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Theoretical News from Neutrinoless Double Beta Decay

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Abstract

Neutrinoless double beta decay is a very sensitive experimental probe for physics beyond the Standard Model. In fact, this process is the only known possibility to ascertain in the foreseeable future whether the neutrino is a Dirac or a Majorana particle. Most theoretical results on this subject, however, have been known for many years. In the advent of the next generation of experiments, it is worthwhile to re-examine old results and provide some new theoretical contributions. This thesis deals with various topics related to neutrinoless double beta decay. In particular, we focus on a discussion of the famous Schechter–Valle (or Black Box) theorem, as well as on a realization of neutrinoless double beta decay in universal extra dimensions, which has not been considered in the literature so far. We find that the Schechter–Valle theorem, although valid, is of merely academic interest, as it generates a neutrino mass which is many orders of magnitude smaller than the one expected. Concerning universal extra dimensions, we are able to give a new bound on their size, which is slightly weaker but complementary to the existing constraints from electroweak precision data. Next generation experiments are expected to improve upon the bounds we obtain.

Zusammenfassung

Der neutrinolose Doppel-Betazerfall ist ein empfindlicher experimenteller Test für Physik jenseits des Standardmodells. Er ist im Moment die einzige denkbare Möglichkeit, in naher Zukunft herauszufinden, ob das Neutrino ein Dirac- oder ein Majorana-Teilchen ist. Die meisten theoretischen Ergebnisse auf diesem Forschungsgebiet sind allerdings schon seit längerer Zeit bekannt. Kurz vor dem Start der nächsten Generation von Experimenten ist es daher sinnvoll, die alten Ergebnisse zu überprüfen und neue Beiträge zu leisten. Diese Arbeit diskutiert verschiedene Themen, die mit dem neutrinolosen Doppel-Betazerfall zusammenhängen. Hauptsächlich geht es um das bekannte Schechter-Valle-Theorem (auch Black-Box-Theorem genannt) und um eine bisher nicht diskutierte Realisierung des Zerfalls in Universal Extra Dimensions. Wir zeigen, dass das Schechter-Valle-Theorem zwar richtig ist, aber eine Neutrinomasse liefert, die um viele Größenordnungen kleiner ist als die erwartete. Die Extradimensionen betreffend berechnen wir eine neue obere Schranke für deren Größe. Diese ist zwar schwächer als diejenige, die man aus Präzisionsmessungen zur elektroschwachen Wechselwirkung erhält, kommt aber aus einer komplementären Klasse von Experimenten. Wir erwarten, dass die Experimente der nächsten Generation diese Schranke noch verbessern.

To my parents
Ingrid and Rudolf Dürr

If we knew what it was we were doing,
it would not be called research, would it?

— Albert Einstein —

Physics is like sex: sure, it may give some practical results,
but that's not why we do it.

— Richard P. Feynman —

Contents

List of Abbreviations and Symbols

$0\nu\beta\beta$ .... neutrinoless double beta decay

$\approx$ .... approximately equal to

$\boldsymbol{a}$ 3-vector

$\equiv$ .... defined as

$\mathcal{L}$ .... Lagrangian

$\mathcal{M}^{0\nu}$ .... nuclear matrix element for neutrinoless double beta decay

H.c. .... Hermitian conjugate

MS .... Minimal Subtraction

$\overline{\text{MS}}$ .... Modified Minimal Subtraction

$\propto$ .... proportional to

$\sim$ .... transforms as

ACD .... Appelquist–Cheng–Dobrescu

ADD .... Arkani-Hamed–Dimopoulos–Dvali

BC(s) .... Boundary Condition(s)

CKM .... Cabibbo–Kobayashi–Maskawa

ED/EDs .... Extra Dimension(s)

EFT .... Effective Field Theory

EW .... Electroweak

KK .... Kaluza–Klein

l.h.s. .... left-hand side

MSM .... Minimal Seesaw Model

MSSM .... Minimal Supersymmetric Standard Model

NME .... Nuclear Matrix Element

NP .... New Physics

NSM .... Nuclear Shell Model

PMNS .... Pontecorvo–Maki–Nakagawa–Sakata

QCD .... Quantum Chromodynamics

QPRA .... Quasi-Particle Random Phase Approximation

r.h.s. .... right-hand side

RS .... Randall–Sundrum

SM .... Standard Model

SSB .... Spontaneous Symmetry Breaking

SUSY .... Supersymmetry

UED/UEDs .... Universal Extra Dimension(s)

VEV .... Vacuum Expectation Value

Chapter 1

Introduction

The last decade was an exciting one for neutrino physics. Predicted by Pontecorvo a long time ago for neutrino–antineutrino systems [1, 2], neutrino oscillations were eventually found in oscillation experiments with atmospheric, solar, reactor, and accelerator neutrinos. These experiments have shown without a doubt that neutrinos have a small but non-zero mass [3–5].

This experimental result definitely exposes a shortcoming of the Standard Model (SM) of particle physics, which was put together in the early 1970's by Glashow, Salam, Weinberg, and many others [6–8]. Until today, it is a remarkably successful theory and describes Nature to a previously unknown precision. In this model, however, neutrinos are assumed to be massless. Although there are more shortcomings of the SM to resolve (the nature of dark matter, the hierarchy problem, unification with gravity, many unexplained parameters, etc.), this experimental result alone forces us to find models for New Physics (NP).

Knowing, however, that neutrinos are massive directly raises the question if neutrinos are Dirac or Majorana particles. It is closely connected to the one if lepton number is or is not a symmetry of Nature, because a Majorana mass term violates lepton number by two units. This issue cannot be solved by neutrino oscillation experiments. Therefore, other experiments have to be performed to determine the nature of neutrinos.

Unfortunately, lepton number violating processes generically have small amplitudes, as they are usually suppressed by the tiny neutrino masses. Therefore, it is very difficult to observe these processes experimentally. At the moment, the most promising attempts to find lepton number violation are the experiments on neutrinoless double beta decay ($0\nu\beta\beta$). Various experiments searching for $0\nu\beta\beta$ were performed (IGEX [9], Heidelberg-Moscow [10], CUORICINO [11], NEMO [12], and others), so far without an unambiguous detection. Currently, the GERDA experiment [13] is under construction and will be running soon, whereas the MAJORANA experiment [14] is in its R&D phase. Both will use enriched Germanium as target nucleus and will hopefully find evidence for the existence of $0\nu\beta\beta$, but at least they will improve the present bounds on the half-life of the isotope under consideration. There has been a lot of previous theoretical work on neutrinoless double beta decay and the Majorana nature of neutrinos. Much of the material, however, only has been reviewed for decades and no new theoretical input has been provided. In the advent of the next generation of experiments, we feel that it is necessary to re-examine some of the old results. We will have a closer look at the well-known Schechter–Valle theorem [15], whose assertion and diagram have been repeated in the literature for almost 30 years.

Concerning possible extensions of the Standard Model, a lot of candidates for New Physics are in the game. All of them are motivated by a solution to at least one of the problems remaining in the SM. It was recognized that the hierarchy problem may be solved in the presence of extra spatial dimensions (EDs) [16, 17], so there has been some theoretical effort concerning their physics in recent times. Mainly, three forms of EDs are under consideration at the moment: The large extra dimensions model by Antoniadis, Arkani-Hamed, Dimopoulos, and Dvali (ADD) [16, 17], the warped extra dimensions model by Randall and Sundrum (RS) [18], and the model by Appelquist, Cheng, and Dobrescu (ACD) [19]. In the ADD and the RS models, at least in their minimal form, the particles of the Standard Model are confined to a four-dimensional brane living in a higher dimensional space (the "bulk"), and only gravity feels the extra dimension(s). In the ACD model, all fields universally propagate in the extra dimension(s), therefore it is called model of universal extra dimensions (UEDs). However, due to this feature, UEDs cannot solve the hierarchy problem.

Nevertheless, UEDs have a very right of existence. Various theories (for example string theory) require the presence of extra spatial dimensions. Already assuming the existence of EDs, we think that it is more natural for all fields to propagate in them. There is another interesting feature of UEDs, a property called KK parity. It leads to a stable dark matter candidate, the lightest KK particle, in complete analogy to the lightest supersymmetric particle in R-parity conserving SUSY. We will see that non-negligible contributions to $0\nu\beta\beta$ arise through KK tower particles in UEDs.

This thesis is organized as follows: We give a short introduction to neutrino theory in chapter 2. Then, the basic facts about neutrinoless double beta decay are reviewed in chapter 3. Our first new contribution is the discussion of the famous Schechter–Valle theorem in chapter 4. After giving an overview of realizations of neutrinoless double beta decay in New Physics models in chapter 5, in chapter 6 the contributions to $0\nu\beta\beta$ in universal extra dimensions are considered. Finally, we summarize and give an outlook in chapter 7.

Chapter 2

Neutrino Theory

This chapter gives a short introduction to neutrino theory. As neutrinos are weakly interacting particles, we start with a concise review of the electroweak theory and then discuss neutrino masses which are of fundamental importance for this thesis.

2.1 Electroweak Theory

The gauge group of the Standard Model (SM) is

$$G^{\mathrm{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{2.1}$$

where C stands for "color", L for "left", and Y is the hypercharge. The Lagrangian density of the SM has to be invariant under all gauge transformations, which strongly restricts its form and the possible interactions. For example, this principle called gauge invariance forces the gauge bosons to be massless before spontaneous symmetry breaking (SSB). The particles of the SM, which transform as different representations of the gauge group, and their transformation properties are listed in table 2.1. Table 2.2 gives some useful parameters.

The SM contains fermions (particles with half-integer spin), the so-called quarks and leptons. They come in three families with identical quantum numbers (an experimental fact which is unexplained from the theoretical point of view), and have a definite chirality structure. Moreover, the SM contains gauge bosons which mediate the strong (the gluons G^a, $a = 1, \ldots, 8$) and electroweak (the W-bosons W^c, $c = 1, 2, 3$, and the B-boson B^0) interactions. Finally, there is the famous Higgs boson, the last particle of the model which has not been detected experimentally yet. It is a scalar boson, that is, it has spin zero.

Type	Spin	Particle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Quarks	$\frac{1}{2}$	$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$+\frac{1}{6}$
		u_R, c_R, t_R	3	1	$+\frac{2}{3}$
		d_R, s_R, b_R	3	1	$-\frac{1}{3}$
Leptons	$\frac{1}{2}$	$l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
		$l_R = e_R, \mu_R, \tau_R$	1	1	-1
Higgs	0	ϕ	1	2	$+\frac{1}{2}$
Gauge Bosons	1	$G^a, \quad a = 1, \ldots, 8$	8	1	0
		$W^c, \quad c = 1, 2, 3$	1	3	0
		B^0	1	1	0

Table 2.1: Particle content of the SM and transformation properties under the different gauge symmetries. Here, we have adopted the Gell-Mann–Nishijima relation $Q = I_3 + Y$ for the electric charge. I_3 is the third component of the isospin.

We can see in table 2.1 that neutrinos (the electrically neutral leptons) do not have a strong charge, and therefore do not underly strong interactions described by the gauge group $SU(3)_C$, the so-called quantum chromodynamics (QCD). Thus we may explain all phenomena concerning neutrinos by the so-called electroweak (EW) theory (or Glashow–Salam–Weinberg model of electroweak interaction), which is based on the gauge group

$$G^{\mathrm{EW}} = SU(2)_L \times U(1)_Y. \tag{2.2}$$

2.1.1 Interactions

The weak interaction is given by

$$\mathcal{L}_I = -\frac{g}{\sqrt{2}} \left[\left(j_l^\alpha + J_h^\alpha \right) W_\alpha^+ + \text{H.c.} \right] - \frac{g}{\cos \theta_W} K_\alpha Z^\alpha, \tag{2.3}$$

which perfectly describes the interactions of neutrinos. Here, the charged leptonic current is defined as

$$j_l^\alpha = \overline{\nu_{eL}} \gamma^\alpha e_L + \overline{\nu_{\mu L}} \gamma^\alpha \mu_L + \overline{\nu_{\tau L}} \gamma^\alpha \tau_L, \tag{2.4}$$

m_u	$=$	1.5 to 3.3 MeV	m_d	$=$	3.5 to 6.0 MeV
m_c	$=$	$1.27^{+0.07}_{-0.11}$ GeV	m_s	$=$	104^{+26}_{-34} MeV
m_t	$=$	(171.2 ± 2.1) GeV	m_b	$=$	$4.20^{+0.17}_{-0.07}$ GeV
m_e	$=$	$(0.510998910 \pm 0.000000013)$ MeV	m_τ	$=$	(1776.84 ± 0.17) MeV
m_μ	$=$	$(105.658367 \pm 0.000004)$ MeV	m_p	$=$	(938.27203 ± 0.00008) MeV
m_W	$=$	(80.398 ± 0.025) GeV	m_Z	$=$	(91.1876 ± 0.0021) GeV
G_F	$=$	$1.16637(1) \times 10^{-5}\,\mathrm{GeV}^{-2}$	g	$=$	0.652

Table 2.2: Masses and constants in the SM (taken from [20]). The value of g is calculated via $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$.

and the charged hadronic current as

$$J_h^\alpha = \overline{u_L}\gamma^\alpha d_L^{\mathrm{mix}} + \overline{c_L}\gamma^\alpha s_L^{\mathrm{mix}} + \overline{t_L}\gamma^\alpha b_L^{\mathrm{mix}}. \tag{2.5}$$

The mixed quark fields are given by

$$d_L^{\mathrm{mix}} = \sum_{q=d,s,b} V_{uq} q_L,\ s_L^{\mathrm{mix}} = \sum_{q=d,s,b} V_{cq} q_L, \text{ and } b_L^{\mathrm{mix}} = \sum_{q=d,s,b} V_{tq} q_L, \tag{2.6}$$

where V is the 3×3 Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix [21, 22]. The neutral current may be written as

$$K_\alpha = \sum_q [\epsilon_L(q)\overline{q_L}\gamma_\alpha q_L + \epsilon_R(q)\overline{q_R}\gamma_\alpha q_R] + \frac{1}{2}\sum_\nu \overline{\nu_L}\gamma_\alpha \nu_L + \frac{1}{2}\sum_l \bar{l}\gamma_\alpha(g_V^l - \gamma_5 g_A^l)l. \tag{2.7}$$

The parameters $\epsilon_L(q)$, $\epsilon_R(q)$, g_V^l, and g_A^l can be calculated in terms of the Weinberg angle θ_W. We do not want to go into more detail concerning neutral current interactions as double beta decay is driven by the charged currents.

2.1.2 Fermion Masses

In table 2.1 we can see that left- and right-handed fermion fields have different quantum numbers. Left-handed fields are doublets under $SU(2)_L$, whereas right-handed fields are singlets. Hence, a bare mass term for fermions is forbidden, as it would not be a singlet of the SM gauge group.[1]

[1] Note that gauge invariance forces all the gauge bosons to be massless, too. This situation is different from what we observe, as we know that the gluons and the photon (a linear combination of W_3

Experiments, however, tell us that quarks and charged leptons (we will talk about the origin of neutrino masses later) are massive. Therefore, a mechanism has to be introduced to generate these masses. In the SM, SSB via the famous Higgs mechanism leads to the observed masses [23–25]. Introducing a scalar boson ϕ, which is a doublet under $SU(2)_L$ (cf. table 2.1), we may form so-called Yukawa terms, for example,

$$\mathcal{L}_Y = -Y^l_{ab}\overline{l^a_L}\phi e^b_R + \text{H.c.}, \tag{2.8}$$

where sums over repeated indices are implied. Y^l is the leptonic Yukawa matrix and $a, b = e, \mu, \tau$ are family indices. When now the neutral component of ϕ acquires a non-zero vacuum expectation value[2] (VEV) $\langle\phi^0\rangle = v/\sqrt{2}$ and $SU(2)_L \times U(1)_Y$ spontaneously breaks down to $U(1)_{\text{em}}$, we obtain a Dirac mass term for the charged leptons:

$$\mathcal{L}_D = -\frac{v}{\sqrt{2}}Y^l_{ab}\overline{e^a_L}e^b_R + \text{H.c.} \tag{2.9}$$

Note that neutrino masses cannot be generated in this way in the SM because right-handed neutrinos are absent. The next section will be dedicated to the question of neutrino masses.

2.2 Neutrino Masses

As was already mentioned in the last section, neutrinos are absolutely massless in the Standard Model because right-handed neutrinos are absent and any mass term has to connect left- and right-handed components. Thus neutrino masses require the introduction of some kind of New Physics. We discuss possible mass terms for neutrinos in this section. This discussion is of a general type, and we only describe possible mass terms without referring to particular New Physics models or scalar sectors generating these mass terms.

and B^0) are massless, but there exist two electrically charged vector bosons ($W^\pm$) and an electrically neutral boson (Z^0) which are massive. They obtain their masses by eating the Goldstone bosons which appear when the gauge symmetry is spontaneously broken. We will not discuss their masses here, as we are mainly interested in neutrino masses.

[2]This happens if the ground state of the Higgs potential is not invariant unter the symmetry group of the theory.

2.2.1 Dirac Mass Term

Although absent in the SM, we may introduce three right-handed singlet neutrinos[3] n_{lR}, in addition to the three left-handed neutrinos ν_{lL}, and form a mass term in analogy to the ones for quarks and charged leptons:

$$\mathcal{L}_D = -\overline{\nu_{kL}} M_D^{kl} n_{lR} + \text{H.c.}, \tag{2.10}$$

where $k, l = e, \mu, \tau$ are flavor indices and M_D is the 3×3 Dirac neutrino mass matrix.

2.2.2 Majorana Mass Term

When we use the charge conjugate field $(\nu_L)^c$, which is right-handed, we may build a mass term without introducing additional right-handed singlet fields. We can write

$$\mathcal{L}_M^L = -\frac{1}{2} \overline{\nu_{kL}} M_L^{kl} \left(\nu_{lL}\right)^c + \text{H.c.} \tag{2.11}$$

Although we did not have to introduce new particles in addition to the particle content of the SM to form this mass term, note that it is not possible to form Majorana mass terms in the SM. $\overline{\nu_L}$ and $(\nu_L)^c$ have the same quantum numbers, as they are in the same representation of the gauge group. Table 2.1 shows that the SM does not contain fermions with zero hypercharge Y. Therefore, a bare Majorana mass term is forbidden by gauge invariance. Moreover, there is no scalar boson with the right quantum numbers to form a gauge singlet with the bilinear $\overline{\nu_L}\,(\nu_L)^c$, so the Majorana mass term cannot be generated via SSB. In addition, a Majorana mass term violates lepton number by two units. Lepton number, however, is only an accidental symmetry of the SM.

