see, e.g., Ref. [62]), we obtain $(T/T_\nu)^3 = \Omega_{\mathrm{DM}} h^2 (94\ \mathrm{eV}/M_1)$ which is equal to Eq. (5.20). By substituting one of these expressions back into Eq. (5.19), we obtain the lower bound on our thermally produced WDM sterile neutrino N_1:

$$M_1 \gtrsim 1.6\ \mathrm{keV} . \tag{5.21}$$

[1] The exact value of the required entropy release $\mathfrak{S}$ may be slightly different, if, for example, some amount of DM sterile neutrino was generated non-thermally after the decoupling from thermal bath. However, in the examples, analysed in this work, this effect is negligible.

5.1.5 Generation of Entropy and BBN Bound

As discussed in Sec. 4.3.4, the entropy (5.17) can be generated by some heavy long-lived particle which goes out of thermal equilibrium at some moment after DM neutrino freeze-out, t_f, and decays after becoming non-relativistic and dominating the energy density of the Universe. The obvious candidates for such particles are the two remaining heavier neutrinos (though other candidates are possible, and can be analysed in a similar way). Let us assume for simplicity that only one of these two neutrinos is responsible for entropy generation, and denote it by N_2. Then, according to Sec. 4.3.4, the entropy release is (cf. Eq. (4.113))

$$\mathfrak{S} \simeq \left(1 + 2.95\left(\frac{2\pi^2\bar{g}_*}{45}\right)^{1/3}\frac{(M_2 Y_i)^{4/3}}{(m_\mathrm{Pl}\Gamma_{N_2})^{2/3}}\right)^{3/4} , \tag{5.22}$$

where M_2 is the mass of N_2, $Y_\mathrm{i} \equiv n_{N_2}(t_\mathrm{i})/s(t_\mathrm{i})$ is the initial abundance of the N_2 particles after decoupling (or, probably more precise, before they start to drive the matter dominated intermediate stage of the Universe expansion), and $\bar{g}_*$ is the properly averaged effective number of d.o.f. during the N_2 decay. The ratio Y_i is maximal if the particle decouples when it is still relativistic, and is equal to

$$Y_i = \frac{g_N}{2}\frac{135\zeta(3)}{4\pi^4 g_*} , \tag{5.23}$$

where $g_N = 2$ is the number of d.o.f. for N_2, and g_* is taken at the N_2 freeze-out. If the entropy generation is large, we can neglect the 1 in Eq. (5.22) and get

$$\mathfrak{S} \simeq 0.76\frac{g_N}{2}\frac{\bar{g}_*^{1/4} M_2}{g_*\sqrt{m_\mathrm{Pl}\Gamma_{N_2}}} . \tag{5.24}$$

By combining Eqs. (5.17) and (5.24), we obtain

$$\Gamma_{N_2} \simeq 0.50 \times 10^{-6}\,\frac{g_N^2}{4}\frac{g_{*\mathrm{f}}^2}{g_*^2}\bar{g}_*^{1/2}\frac{M_2^2}{m_\mathrm{Pl}}\left(\frac{1\ \mathrm{keV}}{M_1}\right)^2 . \tag{5.25}$$

Note that in our case the freeze-out temperatures of the DM sterile neutrino (N_1) and of the entropy generating one (N_2) coincide, such that $g_* = g_{*\mathrm{f}}$. If Eq. (5.25) is satisfied, then we have proper DM abundance in the present Universe. Schematically this is illustrated in Fig. 5.2.

However, Eq. (5.25) is not the only requirement for the lifetime of the heavier sterile neutrino in a realistic model. Entropy generation should finish before the Big Bang Nucleosynthesis (BBN), i.e. the N_2's should decay before it. According to Refs. [86, 87, 88] the temperature after the decay of the sterile neutrino N_2—the so-called "reheating" temperature T_r—should be greater than $0.7 \div 4$ MeV in order not to spoil BBN predictions. This temperature is approximately equal to (see Eq. (4.115))

$$T_r \simeq \frac{1}{2}\left(\frac{2\pi^2\bar{g}_*}{45}\right)^{-1/4}\sqrt{\Gamma_{N_2} m_\mathrm{Pl}} , \tag{5.26}$$

leading to a bound on the N_2 lifetime which should be shorter than approximately $0.1 \div 2$ s. The sterile neutrino with such a lifetime can produce enough entropy, satisfying Eq. (5.25) only if it is sufficiently heavy,

$$M_2 > \left(\frac{M_1}{1\ \mathrm{keV}}\right)(1.7 \div 10)\ \mathrm{GeV} . \tag{5.27}$$

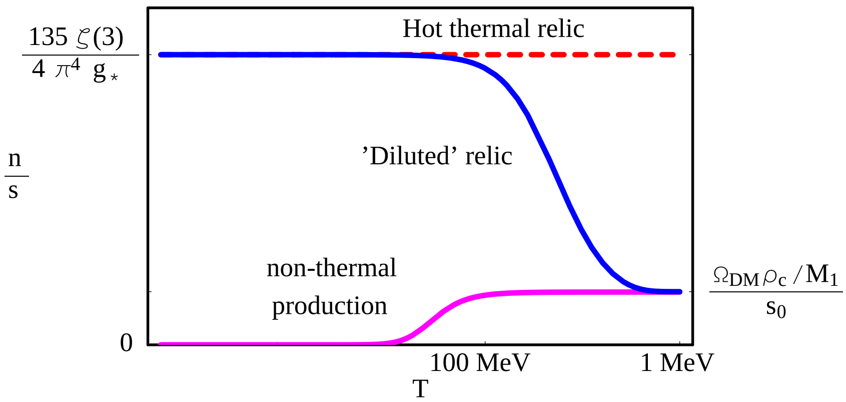


Figure 5.2: Schematic evolution of the light relic abundance in the Universe. The dashed line is a thermal relic decoupled while being relativistic (hot thermal relic), leading to the overclosure of the Universe. The blue line is the same hot thermal relic, but with the abundance diluted by rapid expansion of the Universe (entropy production), leading to correct DM abundance. The lowest magenta line depicts the evolution of the non-thermally produced particle with zero primordial abundance.

Finally, as far as we were assuming that the sterile neutrino N_2 decoupled while still relativistic (otherwise the entropy generation is much less efficient), we should require $T_{\rm f} > M_2$. This, using Eq. (5.12) is translated into a bound for the scale of the right-handed bosons,

$$M > \frac{1}{g_{*\rm f}^{1/8}} \left(\frac{M_2}{1\ \rm GeV}\right)^{3/4} (10 \div 16)\ \rm TeV\ . \tag{5.28}$$

Thus, on the one hand the sufficient entropy generation requires a long-lived neutrino, but on the other hand, the requirement of successful BBN limits its lifetime from above, leading to the lower bounds on its mass and on the mass scale of the additional gauge interactions.