Of course, we may also introduce the right-handed singlet fields n_{lR} as before,[4] and write a Majorana mass term for them, too:

$$\mathcal{L}_M^R = -\frac{1}{2} \overline{(n_{kR})^c} M_R^{kl} n_{lR} + \text{H.c.} \tag{2.12}$$

Here, M_L and M_R are the 3×3 Majorana mass matrices for left- and right-handed neutrinos, respectively.

[3] We use a different symbol for the right-handed neutrinos to make clear that they are additional particles. This becomes more important in the case of a Majorana neutrino mass term, where the Majorana condition leads to the relation $(\nu_L)^c = \nu_R$. Note that ν_L is part of the lepton doublet, whereas n_R is a total singlet under the SM gauge group. We think that a different notation avoids possible confusion.

[4] The right-handed neutrinos have zero hypercharge.

2.2.3 Most General Neutrino Mass Term

Combining Dirac and Majorana mass terms, the most general neutrino mass term may be written as

$$\mathcal{L}_{D+M} = -\frac{1}{2}\overline{\nu_{kL}} M_L^{kl} \left(\nu_{lL}\right)^c - \overline{\nu_{kL}} M_D^{kl} n_{lR} - \frac{1}{2}\overline{(n_{kR})^c} M_R^{kl} n_{lR} + \text{H.c.} \tag{2.13}$$

If we now define

$$N_L = \begin{pmatrix} \nu_L \\ (n_R)^c \end{pmatrix}, \tag{2.14}$$

we can rewrite equation (2.13) in the form

$$\mathcal{L}_{D+M} = -\frac{1}{2}\overline{(N_L)^c} \mathcal{M}_\nu N_L + \text{H.c.}, \tag{2.15}$$

with

$$\mathcal{M}_\nu = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}, \tag{2.16}$$

where M_D is a complex 3×3 matrix, and M_L and M_R are symmetric complex matrices.

2.3 Seesaw Mechanism

The seesaw mechanism provides an interesting possibility to generate small but non-zero neutrino masses. We want to give a short motivation for this section by showing that in the case of Dirac masses generated via the Higgs mechanism, extreme fine tuning would be necessary to obtain neutrino masses of the expected size. This problem is somehow obvious, as experiment tells us that neutrino masses are by many orders of magnitude smaller than the masses of quarks and charged leptons (cf. sections 2.5 and 2.6).

Let us introduce, as before, right-handed neutrinos n_R in addition to the fields of the SM. Then the neutrino mass term could have the following form:

$$\mathcal{L}_{\text{mass}} = -\overline{\nu_L} Y_\nu \phi n_R, \tag{2.17}$$

with Y_ν being the Yukawa coupling, ν_L being the SM left-handed neutrino, and ϕ being the SM Higgs doublet. When ϕ acquires a VEV, we obtain a Dirac neutrino mass of the size $m_D = Y_\nu v/\sqrt{2}$, where $v \approx 246\,\text{GeV}$ is the usual value of the SM Higgs VEV. To obtain a neutrino mass of the desired order of 1 eV (or even less) [26, 27], the Yukawa coupling Y_ν must be at most of the order 10^{-11}. It is obvious that this value is an unnatural one and would require extreme fine tuning. Thus it is not very probable that the tiny neutrino masses are generated by the Higgs mechanism. Let us discuss situations in this section that do not suffer from these problems and provide a natural explanation for small neutrino masses.

2.3.1 Type I Seesaw

Type I seesaw assumes a particular form of the neutrino mass matrix given in equation (2.16). We will do the calculations for the case of one generation of neutrinos first. Let the neutrino mass matrix have the form

$$\mathcal{M}_\nu \equiv \begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}, \tag{2.18}$$

where M_D and M_R are taken to be real numbers, for simplicity. Type I seesaw assumes additionally that M_D is generated by the Higgs mechanism, so that its value is of the order of the electron mass. Furthermore, $M_R \gg M_D$. Then, this matrix can be easily diagonalized using

$$\mathcal{O} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad \text{with} \quad \tan 2\theta = \frac{2M_D}{M_R}. \tag{2.19}$$

One obtains[5]

$$\mathcal{O}\mathcal{M}_\nu\mathcal{O}^T = \begin{pmatrix} -m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \tag{2.20}$$

where the eigenvalues are given as

$$m_{1,2} = \left| \frac{1}{2} \left(\sqrt{M_R^2 + 4M_D^2} \mp M_R \right) \right|. \tag{2.21}$$

For $M_R \gg M_D$ these are approximately

$$m_1 = \frac{M_D^2}{M_R} \quad \text{and} \quad m_2 = M_R. \tag{2.22}$$

The corresponding mass eigenstates are

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu \\ n \end{pmatrix} = \begin{pmatrix} \nu\cos\theta - n\sin\theta \\ \nu\sin\theta + n\cos\theta \end{pmatrix} \approx \begin{pmatrix} \nu \\ n \end{pmatrix}. \tag{2.23}$$

[5]The minus for m_1 can be absorbed into a redefinition of the field ν_1 in the mass basis.

Thus we have for the mass of the light neutrinos

$$m_\nu = m_1 = \frac{M_D^2}{M_R}, \tag{2.24}$$

and for the masses of the heavy neutrinos

$$m_n = m_2 = M_R. \tag{2.25}$$

The case of three generations will be discussed in the next subsection, where we calculate the type II seesaw for three generations. Type I seesaw then is a special case of type II seesaw.

2.3.2 Type II Seesaw

In type II seesaw, we take the neutrino mass matrix to be of the most general form, as given in equation (2.16). However, we make additional assumptions, namely that the eigenvalues of M_L are much smaller than the eigenvalues of M_D, which in turn are much smaller that the eigenvalues of M_R. Our goal then is to block-diagonalize $\mathcal{M}_\nu$ with a unitary matrix $\mathcal{U}$ such that

$$\mathcal{M}_\nu^{\text{diag}} = \mathcal{U}^T \mathcal{M}_\nu \mathcal{U}. \tag{2.26}$$

Consider

$$\mathcal{U} = \begin{pmatrix} 1 & A \\ -A^\dagger & 1 \end{pmatrix}, \tag{2.27}$$

where A is a 3×3 matrix, with all entries $A_{ik} \ll 1$. Then we have

$$\mathcal{U}^\dagger \mathcal{U} = \begin{pmatrix} 1 + AA^\dagger & 0 \\ 0 & 1 + A^\dagger A \end{pmatrix}, \tag{2.28}$$

which is the unit matrix up to terms quadratic in A. Thus $\mathcal{U}$ is unitary to a good approximation. We may then calculate

$$\begin{aligned} \mathcal{U}^T \mathcal{M}_\nu \mathcal{U} &= \begin{pmatrix} 1 & -A^* \\ A^T & 1 \end{pmatrix} \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} 1 & A \\ -A^\dagger & 1 \end{pmatrix} \\ &= \begin{pmatrix} M_L - M_D A^\dagger - A^* M_D^T + A^* M_R A^\dagger & M_L A + M_D - A^* M_D^T A - A^* M_R \\ A^T M_L - A^T M_D A^\dagger + M_D^T - M_R A^\dagger & A^T M_L A + A^T M_D + M_D^T A + M_R \end{pmatrix}. \end{aligned} \tag{2.29}$$

With our initial assumptions on the sizes of the eigenvalues of M_D, M_L, and M_R, and the fact that $A \ll 1$, we may neglect many terms, such that we arrive at

$$\mathcal{U}^T \mathcal{M}_\nu \mathcal{U} = \begin{pmatrix} M_L - M_D A^\dagger - A^*(M_D^T - M_R A^\dagger) & M_D - A^* M_R \\ M_D^T - M_R A^\dagger & M_R \end{pmatrix}. \tag{2.30}$$

We may now choose

$$A^\dagger = M_R^{-1} M_D^T, \tag{2.31}$$

which has entries much smaller than one, and which gives

$$A^* = M_D M_R^{-1}, \tag{2.32}$$

as M_R is symmetric. Thus we finally find

$$\mathcal{U}^T \mathcal{M}_\nu \mathcal{U} = \begin{pmatrix} M_L - M_D M_R^{-1} M_D^T & 0 \\ 0 & M_R \end{pmatrix}. \tag{2.33}$$

The block matrices in this expression are the mass matrices for the light and heavy neutrinos, respectively. It is now easy to see that the three family case of seesaw type I is just a special case of this formula, given by $M_L = 0$. We then have

$$M_\nu = -M_D M_R^{-1} M_D^T \quad \text{and} \quad M_n = M_R \tag{2.34}$$

for the mass matrices of the light and heavy neutrinos.

2.4 Neutrino Mixing

From the fact that neutrinos are massive, we know that the neutrino mass eigenstates can be different from the flavor eigenstates. We say that neutrinos mix. The flavor fields, which enter the charged current, are given by

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x), \quad \text{with} \quad l = e, \mu, \tau. \tag{2.35}$$

U is the 3×3 Pontecorvo–Maki–Nakagawa–Sakata [1, 2, 28] (PMNS) neutrino mixing matrix, and ν_i, $i = 1, 2, 3$, are the light neutrino mass eigenstates.

In a framework containing three light neutrino species, the matrix U is assumed to be unitary. The typical parameterization is [29]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \operatorname{diag}\left(1, e^{i\alpha}, e^{i(\beta+\delta)}\right). \tag{2.36}$$

parameter	best fit	2σ	3σ
Δm^2_{21} [10^{-5} eV^2]	$7.59^{+0.23}_{-0.18}$	$7.22 - 8.03$	$7.03 - 8.27$
$\|\Delta m^2_{31}\|$ [10^{-3} eV^2]	$2.40^{+0.12}_{-0.11}$	$2.18 - 2.64$	$2.07 - 2.75$
$\sin^2\theta_{12}$	$0.318^{+0.019}_{-0.016}$	$0.29 - 0.36$	$0.27 - 0.38$
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	$0.39 - 0.63$	$0.36 - 0.67$
$\sin^2\theta_{13}$	$0.013^{+0.013}_{-0.009}$	≤ 0.039	≤ 0.053

Table 2.3: Three-flavor neutrino oscillations parameters, as calculated in [31].

Here, the usual conventions are used: $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, δ is the Dirac CP-violation phase, and α, β are the two Majorana CP-violation phases.

2.5 Three-Flavor Neutrino Oscillation Parameters

As we already stated before, neutrino mixing leads to neutrino oscillations (and actually these are the only experimental evidence that neutrino mixing takes place). We do not want to enter more deeply into the theory of neutrino oscillations, but just state that by measuring neutrino oscillations of different sources (solar, atmospheric, or reactor neutrinos) we have access to the mass-squared differences and the mixing angles. There have been recent updates of the best-fit values for the three-flavor neutrino oscillation parameters. Gonzalez-Garcia, Maltoni, and Salvado [30], and Schwetz, Tórtola, and Valle [31] have done such calculations. Table 2.3 gives the values of the latter paper, updated in February 2010. We will use these values in the remainder of this thesis.

2.6 Neutrino Mass Observables

The PMNS matrix diagonalizes the neutrino mass matrix whose eigenvalues are the neutrino masses m_i, $i = 1, 2, 3$. There exist three observables related to these masses, which we will shortly describe in this section. All of these observables are measured in different experiments, which are on-going or up-coming. Besides the measurement of an absolute neutrino mass scale, these experiments can (alone or in combination) possibly answer the question if neutrinos are Dirac or Majorana particles.

- Kinematical mass m_β: It is measurable in ordinary beta decay experiments, and is defined as

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 \, m_i^2}. \tag{2.37}$$

Its current upper limit is 2.3 eV [27]. The up-coming experiment KATRIN [32] is expected to push this limit by about one order of magnitude.

- Effective mass $|m_{ee}|$: It is measurable in neutrinoless double beta decay, and is defined as

$$|m_{ee}| = \left| \sum_{i=1}^{3} U_{ei}^2 m_i \right| . \tag{2.38}$$

Note that this mass parameter actually is the modulus of the *ee*-entry of the Majorana neutrino mass matrix. Its current upper limit is about 1 eV (the IGEX experiment gives $|m_{ee}| \leq (0.33 - 1.35)$ eV, depending on the choice of nuclear matrix elements [9]), if one assumes that neutrinoless double beta decay is mediated exclusively by light neutrino exchange. The up-coming experiment GERDA [13] is searching for neutrinoless double beta decay in ^{76}Ge and is expected to be sensitive to $T_{1/2}^{0\nu} = 2 \times 10^{26}$ y, which corresponds to $|m_{ee}| \leq (0.09 - 0.29)$ eV.

- Sum of neutrino masses Σ: It is extracted from cosmological observations, and is defined as

$$\Sigma = \sum_{i=1}^{3} m_i. \tag{2.39}$$

The current upper limit is 0.58 eV from the WMAP seven year data [33].

2.7 Effective Mass $|m_{ee}|$

In this section, we want to discuss the effective Majorana neutrino mass $|m_{ee}|$ a little bit further, as it is the mass parameter measured in neutrinoless double beta decay.

The amplitude of neutrinoless double beta decay is proportional to (cf. chapter 3)

$$\sum_j U_{ej}^2 \frac{m_j}{p^2 - m_j^2}. \tag{2.40}$$

The momentum is given by the nuclear scale, which means that we have for the typical momentum square

$$\langle p^2 \rangle \approx \frac{1}{r^2}, \tag{2.41}$$

where r is the distance between two nucleons. With $r \approx 10^{-13}$ cm, one finds

$$\langle p^2 \rangle \approx (100\,\text{MeV})^2. \tag{2.42}$$

We know that the light neutrino masses are of order 1 eV, so we can expand the fraction inside the sum and arrive at

$$\begin{aligned}
\sum_j U_{ej}^2 \frac{m_j}{p^2 - m_j^2} &= \frac{1}{p^2} \sum_j U_{ej}^2 \frac{m_j}{1 - m_j^2/p^2} \\
&= \frac{1}{p^2} \sum_j U_{ej}^2 m_j \left(1 + \frac{m_j^2}{p^2} + \mathcal{O}\left(\frac{m_j^4}{p^4}\right)\right) \\
&= \frac{1}{p^2} \sum_j U_{ej}^2 m_j + \frac{1}{p^4} \sum_j U_{ej}^2 m_j^3 + \dots
\end{aligned} \tag{2.43}$$

We can see that the leading order of the amplitude indeed contains m_{ee}. Should it vanish for some reason (for example, an accidental cancellation), the next-to-leading order will be proportional to

$$M_{ee}^3 \equiv \sum_j U_{ej}^2 m_j^3. \tag{2.44}$$

Using the parameterization of the PMNS matrix given in equation (2.36), we can rewrite m_{ee} as

$$m_{ee} = c_{13}^2 \left(m_1 c_{12}^2 + e^{i2\alpha} m_2 s_{12}^2\right) + e^{i2\beta} m_3 s_{13}^2. \tag{2.45}$$

The neutrino oscillation data given in table 2.3 shows that the solar mass-squared difference Δm_{21}^2 is much smaller than the modulus of the atmospheric mass-squared difference Δm_{31}^2. Additionally, the sign of Δm_{31}^2 is unknown. Therefore, for three massive neutrinos, basically two different types of neutrino mass spectrum are possible:

- Normal spectrum, that means, $m_1 < m_2 < m_3$. In this case, $\Delta m_{31}^2 > 0$.
- Inverted spectrum, that means, $m_3 < m_1 < m_2$. In this case, $\Delta m_{31}^2 < 0$.

Of course, if the lightest mass eigenvalue (m_1 or m_3, depending on the spectrum) is much bigger than the mass-squared differences, both spectra merge into the so-called degenerate spectrum, where

$$m_1 \approx m_2 \approx m_3. \tag{2.46}$$

Here, we want to discuss the normal spectrum, as it is the only case where m_{ee} may vanish due to accidental cancellation. For normal mass ordering, we may calculate m_2 and m_3 as functions of m_1:

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2} \quad \text{and} \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}. \tag{2.47}$$

Now, we can write m_{ee} as a function of m_1:

$$m_{ee} = c_{13}^2 \left(m_1 c_{12}^2 + e^{i2\alpha} s_{12}^2 \sqrt{m_1^2 + \Delta m_{21}^2}\right) + e^{i2\beta} s_{13}^2 \sqrt{m_1^2 + \Delta m_{31}^2}. \tag{2.48}$$

The same is possible with the expression for the next-to-leading order

$$M_{ee}^3 = c_{12}^2 c_{13}^2 m_1^3 + s_{12}^2 c_{13}^2 e^{i2\alpha}(m_1^2 + \Delta m_{21}^2)^{3/2} + s_{13}^2 e^{i2\beta}(m_1^2 + \Delta m_{31}^2)^{3/2}. \tag{2.49}$$

We can then plot the leading order and the next-to-leading order of the effective mass as a function of the smallest neutrino mass eigenvalue m_1. As we do not know anything about the Majorana phases α and β, we have to vary them over the interval $[-\pi, \pi]$. Moreover, for the neutrino oscillation parameters we will use best fit values and vary them within the 3σ bounds, respectively. Figure 2.1 shows the plots for $|m_{ee}|$ and $\left|M_{ee}^3\right|$ for different values of $\sin^2\theta_{13}$. The data for the plots was generated using a Fortran program with a random number generator. We can see that there is indeed a range of m_1 where $|m_{ee}|$ may vanish (depending on the Majorana phases). This range becomes larger for higher vales of $\sin^2\theta_{13}$. Of course, the situation is similar for $\left|M_{ee}^3\right|$. However, a close look at the plots reveals that the ranges where the two expressions vanish do not necessarily coincide for a given $\sin^2\theta_{13}$. Thus, there is indeed the possibility that neutrinoless double beta decay is driven by $\left|M_{ee}^3\right|$ instead of $|m_{ee}|$.