5.1.6 Constraints from X-ray Observations

A sterile neutrino in the considered class of models is unstable, so it provides a *decaying* DM particle. Through its mixing it dominantly decays via the neutral current into three active neutrinos. To lead to a successful DM scenario, the lifetime of the unstable neutrino N_1 should be greater than the age of the Universe $\tau_{\rm U} \sim 10^{17}$ sec, which constraints its total decay width. However, one obtains significantly stronger restrictions resulting from a subdominant decay channel—the radiative decay $N_1 \to \nu\gamma$, induced at the one loop level (see Fig. 5.3). This process produces a narrow line in the X-ray spectrum of astrophysical objects [9, 89]. In the context of the νMSM the only source of this decay is via the active sterile neutrino mixing θ_1^2 (cf. Eq. (5.7)), and the recent X-ray observations bound it from

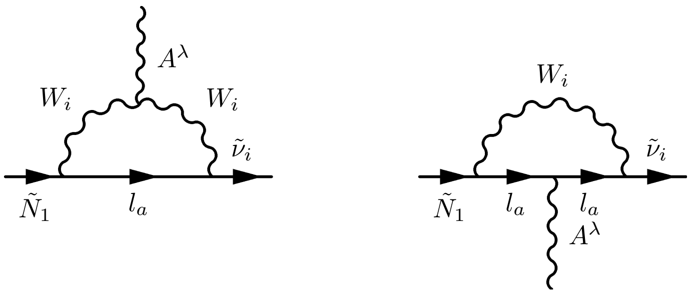


Figure 5.3: Unitary-gauge diagrams contributing to the radiative neutrino decay with charged leptons propagating in the loop.

above. A very rough bound, which will be enough for our purposes, is given in Ref. [79][2]

$$\theta_1^2 \lesssim 1.8 \times 10^{-5} \left(\frac{1\ \mathrm{keV}}{M_1} \right)^5 . \tag{5.29}$$

This restriction corresponds to the following upper bound on the radiative decay width

$$\Gamma_{N_1 \to \gamma\nu} \lesssim 9.9 \times 10^{-27}\ \frac{1}{\mathrm{s}} . \tag{5.30}$$

We also want to mention that there are bounds resulting from supernova cooling. However, they are also much weaker than the diffuse X-ray background limits (5.29) for all possible neutrino masses M_1 (see Ref. [9]).

In the LR symmetric model the X-ray bound (5.30) leads not only to the bound on the mixing angle (5.29), but also bounds the properties of the bosonic sector of the theory. The reason is the possible mixing of the right W_R gauge bosons with the SM W_L ones. Without mixing, the contribution of the W_R bosons to the process $N_1 \to \gamma\nu$ is additionally suppressed by the ratio of the left and right gauge boson masses $(M_W/M)^4$, and can be safely neglected. With the mixing, however, the chiral structure of the diagram changes, and the contribution can be enhanced by a factor $\propto (m_{l_a}/M_1)^2$, where m_{l_a} is the mass of the charged lepton propagating in the loop.

We calculate the total decay width for $N_1 \to \gamma\nu$, summed over the active neutrino flavours, following Refs. [97, 98] (for details see App. B). Supposing from the very beginning, that the right-handed scale is much larger, than the left one, $M \gg M_W$, neglecting the active neutrino masses and assuming small gauge boson mixing, we get

$$\Gamma_{N_1 \to \gamma\nu} \simeq \frac{G_F^2 \alpha M_1^3}{64\pi^4} \sum_{a=e,\mu,\tau} \left| 4 m_{l_a} (V_R)_{a1} \cdot \zeta - \frac{3}{2}\theta_{a1} M_1 \right|^2 . \tag{5.31}$$

with α denoting the fine-structure constant. The second term in the amplitude is proportional to the mass of the sterile neutrino M_1, while the first term to the mass m_{l_a} of the charged intermediate lepton l_a. This can be understood from the following consideration. Because the photon has spin one, there must be a chirality flip on the fermionic line. If, in flavour basis, there is a W_L-W_R mixing, we have the chirality flip[3] on the internal line of

[2]We must note, that careful analysis gives a stronger (in some regions of masses by an order of magnitude) bound. See detailed discussion in Sec. 5.1.2 of [79] and [90, 91, 92, 93, 94, 95, 96]. For our purposes this approximate (weak) bound is sufficient.

[3]As discussed in Sec. 2 a chirality flip is equivalent to mass insertion.

the charged fermion, which produces a term proportional to m_{l_a}. Otherwise, the chirality flip happens on one of the outer lines of the diagram, resulting in a term proportional to the Majorana mass of the sterile neutrino.

If the gauge boson mixing is absent ($\zeta = 0$), only the second term contributes, and we obtain, using Eq. (5.7), the usual result

$$\Gamma_{N_1 \to \gamma\nu}\Big|_{\zeta=0} \simeq \frac{9\, G_F^2\, \alpha\, M_1^5}{256\, \pi^4} \times \theta_1^2 \, . \tag{5.32}$$

If it is present, then, barring the unlikely cancellation between the two terms in Eq. (5.31), we can constrain ζ using the X-ray bound (5.30)

$$\zeta^2 \lesssim 9 \times 10^{-19} \frac{m_{l_\tau}^2}{\sum_{a=e,\mu,\tau} |m_{l_a} (V_R)_{a1}|^2} \left(\frac{1 \text{ keV}}{M_1} \right)^3 \, . \tag{5.33}$$

Thus, the mixing angle of the W-bosons must be vanishingly small.

Finally, we would like to note, that in a LR symmetric model with the Higgs sector as described in Sec. 3.5.1, the W_L-W_R mixing angle ζ is determined by Eq. (3.98):

$$\tan 2\zeta = -\frac{2\kappa\kappa'}{v_R^2 - v_L^2} \, . \tag{5.34}$$

where κ and κ' are bidoublet VEVs and v_L and v_R correspond to the left- and right-handed Higgs triplet VEVs, respectively. Thus, Eq. (5.33) strongly restricts the ratio $\kappa\kappa'/(v_R^2 - v_L^2)$.

5.1.7 Summary of Constraints

Let us summarise this section. A theory where the DM sterile neutrino was in thermal equilibrium at some moment during the evolution of the Universe, should satisfy the following set of constraints:

- From X/γ-ray observations, we have the model independent upper limit on the radiative decay width of the DM sterile neutrino N_1 (see Eq. (5.30))

$$\Gamma_{N_1 \to \gamma\nu} \lesssim 9.9 \times 10^{-27} \sec^{-1} \, . \tag{5.35}$$

  Note that this is a conservative value, c.f. the footnote on page 58.

  This constraint translates to the limit on the sterile-active neutrino mixing angle (cf. Eq. (5.29))

$$\theta_1^2 \lesssim 1.8 \times 10^{-5} \left(\frac{1 \text{ keV}}{M_1} \right)^5 \, , \tag{5.36}$$

  and to the bound on the mixing between the left and right gauge bosons (cf. Eq. (5.33))

$$\zeta^2 \lesssim 9 \times 10^{-19} \frac{m_{l_\tau}^2}{\sum_{a=e,\mu,\tau} |m_{l_a} (V_R)_{a1}|^2} \left(\frac{1 \text{ keV}}{M_1} \right)^3 \, . \tag{5.37}$$

- From the structure formation requirements (Ly-α bound), the mass of the sterile neutrino is constrained in the same way as the mass of a thermal relic, i.e.

$$M_1 \gtrsim 1.6 \text{ keV} \, . \tag{5.38}$$

- The right abundance of the sterile neutrino can be achieved by an out-of-equilibrium decay of a long-lived heavy particle. We will use another sterile neutrino of the model, N_2, for this purpose, but most considerations here can be also applied to another long-lived particle present in the theory.

  To provide the proper entropy release (5.17), N_2 should decouple while relativistic and its total decay width should be

$$\Gamma_{N_2} \simeq 0.50 \times 10^{-6} \frac{g_N^2}{4} \frac{g_{*\mathrm{f}}^2}{g_*^2} \bar{g}_*^{1/2} \frac{M_2^2}{m_{\mathrm{Pl}}} \left(\frac{1\ \mathrm{keV}}{M_1}\right)^2 . \tag{5.39}$$

- At the same time, this heavy neutrino N_2 should decay before BBN, which bounds its lifetime to be shorter than approximately $0.1 \div 2$ s. Then, the proper entropy can be generated only if its mass is larger than

$$M_2 > \left(\frac{M_1}{1\ \mathrm{keV}}\right) (1.7 \div 10)\ \mathrm{GeV} . \tag{5.40}$$

- The entropy is effectively generated by out-of-equilibrium decay 5.1.5, when the particle decoupled while still relativistic. If this particle is N_2, i.e. one of the sterile neutrinos, then its decoupling happens at temperature (5.12), and $T_f > M_2$ requires the bound on the right-handed gauge boson mass

$$M > \frac{1}{g_{*\mathrm{f}}^{1/8}} \left(\frac{M_2}{1\ \mathrm{GeV}}\right)^{3/4} (10 \div 16)\ \mathrm{TeV} . \tag{5.41}$$

  Note, that this is the only requirement which changes in the case of entropy generated by some other particle instead of the heavy sterile neutrino.

5.2 Models with Low Scale Type I See-Saw

Let us start from the analysis of models where the active neutrino masses are generated by a "type I" see-saw formula. This means that in the neutrino mass matrix (5.2), M_L vanishes. The values for the mixing angles θ_I^2 (cf. Eq. (5.7)) are restricted by the requirements on the decay widths of the sterile neutrinos. As discussed in the previous chapter, θ_1 is bounded from above by X-ray observations (see Eq. (5.29)), whereas θ_2, regulating the decay width of the entropy generating neutrino,[4] is bounded by requiring a long enough life-time.

A convenient way to parametrize the Dirac mass matrix m_D, separating high- and low-energy parameters, is provided by the Casas & Ibarra parametrization [48] reviewed in Sec. 3.4.2. By using Eq. (3.61):[5]

$$m_D = \pm i\, U^* \sqrt{m_\nu^{\mathrm{diag}}}\, R \sqrt{M_R^{\mathrm{diag}}} , \tag{5.42}$$

[4] Note that additional generation of the entropy by the third neutrino does not change conclusions.

[5] As far as we are using in this section the basis with diagonal $M_R = M_R^{\mathrm{diag}}$, the right handed mixing matrix is trivial, $V_R = I$.

we get, with Eq. (5.7),

$$\theta_I^2 = \frac{\left[\sqrt{M_R^{\rm diag}}\, R^T\, m_\nu^{\rm diag}\, R^*\, \sqrt{M_R^{\rm diag}}\right]_{II}}{M_I^2}\,, \tag{5.43}$$

where

$$m_\nu^{\rm diag} = {\rm diag}(m_1, m_2, m_3)\,, \quad \text{and} \quad M_R^{\rm diag} = {\rm diag}(M_1, M_2, M_3)\,. \tag{5.44}$$

Here R is a complex orthogonal matrix parametrized by three complex angles ω_{12}, ω_{13}, and ω_{23} (cf. Eq. (3.60)).

Let us check, whether we can satisfy the bounds on the mixing angles if the active masses m_i are consistent with the observed neutrino oscillation mass differences, summarised below. The current best-fit and 3σ ranges are (see Ref. [99]):

$$\Delta m^2_{\rm sol} = \left(7.65^{+0.69}_{-0.6}\right) \times 10^{-5}\ {\rm eV}^2\,, \tag{5.45a}$$

$$\Delta m^2_{\rm atm} = \left(2.4^{+0.35}_{-0.33}\right) \times 10^{-3}\ {\rm eV}^2\,. \tag{5.45b}$$

In the following analysis we will for convenience order the active neutrino masses as

$$m_1 < m_2 < m_3\,. \tag{5.46}$$

From Eq. (5.43), we get, using the parametrization in Eq. (3.60), for the first two sterile neutrinos

$$\begin{aligned} M_1\theta_1^2 = {} & m_3\,|\sin\omega_{13}|^2 + m_2|\cos\omega_{13}|^2|\sin\omega_{12}|^2 \\ & + m_1|\cos\omega_{13}|^2|\cos\omega_{12}|^2\,, \end{aligned} \tag{5.47a}$$

$$\begin{aligned} M_2\theta_2^2 = {} & m_3|\cos\omega_{13}|^2|\sin\omega_{23}|^2 \\ & + m_2|\cos\omega_{23}\cos\omega_{12} - \sin\omega_{23}\sin\omega_{13}\sin\omega_{12}|^2 \\ & + m_1|\cos\omega_{23}\sin\omega_{12} + \sin\omega_{23}\sin\omega_{13}\cos\omega_{12}|^2\,. \end{aligned} \tag{5.47b}$$

Note that as far as we ordered the active neutrino masses, if we change m_1 to zero, and replace m_3 by m_2, the right hand sides of Eqs. (5.47) can only become smaller. We can also confine ourselves to the real values of the mixing angles, as far as the sine and cosine absolute values only become larger for complex angles, and the inequality $|z - w| \geq ||z| - |w||$ is used to transform the square of the difference of the angles in Eq. (5.47b). Thus, the following inequalities should be satisfied

$$M_1\theta_1^2 \geq m_2\left\{\sin^2\omega_{13} + \cos^2\omega_{13}\sin^2\omega_{12}\right\}\,, \tag{5.48a}$$

$$\begin{aligned} M_2\theta_2^2 \geq m_2\Big\{ & \cos^2\omega_{13}\sin^2\omega_{23} + \big(|\cos\omega_{23}||\cos\omega_{12}| \\ & - |\sin\omega_{23}||\sin\omega_{13}||\sin\omega_{12}|\big)^2\Big\}\,. \end{aligned} \tag{5.48b}$$

The minimum of the sum of the right hand sides is m_2, and therefore the following very simple inequality always holds

$$M_1\theta_1^2 + M_2\theta_2^2 \geq m_2 \geq \Delta m_{\rm sol}\,. \tag{5.49}$$

The second inequality is trivially fulfilled, since in all possible mass hierarchies the mass of the second (in mass) active neutrino is larger than $\Delta m_{\rm sol}$. The meaning of the inequality

(5.49) is very simple—one can not generate active neutrino masses with the "type I" see-saw formula without sufficient mixing between the active and sterile neutrino sectors. Note in passing, that the cancellation is possible in another direction—one can have very small active neutrino masses and large active-sterile mixing.

Now, we are ready to compare the requirement from the observed active neutrino masses, Eq. (5.49), with the DM bounds on the mixing angles. The angle θ_2 can be bound from the width (5.25) required to generate sufficient entropy. Estimating the width of the heavy neutrino as (cf. App. A)

$$\Gamma_{N_2} \geq \frac{G_F^2 M_2^5}{192\pi^3} \cdot \theta_2^2 \,, \tag{5.50}$$

we have

$$M_2\theta_2^2 \lesssim 1.8 \times 10^{-3} \bar{g}_*^{1/2} \left(\frac{1\ \mathrm{GeV}}{M_2}\right)^2 \left(\frac{1\ \mathrm{keV}}{M_1}\right)^2 \,. \tag{5.51}$$

It can be clearly seen that for all allowed masses M_1 and M_2 this is much smaller than Δm_{sol}.

The contribution of the DM sterile neutrino itself can be larger than $M_2\theta_2^2$. From Eq. (5.29), we have

$$M_1\theta_1^2 \lesssim 1.8 \times 10^{-2} \left(\frac{1\ \mathrm{keV}}{M_1}\right)^4 \,. \tag{5.52}$$

Together with the Ly-α bound on the WDM mass, Eq. (5.21), this contribution again violates Eq. (5.49). Thus, we conclude that the small mixing angles, required by the proper DM abundance and good DM properties in the model, prevent generation of the observed active neutrino masses by the "type I" see-saw formula.