Let us calculate the expected half-life, assuming that the next-to-leading order is responsible for neutrinoless double beta decay. We will consider the case of $\sin^2\theta_{13} = 0$, the only case for which we may explicitly calculate the value of m_1 when $|m_{ee}|$ is zero. From equation (2.48) we obtain in this case

$$m_1 = \left|\frac{\Delta m_{21}^2}{c_{12}^2 - s_{12}^2}\right|^{1/2}. \tag{2.50}$$

Using the best fit values from table 2.3, we can calculate

$$m_1 = 0.0109\,\mathrm{eV}. \tag{2.51}$$

From figure 2.1, we can see that in this case $\left|M_{ee}^3\right|$ is of the order 10^{-7}. We have to correct with a factor of $\langle p^2\rangle$, an thus obtain

$$T_{1/2}^{0\nu} = \left(\left|\frac{M_{ee}^3}{\langle p^2\rangle}\right|^2 \left|\mathcal{M}^{0\nu}\right|^2 G^{0\nu}\right)^{-1} \approx 10^{70}\,\mathrm{y}, \tag{2.52}$$

a value which is far beyond experimental reach. Should the leading order of the effective mass indeed vanish, we have almost no chance to probe neutrinoless double beta decay experimentally (as long as there is no other mechanism giving a sizeable contribution, cf. chapter 5).

2.8 Spinors in Four Dimensions

In this section, we will discuss basic facts about spinors in four dimensions. We will work in the chiral basis of gamma matrices as is used for example in the textbook by

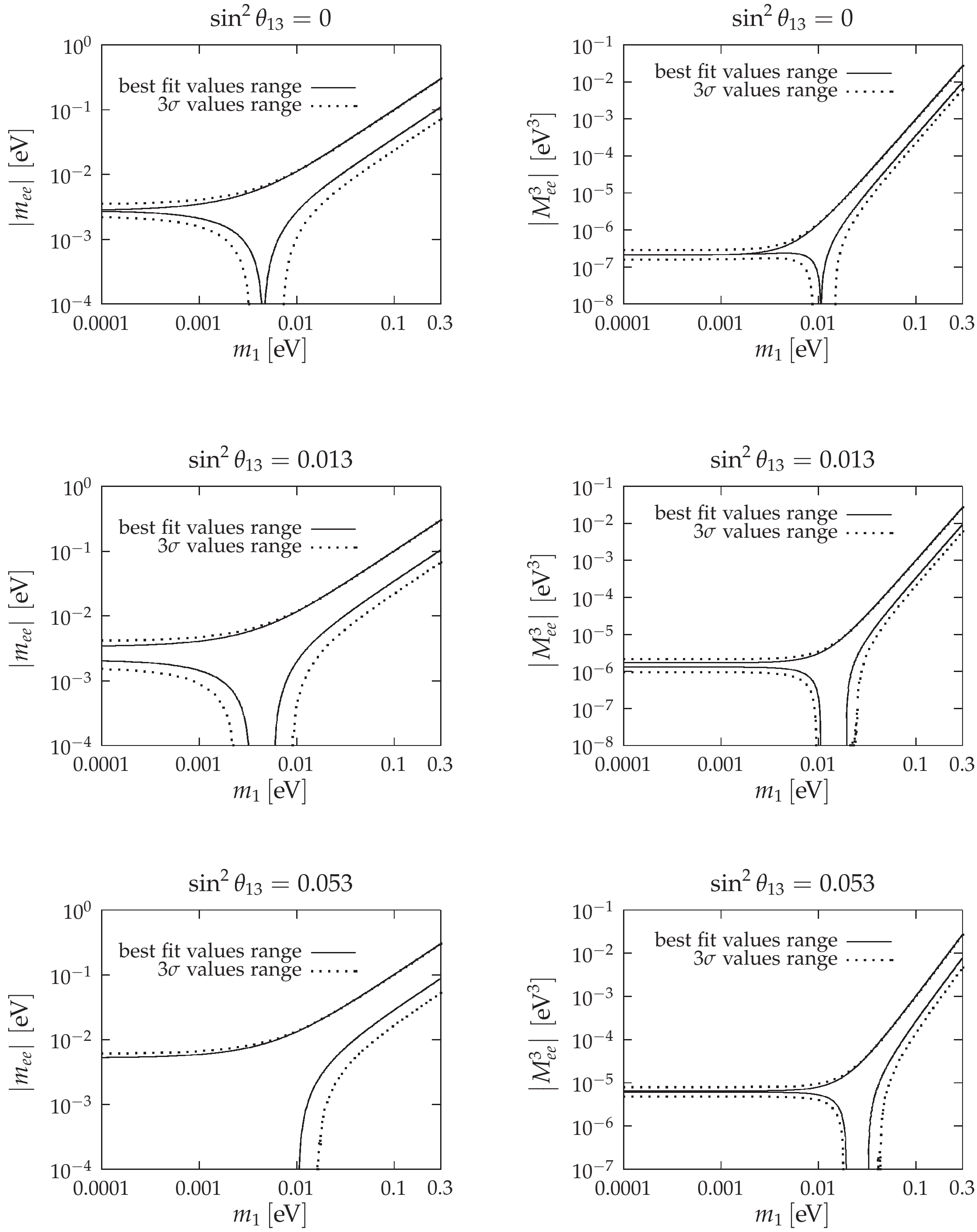


Figure 2.1: Plot of $|m_{ee}|$ and $\left|M^3_{ee}\right|$ for different values of $\sin^2\theta_{13}$.

Peskin and Schroeder [34]. However, note that the results of the discussion will be independent of the particular basis used.

The gamma matrices γ^μ ($\mu = 0, \ldots, 3$) in the chiral basis are given by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \quad \text{where} \quad \sigma^\mu = (\mathbb{1}, \boldsymbol{\sigma}) \quad \text{and} \quad \overline{\sigma}^\mu = (\mathbb{1}, -\boldsymbol{\sigma}) \,. \tag{2.53}$$

Here, $\mathbb{1}$ is the 2×2 identity matrix and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the conventional Pauli matrices given in appendix A.4. As usual, we define

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}. \tag{2.54}$$

The projection operators for left- and right-handed spinors have the well-known form

$$P_L = \frac{1}{2}(1 - \gamma^5) \quad \text{and} \quad P_R = \frac{1}{2}(1 + \gamma^5). \tag{2.55}$$

Now one can decompose an arbitrary Dirac spinor ψ into two spinors ψ_L and ψ_R as

$$\psi = \psi_L + \psi_R = \frac{1}{2}(1 - \gamma^5)\psi + \frac{1}{2}(1 + \gamma^5)\psi. \tag{2.56}$$

Another way to write an arbitrary 4-component (Dirac) spinor is

$$\psi = \begin{pmatrix} \xi \\ \overline{\eta} \end{pmatrix}, \tag{2.57}$$

where ξ and η are 2-component spinors, and $\overline{\eta}$ is defined as

$$\overline{\eta} \equiv i\sigma_2\eta^*. \tag{2.58}$$

Note that this definition is different from the one of the adjoint spinor, which is given by

$$\overline{\psi} = \psi^\dagger\gamma_0. \tag{2.59}$$

In this formalism, a conventional Dirac mass term can be written as

$$\begin{aligned} \mathcal{L}_D^{\text{mass}} = m_D\overline{\psi}\psi &= m_D\left(\overline{\psi_R}\psi_L + \overline{\psi_L}\psi_R\right) \\ &= m_D\left(\eta^T i\sigma_2\xi - \xi^\dagger i\sigma_2\eta^*\right) = m_D\left(\xi^\dagger\overline{\eta} - \eta^T\overline{\xi}^*\right). \end{aligned} \tag{2.60}$$

From the definition in equation (2.57) one can see that ξ and $\overline{\eta}$ must carry the same lepton number. Therefore, as one may easily check, this Dirac mass term preserves lepton number.

As in this thesis we are mainly interested in Majorana neutrinos, we have to introduce a charge conjugation operation. Let us define

$$\psi^c = C\overline{\psi}^T = C\gamma_0\psi^*, \tag{2.61}$$

where the charge conjugation matrix C satisfies

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T. \tag{2.62}$$

The Majorana condition

$$\psi = \psi^c \tag{2.63}$$

then defines a Majorana spinor.

It is clear that ψ^c should be properly normalized whenever ψ is. Therefore $C\gamma_0$ should be unitary, and from the unitarity of γ_0 the unitarity of C follows directly. So we have

$$C^\dagger = C^{-1}. \tag{2.64}$$

Furthermore, for the definition of the charge conjugation being sensible, we must require

$$(\psi^c)^c = \psi. \tag{2.65}$$

This condition gives $CC^* = -1$, or alternatively

$$C^T = -C. \tag{2.66}$$

A widely used representation for the charge conjugation matrix in four dimensions is

$$C = i\gamma_2\gamma_0 = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}. \tag{2.67}$$

Note that an additional minus comes from the fact that in this definition γ_2 has a lower index ($\gamma^0 = \gamma_0$, but $\gamma^i = -\gamma_i$, $i = 1, 2, 3$).

In constructing Majorana mass terms we need the charge conjugate spinor $\overline{\psi^c}$. We may write it in a slightly different way:

$$\overline{\psi^c} = (\psi^c)^\dagger\gamma_0 = (-\gamma_0 C\psi^*)^\dagger\gamma_0 = -\psi^T C^{-1}\gamma_0^\dagger\gamma_0 = -\psi^T C^{-1}. \tag{2.68}$$

Using now the definitions in equations (2.57) and (2.67), we may write the Majorana mass term as

$$\begin{aligned}\mathcal{L}_M^{\text{mass}} &= m_M\overline{\psi^c}\psi = -m_M\psi^T C^{-1}\psi \\ &= m_M\left(\eta^\dagger i\sigma_2\eta^* - \xi^T i\sigma_2\xi\right) = m_M\left(\xi^T\overline{\xi}^* - \eta^\dagger\overline{\eta}\right).\end{aligned} \tag{2.69}$$

It can now be seen easily that the Majorana mass term violates lepton number, in contrast to the Dirac mass term given before.

Chapter 3

Basics of Neutrinoless Double Beta Decay

This chapter is aimed to be a short review of important aspects of neutrinoless double beta decay.

3.1 Different Beta Decays

In some special arrangement of nuclei with values of atomic number Z differing by one, it is possible that single beta decay, that is, the transition

$$(A, Z) \rightarrow (A, Z+1) + e^- + \overline{\nu}_e \tag{3.1}$$

is energetically forbidden. Here, A is the total number of nucleons in the nucleus and $\overline{\nu}_e$ denotes the electron anti-neutrino. In this case, another process may be allowed, which is called double beta decay:

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \overline{\nu}_e + \overline{\nu}_e. \tag{3.2}$$

It is denoted by $2\nu\beta\beta$. This process is second order in the weak Hamiltonian, and thus occurs rarely. Nevertheless, it has been detected experimentally and half-lives have been measured. Depending on the nucleus involved, half-lives vary roughly between 10^{19} and 10^{21} y. Single beta decay, as well as double beta decay, conserves lepton number and is therefore possible in the SM. Many models for physics beyond the SM predict lepton number violation. Should lepton number indeed be broken in Nature, another form of double beta decay is possible, the so-called neutrinoless double beta decay (denoted by $0\nu\beta\beta$). It is the transition

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-. \tag{3.3}$$

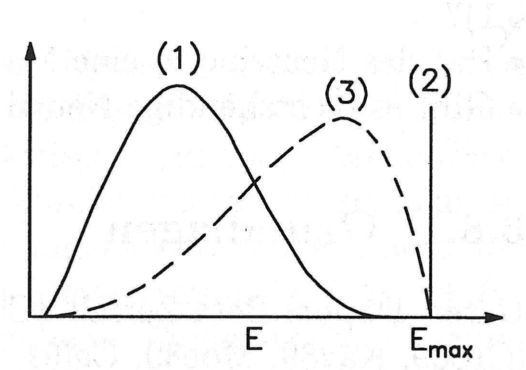


Figure 3.1: Energy distribution for the emitted electrons in double beta decays: (1) $2\nu\beta\beta$, (2) $0\nu\beta\beta$, (3) $0\nu\beta\beta$ with majoron emission (Figure taken from [35]).

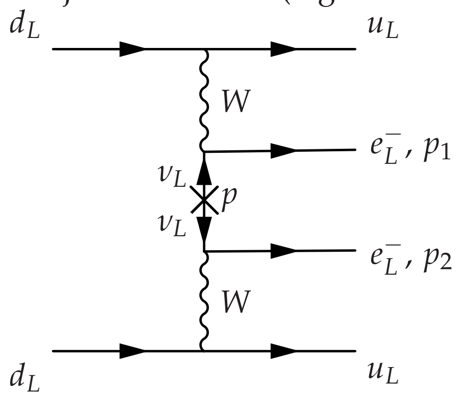


Figure 3.2: Momentum assignment for the calculation of the amplitude of neutrinoless double beta decay mediated by neutrino exchange.

The two different double beta decays may be distinguished by the energy distribution of the emitted electrons. Figure 3.1 gives more details.

Until today, $0\nu\beta\beta$ has not been observed experimentally.[1] Nevertheless, stringent bounds on the half-lives of different elements have been extracted from experiments. Limits are of the order 10^{25} y (table 3.2 gives an overview). Thus it is clear that this process is much rarer than $2\nu\beta\beta$.

3.2 Amplitudes for the Neutrino Mass Mechanism

The most obvious realization of neutrinoless double beta decay of course is neutrino exchange. However, as we will see later, this realization is not the only one which

[1]Note that one claim for a positive signal of $0\nu\beta\beta$ exists. A subgroup of the Heidelberg-Moscow collaboration gives the half-life $T_{1/2}^{0\nu} = 1.98 \times 10^{25}$ y [10].

may give $0\nu\beta\beta$. We must find a possibility to compare different contributions, and to decide which is the dominant one in a specific model. This is done best by comparing the amplitudes of the decay in different realizations. In this section, we calculate and compare the amplitudes of light and heavy neutrino exchange, which may occur in almost any extension of the Standard Model. A diagram with the momentum assignments used is given in figure 3.2. The matrix element for neutrino exchange (light or heavy) is

$$\mathcal{M}_{\mu\lambda} = \sum_i \left(\overline{e_L}(p_1)\gamma_\mu U_{ei}\right)_\alpha \left(\overline{e_L}(p_2)\gamma_\lambda U_{ei}\right)_\beta S'_{Fi}(p)^{\alpha\beta} - (p_1 \leftrightarrow p_2)\,, \tag{3.4}$$

where p_1 and p_2 are the momenta of the outgoing electrons, and p is the momentum of the virtually exchanged neutrino. $S'_F(p)$ is the propagator for the Majorana neutrino and can be written as

$$S'_F(p) = \lambda^* S_F(p) C, \tag{3.5}$$

where C is the charge conjugation matrix, λ is a phase, and $S_F(p)$ is the usual (Dirac) propagator for leptons given by

$$S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + i\epsilon}. \tag{3.6}$$

So we can write

$$\mathcal{M}_{\mu\lambda} = \sum_i \lambda^* \left(\overline{e_L}(p_1)\gamma_\mu U_{ei}\right) S_{Fi}(p) C \left(\overline{e_L}(p_2)\gamma_\lambda U_{ei}\right)^T - (p_1 \leftrightarrow p_2)\,. \tag{3.7}$$

After some algebra we arrive at

$$\mathcal{M}_{\mu\lambda} = -\lambda^* g_{\mu\lambda} \sum_i \frac{U_{ei}^2 m_i}{p^2 - m_i^2} \overline{e_L}(p_1) C \overline{e_L}^T(p_2). \tag{3.8}$$

As before, we may take $\langle p^2 \rangle \approx (100\,\text{MeV})^2$ for the typical momentum square [see equation (2.42)]. So we can approximate this formula for the two extreme cases of light ($m_i^2 \ll \langle p^2 \rangle$) and heavy ($m_i^2 \gg \langle p^2 \rangle$) neutrinos, and finally calculate the amplitude for neutrinoless double beta decay by integrating out the heavy W boson fields.

3.2.1 Light Neutrino Exchange

In the case of $m_i^2 \ll \langle p^2 \rangle$ we can write

$$\mathcal{M}_{\mu\lambda} = -\lambda^* g_{\mu\lambda} \sum_i \frac{U_{ei}^2 m_i}{\langle p^2 \rangle} \overline{e_L}(p_1) C \overline{e_L}^T(p_2). \tag{3.9}$$

Thus we obtain for the amplitude

$$\mathcal{A}_\nu = \left(\frac{g}{\sqrt{2}}\right)^4 \frac{1}{M_W^4} \sum_i \frac{U_{ei}^2 m_i}{\langle p^2 \rangle}. \tag{3.10}$$

Nucleus	$G^{0\nu}(Q,Z)/10^{-25}\,\mathrm{y}^{-1}\mathrm{eV}^{-2}$
^{76}Ge	0.30
^{100}Mo	2.19
^{130}Te	2.12
^{136}Xe	2.26

Table 3.1: Some phase space integrals $G^{0\nu}(Q,Z)$ for neutrinoless double beta decay (Values taken from [38]).

3.2.2 Heavy Neutrino Exchange

In the case of $m_i^2 \gg \langle p^2 \rangle$ we can write

$$\mathcal{M}_{\mu\lambda} = \lambda^* g_{\mu\lambda} \sum_i \frac{U_{ei}^2}{m_i} \overline{e_L}(p_1) C \overline{e_L}^T(p_2). \tag{3.11}$$

Thus we obtain for the amplitude

$$\mathcal{A}_N = \left(\frac{g}{\sqrt{2}}\right)^4 \frac{1}{M_W^4} \sum_i \frac{U_{ei}^2}{m_i}. \tag{3.12}$$

3.3 Decay Rate

3.3.1 Light Neutrino Exchange

The total decay rate of neutrinoless double beta decay (if mediated by light neutrino exchange) is given by [36]

$$\frac{\Gamma^{0\nu}}{\ln 2} = \frac{1}{T_{1/2}^{0\nu}} = |m_{ee}|^2 \left|\mathcal{M}^{0\nu}\right|^2 G^{0\nu}(Q,Z). \tag{3.13}$$

Here, $T_{1/2}^{0\nu}$ is the half-life of $0\nu\beta\beta$, and $|m_{ee}|$ is the effective Majorana mass of the electron neutrino, which was discussed in section 2.6. $\mathcal{M}^{0\nu}$ is the nuclear matrix element, which depends on the nucleus under consideration, and $G^{0\nu}(Q,Z)$ is the corresponding phase space integral. The calculation of the total decay rate for neutrinoless double beta decay is a rather lengthy process. It can be found in the reviews by Bilenky and Petcov [37] and Bilenky [36]. Some phase space integrals $G^{0\nu}(Q,Z)$ are given in table 3.1. Limits on the half-lives $T_{1/2}^{0\nu}$ for different nuclei are given in table 3.2.