Note, however, that without the Ly-α bound it would have been possible for very light WDM, with

$$M_1 \lesssim 1.2\ \mathrm{keV} \,. \tag{5.53}$$

5.3 Type II See-Saw—working Example

In the previous section we have seen that if one of the not DM-like sterile neutrinos is responsible for entropy production, it is impossible to get the observed active neutrino masses with a "type I" like see-saw. Here we will present a working example of the sterile neutrino DM in the framework of a LR symmetric model, where the active neutrino masses are generated by the contribution of the type II see-saw.

We will continue to work in the framework of the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, sketched in pieces in Sec. 5.1.1 and reviewed in more detail in Sec. 3.5. Let us do a short repetition of important facts. As already discussed, in a properly LR symmetric model, the left- and right-handed leptons are treated symmetrically. One has the usual SM doublets Ψ_L^i with generation index $i \in \{1, 2, 3\}$, and three additional right-handed neutrinos which form together with the three charged right-handed lepton fields the $SU(2)_R$ doublets Ψ_R^i (cf. Eq. (3.66)). The Higgs sector consists of one $SU(2)_L$ triplet Δ_L, one $SU(2)_R$ triplet Δ_R and one bidoublet ϕ. If their neutral components acquire a non-zero VEV the gauge symmetry from above will be spontaneously broken and a neutrino mass matrix of the following pattern will result, (cf. Sec. 3.5.3):

$$\mathcal{M} = \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \equiv \begin{pmatrix} f_L\, v_L & y\, \kappa_+ \\ y^T\, \kappa_+ & f_R\, v_R \end{pmatrix} , \tag{5.54}$$

The Majorana blocks on the diagonal come from the coupling of the bilinears $\psi^{i\,T}_{L,R}\mathcal{C}\psi_{L,R}{}^j$ to the triplets $\Delta_{L,R}$, respectively, and the Dirac-type ones from the coupling of $\overline{\psi_L}\psi_R$ to the bidoublet ϕ and its complex conjugate $\tilde{\phi} = \sigma_2\phi^*\sigma_2$. The VEVs of the neutral components in $\Delta_{L,R}$ are denoted by $v_{L,R}$, whereas the SM scale $\kappa_+ = \sqrt{\kappa^2 + \kappa'^2} = 174$ GeV is a combination of the bidoublet VEVs κ and κ' (see Sec. 3.5.2). These quantities are related by Eq. (3.80),

$$\gamma = \frac{v_L v_R}{\kappa_+^2} , \tag{5.55}$$

where γ is a function of parameters in the Higgs potential, and naturally of order one (for more details see Sec. 3.5.2).

In the following we will postulate the *exact* discrete LR symmetry (3.73), and assume for convenience real Yukawa couplings. This requires

$$f_L = f_R , \quad y = y^T . \tag{5.56}$$

As we will now show, with such a model, it is possible to satisfy all the requirements form Sec. 5.1. Let us consider the type II see-saw formula, following from block diagonalization of Eq. (5.54) with the assumption $\mathcal{O}(M_R) \gg \mathcal{O}(m_D) \gg \mathcal{O}(M_L)$:

$$m_\nu = v_L f_L - \frac{\kappa_+^2}{v_R} y f_R^{-1} y^T . \tag{5.57}$$

After applying the conditions of discrete left-right symmetry, Eq. (5.56), we find

$$m_\nu = v_L f - \frac{\kappa_+^2}{v_R} y f^{-1} y , \tag{5.58}$$

To simplify the calculations, we further assume for illustration, proportionality of the Dirac-Yukawa y to the triplet Yukawa f, i.e. $y = \mathfrak{P} f$, where $\mathfrak{P}$ is a numerical number. Equation (5.58) then goes into

$$m_\nu = \left(v_L - \frac{\kappa_+^2 \mathfrak{P}^2}{v_R} \right) f . \tag{5.59}$$

In this case all Yukawas are diagonalised by the same transformation—the transformation which brings m_ν into diagonal form, i.e. the PMNS matrix U. The ratios of the eigenvalues of the matrices on both sides of the equality are then the same

$$\frac{m_1}{m_2} = \frac{f_1}{f_2} = \frac{M_1}{M_2} . \tag{5.60}$$

Thus, the mass spectrum of the sterile neutrinos (or, specifically, the BBN requirement (5.27)) leads to the same hierarchical active neutrino spectrum

$$\frac{m_1}{m_2} \lesssim 5.9 \times 10^{-7} . \tag{5.61}$$

This implies that the lightest active neutrino should be very light, and we can have either normal or inverse hierarchy. For definiteness, we will use the normal hierarchy in our example, though the inverse one works equally well (one should only take into account that in the latter case $M_2 \simeq M_3$, and thus—because of equal mixing angles (cf. Eq. (5.63))—$\Gamma_{N_2} \simeq \Gamma_{N_3}$, such that both N_1 and N_2 generate the same amount of entropy (5.24)). As far as the active neutrino mass hierarchy is fixed, we have $m_2 \simeq \Delta m_{\rm sol}$ and $m_3 \simeq \Delta m_{\rm atm}$. We then obtain for the mass of the third sterile neutrino

$$M_3 = \frac{m_3}{m_2} M_2 \, . \tag{5.62}$$

Furthermore, since the following relation holds (use Def. (5.7) together with (5.6))

$$\theta_I^2 = \sum_a \frac{\left|\left(U^* m_D^{\rm diag}\right)_{aI}\right|^2}{|M_I|^2} = \frac{\left|\left(m_D^{\rm diag}\right)_{II}\right|^2}{|M_I|^2} = \frac{\kappa_+^2 \mathfrak{P}^2}{v_R^2} \, , \tag{5.63}$$

the active-sterile mixing angles in the case of proportional Yukawa constants are all the same, while the mixing angles for individual flavours are proportional to the PMNS matrix U:

$$\theta_{aI} = (U^*)_{aI} \frac{\kappa_+ \mathfrak{P}}{v_R} = (U^*)_{aI} \sqrt{\theta_I^2} \, . \tag{5.64}$$

Thus, the total decay width Γ_{N_2} is proportional to θ_2^2 (see App. A). The value of θ_2^2 is then defined from the requirement of sufficient entropy production, Eq. (5.25), and depends only on M_1 and M_2.

At this moment the only free parameter left is the VEV ratio γ, and everything can be expressed via γ, M_1, M_2 and $m_2 \simeq \Delta m_{\rm sol}$, $m_3 \simeq \Delta m_{\rm atm}$. From Eqs. (5.59) and (5.55), we get

$$v_R = \sqrt{\frac{\kappa_+^2 \gamma}{\frac{m_2}{M_2} + \theta_2^2}} \, . \tag{5.65}$$

The VEV of the left-handed triplet Δ_L is then given by

$$v_L = \sqrt{\kappa_+^2 \gamma \left(\frac{m_2}{M_2} + \theta_2^2\right)} \, . \tag{5.66}$$

Together with Eq. (5.65), Eq. (5.63) determines the proportionality constant $\mathfrak{P}$

$$\mathfrak{P} = \sqrt{\theta_2^2 v_R^2 / \kappa_+^2} \, . \tag{5.67}$$

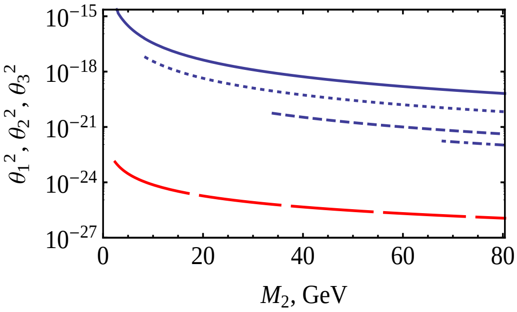
The full mass matrix (5.54) is as follows

$$\mathcal{M} = \begin{pmatrix} U^* & 0 \\ 0 & U^* \end{pmatrix} \begin{pmatrix} M_L^{\rm diag} & m_D^{\rm diag} \\ m_D^{\rm diag} & M_R^{\rm diag} \end{pmatrix} \begin{pmatrix} U^\dagger & 0 \\ 0 & U^\dagger \end{pmatrix} , \tag{5.68}$$

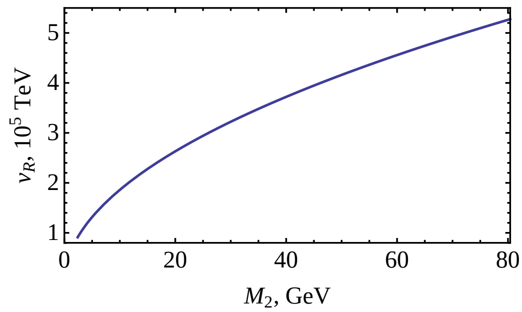
where

$$m_D^{\rm diag} = \mathfrak{P} \frac{\kappa_+}{v_L} M_L^{\rm diag} = \mathfrak{P} \frac{\kappa_+}{v_R} M_R^{\rm diag} \, , \tag{5.69a}$$

$$M_L^{\rm diag} = \frac{v_L}{v_R} M_R^{\rm diag} \, , \tag{5.69b}$$



(a) θ_2^2 as a function of M_2. $M_1 = 1.6$ keV: (continuous); $M_1 = 5$ keV: (dotted); $M_1 = 20$ keV: (dashed) and $M_1 = 40$ keV: (dashed-dotted). It is accounted for the lower bound, Eq. (5.27). The long-dashed red line shows the ratio $(M_W/M)^4$ and illustrates that processes mediated by W_R-bosons can be neglected in the decay rate of N_2.



(b) v_R as a function of M_2. The dependence on M_1 is very weak and therefore invisible in this plot. However, if one goes to larger M_1, one has to take into account the lower BBN bound (5.27) on M_2.

Figure 5.4: LR symmetric model with keV scale sterile neutrino WDM and an entropy generating heavy sterile neutrino of mass M_2.

and

$$M_R^{\text{diag}} = \text{diag}\left(M_1, M_2, M_3\right) \ . \tag{5.70}$$

Let us fix the input values. It was mentioned before, that $\gamma \sim \mathcal{O}(1)$ is natural in the LR symmetric model, therefore we simply choose $\gamma = 1$. For the masses of the DM and the entropy producing sterile neutrinos, we choose the smallest possible ones (see Sec. 5.1):

$$M_1 = 1.6 \text{ keV} \ , \qquad M_2 = 2.7 \text{ GeV} \ . \tag{5.71}$$

With such an input, we obtain:

$$\begin{aligned} m_1 &= 5.2 \times 10^{-9} \text{ eV} \ , & v_L &= 313 \text{ keV} \ , \\ m_2 &\simeq \sqrt{\Delta m^2_{\text{sol}}} = 8.7 \times 10^{-3} \text{ eV} \ , & v_R &= 9.67 \times 10^4 \text{ TeV} \ , \\ m_3 &\simeq \sqrt{\Delta m^2_{\text{atm}}} = 4.9 \times 10^{-2} \text{ eV} \ , & \mathfrak{P} &= 0.027 \ , \\ M_3 &= 15.1 \text{ GeV} \ , & \theta_1^2 = \theta_2^2 = \theta_3^2 &= 2.3 \times 10^{-15} \ . \end{aligned} \tag{5.72}$$

The Figures 5.4(a) and 5.4(b), respectively show the values of θ_1^2 and v_R for several M_1 and $M_2 \gtrsim 2.7$ GeV. Because of the smallness of θ_2^2 compared to m_2/M_2 and its suppression with M_1^{-2} and M_2^{-3} (see Eq. (5.51)), v_R given by Eq. (5.65) is effectively independent of θ_2^2. Therefore, the curve of v_R has a very weak M_1 dependence (invisible in Fig. 5.4(b)). For larger M_1 it is accounted for the BBN bound (5.27) on M_2.

One can check that none of the constraints, summarized in Sec. 5.1.7, is violated. Even the mixing angle θ_1^2, corresponding to our DM neutrino, is much smaller than its upper bound (5.29). Furthermore because of the large v_R scale, the additional non-SM gauge and Higgs bosons are not observable (they all have masses $\propto v_R$ [56]). However, we should choose the Higgs potential to have very small mixing between the left and right gauge bosons (see Eqs. (5.33) and (5.34)).

To conclude, we demonstrate that the obtained VEV scales are in no conflict with the ρ parameter, introduced at the end of Sec. 3.3.2. Equation (3.36) applied on the LR symmetric model gives

$$\rho = \frac{\kappa_+^2 + 2\,|v_L|^2}{\kappa_+^2 + 4\,|v_L|^2}\,, \tag{5.73}$$

For v_L of the order of MeV the deviations are well below the current experimentally allowed ones, which have $\mathcal{O}(10^{-4})$ [36].

Chapter 6

Conclusions

Introducing some basics of the theory of massive neutrinos has been the starting point of this work. In particular we have explained the difference between Dirac and Majorana mass terms, motivated the introduction of sterile (right-handed) neutrinos and demonstrated that the see-saw mechanism in its type I or II form could be an aesthetic explanation of the small active neutrino masses. Then, after a short review of a useful parametrization, proposed by Casas and Ibarra, we have closed Chap. 3 by describing a Left-Right symmetric model, which in a very natural way implements the see-saw mechanism of type II.

In chapter 4 we have given an introduction of some fundamentals of cosmology, which are of essential relevance for this work. We have outlined the derivation of the fundamental Friedmann and fluid equations and determined their solutions in specific cosmological scenarios. In Sec. 4.3 we have sketched early Universe thermodynamics and discussed the freeze-out process of thermal relics as well as the entropy release due to an out-of-equilibrium decaying particle. With Sec. 4.4, providing basic facts of Dark Matter in cosmology, we have concluded this Chapter and passed on to our main goal—the analysis of the possibility of thermally produced keV scale sterile neutrino Warm Dark Matter in gauge extensions of the Standard Model.

During the analysis of such a realisation in Chap. 5, we have found three ways capable to circumvent the naïve expectation of significant overproduction of Dark Matter in case of a keV scale sterile neutrino, decoupling from the thermal bath while still relativistic. They include a low reheating temperature so that the thermal equilibrium is never reached by the would be Dark Matter sterile neutrino, (very) large number of degrees of freedom in the early Universe at the Dark Matter neutrino freezeout, or sufficient dilution of its density by out-of-equilibrium decay of a massive particle like another heavy sterile neutrino (cf. Fig. 5.1). The last possibility, assumed to be the most natural one, has been further analysed[1] and a set of requirements for this scenario has been formulated. Summarized in short, these requirements bound the mass of the Dark Matter sterile neutrino from below from structure formation considerations; limit its mixing angle with active neutrinos and constrain mixing between the ordinary Standard Model (left) and additional (right) gauge bosons from the radiative decay of the Dark Matter sterile neutrino; fix the lifetime of the heavier sterile neutrino from the requirement to have sufficient entropy release, diluting the Dark Matter abundance down to the observed value; and finally constrain the mass of these heavier sterile neutrinos from considerations of Big Bang Nucleosynthesis.