Experiment	Nucleus	$T^{0\nu}_{1/2}$
Heidelberg-Moscow [39]	^{76}Ge	$\geq 1.9 \times 10^{25}$ y
IGEX [9]	^{76}Ge	$\geq 1.57 \times 10^{25}$ y
CUORICINO [11]	^{130}Te	$\geq 0.3 \times 10^{25}$ y
NEMO-3 [12]	^{48}Ca	$\geq 1.3 \times 10^{22}$ y
NEMO-3 [12]	^{82}Se	$\geq 2.1 \times 10^{23}$ y
NEMO-3 [12]	^{96}Zr	$\geq 8.6 \times 10^{21}$ y
NEMO-3 [12]	^{100}Mo	$\geq 5.8 \times 10^{23}$ y
NEMO-3 [12]	^{150}Nd	$\geq 1.8 \times 10^{22}$ y

Table 3.2: Limits on the half-lives $T^{0\nu}_{1/2}$ of $0\nu\beta\beta$ for different nuclei.

3.3.2 Heavy Neutrino Exchange

If neutrinoless double beta decay is mediated by heavy neutrino exchange, the formula for the decay rate has to be modified slightly. In the case of mediation by light neutrinos, we have

$$\mathcal{A}_\nu \propto \frac{m_{ee}}{\langle p^2 \rangle}, \tag{3.14}$$

where the typical momentum square $\langle p^2 \rangle$ can be taken to be $\langle p^2 \rangle \approx (100\,\text{MeV})^2$ as before. In the case of mediation by heavy neutrinos, the amplitude of the process is

$$\mathcal{A}_N \propto \frac{1}{M_N} = \sum_i \frac{U_{ei}^2}{m_i}. \tag{3.15}$$

So taking the same phase space integral and nuclear matrix element as in the case of light neutrino exchange [cf. equation (3.13)] for simplicity, we can get an approximate formula for the total decay rate in the case of heavy neutrino exchange:

$$\frac{\Gamma^{0\nu}}{\ln 2} = \frac{1}{T^{0\nu}_{1/2}} = \frac{(100\,\text{MeV})^4}{|M_N|^2} \left|\mathcal{M}^{0\nu}\right|^2 G^{0\nu}(Q, Z)\,. \tag{3.16}$$

3.3.3 New Physics Processes

Usually the contribution of light neutrino exchange is taken to be the leading one. However, there are various possible contributions from New Physics beyond the Standard Model (right-handed currents, Higgs triplets, supersymmetric particles, etc.), for which the formula for the measured half-life of the decay is given by (see Deppisch and Päs [40] for a more detailed discussion):

$$\left[T_{1/2}^{\mathrm{NP}}\right]^{-1} = \varepsilon_{\mathrm{NP}}^{2} G_{\mathrm{NP}} \left|\mathcal{M}^{\mathrm{NP}}\right|^{2}. \tag{3.17}$$

In this equation, $\mathcal{M}^{\mathrm{NP}}$ is the nuclear matrix element for the underlying New Physics mechanism and G_{NP} is the corresponding phase space integral. $\varepsilon_{\mathrm{NP}}$ denotes the effective coupling strength of the New Physics process. Therefore, results of the on-going experiments will crucially depend on the determination of the underlying mechanism.

3.4 Nuclear Matrix Elements

As is easy to see from equation (3.13), a measurement of only the half-life $T_{1/2}^{0\nu}$ does not allow to extract the effective Majorana neutrino mass $|m_{ee}|$. Additionally, the so-called nuclear matrix elements (NMEs) $\mathcal{M}^{0\nu}$ have to be calculated. Their calculation is a problem of nuclear physics, and a many-body problem. Hence approximations have to be used. At the moment, there are two approaches being under consideration: The Nuclear Shell Model (NSM) [41] and the Quasi-Particle Random Phase Approximation (QPRA) [42, 43]. The outcomes of the two approaches differ significantly, but it is difficult to say which one gives the better values. A nice review of the different approaches and an informative discussion can be found in [44].

Chapter 4

The Schechter–Valle Theorem

In this chapter, we re-examine the well-known Schechter–Valle theorem (often called Black Box theorem) [15]. We find that the theorem is of merely academic interest, as the mass generated via the well-known diagram (see figure 4.1) is many orders of magnitude smaller than the expected size of the neutrino masses. We also comment on the extended Black Box theorem, which is an extension of the usual Black Box theorem in the presence of arbitrary lepton number and lepton flavor violating processes.

4.1 Schechter–Valle Theorem (Black Box Theorem)

The famous Schechter–Valle theorem (Black Box theorem) [15], which was established in 1982, relates neutrinoless double beta decay and a non-zero effective Majorana electron neutrino mass. The argument of Schechter and Valle can easily be explained using figure 4.1.

Effectively, neutrinoless double beta decay may be seen as a scattering amplitude for $0 \to \overline{d}\,\overline{d}\,u\,u\,e\,e$, as under the assumption that the weak interaction is described

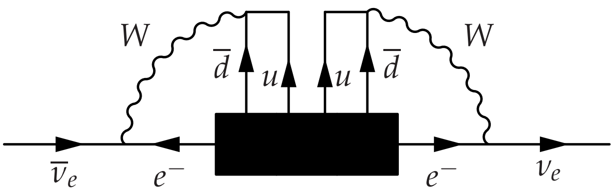


Figure 4.1: Contribution to the Majorana neutrino mass, as suggested in [15].

by a local gauge theory, crossing symmetry will hold (this result is a rather general one from quantum field theory, a proof can be found in the textbook by Peskin and Schroeder [34], p. 222 ff.). One only has to assume additionally that the gauge theory includes interactions of W gauge bosons with left-handed electrons and neutrinos, as well as with left-handed u and d quarks. Then, it will be possible to draw the diagram in figure 4.1, so that neutrinoless double beta decay can be used to generate a non-zero effective Majorana mass for the electron neutrino, no matter which is the underlying mechanism of the decay. The Black Box may contain any mechanism imaginable in New Physics scenarios.

There is one subtlety: The diagram in figure 4.1 is certainly not the only one that could generate a non-zero effective Majorana mass for the electron neutrino. Other diagrams (albeit including more loops) do exist. So there is the possibility of a cancellation. Clearly, the precise cancellation of all diagrams which include the Black Box would require some form of fine tuning of parameters, and would therefore be unnatural. There is, however, the possibility of a symmetry protecting the cancellation to all orders in perturbation theory.

Taking into account this possibility of cancellations, Takasugi [45] and Nieves [46] improved the argument of Schechter and Valle [15], and showed that there cannot be a continuous or discrete symmetry protecting a vanishing Majorana mass to all orders in perturbation theory. We will follow the arguments of Takasugi [45] here. He assumed an unbroken discrete symmetry protecting the Majorana neutrino mass together with the two additional assumptions that

1. the u and d quarks and the electron are massive and
2. the standard left-handed interaction $\left(\overline{\nu_{eL}}\gamma_\mu e_L + \overline{u_L}\gamma_\mu d_L\right) W^\mu$ exists.

These two assumptions are necessary to assure that two identical neutrinos are created. This can be seen in the following way: We do not know anything about the chirality of the electrons and quarks produced by neutrinoless double beta decay. The first assumption, however, assures that we can make the particles running in the loops in figure 4.1 left-handed, by mass insertion if necessary. Thus the standard left-handed interaction from the second assumption produces the same neutrino at both vertices. Otherwise it would be possible that a neutrino and an antineutrino would be created, which would give a Dirac mass term.

We may then check whether the discrete symmetry introduced by Takasugi [45] is compatible with $0\nu\beta\beta$ decay. The symmetry is the following (the η's are global phase factors):

$$\nu_{eL} \to \eta_\nu \nu_{eL}, \quad e_L \to \eta_e e_L, \quad q_L \to \eta_q q_L (q = u, d), \quad W_L^{+\mu} \to \eta_W W_L^{+\mu}. \tag{4.1}$$

To forbid the Majorana mass term, one needs to have

$$\eta_\nu^2 \neq 1 \tag{4.2}$$

and the invariance of the left-handed interaction requires

$$\eta_\nu^* \eta_e \eta_W = \eta_u^* \eta_d \eta_W = 1. \tag{4.3}$$

However, the existence of $0\nu\beta\beta$ (that is, the process $d_L + d_L \rightarrow u_L + u_L + e_L + e_L$) implies

$$\eta_u^2 \eta_d^{*2} \eta_e^2 = 1. \tag{4.4}$$

It is easy to see that the equations (4.2), (4.3), and (4.4) cannot be solved simultaneously. Thus, if the Majorana mass term is forbidden by an unbroken discrete symmetry, there will be no neutrinoles double beta decay. On the other hand, if neutrinoless double beta decay exists, there cannot be a symmetry protecting the Majorana mass and the term will be induced (the possibility of accidental cancellation to all orders in perturbation theory is not excluded, but appears to be very unlikely).

The statement of this theorem is a strong one. This becomes particularly clear when we think of contributions to neutrinoless double beta decay from New Physics beyond the Standard Model. Besides the usual and well-known mass mechanism involving a virtual neutrino, in almost all New Physics models neutrinoless diagrams giving a contribution to neutrinoless double beta decay exist. Thus the theorem states that it is not possible to construct models without massive neutrinos but with $0\nu\beta\beta$ decay.

However, it cannot be overemphasized that the theorem so far is only a qualitative one and does not say anything about the size of the Majorana neutrino mass. Therefore, it is an interesting question what the mass generated by the diagram in figure 4.1 will be for different realizations of the Black Box. We will do such a calculation for different operators using an effective field theory (EFT) approach independent of the underlying model in section 4.3. The results will be interesting, as they reveal some problems of the Black Box theorem.

4.2 Extended Black Box Theorem

As the classical Black Box theorem only establishes a relation between neutrinoless double beta decay and the effective Majorana mass of the electron neutrino, but does not take into account the mixing of ν_e with ν_μ and ν_τ, an extension of the theorem to the three-generation Majorana neutrino mass matrix is needed. This was done by Hirsch et al. [47]. Moreover, they extended the theorem to arbitrary lepton number and lepton flavor violating processes. They found that there exists a general set of one-to-one correspondence relations between the effective operators generating these processes and the elements of the neutrino mass matrix.

Especially, they discussed $\Delta L = 2$ processes described by $\Phi_k \rightarrow l_\alpha l_\beta$ conserving baryon number, which means that lepton number violation manifests itself via two external charged leptons in the final state. Φ_k is a set of external particles with $B = L = 0$ and electrical charge $Q = -2$. Note that, for a lepton number violating

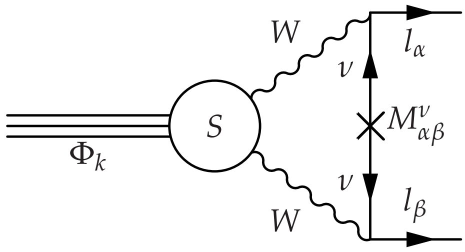


Figure 4.2: Contribution of the $M^\nu_{\alpha\beta}$ entry of the Majorana neutrino mass matrix to the effective lepton number and lepton flavor violating vertex $\Gamma_{\alpha\beta}$ (Figure as in [47]).

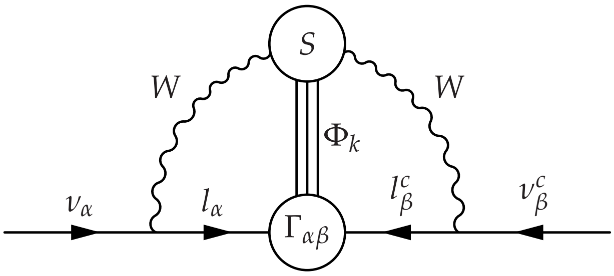


Figure 4.3: Contribution of the effective lepton number and lepton flavor violating vertex $\Gamma_{\alpha\beta}$ to the $M^\nu_{\alpha\beta}$ entry of the Majorana neutrino mass matrix (Figure as in [47]).

process with the same flavor states l_α and l_β different sets of external particles Φ_k are possible. With a symmetry argument, similar to the one used before to prove the classical Black Box theorem, they have shown that the following relation between the effective lepton number and lepton flavor violating vertex $\Gamma_{\alpha\beta}$ and the entry $M^\nu_{\alpha\beta}$ of the Majorana neutrino mass matrix exists:

$$M^\nu_{\alpha\beta} = 0 \Leftrightarrow \Gamma_{\alpha\beta} = 0. \tag{4.5}$$

The corresponding diagrams are shown in figures 4.2 and 4.3.

There has been some more work on the Black Box theorem: Hirsch et al. proved a supersymmetric version of it in [48, 49]. This version extends the relation of the theorem to the lepton number violating scalar neutrino mass. We will not discuss their work further, as it is not of great importance to us.

4.3 Neutrino Mass Generated by the Black Box Diagram

Now we want to calculate the mass correction which is induced by the Black Box diagram shown in figure 4.1. We will first give a general parameterization of $0\nu\beta\beta$ decay in terms of effective operators, and then calculate the diagram in dimensional regularization. We will find that the Schechter–Valle theorem is not as useful as it might seem: The mass which is generated by the diagram in figure 4.1 is many orders

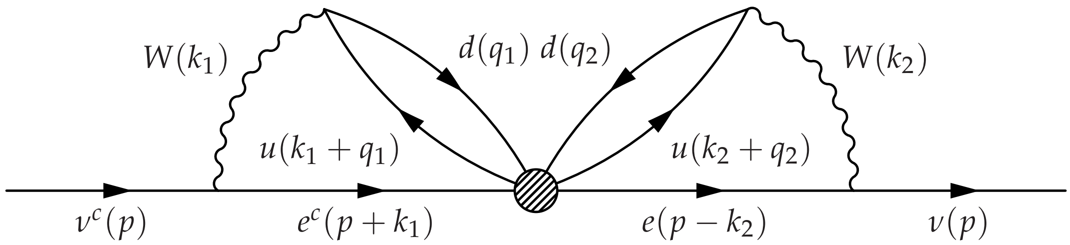


Figure 4.4: Diagram to be calculated in section 4.3.

of magnitude smaller than what is expected for the neutrino mass. Moreover, there exist operators generating $0\nu\beta\beta$ decay, but giving zero contribution to the Majorana neutrino mass via this particular diagram. Of course, other diagrams, although more stronly suppressed, may give a nonzero contribution also for these operators.

4.3.1 Parameterization of Neutrinoless Double Beta Decay

Reference [50] gives the most general Lorentz invariant Lagrangian contributing to neutrinoless double beta decay, which has the following form:

$$\mathcal{L} = \frac{G_F^2}{2} m_p^{-1} \left(\epsilon_1 JJj + \epsilon_2 J^{\mu\nu} J_{\mu\nu} j + \epsilon_3 J^\mu J_\mu j + \epsilon_4 J^\mu J_{\mu\nu} j^\nu + \epsilon_5 J^\mu J j_\mu \right), \tag{4.6}$$

where G_F is the Fermi coupling constant, and m_p is the proton mass. The hadronic currents required are given by

$$J = \overline{u}\left(1 \pm \gamma_5\right) d, \; J^\mu = \overline{u}\gamma^\mu \left(1 \pm \gamma_5\right) d, \; J^{\mu\nu} = \overline{u}\frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]\left(1 \pm \gamma_5\right) d, \tag{4.7}$$

and the leptonic currents required are given by

$$j = \overline{e}\left(1 \pm \gamma_5\right) e^c, \; j^\mu = \overline{e}\gamma^\mu \left(1 \pm \gamma_5\right) e^c. \tag{4.8}$$

For all currents, different chirality structure is permitted. Note that in equation (4.6) we have already left out the following terms which are Lorentz invariant, too:

$$\mathcal{L}' = \frac{G_F^2}{2} m_p^{-1} \left(\epsilon_6 J^\mu J^\nu j_{\mu\nu} + \epsilon_7 J J^{\mu\nu} j_{\mu\nu} + \epsilon_8 J_{\mu\alpha} J^{\nu\alpha} j^\mu_\nu \right), \tag{4.9}$$

where the leptonic tensor current is given by

$$j^{\mu\nu} = \overline{e}\frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]\left(1 \pm \gamma_5\right) e^c. \tag{4.10}$$

These terms were included in the Lagrangian density in [50], but neglected in the final analysis, as the authors worked in the s-wave approximation where these contributions vanish. In [51], it was pointed out that all operators proportional to

$$\overline{e}\gamma_\mu e^c, \; \overline{e}\frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right] e^c, \text{ and } \overline{e}\gamma_5 \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right] e^c \tag{4.11}$$

$\|\epsilon_1\|$	$\|\epsilon_2\|$	$\|\epsilon_3^{LLz}\|$, $\|\epsilon_3^{RRz}\|$	$\|\epsilon_3^{LRz}\|$, $\|\epsilon_3^{RLz}\|$	$\|\epsilon_4\|$	$\|\epsilon_5\|$
3×10^{-7}	2×10^{-9}	4×10^{-8}	1×10^{-8}	2×10^{-8}	2×10^{-7}

Table 4.1: Upper bounds for the coupling parameters in equation (4.6), calculated in [50]. They were evaluated "on axis," meaning that all other contributions were set to zero to extract the limits on one of the parameters. Different chiralities in the hadronic currents lead to different values in the case of ϵ_3 only.

vanish identically because the electron fields are Grassman variables. Therefore, the terms in equation (4.9) are not relevant for neutrinoless double beta decay.

We may have to distinguish different chiralities of the involved currents, as in some cases the decay rate depends on them. Therefore, the authors of [50] used the parameters ϵ_i^{xyz} with $x, y, z = L/R$ to define the chiralities of the hadronic and leptonic currents in order of appearance in equation (4.6). A suppressed chirality index indicates that it is not necessary to distinguish different chiralities. The limits obtained for the parameters ϵ_i are given in table 4.1.