We have demonstrated in this framework, that the type I low scale see-saw mechanism of generating masses for the active neutrinos can not lead to sufficient dilution of the Dark Matter abundance due to the very small mixings between the active and sterile neutrino sectors. At the same time, we have provided a working example, where the active neutrinos

[1] Note that another natural possibility is achieved in the νMSM model [12, 13], where the introduction of sterile neutrinos is the *only* extension of the Standard Model, and the keV sterile neutrino does not enter thermal equilibrium up to Planck scale temperatures.

are generated by a type II style see-saw in the context of an exactly Left-Right symmetric theory. The obtained general constraints and observations can serve as a basis for the search of a Grand Unified Theory with Warm Dark Matter sterile neutrinos.

Appendix A

Decay Widths of sterile Neutrinos

In the mass range 2.7 GeV $\leq M_2 < M_W$, a sterile neutrino N_2 dominantly decays into leptons and spectator quarks. The corresponding partial widths can be calculated in the νMSM because additional boson-interactions, which usually appear in more complicated models, are of high scale—of order $\mathcal{O}(M)$—compared to that of electroweak scale. To make use of the νMSM results, we have, for being on the safe side, to compare the suppression factors in the νMSM $\sim \mathcal{O}(|\theta_i|^2)$ with that of additional interactions $\sim \mathcal{O}(M_W/M)^4$, appearing in the models we are interested in. Furthermore, the mixing of the new bosons should be small compared to $\mathcal{O}(|\theta_i|)$, otherwise there could be significant contributions from processes where new bosons mix with the SM ones (see, for example, App. B). These effects are neglected in the following calculations. One can see, that for most practical purposes it is the case, as far as the bound on the gauge boson mixings (5.33) is much stronger than those for the active-sterile neutrino mixings.

Moreover, such additional contributions do not affect the conclusions in the main part of this work. Indeed, in Sec. 5.2 additional interactions can only result in a stronger bound and therefore the conclusion remains the same. In the type II see-saw model discussed in Sec. 5.3, we need the exact value of the width. However, if we assume no mixing of the W-bosons ($\zeta = 0$), the contributions from W_R-boson mediated processes are in the considered mass range negligible small (cf. Fig. 5.4(a)).

In what follows we give all relevant decay rate formulae (at tree-level) of a sterile neutrino N_2, with a mass M_2 located above the BBN bound 2.7 GeV (cf. Eq. (5.27)) and below the SM W-boson mass $M_W \simeq 80$ GeV [100].

$$\Gamma_1\left(N_2 \to \sum_{\alpha,\beta} \nu_\alpha \bar{\nu}_\beta \nu_\beta\right) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot \sum_\alpha |\theta_{2\alpha}|^2 \,, \tag{A.1a}$$

$$\Gamma_2\left(N_2 \to l^-_{\alpha\neq\beta} l^+_\beta \nu_\beta\right) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot |\theta_{2\alpha}|^2 \left(1 - 8x_l^2 + 8x_l^6 - x_l^8 - 12x_l^4 \log x_l^2\right) \,, \quad x_l = \frac{\max\left[M_{l_\alpha},\ M_{l_\beta}\right]}{M_2} \,, \tag{A.1b}$$

$$\begin{aligned}\Gamma_3\left(N_2 \to \nu_\alpha l^+_\beta l^-_\beta\right) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot |\theta_{2\alpha}|^2 \Bigg[&(C_1(1-\delta_{\alpha\beta}) + C_3\delta_{\alpha\beta}) \left(\left(1 - 14x_l^2 - 2x_l^4 - 12x_l^6\right)\sqrt{1-4x_l^2}\right. \\ &\left. + 12x_l^4\left(x_l^4 - 1\right) L\right) + 4\left(C_2(1-\delta_{\alpha\beta}) + C_4\delta_{\alpha\beta}\right)\left(x_l^2\left(2 + 10x_l^2 - 12x_l^4\right)\sqrt{1-4x_l^2}\right. \\ &\left. + 6x_l^4\left(1 - 2x_l^2 + 2x_l^4\right) L\right)\Bigg] \,,\end{aligned} \tag{A.1c}$$

with

$$L = \log\left[\frac{1 - 3x_l^2 - \left(1 - x_l^2\right)\sqrt{1-4x_l^2}}{x_l^2\left(1 + \sqrt{1-4x_l^2}\right)}\right] \,, \qquad x_l \equiv \frac{M_l}{M_2} \,,$$

fermions	g_L^q	g_R^q
$q = \nu_e, \nu_\mu, \nu_\tau$	$\frac{1}{2}$	0
$q = u, c, t$	$\frac{1}{2} - \frac{2}{3}s_W^2$	$-\frac{2}{3}s_W^2$
$q = d, s, b$	$-\frac{1}{2} + \frac{1}{3}s_W^2$	$\frac{1}{3}s_W^2$

Table A.1: Coupling constants; $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$.

and

$$C_1 = \frac{1}{4}\left(1 - 4\sin^2\theta_w + 8\sin^4\theta_w\right) , \qquad C_2 = \frac{1}{2}\sin^2\theta_w\left(2\sin^2\theta_w - 1\right) ,$$
$$C_3 = \frac{1}{4}\left(1 + 4\sin^2\theta_w + 8\sin^4\theta_w\right) , \qquad C_4 = \frac{1}{2}\sin^2\theta_w\left(2\sin^2\theta_w + 1\right) .$$

The formulae for the decay modes into quarks are presented below. In the range 2.7 GeV $\leq M_2 < M_W$ it is sufficient to use the free quark approach for the decay products. We give these formulae in the approximation where M_2 is much heavier than the decay product masses (unlike above, Eqs. (A.1), for the lepton decays). The corrections are important at the threshold, when new decay channels open. However, at high mass M_2 this introduces a rather small relative error, because the number of open channels into light particles is significant, and provide the main part of the decay width. The exact analysis would smooth the discontinuities of the decay width at the mass thresholds (see Fig. A.1). We have

$$\Gamma_4\left(N_2 \to l_\alpha^- U\bar{D}\right) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot 3 \cdot |V_{UD}|^2 \cdot |\theta_{2\alpha}|^2 , \tag{A.2a}$$

$$\Gamma_5\left(N_2 \to \nu_\alpha q\bar{q}\right) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot 3 \cdot \Xi^q \cdot |\theta_{2\alpha}|^2 , \tag{A.2b}$$

with

$$\Xi^q = g_L^{\nu\,2} \cdot \left(g_L^{q\,2} + g_R^{q\,2}\right) .$$

The factor 3 is the colour factor and V_{UD} are the CKM-matrix elements. The coupling constants g_L and g_R correspond to the coupling of the Z-boson to left- or right-handed particles, respectively. For a fermion f, with weak isospin component I_3^f and charge q_f, one has:

$$g_L^f = I_3^f - q_f \sin^2\theta_W , \tag{A.3a}$$

$$g_R^f = -q_f \sin^2\theta_W . \tag{A.3b}$$

Table A.1 gives the required values of the charges. In the mass range of M_2 mentioned above, the Majorana neutrino total decay rate Γ_{N_2} is a sum of all rates presented above multiplied by a factor of 2, which accounts for charge-conjugated decay modes.

When M_2 exceeds the SM Z-boson mass $M_Z = 91$ GeV, and if contributions of new interactions are negligible the sterile neutrino predominantly decays into a SM gauge boson and a lepton. Then its total decay width is given by

$$\Gamma_{N_2} = 2\frac{G_F M_2^3}{8\sqrt{2}\pi}\left[\left(1 + \frac{2M_W^2}{M_2^2}\right)\left(1 - \frac{M_W^2}{M_2^2}\right)^2 + \frac{1}{2}\left(1 + \frac{2M_Z^2}{M_2^2}\right)\left(1 - \frac{M_Z^2}{M_2^2}\right)^2\right] \cdot \sum_\alpha |\theta_{2\alpha}|^2 . \tag{A.4}$$

In between, where $M_W \leq M_2 < M_Z$, one can approximate the width by the first term in Eq. (A.4).