4.3.2 Vertices and Propagators

The diagram we have to calculate (figure 4.4) is a non-standard one. At the effective vertex, lepton number is violated and two electrons are produced, which leads to two outgoing neutrinos. We want to have a continuous fermion line in our diagram, so we have to rewrite some of the outgoing fields as incoming charge conjugate fields. The leptonic part of the standard electroweak vertex is given by

$$\frac{g}{\sqrt{2}} \left(\bar{e}\gamma^\mu P_L \nu W_\mu + \text{H.c.}\right). \tag{4.12}$$

We are interested in the Hermitian conjugate part, which is responsible for the annihilation of an incoming electron and the creation of an outgoing neutrino, and can be written as

$$\left(\bar{e}\gamma^\mu P_L \nu W_\mu\right)^\dagger = \overline{\nu}\gamma^\mu P_L e W_\mu. \tag{4.13}$$

To rewrite it in terms of incoming charge conjugate fields, we have to transpose this expression. We obtain

$$\left(\overline{\nu}\gamma^\mu P_L e\right)^T = \overline{e^c}\gamma^\mu P_R \nu^c, \tag{4.14}$$

using the relations

$$C\gamma^{\mu T} C^{-1} = -\gamma^\mu,\ C\gamma_5^T C^{-1} = \gamma_5, \tag{4.15}$$

and

$$\overline{\psi^c} = -\psi^T C^{-1}. \tag{4.16}$$

We additionally have to calculate the propagator of the charge conjugate electron fields:

$$e^c(y)\overline{e^c}(x). \tag{4.17}$$

Starting from the usual electron propagator

$$e(x)\overline{e}(y) = iS_F(x-y) \tag{4.18}$$

with

$$S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{\not{p}+m}{p^2-m^2+i\epsilon} e^{-ip(x-y)}, \tag{4.19}$$

it is easy to see that

$$\overline{e}^T(y)e^T(x) = iS_F^T(x-y). \tag{4.20}$$

If we now rewrite expression (4.17) with the help of

$$e^c(y) = C\overline{e}^T(y) \tag{4.21}$$

and the relation

$$CS_F^T(x-y)C^{-1} = S_F(y-x), \tag{4.22}$$

which follows directly from equation (4.15), we obtain

$$e^c(y)\overline{e^c}(x) = iS_F(y-x). \tag{4.23}$$

In the remainder of this section, we will work in momentum space. Therefore, a look at equation (4.19) reveals that the change of sign in the argument of S_F changes the sign of the momentum $\not{p}$. We thus have in momentum space

$$e^c(y)\overline{e^c}(x) \sim i\frac{-\not{p}+m}{p^2-m^2+i\epsilon}. \tag{4.24}$$

4.3.3 Decay Mediated by the Operator $J_L J_L j_L$

To calculate the diagram in figure 4.4, we have to write down the matrix element. Let us define the weak leptonic current

$$j_l^\alpha = \overline{\nu}\gamma^\alpha P_L e \tag{4.25}$$

and the weak hadronic current

$$J_h^\mu = \overline{d}\gamma^\mu P_L u. \tag{4.26}$$

We thus have to find all possible contractions in

$$\langle f|J_L J_L j_L j_l^{\nu T} W_\nu J_h^\mu W_\mu j_l^\alpha W_\alpha J_h^\beta W_\beta|i\rangle. \tag{4.27}$$

It is easy to see that all contractions lead to the same diagram given in figure 4.4. We find

$$\langle f|\overline{\nu}\gamma^\alpha P_L e\overline{e} P_L e^c\overline{e^c}\gamma^\nu P_R \nu^c \overline{u} P_L d\overline{d}\gamma^\mu P_L u\overline{u} P_L d\overline{d}\gamma^\beta P_L u W_\nu W_\mu W_\alpha W_\beta|i\rangle. \tag{4.28}$$

Different contractions lead to 8 diagrams (there are 4 ways to contract the quark fields, and independently there are 2 ways to contract the W boson fields). Using the information from the last subsection and including all necessary factors, we directly can write down the matrix element of the decay:

$$\begin{aligned}
&\frac{8g^4G_F^2\epsilon_1}{m_p}\int d\tilde{k}_1 d\tilde{k}_2 d\tilde{q}_1 d\tilde{q}_2 \overline{\nu}(p)\gamma^\alpha P_L i\frac{\not{p}-\not{k}_2+m_e}{(p-k_2)^2-m_e^2}P_L i\frac{-\not{p}-\not{k}_1+m_e}{(p+k_1)^2-m_e^2}\gamma^\nu P_R\nu^c(p)\\
&\quad\mathrm{tr}\left(i\frac{\not{k}_1+\not{q}_1+m_u}{(k_1+q_1)^2-m_u^2}P_L i\frac{\not{q}_1+m_d}{q_1^2-m_d^2}\gamma^\beta P_L\right)\mathrm{tr}\left(i\frac{\not{k}_2+\not{q}_2+m_u}{(k_2+q_2)^2-m_u^2}P_L i\frac{\not{q}_2+m_d}{q_2^2-m_d^2}\gamma^\mu P_L\right)\\
&\qquad\qquad\qquad\frac{-ig_{\alpha\beta}}{(k_1^2-M_W^2)}\frac{-ig_{\mu\nu}}{(k_2^2-M_W^2)},
\end{aligned}\tag{4.29}$$

where we have used the short-hand notation $d\tilde{k}=\frac{d^4k}{(2\pi)^4}$. Let us calculate the traces first, where we have to simplify the expression

$$\mathrm{tr}\left\{\left[(\not{k}_1+\not{q}_1)+m_u\right]P_L\left(\not{q}_1+m_d\right)\gamma^\beta P_L\right\}. \tag{4.30}$$

Moving P_L to the right and splitting up the expression in square brackets, we obtain

$$\mathrm{tr}\left\{\left[(\not{k}_1+\not{q}_1)+m_u\right]\not{q}_1\gamma^\beta P_L\right\}=\mathrm{tr}\left[(\not{k}_1+\not{q}_1)\not{q}_1\gamma^\beta P_L\right]+\mathrm{tr}\left[m_u\not{q}_1\gamma^\beta P_L\right]. \tag{4.31}$$

The first term on the r.h.s. contains an odd number of gamma matrices (as γ_5, which is contained in P_L, is a product of four gamma matrices), so it vanishes. For the second term on the r.h.s. we use the relations

$$\mathrm{tr}\left[\gamma^\alpha\gamma^\beta\right]=4g^{\alpha\beta} \tag{4.32}$$

and

$$\mathrm{tr}\left[\gamma^\alpha\gamma^\beta\gamma_5\right]=0, \tag{4.33}$$

so that we find

$$\mathrm{tr}\left(i\frac{\not{k}_1+\not{q}_1+m_u}{(k_1+q_1)^2-m_u^2}P_L i\frac{\not{q}_1+m_d}{q_1^2-m_d^2}\gamma^\beta P_L\right)=-\frac{2(q_1)^\beta m_u}{[(k_1+q_1)^2-m_u^2](q_1^2-m_d^2)}. \tag{4.34}$$

In an analogous manner we can calculate

$$\mathrm{tr}\left(i\frac{\not{k}_2+\not{q}_2+m_u}{(k_2+q_2)^2-m_u^2}P_L i\frac{\not{q}_2+m_d}{q_2^2-m_d^2}\gamma^\mu P_L\right)=-\frac{2(q_2)^\mu m_u}{[(k_2+q_2)^2-m_u^2](q_2^2-m_d^2)}. \tag{4.35}$$

We may simplify the first line of equation (4.29) by moving the P_L's to the right. We thus project out the terms containing m_e. We finally obtain the expression

$$\Sigma(p) = \frac{32 g^4 G_F^2 \epsilon_1 m_u^2 m_e^2}{m_p} \mathcal{I}, \tag{4.36}$$

where the integral $\mathcal{I}$ is given by

$$\mathcal{I} = \mathcal{I}_1 \times \mathcal{I}_2, \tag{4.37}$$

with

$$\mathcal{I}_1 = \int d\tilde{k}_1 d\tilde{q}_1 \frac{\not{q}_1}{[(p+k_1)^2 - m_e^2][(k_1+q_1)^2 - m_u^2](q_1^2 - m_d^2)(k_1^2 - M_W^2)} \tag{4.38}$$

and

$$\mathcal{I}_2 = \int d\tilde{k}_2 d\tilde{q}_2 \frac{\not{q}_2}{[(p-k_2)^2 - m_e^2][(k_2+q_2)^2 - m_u^2](q_2^2 - m_d^2)(k_2^2 - M_W^2)}. \tag{4.39}$$

Integrals of this kind typically arise in two-loop calculations, which is not surprising. Our four-loop diagram is symmetric and basically consists of two two-loop diagrams. These integrals may be calculated analytically in terms of Spence functions, for which extensive literature exists. The calculations have for example been done in [52–54], in the framework of Higgs physics. For us it suffices to know that the integrals are well behaved. To extract the dependence on masses and momenta, we will go another way: We can calculate $\mathcal{I}_1$ and $\mathcal{I}_2$ separately with the help of Feynman parameters and dimensional regularization (some basic formulae are given in appendix A). As an example, let us calculate the integral

$$\mathcal{I}_1 = \int d\tilde{q}_1 \frac{\not{q}_1}{(q_1^2 - m_d^2)} \mathcal{I}_1', \tag{4.40}$$

where

$$\mathcal{I}_1' = \int d\tilde{k}_1 \frac{1}{[(p+k_1)^2 - m_e^2][(k_1+q_1)^2 - m_u^2](k_1^2 - M_W^2)}. \tag{4.41}$$

Using Feynman parameterization, we can rewrite

$$\begin{aligned} \mathcal{I}_1' = & \int_0^1 dx \int_0^{1-x} dy \\ & \times \int d\tilde{k}_1 \frac{2}{\{x[(p+k_1)^2 - m_e^2] + y[(k_1+q_1)^2 - m_u^2] + (1-x-y)(k_1^2 - M_W^2)\}^3}. \end{aligned} \tag{4.42}$$

We may rewrite the denominator

$$
\begin{aligned}
x[(p+k_1)^2 - m_e^2] + y[(k_1+q_1)^2 - m_u^2] + (1-x-y)(k_1^2 - M_W^2)& \\
= [k_1 + (xp + yq_1)]^2 - \Delta_1',& \quad (4.43)
\end{aligned}
$$

where

$$
\Delta_1' = (xp+yq_1)^2 - xp^2 - yq_1^2 + (1-x-y)M_W^2. \quad (4.44)
$$

Here, we have neglected m_e^2 and m_u^2 in comparison to M_W^2. Thus we can write

$$
\mathcal{I}_1' = \int_0^1 dx \int_0^{1-x} dy \int d\tilde{k}_1 \frac{2}{\{[k_1 + (xp+yq_1)]^2 - \Delta_1'\}^3}. \quad (4.45)
$$

Performing a linear substitution $k_1 \to k_1 - (xp + yq_1)$, we arrive at

$$
\begin{aligned}
\mathcal{I}_1' &= \int_0^1 dx \int_0^{1-x} dy \int d\tilde{k}_1 \frac{2}{(k_1^2 - \Delta_1')^3} \\
&= \frac{-i}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{1}{(xp+yq_1)^2 - xp^2 - yq_1^2 + (1-x-y)M_W^2}.
\end{aligned} \quad (4.46)
$$

We may now plug this result back into the expression for $\mathcal{I}_1$ and obtain

$$
\begin{aligned}
\mathcal{I}_1 = \frac{-i}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \int d\tilde{q}_1 \frac{\not{q}_1}{(q_1^2 - m_d^2)}& \\
\times \frac{1}{[(xp+yq_1)^2 - xp^2 - yq_1^2 + (1-x-y)M_W^2]}.& \quad (4.47)
\end{aligned}
$$

If we introduce a third Feynman parameter, we get

$$
\begin{aligned}
\mathcal{I}_1 &= \frac{-i}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \int_0^1 dz \\
&\times \int d\tilde{q}_1 \frac{\not{q}_1}{\{(1-z)(q_1^2 - m_d^2) + z[(xp+yq_1)^2 - xp^2 - yq_1^2 + (1-x-y)M_W^2]\}^2}. \quad (4.48)
\end{aligned}
$$

After completing the square and neglecting $m_d \ll M_W$, we may write the denominator as

$$
(1 - z + zy^2 - zy) \left[\left(q_1 + \frac{xyz}{1 - z + zy^2 - zy} p \right)^2 - \Delta_1 \right], \quad (4.49)
$$

where

$$
\begin{aligned}
\Delta_1(x,y,z) &= \left(\frac{xyz}{(1 - z + zy^2 - zy)} p \right)^2 \\
&- \frac{1}{(1 - z + zy^2 - zy)} \left(zx^2p^2 - zxp^2 + (1-x-y)zM_W^2 - m_d^2 \right).
\end{aligned} \quad (4.50)
$$

Plugging this result back into the expression for $\mathcal{I}_1$, performing again a linear substitution, and kicking out the terms linear in q_1 (as they vanish under symmetrical integration), we obtain

$$\mathcal{I}_1 = \frac{i}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \int_0^1 dz \frac{xyz}{(1-z+zy^2-zy)^3} \not{p} \int d\tilde{q}_1 \frac{1}{(q_1^2-\Delta_1)^2}. \tag{4.51}$$

Performing the integral over q_1, we arrive at

$$\mathcal{I}_1 = \frac{\not{p}}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \int_0^1 dz \frac{xyz}{(1-z+zy^2-zy)^3} \times \left(\frac{2}{\epsilon} - \log \Delta_1 - \gamma + \log 4\pi + \mathcal{O}(\epsilon) \right). \tag{4.52}$$

Calculating the integral $\mathcal{I}_2$ goes along the same lines. We finally obtain

$$\begin{aligned} \mathcal{I} = &- \frac{p^2}{(16\pi^2)^4} \\ &\times \int_0^1 dx \int_0^{1-x} dy \int_0^1 dz \frac{xyz}{(1-z+zy^2-zy)^3} \left(\frac{2}{\epsilon} - \log \Delta_1 - \gamma + \log 4\pi + \mathcal{O}(\epsilon) \right) \\ &\times \int_0^1 da \int_0^{1-a} db \int_0^1 dc \frac{abc}{(1-c+cb^2-cb)^3} \left(\frac{2}{\epsilon} - \log \Delta_2 - \gamma + \log 4\pi + \mathcal{O}(\epsilon) \right). \end{aligned} \tag{4.53}$$

In this expression, $\epsilon = 4 - d$ and γ is Euler's constant. Δ_1 was given before [see equation (4.50)], and for Δ_2 we have

$$\begin{aligned} \Delta_2(a,b,c) = &\left(\frac{abc}{(1-c+cb^2-cb)} p \right)^2 \\ &- \frac{1}{(1-c+cb^2-cb)} \left(ca^2p^2 - cap^2 + (1-a-b)cM_W^2 - m_d^2 \right). \end{aligned} \tag{4.54}$$

In the case of $p^2 = 0$, these expressions simplify to

$$\Delta_1(x,y,z) = -\frac{1}{(1-z+zy^2-zy)} \left((1-x-y)zM_W^2 - m_d^2 \right) \tag{4.55}$$

and

$$\Delta_2(a,b,c) = -\frac{1}{(1-c+cb^2-cb)} \left((1-a-b)cM_W^2 - m_d^2 \right). \tag{4.56}$$

The expression in equation (4.53) can be renormalized via the $\overline{\text{MS}}$ scheme, where we set

$$\frac{2}{\epsilon} - \log \Delta - \gamma + \log 4\pi + \mathcal{O}(\epsilon) \rightarrow -\log \frac{\Delta}{M^2}. \tag{4.57}$$

Here, M is the renormalization scale.

We are now able to give an expression for the correction to the neutrino mass generated by the diagram in figure 4.4 assuming that there is no bare Majorana mass term in the Lagrangian.[1] Plugging equation (4.53) into equation (4.36), we get

$$\delta m_\nu = \Sigma(\not{p} = m_\nu) \propto m_u^2 m_e^2 p^2 \Big|_{\not{p}=m_\nu}, \tag{4.58}$$

where m_ν is the physical neutrino mass. We have neglected the dependence on p^2 inside the logarithms, but as the physical neutrino mass m_ν is much smaller than M_W, this should be no problem. The physical mass m_ν is the solution to the equation

$$\not{p} - \Sigma(\not{p})\big|_{\not{p}=m_\nu} = 0. \tag{4.59}$$

One solution of this equation is $m_\nu = 0$. The second solution $m_\nu = \alpha^{-1}$, where α is the proportionality factor from equation (4.58), is huge due to the smallness of α and is therefore unphysical. Thus we obtain

$$\delta m_\nu = 0. \tag{4.60}$$

This means that the operator $J_L J_L j_l$ does not give a contribution via the diagram shown in figure 4.4. One may now wonder if there exists an operator generating a non-zero contribution via the well-known Black Box diagram at all. It does, and we want to give an example for such a diagram in the next subsection. The result, however, will be many orders of magnitude smaller than the expected light neutrino mass.

Let us point out that the result of this subsection does not mean that all thinkable diagrams involving the operator $J_L J_L j_l$ will give a zero contribution like the one we calculated. Any other diagram, however, will include more loops and therefore be suppressed more strongly. Thus, the mass we expect to find will be even smaller than the one we find in the next section for a different operator. However, one should keep the result of this subsection in mind when arguing for the Majorana nature of the electron neutrino via the diagram in figure 4.4 (something that is regularly done in review papers and talks). Depending on the operator (that is, the underlying mechanism) of $0\nu\beta\beta$, one possibly draws a zero mass. Clearly, this does not support the hypothesis of a Majorana neutrino (although there might be other diagrams giving a non-zero contribution for the same operator).

[1] Note that our calculation is only useful in the case where no bare Majorana mass term for the electron neutrino in the Lagrangian exists. Would there be such a term, then necessarily also a counterterm, which cancels all loop divergences, would exist. The neutrino mass then would be a parameter to be determined by experiment and put into the theory by hand.

4.3.4 Decay Mediated by the Operator $J_R^\mu J_{\mu R} j_L$

In the previous subsection we have seen that the diagram calculated vanishes. However, there are of course operators responsible for neutrinoless double beta decay which give non-zero contributions to the mass correction. In this subsection, we want to calculate the diagram assuming that the vertex is proportional to ϵ_3 from equation (4.6). Additionally, we may choose the chirality structure in the following way:

$$J_R^\sigma J_{R\sigma} j_L = \overline{u}\gamma^\sigma P_R d \overline{u}\gamma_\sigma P_R d \overline{e} P_L e^c. \tag{4.61}$$

As in the former case, we have to find all contractions possible. So we can directly write down the matrix element of the diagram:

$$\begin{aligned}&\frac{8g^4G_F^2\epsilon_3}{m_p}\int d\tilde{k}_1 d\tilde{k}_2 d\tilde{q}_1 d\tilde{q}_2 \overline{\nu}(p)\gamma^\alpha P_L i\frac{\not{p}-\not{k}_2+m_e}{(p-k_2)^2-m_e^2}P_L i\frac{-\not{p}-\not{k}_1+m_e}{(p+k_1)^2-m_e^2}\gamma^\nu P_R \nu^c(p)\\ &\mathrm{tr}\left(i\frac{\not{k}_1+\not{q}_1+m_u}{(k_1+q_1)^2-m_u^2}\gamma^\sigma P_R i\frac{\not{q}_1+m_d}{q_1^2-m_d^2}\gamma^\beta P_L\right)\mathrm{tr}\left(i\frac{\not{k}_2+\not{q}_2+m_u}{(k_2+q_2)^2-m_u^2}\gamma_\sigma P_R i\frac{\not{q}_2+m_d}{q_2^2-m_d^2}\gamma^\mu P_L\right)\\ &\qquad\qquad\frac{-ig_{\alpha\beta}}{(k_1^2-M_W^2)}\frac{-ig_{\mu\nu}}{(k_2^2-M_W^2)}.\end{aligned} \tag{4.62}$$

Calculating the traces in a similar manner as before, we obtain

$$\mathrm{tr}\left(i\frac{\not{k}_1+\not{q}_1+m_u}{(k_1+q_1)^2-m_u^2}\gamma^\sigma P_R i\frac{\not{q}_1+m_d}{q_1^2-m_d^2}\gamma^\beta P_L\right) = -\frac{2m_u m_d g^{\sigma\beta}}{[(k_1+q_1)^2-m_u^2][q_1^2-m_d^2]} \tag{4.63}$$

and

$$\mathrm{tr}\left(i\frac{\not{k}_2+\not{q}_2+m_u}{(k_2+q_2)^2-m_u^2}\gamma_\sigma P_R i\frac{\not{q}_2+m_d}{q_2^2-m_d^2}\gamma^\mu P_L\right) = -\frac{2m_u m_d g^\mu_\sigma}{[(k_2+q_2)^2-m_u^2][q_2^2-m_d^2]}. \tag{4.64}$$

We can now rewrite the expression for the diagram as before

$$\Sigma(p) = \frac{128g^4G_F^2\epsilon_3 m_u^2 m_e^2 m_d^2}{m_p}\tilde{\mathcal{I}}, \tag{4.65}$$

where the integral $\tilde{\mathcal{I}}$ is given by

$$\tilde{\mathcal{I}} = \tilde{\mathcal{I}}_1 \times \tilde{\mathcal{I}}_2, \tag{4.66}$$

with

$$\tilde{\mathcal{I}}_1 = \int d\tilde{k}_1 d\tilde{q}_1 \frac{1}{[(p+k_1)^2-m_e^2][(k_1+q_1)^2-m_u^2](q_1^2-m_d^2)(k_1^2-M_W^2)} \tag{4.67}$$

and

$$\tilde{\mathcal{I}}_2 = \int d\tilde{k}_2 d\tilde{q}_2 \frac{1}{[(p-k_2)^2 - m_e^2][(k_2+q_2)^2 - m_u^2](q_2^2 - m_d^2)(k_2^2 - M_W^2)}. \tag{4.68}$$

The factor 4, which appears in equation (4.65) but not in equation (4.36) arises from a contraction

$$\gamma^\mu \gamma_\mu = 4. \tag{4.69}$$

The numerators in equations (4.67) and (4.68) do not contain loop momenta, which is a difference to the case discussed in the last subsection. Therefore, the only dependence on p is in the logarithms inside the integral (compare the last subsection), which may be neglected. The integral $\tilde{\mathcal{I}}$ is well behaved, as already stated before, and therefore we expect a non-zero mass contribution in this case:

$$\delta m_\nu = \Sigma(\not{p} = m_\nu) \propto m_u^2 m_d^2 m_e^2. \tag{4.70}$$

It is interesting to estimate the order of magnitude of this mass correction. Comparison with the integrals calculated for the Higgs propagator in [53] shows that we may expect $\tilde{\mathcal{I}}$ to be of the order $\frac{1}{(16\pi^2)^4}$. So incorporating all prefactors from equation (4.65), we obtain a neutrino mass $m_\nu = \delta m_\nu$ of the order 10^{-32} eV. Here, we used the values for masses and constants given in table 2.2. It is obvious that this correction is far too small to be the main contribution to the neutrino masses we expect to have. Therefore, another mechanism must give the leading contribution of neutrino masses. However, if we know that a Majorana neutrino mass is generated in a New Physics model anyway, we do not need the Schechter–Valle theorem anymore.

Chapter 5

Neutrinoless Double Beta Decay in New Physics Models

In this chapter, we give a short overview of mechanisms contributing to neutrinoless double beta decay in New Physics models. All of them are discussed in the literature. The aim of this survey is merely to show that contributions to $0\nu\beta\beta$ arise in many models for physics beyond the SM. This chapter may be seen as a preparation for the following, where the realizations of $0\nu\beta\beta$ in universal extra dimensions, which have not been considered in the literature so far, are discussed.

5.1 $0\nu\beta\beta$ in Seesaw Type I Models

5.1.1 The Models

To account for the small but non-zero neutrino masses, one may introduce $n \geq 2$ heavy right-handed Majorana neutrinos n_{iR}, which are $SU(2)_L$ singlets, in addition to the SM fields. For example, in the minimal seesaw model (MSM), which was proposed by Frampton, Glashow, and Yanagida [55], $n = 2$ right-handed neutrinos are added. This way is the most efficient to address neutrino masses. In the MSM, one light neutrino will be massless which is perfectly allowed by oscillation data. Note that one could introduce only one right-handed neutrino, but then two of three light neutrinos would be massless, a situation excluded by neutrino oscillation experiments. Adding $n = 3$ right-handed neutrinos leads to the standard seesaw scenario, but in principle one could include an arbitrary number $n > 3$ of right-handed neutrinos into the model to account for the small neutrino masses.

Let us denote the right-handed charged leptons by l_R, then the part of the Lagrangian containing the mass terms can be written as

$$-\mathcal{L}^{\text{mass}} = \overline{l_L} Y_l l_R \phi + \overline{l_L} Y_\nu n_R \tilde{\phi} + \frac{1}{2}\overline{(n_R)^c} M_R n_R + \text{H.c.} \tag{5.1}$$

In this formula, $\tilde{\phi} = i\sigma_2 \phi^*$. After the spontaneous breaking of gauge symmetry, we obtain the mass term

$$-\mathcal{L}^{\text{mass}} = \overline{e_L} M_l e_R + \frac{1}{2}\overline{(\nu_L \ (n_R)^c)} \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ n_R \end{pmatrix} + \text{H.c.}, \tag{5.2}$$

where e symbolically stands for $e = e, \mu, \tau$. Here, $M_l = vY_l/\sqrt{2}$ is the charged lepton mass matrix and $M_D = vY_\nu\sqrt{2}$ is the Dirac neutrino mass matrix, with $v \approx 246\,\text{GeV}$ being the VEV of the neutral component of the Higgs doublet. M_R ist the heavy right-handed Majorana neutrino mass matrix, a complex and symmetric $n \times n$ matrix. Without loss of generality, we may work in the basis where M_l and M_R are diagonal:

$$M_l = \text{diag}(m_e, m_\mu, m_\tau) \ \text{ and } \ M_R = \text{diag}(M_1, \ldots, M_n). \tag{5.3}$$

The general form of M_D is

$$M_D = \begin{pmatrix} a_1 & \ldots & a_n \\ b_1 & \ldots & b_n \\ c_1 & \ldots & c_n \end{pmatrix}, \tag{5.4}$$

where a_i, b_i, c_i $(i = 1, \ldots, n)$ are complex.

This model gives an elegant realization of seesaw type I: The masses of the right-handed neutrinos are not given by electroweak symmetry breaking. Thus, they may be arbitrarily high and generate the small masses for the left-handed neutrinos.

5.1.2 Possible Realizations

When we add heavy right-handed neutrinos, neutrinoless double beta decay can procede via two different realizations, light neutrino exchange and heavy neutrino exchange. This observation can easily be made from the charged current Lagrangian, which is given by

$$-\mathcal{L} = \frac{g}{\sqrt{2}}\overline{(e\ \mu\ \tau)_L}\gamma^\mu \left[V \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} + R \begin{pmatrix} (n_{1R})^c \\ \vdots \\ (n_{nR})^c \end{pmatrix} \right] W_\mu^-. \tag{5.5}$$

Here, the $(3+n)\times(3+n)$ mixing matrix is parameterized as

$$\mathcal{U} = \begin{pmatrix} V & R \\ S & T \end{pmatrix}. \tag{5.6}$$

There is nothing new in the decay amplitudes, they have already been given in chapter 3, together with the corresponding Feynman diagram.

5.1.3 Leading Contribution

Unitarity of the total Lagrangian requires

$$\mathbb{1} = \mathcal{U}\mathcal{U}^\dagger = \begin{pmatrix} VV^\dagger + RR^\dagger & VS^\dagger + RT^\dagger \\ SV^\dagger + TR^\dagger & SS^\dagger + TT^\dagger \end{pmatrix}. \tag{5.7}$$

This condition directly translates into

$$VV^\dagger + RR^\dagger = \mathbb{1} \tag{5.8}$$

or, expanding in components,

$$|V_{e1}|^2 + |V_{e2}|^2 + |V_{e3}|^2 + |R_{e1}|^2 + \ldots + |R_{en}|^2 = 1. \tag{5.9}$$

Antusch et al. [56] showed that unitarity bounds on the light neutrino mixing matrix provide the constraints

$$|V_{e1}|^2 + |V_{e2}|^2 + |V_{e3}|^2 = 0.994 \pm 0.005, \tag{5.10}$$

so we conclude

$$|R_{e1}|^2 + \ldots + |R_{en}|^2 = 0.006 \pm 0.005. \tag{5.11}$$

Let $\mathcal{A}_\nu$ be the amplitude of light neutrino exchange and $\mathcal{A}_n$ be the amplitude of heavy neutrino exchange. We then obtain for the total amplitude of neutrinoless double beta decay:

$$\begin{aligned} \mathcal{A}_{0\nu\beta\beta} &= \mathcal{A}_\nu + \mathcal{A}_n \\ &\propto \sum_{i=1}^{3} V_{ei}^2 \frac{m_i}{\langle p^2 \rangle} + \sum_{i=1}^{n} R_{ei}^2 \frac{1}{M_i}, \end{aligned} \tag{5.12}$$

where m_i are the light neutrino mass eigenvalues and M_i are the heavy neutrino mass eigenvalues. We can see from this formula that there is a possibility of cancellations, as the V_{ei}'s and the R_{ei}'s include phases.

Let us check if the two contributions may become comparable in size. Any seesaw model is aiming to reproduce the light neutrino masses of order 1 eV. To achieve this goal, a mass of the right-handed neutrinos of at least 10^{12} GeV is needed if the Yukawa coupling is not much smaller than 1. Using a typical value $\langle p^2\rangle \approx (100\,\mathrm{MeV})^2$, we then obtain

$$\frac{\langle p^2\rangle}{M_i}/m_i = \frac{\langle p^2\rangle}{m_i M_i} \approx 10^{-5}. \tag{5.13}$$

As $|V_{e1}|^2 + |V_{e2}|^2 + |V_{e3}|^2 \approx 1$, we would need $|R_{e1}|^2 + |R_{e2}|^2 + |R_{e3}|^2 \approx 10^5$ for the two contributions to cancel. A value that big, however, is not allowed by the unitarity bounds given above.

Note that in this case not only there is no possibility of cancellations, but also because of the small value of $\frac{1}{M_i}$ there is approximately no contribution of $\mathcal{A}_n$ and the relevant mechanism for neutrinoless double beta decay in this model is the light neutrino exchange. Of course, the improbable case of an accidental cancellation, so that $\sum V_{ei}^2 m_i = 0$, is not excluded. Then, the leading contribution would indeed be the one from heavy neutrinos, which however would be small and difficult to detect experimentally.

5.2 $0\nu\beta\beta$ in the Higgs Triplet Model

5.2.1 The Model

The salient feature of this model is the enlargement of the Higgs sector of the SM consisting of one Higgs doublet ϕ by a Higgs triplet Δ which can be parameterized as follows:

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 1). \tag{5.14}$$

The Lagrangian responsible for the generation of neutrino masses can then be written

$$\mathcal{L}^{\text{mass}} = (D_\mu\phi)^T(D^\mu\phi) + \mathrm{Tr}\left[(D_\mu\Delta)^\dagger(D^\mu\Delta)\right] + \mathcal{L}_Y - V(\phi,\Delta), \tag{5.15}$$

where

$$\mathcal{L}_Y = -Y^{ij}\overline{(l_L)_i^c} i\sigma_2 \Delta l_{Lj} + \text{H.c.} \tag{5.16}$$

and

$$\begin{aligned} V(\phi,\Delta) = &-m_\phi^2\phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2 + M_\Delta^2\mathrm{Tr}\left(\Delta^\dagger\Delta\right) + (\alpha\phi^T i\sigma_2\Delta^\dagger\phi + \text{H.c.}) \\ &+ \lambda_1(\phi^\dagger\phi)\mathrm{Tr}\left(\Delta^\dagger\Delta\right) + \lambda_2\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 + \lambda_3\mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right] + \lambda_4\phi^\dagger\Delta\Delta^\dagger\phi. \end{aligned} \tag{5.17}$$

One can see that lepton number is explicitly broken in this model: It is not possible to assign consistent lepton numbers to all fields because of the coexistence of $\mathcal{L}_Y$ and the α-term in $V(\phi, \Delta)$.

Usually, it is assumed that the Higgs triplet is heavy ($M_\Delta^2 > v_0^2/2$). Then we may neglect the terms proportional to $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ in the Higgs potential. When now the Higgses acquire VEVs,

$$\langle H \rangle = \begin{pmatrix} 0 \\ v_0/\sqrt{2} \end{pmatrix} \quad \text{and} \quad \langle \Delta \rangle = \begin{pmatrix} 0 & 0 \\ v_\Delta/\sqrt{2} & 0 \end{pmatrix}, \tag{5.18}$$

we arrive at

$$V(v_0, v_\Delta) = -m_H^2 \frac{v_0^2}{2} + \frac{\lambda}{4}\frac{v_0^4}{4} + M_\Delta^2 \frac{v_\Delta^2}{2} - \frac{1}{2\sqrt{2}} \alpha v_0^2 v_\Delta. \tag{5.19}$$

From the minimization of this potential we obtain the relation

$$v_\Delta = \frac{\alpha v_0^2}{\sqrt{2} M_\Delta^2}. \tag{5.20}$$

Thus, the neutrinos acquire a Majorana mass matrix

$$M_L = \sqrt{2} Y v_\Delta = Y \frac{\alpha v_0^2}{M_\Delta^2}. \tag{5.21}$$

5.2.2 Possible Realizations

Two possible decay modes exist in the Higgs triplet model: light neutrino exchange and Higgs triplet exchange (in which no neutrinos are involved). The Feynman diagram for Higgs triplet exchange is shown in figure 5.1.

The amplitude for light neutrino exchange is the same as given before. For the Higgs triplet exchange, after integrating out the heavy fields, one obtains for the amplitude

$$\mathcal{A}_\Delta = \frac{g^4}{\sqrt{2}} \frac{Y v_\Delta}{M_W^4 M_\Delta^2}. \tag{5.22}$$

5.2.3 Leading Contribution

In the Higgs triplet model, there are no cancellations possible between the diagrams from light neutrino exchange and Higgs triplet exchange because there are no phases involved. If the Higgs triplet is responsible for the light neutrino masses via the formula in equation (5.21), its contribution will be negligible due to its small VEV and its huge mass.

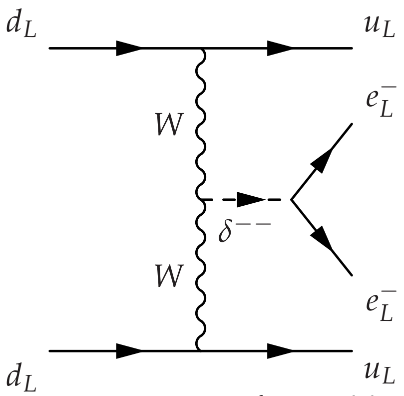


Figure 5.1: Diagram for $0\nu\beta\beta$ mediated via Higgs triplet exchange in the Higgs triplet model.

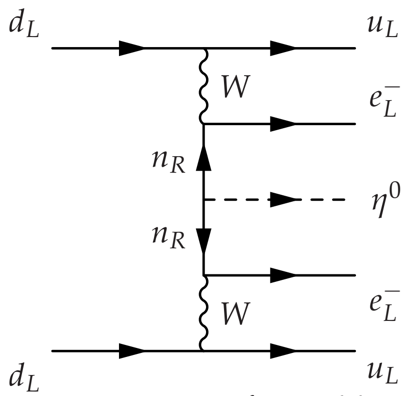


Figure 5.2: Diagram for $0\nu\beta\beta$ accompanied by majoron emission in the singlet majoron model.

5.3 $0\nu\beta\beta$ in the Singlet Majoron Model

5.3.1 The Model

This model was first discussed by Chikashige, Mohapatra, and Peccei [57]. In addition to the SM field content, we introduce right-handed neutrinos n_R and a singlet scalar field χ which will eventually drive the spontaneous breakdown of lepton number. We do not want to consider all the details of this model, just note that a neutrino mass matrix similar to the seesaw type I case is generated after SSB.

The singlet field χ obtains a vev, $\langle\chi\rangle = f$, so that we may write

$$\chi = \frac{1}{\sqrt{2}}\left(f + \sigma + i\eta^0\right), \tag{5.23}$$

where σ is an additional massive boson and η^0 is the massless majoron. χ only couples to the right-handed neutrinos n_R, not to the left-handed ones, and so does η^0.

The singlet majoron is only one possible model leading to spontaneous breaking of lepton number as a global symmetry. One could also introduce doublet majorons and triplet majorons, both possibilities were discussed in the literature. However, there are stronger bounds on these models than on the singlet majoron model. The triplet majoron model is basically ruled out by the Z width, whereas a mixture of singlet and doublet majoron still would be viable.