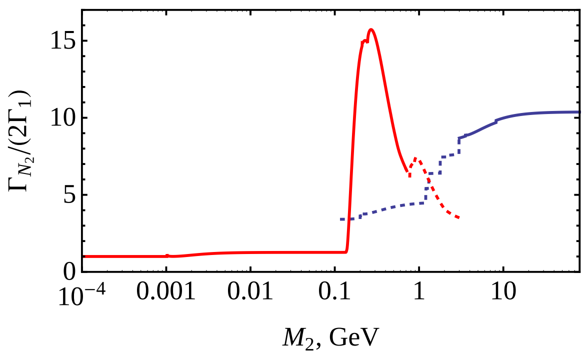


Figure A.1: The ratio $\Gamma_{N_2}/2\Gamma_1$ calculated in the specific LR model of Sec. 5.3. We used Eqs. (A.1) together with the formulas for the two-body decays into mesons [100] for the red curve and Eqs. (A.1) together with Eqs. (A.2) for the blue curve. The dotted curves correspond to the region, where both approximations are not entirely reliable.

Below $M_2 \sim 2$ GeV it is important to consider mesons instead of quarks as final states for the sterile neutrino decay. Therefore, instead of the three-body decay modes into spectator quarks, Eqs. (A.2), one has to use the corresponding two-body ones into mesons (see [100] for the decay width formulas). In our case this mass range of M_2 is forbidden by the BBN-bound (5.27) and therefore we do not list them here. Nevertheless, to get a feeling of the behaviour of the total width Γ_{N_2}, we show in Fig. A.1 the ratio $\Gamma_{N_2}/2\Gamma_1$ calculated in the specific LR model described in Sec. 5.3, using both the free quark and chiral meson approximations. It is clearly seen, that the transition between two approximations happen around $M_2 \sim \mathcal{O}(1)$ GeV. In this region (dotted in Fig. A.1) decays into spectator quarks more and more replace decays into mesons and therefore one has to carefully reanalyse the given formulas, if one is interested in this mass range. Furthermore note that because of Eq. (5.64), the total decay width is proportional to θ_2^2 and therefore the plotted ratio is independent of this quantity. In more general models, Eq. (5.64) will no longer be valid. However, as one recognizes by considering the formulas together with the definition of θ_2 (cf. Eq. (5.7)), there will be no significant difference, especially for heavy masses M_2, so that Fig. A.1 can be used as a good estimate in such models.

Appendix B

Radiative Decay Width

Here we will provide some details of calculation of the width for the radiative decay $N_1 \to \gamma\nu_i$ shown in Fig. B.1. We will follow Ref. [97], where general formulae for this type of process are given. In our case N_1 denotes a heavy sterile neutrino with mass M_1, ν_i one of the active neutrinos with mass m_i and γ a photon. The neutrinos are considered as Majorana particles.

The amplitude for such a decay is $e\varepsilon^*_\mu(q)\mathcal{M}^\mu$, where e is the electric charge of the positron and $\varepsilon^*_\mu(q)$ the polarisation vector of the outgoing photon. The Ward identity for the electromagnetic current implies that $q_\mu\mathcal{M}^\mu$ must be zero; therefore $\mathcal{M}^\mu$ must have the form

$$\mathcal{M}^\mu = \bar{u}_i \left[i\sigma^{\mu\nu}q_\nu \left(\sigma_L P_L + \sigma_R P_R\right)\right] u_1 \ , \tag{B.1}$$

where $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$ and $P_L = (1-\gamma_5)/2$ and $P_R = (1+\gamma_5)/2$ are the projectors of chirality, introduced in Sec. 2.1. σ_L and σ_R are numerical coefficients with dimension of inverse mass. The partial decay width for $N_1 \to \gamma\nu_i$ is then given by

$$\Gamma_{N_1\to\gamma\nu_i} = \frac{(M_1^2 - m_i^2)^3}{16\pi M_1^3}\left(|\sigma_L|^2 + |\sigma_R|^2\right) \ . \tag{B.2}$$

By comparing the Lagrange-term for the charged current (5.1) combined with Eqs. (5.8) and the transformation rule which diagonalizes the neutrino mass matrix in Eq. (5.2)

$$\begin{pmatrix} \tilde{\nu}_{aL} \\ \tilde{N}^c_{aR} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} \nu_{iL} \\ N^c_{IR} \end{pmatrix} , \tag{B.3}$$

with that given in chapter 5 of [97], we can easily calculate the coefficients σ_L and σ_R. Supposing from the very beginning, that the right-handed scale is much larger, than the

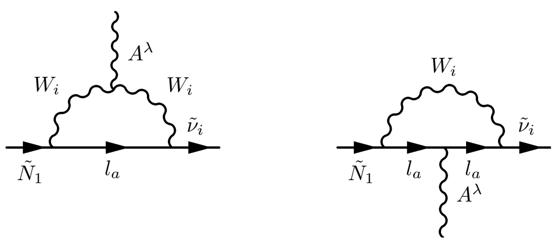


Figure B.1: Unitary-gauge diagrams contributing to the radiative neutrino decay with charged leptons propagating in the loop.

left one, $M \gg M_W \simeq 80.4$ GeV, and neglecting the active neutrino masses, we get[1]

$$i\sigma_R = \frac{g^2 e}{32 M_W^2 \pi^2} \times \sum_{a=e,\mu,\tau} \left\{ \cos\zeta \sin\zeta \, A^*_{ai} D^*_{a1} \, m_{l_a} \mathcal{F}(r_a) + \cos^2\zeta \, A^*_{ai} B_{a1} M_1 F(r_a) \right\} , \tag{B.4a}$$

$$i\sigma_L = \frac{g^2 e}{32 M_W^2 \pi^2} \times \sum_{a=e,\mu,\tau} \left\{ \cos\zeta \sin\zeta \, C_{ai} B_{a1} \, m_{l_a} \mathcal{F}(r_a) \right\} , \tag{B.4b}$$

where $F(r_a)$ and $\mathcal{F}(r_a)$ are functions of $r_a \equiv m_{l_a}^2 / M_W^2$. In our case, we have in good approximation $F(r_a) \simeq -3/2$ and $\mathcal{F}(r_a) \simeq 4$. The exact expressions for these functions were calculated by us and do agree with that given in Ref. [101].