5.3.2 Possible Realizations

There are three possible realizations of neutrinoless double beta decay in the singlet majoron model, which are light neutrino exchange, heavy neutrino exchange, and heavy neutrino exchange with simultaneous emission of a majoron. The first two realizations will lead to the usual $0\nu\beta\beta$ peak at $2E_0$, whereas the third one will lead to an continuous spectrum, as the majoron can take away an arbitrary amount of energy and leave undetected. However, the majoron emitting mode will be distinguishable from the two neutrino decay mode of double beta decay, as the peak of the continuous spectrum will be at higher energies compared to the two neutrino mode. See figure 3.1 in chapter 3 for more details. The Feynman diagram for the majoron accompanied realization is shown in figure 5.2. The diagrams for light and heavy neutrino exchange are the same as before. The half-life for majoron emission is given by

$$\left(T_{1/2}^{0\nu,\eta^0}\right) = \epsilon^2_{\nu\eta^0} \left|\mathcal{M}^{0\nu}\right|^2 G^{0\nu,\eta^0}, \tag{5.24}$$

where $\epsilon_{\nu\eta^0}$ is the effective coupling strength between neutrinos and the majoron, $\mathcal{M}^{0\nu}$ is the same matrix element as for the light neutrino exchange, but the phase space integral $G^{0\nu,\eta^0}$ may differ from the one for the light neutrino case. As we deal with three particles in the final state, whereas in the case of light neutrino exchange only two particles are in the final state, this conclusion is obvious.

Limits on the effective neutrino-majoron coupling constants have been obtained from the NEMO-3 experiment [58]: For the majoron accompanied decay of ^{100}Mo ($T_{1/2} > 2.7 \times 10^{22}$ y) and ^{82}Se ($T_{1/2} > 1.5 \times 10^{22}$ y), the bounds are $\left|\epsilon_{\nu\eta^0}\right| < (0.4 - 1.9) \times 10^{-4}$ and $\left|\epsilon_{\nu\eta^0}\right| < (0.66 - 1.7) \times 10^{-4}$, respectively.

5.4 $0\nu\beta\beta$ in Left–Right Symmetric Models

5.4.1 The Model

Left–right symmetric models [59–61] are especially interesting, because they provide a natural realization of the seesaw mechanism.

In the left–right symmetric model with the gauge group

$$G^{\mathrm{LR}} \equiv SU(2)_L \times SU(2)_R \times U(1)_{B-L}, \tag{5.25}$$

we have left-handed and right-handed fermion doublets (family indices will be suppressed throughout the whole section):

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (\underline{\mathbf{2}}, \underline{\mathbf{1}}, -1) \ \text{ and } \ l_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \sim (\underline{\mathbf{1}}, \underline{\mathbf{2}}, -1). \tag{5.26}$$

Here, we use the electric charge formula

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2}, \tag{5.27}$$

where I_{3L} and I_{3R} are the third component of the weak isospin of $SU(2)_L$ and $SU(2)_R$, respectively.

The Higgs sector of this model contains one complex bidoublet

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \sim (\underline{\mathbf{2}}, \underline{\mathbf{2}}^*, 0) \tag{5.28}$$

and two complex triplets

$$\Delta_L = \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix} \sim (\underline{\mathbf{3}}, \underline{\mathbf{1}}, 2) \text{ and } \Delta_R = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix} \sim (\underline{\mathbf{1}}, \underline{\mathbf{3}}, 2). \tag{5.29}$$

The Higgs fields eventually obtain VEVs:

$$\langle\phi\rangle = \begin{pmatrix} \kappa_1/\sqrt{2} & 0 \\ 0 & \kappa_2/\sqrt{2} \end{pmatrix} \text{ and } \langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R}/\sqrt{2} & 0 \end{pmatrix}. \tag{5.30}$$

The original gauge symmetry is broken in two steps down to $U(1)_{\text{em}}$:

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle\Delta_R\rangle\neq 0} SU(2)_L \times U(1)_Y \xrightarrow{\langle\phi\rangle\neq 0} U(1)_{\text{em}}. \tag{5.31}$$

The terms in the Lagrangian responsible for the lepton masses are

$$\begin{aligned} \mathcal{L}_{\text{mass}} =& f\overline{l_L}\phi l_R + h\overline{l_L}\tilde{\phi} l_R + \text{H.c.} \\ &+ ik\left(l_L^T C^{-1}\sigma_2\Delta_L l_L + l_R^T C^{-1}\sigma_2\Delta_R l_R\right) + \text{H.c.}, \end{aligned} \tag{5.32}$$

where $\tilde{\phi} = \sigma_2\phi^*\sigma_2$. We assume f, h, k to be real for simplicity (keep in mind that we suppress family indices, so in the three family version f, h, k would be real 3×3 matrices).

With the Higgs fields acquiring their VEVs, we finally obtain

$$\begin{aligned} \mathcal{L}_{\text{mass}} =& \left(f\frac{\kappa_1}{\sqrt{2}} + h\frac{\kappa_2}{\sqrt{2}}\right)\overline{\nu_L}\nu_R + \left(h\frac{\kappa_1}{\sqrt{2}} + f\frac{\kappa_2}{\sqrt{2}}\right)\overline{e_L}e_R + \text{H.c.} \\ &+ k\left(\frac{v_L}{\sqrt{2}}\overline{(\nu_L)^c}\nu_L + \frac{v_R}{\sqrt{2}}\overline{(\nu_R)^c}\nu_R\right) + \text{H.c.} \end{aligned} \tag{5.33}$$

Noting that $\nu_L C^{-1} = \overline{(\nu_L)^c}$, the mass term for the neutrinos is

$$\mathcal{L}_{\text{mass}}^{\text{neutrino}} = \left(f\frac{\kappa_1}{\sqrt{2}} + h\frac{\kappa_2}{\sqrt{2}}\right)\overline{\nu_L}\nu_R + \frac{kv_L}{\sqrt{2}}\overline{(\nu_L)^c}\nu_L + \frac{kv_R}{\sqrt{2}}\overline{(\nu_R)^c}\nu_R + \text{H.c.} \tag{5.34}$$

Define now the Dirac neutrino mass as

$$M_D \equiv f\frac{\kappa_1}{\sqrt{2}} + h\frac{\kappa_2}{\sqrt{2}} \tag{5.35}$$

and the Majorana masses as

$$M_{L,R} \equiv \sqrt{2}kv_{L,R}, \tag{5.36}$$

and define the self-conjugate Majorana fields

$$\nu = \frac{1}{\sqrt{2}}\left[\nu_L + (\nu_L)^c\right] \quad \text{and} \quad N = \frac{1}{\sqrt{2}}\left[\nu_R + (\nu_R)^c\right]. \tag{5.37}$$

We then can write

$$\mathcal{L}^{\text{neutrino}}_{\text{mass}} = (\overline{\nu}\ \overline{N})\begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}\begin{pmatrix} \nu \\ N \end{pmatrix}. \tag{5.38}$$

Comparison with equation (2.16) shows that in this model the most general neutrino mass term is generated.

5.4.2 Possible Realizations

In the left–right symmetric model, there are five possible diagrams for neutrinoless double beta decay, three of which are shown in figure 5.3.

For the light neutrino exchange, there is no difference to the cases discussed before. In the left–right symmetric model, right-handed neutrinos couple mainly to "right-handed" W bosons (as it is always the case when right-handed currents are present). We then have for the amplitude for heavy neutrino exchange

$$\mathcal{A}_N = \left(\frac{g}{\sqrt{2}}\right)^4 \left(\frac{1}{M_{W_R}}\right)^4 \frac{1}{M_N}. \tag{5.39}$$

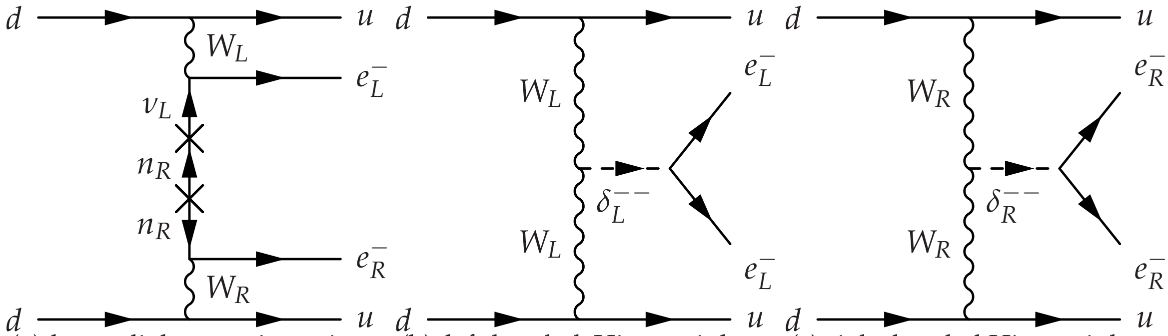
For possible values and a discussion thereof, see Mohapatra [62].

A new realization is the heavy-light neutrino mixing. Recall that we have neutrino mixing in the following form

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \nu \\ N \end{pmatrix} = \begin{pmatrix} \nu\cos\theta - N\sin\theta \\ \nu\sin\theta + N\cos\theta \end{pmatrix}. \tag{5.40}$$

Only mass eigenstates propagate, and couple according to their mixing at the different vertices. Assume that the lightest neutrino $\nu \approx \nu_1$ is propagating. We then find for the matrix element

$$\begin{aligned} \mathcal{M}_{\mu\lambda} &= \sin\theta\cos\theta\left[\overline{e}(p_1)\gamma_\mu P_L \frac{\not{q} + m_\nu}{q^2 - m_\nu^2}\gamma_\lambda P_L e(p_2) - (1 \leftrightarrow 2)\right] \\ &\approx \sin\theta\left[\overline{e}(p_1)\gamma_\mu P_L \frac{\not{q}}{q^2 - m_\nu^2}\gamma_\lambda P_L e(p_2) - (1 \leftrightarrow 2)\right]. \end{aligned} \tag{5.41}$$



(a) heavy-light neutrino mixing (b) left-handed Higgs triplet exchange (c) right-handed Higgs triplet exchange

Figure 5.3: Diagrams for $0\nu\beta\beta$ decay in the left–right symmetric model.

Comparison with the light and heavy neutrino exchange gives

$$\mathcal{A}_{\text{mix}} = \left(\frac{g}{\sqrt{2}}\right)^4 \frac{1}{M_{W_L}^2 M_{W_R}^2} \sin\theta \frac{1}{\langle \not{q} \rangle}. \tag{5.42}$$

The other new contribution is the Higgs triplet exchange. The contribution of the left-handed triplet is negligible, due to its small VEV. For the amplitude of the right-handed triplet we obtain

$$\mathcal{A}_{\Delta} = \frac{g^4}{2} \left(\frac{1}{M_{W_R}}\right)^4 \frac{M_{W_R}}{g M_{\Delta_R}^2}. \tag{5.43}$$

The factor g in the denominator arises because of the fact that $M_{W_R} = g v_R$.

5.5 $0\nu\beta\beta$ in Supersymmetric Models

At the end of this chapter about neutrinoless double beta decay in New Physics models let us just shortly mention that of course contributions to neutrinoless double beta decay exist in supersymmetric models. If R-parity is violated, lepton number violating processes may occur. Therefore, observable amplitudes for $0\nu\beta\beta$ may arise, even for a very small neutrino mass. Unfortunately, an introduction to the minimal supersymmetric standard model (MSSM) or to R-parity violation is far beyond the scope of this thesis. Therefore, we will not give explicit diagrams or amplitudes.

However, stringent bounds on R-parity violating couplings have been deduced from data on neutrinoless double beta decay in ^{76}Ge by the Heidelberg-Moscow group [10]. A discussion of various realizations may be found in [63]. The current bounds on SUSY accompanied neutrinoless double beta decay may be found in [64].

Chapter 6

Neutrinoless Double Beta Decay in Extra Dimensions

We have seen in the last chapter that many New Physics scenarios will give considerable contributions to $0\nu\beta\beta$ decay, so limits on half-lives obtained from experiment may give constraints on the parameters of these models. In scenarios with extra dimensions, the only free parameter is the size (the radius R) of the extra dimension(s). So, if a model of extra dimensions gives relevant contributions to $0\nu\beta\beta$, one can find bounds on R.

There has been some previous work concerning the relation between neutrinoless double beta decay and extra dimensions. Gozdz et al. [65, 66] worked in the ADD model, whereas Bhattacharyya et al. [67] used a particular model of one sterile right-handed neutrino in one extra spatial dimension to account for the observed half-lives of $0\nu\beta\beta$. However, the model of universal extra dimensions has not been considered yet, and we will see in this chapter that non-negligible contributions to $0\nu\beta\beta$ arise through Kaluza–Klein (KK) tower particles in UEDs. This is not as surprising as it may seem, because in many models of extra dimensions such contributions may be generated. Dienes et al. [68] implement a seesaw model in extra dimensions which allows for $0\nu\beta\beta$, too.

Using present data on the half-life of $0\nu\beta\beta$, the bounds we calculate are slightly weaker than these from electroweak precision data. However, future experiments are expected to push the bounds on the half-life of $0\nu\beta\beta$ by at least one order of magnitude, so our limits may become competitive to or even stronger than the bounds from electroweak precision data.

The chapter is organized as follows: In section 6.1 we review shortly the basic facts on universal extra dimensions used in this thesis. In section 6.2 we discuss the issue of neutrinos and neutrino masses in UEDs. In section 6.3 we will calculate the influence of universal extra dimensions on neutrinoless double beta decay.

6.1 Universal Extra Dimensions

The basic feature of extra dimensions we will use in this chapter is that, from a $4d$ point of view, extra dimensions appear as towers of new particles, the so-called KK towers. One can compare this feature to ordinary quantum mechanics, where a $1d$ box leads to an infinite number of modes for the particle confined in it. In this section, we will review the relevant facts of universal extra dimensions and derive the mode expansions for scalar particles, fermions, and gauge bosons.

6.1.1 Why an Extra Spatial Dimension Must Be Compactified

The first thing we need to think about is the question if extra spatial dimensions are compactified or not. Assume we would have an extra spatial dimension that extends from $-\infty$ to $+\infty$, that means, it is not compactified in any way. The differential equation for the gravitational potential $\Phi(\boldsymbol{r})$ is Poisson's equation

$$\Delta\Phi(\boldsymbol{r}) = 4\pi G\varrho(\boldsymbol{r}), \tag{6.1}$$

where G is Newton's gravitational constant and ϱ is the mass distribution in the problem under consideration. Here $\boldsymbol{r}$ is a spatial vector in d dimensions, as all the other vectorial entities in this subsection are.

We can define the gravitational field (which is equal to the gravitational acceleration) by

$$\boldsymbol{g} = -\nabla\Phi. \tag{6.2}$$

Thus we arrive at the differential equation for $\boldsymbol{g}$

$$\nabla \cdot \boldsymbol{g} = -4\pi G\varrho. \tag{6.3}$$

Integrating this equation over the volume V, we obtain for the r.h.s.

$$-4\pi G \int_V dV \varrho = -4\pi G M, \tag{6.4}$$

with M being the total mass in V. For the l.h.s. we find

$$\int_V dV \nabla \cdot \boldsymbol{g} = \oint_{\partial V} d\boldsymbol{A} \cdot \boldsymbol{g}, \tag{6.5}$$

by Gauss' law. We may assume that the gravitational field is spherically symmetric, so that we may write

$$\boldsymbol{g}(r) = g(r)\boldsymbol{e}_r, \tag{6.6}$$

where $\boldsymbol{e}_r$ is the unit vector in radial direction. As the surface element $d\boldsymbol{A}$ has radial direction, we have

$$\boldsymbol{e}_r \cdot d\boldsymbol{A} = dA. \tag{6.7}$$

dA is given by

$$dA = \frac{dV}{dr} = r^{d-1} d\Omega_d, \tag{6.8}$$

where $d\Omega_d$ is the d-dimensional unit surface element and $dV = r^{d-1} dr d\Omega_d$. Thus we arrive at

$$\int_V dV \nabla \cdot \boldsymbol{g} = g(r) r^{n-1} \oint_{\partial V} d\Omega_d. \tag{6.9}$$

The unit sphere in d dimensions is given by

$$\oint d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}. \tag{6.10}$$

Putting all together, we obtain for the gravitational field

$$g(r) = -\frac{2\pi g M \Gamma(d/2)}{\pi^{d/2}} \frac{1}{r^{d-1}}. \tag{6.11}$$

In three (spatial) dimensions, this formula reduces to the well-known expression

$$g(r) = -\frac{GM}{r^2}. \tag{6.12}$$

It is clear now that any extra spatial dimension must be compactified, as for large distances the gravitational force law falls off as $1/r^2$ and not as $1/r^3$, or some other power related to the number of extra dimensions.

6.1.2 Compactification and Orbifolding

As we have seen in the last subsection, any extra spatial dimension must be compactified. For one extra dimension, this compactification is often done on a circle S^1 with radius R. It is possible to describe this compactification in two equivalent ways (here we follow closely the presentation of Agashe in Chapter 1 of [69]):

1. y (the variable of the extra dimension) unrestricted ($-\infty < y < +\infty$) but imposing periodic boundary conditions on all fields, $F(y) = F(y + 2\pi R)$, or

2. the range of y restricted to $0 \leq y \leq 2\pi R$ and imposing boundary conditions $F(y = 0) = F(y = 2\pi R)$ on all fields.

In this thesis we will work with the first description, so that we can Fourier expand any $5d$ field F as follows (with x^μ being the ordinary $4d$ space-time coordinate, and y being the coordinate of the extra dimension):

$$F(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} f^{(n)}(x) e^{iny/R}. \tag{6.13}$$

It turns out that such a compactification does not lead to the chiral fermions known from the Standard Model. In addition to the identification $y \equiv y + 2\pi R$ we impose a Z_2 symmetry on the circle S^1, identifying $-y \equiv y$. Thus, the physical domain extends only within $0 \leq y \leq \pi R$. This compactification scheme is denoted as S^1/Z_2 and called an orbifold. We can see that the end points of the orbifold ($y = 0$, $y = \pi R$) do not transform under Z_2, which means they are fixed points of this orbifold. Moreover, they are *not* identified (as opposed to $y = 0$ and $y = 2\pi R$).