Because of the Majorana nature of our in- and outgoing neutrinos, we also have to add the contribution of the complex conjugated process to our amplitude. This is easily obtained out of Eqs. (B.4) by putting in the substitutions

$$\begin{array}{ccc} A, B \to A^*, B^* & \text{and} & C, D \to C^*, D^* \, , \\ \gamma_5 \to -\gamma_5 & \Rightarrow & L, R \to R, L \, , \end{array} \tag{B.5}$$

and an overall negative sign coming from the photon vertex. After adding the derived σ_L and σ_R, it is easy to see that $|\sigma_L|^2 = |\sigma_R|^2$, where

$$|\sigma_L|^2 = \left(\frac{g^2 e}{32 M_W^2 \pi^2} \right)^2 \times \Bigg| 4 \cos\zeta \sin\zeta \sum_{a=e,\mu,\tau} (A_{ai} D_{a1} - C_{ai} B_{a1}) \, m_{l_a} - \frac{3}{2} \cos^2\zeta \left(\sum_{a=e,\mu,\tau} A_{ai} B^*_{a1} \right) M_1 \Bigg|^2 . \tag{B.6}$$

Using this expression in Eq. (B.2) we obtain

$$\Gamma_{N_1 \to \gamma \nu_i} \simeq \frac{G_F^2 \alpha M_1^3}{64 \pi^4} \times \Bigg| 4 \cos\zeta \, \sin\zeta \sum_{a=e,\mu,\tau} (A_{ai} \, D_{a1} - C_{ai} \, B_{a1}) \, m_{l_a} - \frac{3}{2} \cos^2\zeta \left(\sum_{a=e,\mu,\tau} A_{ai} \, B^*_{a1} \right) M_1 \Bigg|^2 . \tag{B.7}$$

Here G_F is the Fermi constant, α the fine-structure constant, and m_{l_a} is the mass of the charged lepton propagating in the loop.

The total width of the radiative decay is then given by the sum over the outgoing states

$$\Gamma_{N_1 \to \gamma \nu} = \sum_{i=1}^{3} \Gamma_{N_1 \to \gamma \nu_i} \, . \tag{B.8}$$

In a model where a see-saw mechanism of type I or II is responsible for the small active neutrino masses, the transformation (B.3) corresponds to Eq. (5.3). Putting this into our formulas, we get out of Eq. (B.7) the expression (5.31).

[1] Note, that our results do not coincide with the formulas in [101]. This is because of a mistake in the second term of the third line of equation (10) in [101]. The correct labelling of the transformation matrices should be $P_{aB} Q_{aA}$ instead of $P_{aA} Q_{aB}$. In our notations, where a sterile neutrino (with mass eigenstate index 1) decays through the radiative process into an active neutrino (with mass eigenstate index i) the expression $P_{aB} Q_{aA}$ translates into $B_{a1} C_{ai}$ which is contained in Eq. (B.4b).

Appendix C

The Higgs Potential in the LR symmetric Model

The most general renormalizable expression for the Higgs potential satisfying the gauge symmetry $G_{\mathrm{LR}} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ of the left-right symmetric model and in addition the discrete parity transformation defined by (cf. Eq. (3.73))

$$\Psi_L^i,\ q_L^i \leftrightarrow \Psi_R^i,\ q_R^i\,, \quad \phi \leftrightarrow \phi^\dagger\,, \quad \Delta_L \leftrightarrow \Delta_R\,, \tag{C.1}$$

is given by [56]

$$\begin{aligned}
\mathcal{V}(\phi, \Delta_L, \Delta_R) = &- \mu_1^2 \left[\mathrm{Tr}\left(\phi^\dagger \phi\right)\right] - \mu_2^2 \left[\mathrm{Tr}\left(\tilde{\phi}^\dagger \phi\right)\right] - \mu_3^2 \left[\mathrm{Tr}\left(\Delta_L \Delta_L^\dagger\right) + \mathrm{Tr}\left(\Delta_R \Delta_R^\dagger\right)\right] \\
&+ \lambda_1 \left[\mathrm{Tr}\left(\phi\phi^\dagger\right)\right]^2 + \lambda_2 \left\{\left[\mathrm{Tr}\left(\tilde{\phi}\phi^\dagger\right)\right]^2 + \left[\mathrm{Tr}\left(\tilde{\phi}^\dagger\phi\right)\right]^2\right\} \\
&+ \lambda_3 \left[\mathrm{Tr}\left(\tilde{\phi}\phi^\dagger\right) \mathrm{Tr}\left(\tilde{\phi}^\dagger\phi\right)\right] + \lambda_4 \mathrm{Tr}\left(\phi\phi^\dagger\right)\left[\mathrm{Tr}\left(\tilde{\phi}\phi^\dagger\right) + \mathrm{Tr}\left(\tilde{\phi}^\dagger\phi\right)\right] \\
&+ \rho_1 \left\{\left[\mathrm{Tr}\left(\Delta_L\Delta_L^\dagger\right)\right]^2 + \left[\mathrm{Tr}\left(\Delta_R\Delta_R^\dagger\right)\right]^2\right\} \\
&+ \rho_2 \left[\mathrm{Tr}\left(\Delta_L^2\right) \mathrm{Tr}\left(\Delta_L^{\dagger\,2}\right) + \mathrm{Tr}\left(\Delta_R^2\right) \mathrm{Tr}\left(\Delta_R^{\dagger\,2}\right)\right] \\
&+ \rho_3 \left[\mathrm{Tr}\left(\Delta_L\Delta_L^\dagger\right) \mathrm{Tr}\left(\Delta_R\Delta_R^\dagger\right)\right] \\
&+ \rho_4 \left[\mathrm{Tr}\left(\Delta_L^2\right) \mathrm{Tr}\left(\Delta_R^{\dagger\,2}\right) + \mathrm{Tr}\left(\Delta_L^{\dagger\,2}\right) \mathrm{Tr}\left(\Delta_R^2\right)\right] \\
&+ \alpha_1 \left\{\mathrm{Tr}\left(\phi\phi^\dagger\right)\left[\mathrm{Tr}\left(\Delta_L\Delta_L^\dagger\right) + \mathrm{Tr}\left(\Delta_R\Delta_R^\dagger\right)\right]\right\} \\
&+ \alpha_2 \left[\mathrm{Tr}\left(\phi\tilde{\phi}^\dagger\right) \mathrm{Tr}\left(\Delta_R\Delta_R^\dagger\right) + \mathrm{Tr}\left(\phi^\dagger\tilde{\phi}\right) \mathrm{Tr}\left(\Delta_L\Delta_L^\dagger\right)\right] \\
&+ \alpha_2^* \left[\mathrm{Tr}\left(\phi^\dagger\tilde{\phi}\right) \mathrm{Tr}\left(\Delta_R\Delta_R^\dagger\right) + \mathrm{Tr}\left(\tilde{\phi}^\dagger\phi\right) \mathrm{Tr}\left(\Delta_L\Delta_L^\dagger\right)\right] \\
&+ \alpha_3 \left[\mathrm{Tr}\left(\phi\phi^\dagger\Delta_L\Delta_L^\dagger\right) + \mathrm{Tr}\left(\phi\phi^\dagger\Delta_R\Delta_R^\dagger\right)\right] \\
&+ \beta_1 \left[\mathrm{Tr}\left(\phi\Delta_R\phi^\dagger\Delta_L^\dagger\right) + \mathrm{Tr}\left(\phi^\dagger\Delta_L\phi\Delta_R^\dagger\right)\right] \\
&+ \beta_2 \left[\mathrm{Tr}\left(\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger\right) + \mathrm{Tr}\left(\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger\right)\right] \\
&+ \beta_3 \left[\mathrm{Tr}\left(\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger\right) + \mathrm{Tr}\left(\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger\right)\right]\,.
\end{aligned} \tag{C.2}$$

This potential is a possible source of CP violation. First of all, every term in expression (C.2) is self-conjugate *except* that of α_2. This parameter is therefore the only one in the Higgs potential which may be complex, leading to *explicit* CP violation. Secondly, the CP symmetry can be broken *spontaneously* if the Higgs particles in the LR symmetric model acquire complex VEVs. In Ref. [56] it is demonstrated, that if we remove explicit CP violation from the potential (C.2) by taking α_2 to be real, then, in the absence of extreme fine-tuning, spontaneous CP violation is excluded, meaning that in this case the vacuum expectation values $v_{L,R}$ and the κ's can be chosen real.

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Erklärung:

Ich versichere, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

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