On this orbifold, we can rewrite the decomposition of our field F in terms of functions that are even and odd under the Z_2 as follows

$$\begin{aligned}
F(x,y) &= \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} f^{(n)}(x) e^{iny/R} \\
&= \frac{1}{\sqrt{2\pi R}} f^{(0)}(x) + \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \left(f^{(n)}(x) e^{iny/R} + f^{(-n)}(x) e^{-iny/R} \right) \\
&= \frac{1}{\sqrt{2\pi R}} f^{(0)}(x) + \frac{1}{2\sqrt{2\pi R}} \sum_{n=1}^{\infty} \Big(f^{(n)}(x) e^{iny/R} + f^{(n)}(x) e^{-iny/R} \\
&\qquad + f^{(-n)}(x) e^{iny/R} + f^{(-n)}(x) e^{-iny/R} \\
&\qquad + f^{(n)}(x) e^{iny/R} - f^{(n)}(x) e^{-iny/R} \\
&\qquad - f^{(-n)}(x) e^{iny/R} + f^{(-n)}(x) e^{-iny/R} \Big) \\
&= \frac{1}{\sqrt{2\pi R}} f^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[\frac{1}{\sqrt{2}} \left(f^{(n)}(x) + f^{(-n)}(x) \right) \frac{e^{iny/R} + e^{-iny/R}}{2} \right. \\
&\qquad \left. + \frac{i}{\sqrt{2}} \left(f^{(n)}(x) - f^{(-n)}(x) \right) \frac{e^{iny/R} - e^{-iny/R}}{2i} \right].
\end{aligned} \tag{6.14}$$

If for $n > 0$ we define

$$f_+^{(n)} = \frac{1}{\sqrt{2}} \left(f^{(n)} + f^{(-n)} \right) \quad \text{and} \quad f_-^{(n)} = \frac{i}{\sqrt{2}} \left(f^{(n)} - f^{(-n)} \right), \tag{6.15}$$

we may finally write

$$F(x,y) = \frac{1}{\sqrt{2\pi R}} f^{(0)} + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \left[f_+^{(n)} \cos \frac{ny}{R} + f_-^{(n)} \sin \frac{ny}{R} \right]. \tag{6.16}$$

Physics must be invariant under Z_2, so we can define a parity transformation

$$F(x, -y) = P_y F(x, y), \tag{6.17}$$

where $P_y = +1$ for F even and $P_y = -1$ for F odd. Thus we have $f_-^{(n)} = 0$ for $P_y = +1$ and $f_+^{(n)}, f^{(0)} = 0$ for $P_y = -1$. Therefore, the compactification on an orbifold leads to two effects that will prove useful in the further analysis:

- reduction of the mode number by a factor of two and
- removal of the zero mode for odd 5d fields.

Let us discuss the relation between orbifolds and boundary conditions (BCs) in some more detail. Assume that F is a scalar field living in an extra dimension which is an orbifold, so that the physical domain is $y \in [0, \pi R]$. The discussion for the other fields (fermions, gauge fields) would be the same. The current in y direction is given as

$$J_y = iF^\dagger \partial_y F. \tag{6.18}$$

As long as there are no additional sources at the boundaries, the current should vanish there. This constraint can be achieved by the boundary condition

$$\partial_y F|_{y=0,\pi R} = 0, \tag{6.19}$$

which is called Neumann[1] boundary condition, or by

$$F|_{y=0,\pi R} = 0, \tag{6.20}$$

which is called Dirichlet[2] boundary condition. Imposing one of these boundary conditions at the fixed points is equivalent to the field being even or odd under the Z_2 symmetry. We will prove this fact now. Let us call the field F_+ if it is even under the Z_2 and F_- if it is odd. From

$$F_+(-y) = F_+(y) \tag{6.21}$$

it follows directly that

$$\partial_y F_+(-y) = -\partial_y F_+(y), \tag{6.22}$$

such that we must have

$$\partial_y F_+(y=0) = \partial_y F_+(y = \pi R) = 0, \tag{6.23}$$

which are Neumann BCs. On the other hand, from

$$F_-(-y) = -F_-(y) \tag{6.24}$$

[1] After Carl Neumann, a German mathematician.
[2] After Johann Peter Gustav Lejeune Dirichlet, a German mathematician, too.

it follows directly that we must have

$$F_{-}(y=0) = F_{-}(y=\pi R) = 0, \tag{6.25}$$

which are Dirichlet BCs.

In the following subsections, we will shortly give the boundary conditions and mode expansions for all types of fields. These can be found in a similar manner for example in [70].

6.1.3 Scalar Fields

As a scalar field can be even or odd under the orbifold Z_2 symmetry, it has to fullfill the following boundary conditions at $y = 0, \pi R$:

$$\begin{aligned} \partial_y \phi_+ &= 0 \qquad \text{for even fields,} \\ \phi_- &= 0 \qquad \text{for odd fields.} \end{aligned} \tag{6.26}$$

Then we have the corresponding KK mode expansions,

$$\begin{aligned} \phi_+(x,y) &= \frac{1}{\sqrt{2\pi R}} \phi_+^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_+^{(n)}(x) \cos \frac{ny}{R}, \\ \phi_-(x,y) &= \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_-^{(n)}(x) \sin \frac{ny}{R}. \end{aligned} \tag{6.27}$$

It is easy to check that these fields indeed fulfill equation (6.26).

6.1.4 Fermion Fields

Dirac spinors must satisfy at $y = 0, \pi R$ either the conditions

$$\begin{aligned} \partial_y \psi_{+R} &= 0, \\ \psi_{+L} &= 0, \end{aligned} \tag{6.28}$$

or

$$\begin{aligned} \partial_y \psi_{-L} &= 0, \\ \psi_{-R} &= 0. \end{aligned} \tag{6.29}$$

Then the KK mode expansions for fermions are

$$\begin{aligned} \psi_+(x,y) &= \frac{1}{\sqrt{2\pi R}} \psi_R^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left(\psi_R^{(n)}(x) \cos \frac{ny}{R} + \psi_L^{(n)}(x) \sin \frac{ny}{R} \right), \\ \psi_-(x,y) &= \frac{1}{\sqrt{2\pi R}} \psi_L^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left(\psi_L^{(n)}(x) \cos \frac{ny}{R} + \psi_R^{(n)}(x) \sin \frac{ny}{R} \right). \end{aligned} \tag{6.30}$$

It can easily be seen that the zero-mode is either left-handed or right-handed. Therefore, this model reproduces the chiral fermions of the SM (chiral means that left-handed and right-handed fields transform differently under the gauge group of the SM). In higher modes, left-handed and right-handed fields mix. We have vector-like fermions (meaning that left-handed and right-handed fields transform in the same way under the gauge group of the SM).

6.1.5 Gauge Fields

For the vector fields, we choose A_μ to be even under the orbifold Z_2 symmetry and A_5 to be odd, which is necessary to get the correct behavior of the gauge bosons. We then have the following boundary conditions at $y = 0, \pi R$:

$$\begin{aligned} \partial_y A_\mu &= 0, \\ A_5 &= 0. \end{aligned} \tag{6.31}$$

These conditions give the mode expansions

$$\begin{aligned} A_\mu(x,y) &= \frac{1}{\sqrt{2\pi R}} A_\mu^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \cos \frac{ny}{R}, \\ A_5(x,y) &= \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_5^{(n)}(x) \sin \frac{ny}{R}. \end{aligned} \tag{6.32}$$

6.1.6 KK Number Conservation

Having a massless particle propagating in a compactified (spacelike) 5th dimension looks like a massive particle in 4*d*. That can easily be seen in the following way: Assuming that 5*d* Lorentz invariance holds, then the 5*d* momentum for this particle is given by

$$0 = p^2 = p_0^2 - \boldsymbol{p}^2 - p_5^2. \tag{6.33}$$

Here p_0 is the energy of the particle, $\boldsymbol{p}^2$ is the square of the particle three-momentum, and p_5 is its momentum along the fifth dimension. Rewriting this equation, one obtains:

$$p_0^2 - \boldsymbol{p}^2 = p_5^2. \tag{6.34}$$

Comparing this equation to the well-known equality

$$p_0^2 - \boldsymbol{p}^2 = m^2, \tag{6.35}$$

the above statement is obvious. Note that it is crucial for the extra dimension to be spacelike, as otherwise there would be particles with negative mass square, also known as “tachyons”. It is not desirable for different reasons to have a theory containing tachyons, so we will take the extra dimension to be spacelike in the remainder of this thesis.

As p_5 will be quantized in the compactified extra dimension (compare the particle in a box from ordinary quantum mechanics), one obtains a tower of states with masses

$$m_n = \frac{n}{R}. \tag{6.36}$$

These states are called KK excitations, and the integer quantum number n is usually referred to as KK number. As long as $5d$ Lorentz invariance holds, the KK number is a good quantum number and needs to be conserved in all processes seen in $4d$. Compactifying the extra dimension on an orbifold with fixed points, this conclusion is no longer true. Having fixed points destroys translation invariance in the extra dimension and therefore necessarily violates 5-momentum conservation, too. A subgroup of S^1/Z_2, however, remains unbroken, which is called KK parity. In $4d$, it is a discrete Z_2 symmetry under which only KK modes with odd quantum number are charged, while KK modes with even quantum numbers remain uncharged.

This result can easily be proven in the following way: Taking into account the fixed points, translation by πR remains a symmetry of the orbifold. Using the mode expansions for fermions given in equation (6.30) and translating y by πR, we obtain

$$\begin{aligned}\psi_+(x, y+\pi R) &= \frac{1}{\sqrt{2\pi R}}\psi_R^{(0)}(x) \\ &\quad + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\left(\psi_R^{(n)}(x)\cos\frac{n(y+\pi R)}{R} + \psi_L^{(n)}(x)\sin\frac{n(y+\pi R)}{R}\right).\end{aligned} \tag{6.37}$$

Due to the translational properties of the trigonometric functions, KK modes with even KK number n are invariant under this translation, whereas the KK modes with odd KK number change sign. Thus mixing between different KK modes is possible. Only KK modes with odd numbers need to be produced in pairs at vertices, whereas KK modes with even number may be produced in arbitrary numbers. This result can be stated in a slightly different form: With n_i being the KK number of particle i interacting at a vertex, at this vertex the following relation has to hold,

$$\sum_i n_i \equiv 0 \bmod 2. \tag{6.38}$$

A short example should make this point clear: We are interested in heavy KK neutrinos contributing to neutrinoless double beta decay. Therefore we are interested in the vertex $\nu^{(n)}$-W^--e^-. We will see later that it vanishes, but it may or may not be allowed by KK parity, depending on the KK number of the neutrino. See figure 6.1 for more details.

Figure 6.1: Left: Vertex involving only one odd KK mode. This vertex is forbidden by KK parity. Odd KK modes may only interact in pairs at vertices. Right: Vertex involving a single even KK mode. This vertex is perfectly allowed by KK parity. Even KK modes may be produced in arbitrary numbers at vertices.

6.2 Fermions and Fermion Masses in Universal Extra Dimensions

6.2.1 Spinors in Five Dimensions

In this subsection, we want to generalize the spinor from four dimensions (cf. section 2.8) to five dimensions and carry over as many properties as possible. First note that we have to introduce one more gamma matrix to satisfy the Dirac-Clifford algebra in five dimensions

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}, \quad M, N = 0, 1, 2, 3, 5. \tag{6.39}$$

Therefore, we define the gamma matrices

$$\Gamma^\mu = \gamma^\mu, \quad \mu = 0, 1, 2, 3, \quad \text{and} \quad \Gamma^5 = i\gamma^5. \tag{6.40}$$

The construction of Majorana spinors is not directly possible in five dimensions. To understand this statement, have a look at the defining equation for the charge conjugation matrix in odd dimensions

$$C^{-1}\Gamma_\mu C = (-1)^{(d-1)/2}\Gamma_\mu^T. \tag{6.41}$$

It is clear that on the r.h.s. we obtain a $+$ in five dimensions instead of $-$, so the Majorana condition cannot be consistently imposed. However, we may introduce the so called symplectic Majorana condition between a pair of Dirac spinors,

$$\psi_1^c = -\psi_2 \quad \text{and} \quad \psi_2^c = \psi_1. \tag{6.42}$$

To calculate the relations which this equation imposes on the spinor components, we should find a representation of the charge conjugation matrix in five dimensions. One possibility is

$$C_5 = \Gamma^0\Gamma^2\Gamma^5 = i\gamma^0\gamma^2\gamma^5 = \begin{pmatrix} -\epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}, \text{ where } \epsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{6.43}$$

As in the four-dimensional case, we may write the spinors

$$\psi_1 = \begin{pmatrix} \xi_1 \\ \overline{\eta}_1 \end{pmatrix} = \begin{pmatrix} \xi_1 \\ i\sigma_2 \eta_1^* \end{pmatrix} \quad \text{and} \quad \psi_2 = \begin{pmatrix} \xi_2 \\ \overline{\eta}_2 \end{pmatrix} = \begin{pmatrix} \xi_2 \\ i\sigma_2 \eta_2^* \end{pmatrix}. \tag{6.44}$$

It is now straightforward to check that the symplectic Majorana condition gives the relations

$$\xi_2 = \eta_1 \quad \text{and} \quad \eta_2 = -\xi_1. \tag{6.45}$$

With these alterations in mind, we may now write a so called symplectic Majorana mass term, using the symplectic Majorana condition. It is given by

$$\psi_1^T C_5^{-1} \psi_2 = -\xi_1^T \overline{\eta}_1^* - \eta_1^\dagger \overline{\xi}_1. \tag{6.46}$$

6.2.2 Toy Model for Neutrinos in Universal Extra Dimensions

In this section, we want to show (as a proof of principle) that it is possible to construct a model of neutrinos in one universal extra dimension which includes lepton number violation and therefore allows for processes such as neutrinoless double beta decay.

For simplicity, we stick to one generation of SM neutrinos. As in UEDs all fields propagate in the higher dimensional space (called the "bulk"), we do not need to introduce additional fields to see effects beyond the SM. Remember that the extra spatial dimension is compactified on an S^1/Z_2 orbifold. This model is inspired by the one discussed in [71], but differs in many important points: The authors of [71] have discussed an iso-singlet right-handed neutrino propagating in the bulk, with all SM fields confined to a 3-brane. Nevertheless, the formalism is somehow similar.

Let us introduce the neutrino

$$\nu(x,y) = \begin{pmatrix} \xi(x,y) \\ \overline{\eta}(x,y) \end{pmatrix}, \tag{6.47}$$

where, as before, ξ and η are 2-component spinors and $\overline{\eta} = i\sigma_2\eta^*$. x is the 4-dimensional coordinate, and the y-coordinate is compactified such that we have

$$\nu(x,y) = \nu(x, y + 2\pi R), \tag{6.48}$$

with R being the radius of the extra dimension. The 2-component spinors may be chosen to have different parity under the Z_2, such that

$$\xi(x,y) = \xi(x,-y) \quad \text{and} \quad \eta(x,y) = -\eta(x,-y). \tag{6.49}$$

Note that this neutrino is the usual SM neutrino and therefore is *not* an SM singlet. In this way our model differs strongly from these considered elsewhere [67, 68, 72].

Note also that we work in UEDs, so that all SM fields and gravity experience the extra dimension. Therefore there is no need for the neutrino to be a singlet, which in other models is taken to be the reason why it may propagate in the extra dimension.

A usual Dirac mass term for ν then would have the form

$$m_D \overline{\nu}\nu = m_D \left(\eta^T i\sigma_2 \xi - \xi^\dagger i\sigma_2 \eta^* \right). \tag{6.50}$$

It can easily be seen that this term is forbidden by the Z_2, as under parity transformation it transforms into

$$m_D \left(\xi^\dagger i\sigma_2 \eta^* - \eta^T i\sigma_2 \xi \right) = -m_D \overline{\nu}\nu. \tag{6.51}$$

So for the Lagrangian of the neutrino we may write (including a Majorana mass term and A running over $0, 1, 2, 3, 5$)

$$\mathcal{L}_\nu = \int_0^{2\pi R} dy \left[\overline{\nu} i\Gamma^A \partial_A \nu - \frac{1}{2} m_\nu \left(\nu^T C_5^{-1} \nu + \text{H.c.} \right) \right]. \tag{6.52}$$

We will use the following KK expansions for even and odd fields:

$$\begin{aligned} \xi(x,y) &= \frac{1}{\sqrt{2\pi R}} \xi^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \xi^{(n)}(x) \cos \frac{ny}{R}, \\ \eta(x,y) &= \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \eta^{(n)}(x) \sin \frac{ny}{R}. \end{aligned} \tag{6.53}$$

With these expansions and after integrating out the y-coordinate, we obtain (a detailed calculation is given in appendix B.2)

$$\begin{aligned} \mathcal{L}_\nu = {} & \xi^{(0)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(0)} - \frac{1}{2} m_\nu \left(\xi^{(0)} \xi^{(0)} + \text{H.c.} \right) + \sum_{n=1}^{\infty} \Big[\xi^{(n)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(n)} + \eta^{(n)\dagger} i\overline{\sigma}^\mu \partial_\mu \eta^{(n)} \\ & + \frac{n}{R} \left(\xi^{(n)} \eta^{(n)} + \overline{\xi}^{(n)} \overline{\eta}^{(n)} \right) - \frac{1}{2} m_\nu \left(\xi^{(n)} \xi^{(n)} + \eta^{(n)} \eta^{(n)} + \text{H.c.} \right) \Big]. \end{aligned} \tag{6.54}$$

From the definition of $\nu(x,y)$ it is clear that ξ and $\overline{\eta}$ have the same lepton number. Thus in this Lagrangian, lepton number is explicitly broken and lepton number violating processes are allowed. We now can see that the part of the Lagrangian giving mass to the particles is given by

$$\begin{aligned} -\mathcal{L}_{\text{mass}}^{\text{KK}} &= \frac{1}{2} m_\nu \xi^{(0)} \xi^{(0)} + \text{H.c.} \\ &\quad - \sum_{n=1}^{\infty} \left[\frac{n}{R} \left(\xi^{(n)} \eta^{(n)} + \overline{\xi^{(n)}}\, \overline{\eta^{(n)}} \right) + \frac{1}{2} m_\nu \left(\xi^{(n)} \xi^{(n)} + \eta^{(n)} \eta^{(n)} + \text{H.c.} \right) \right] \\ &= \frac{1}{2} m_\nu \xi^{(0)} \xi^{(0)} + \frac{1}{2} \sum_{n=1}^{\infty} \left(\xi^{(n)}, \eta^{(n)} \right) \begin{pmatrix} m_\nu & -n/R \\ -n/R & m_\nu \end{pmatrix} \begin{pmatrix} \xi^{(n)} \\ \eta^{(n)} \end{pmatrix} + \text{H.c.} \end{aligned} \tag{6.55}$$

If we now define

$$\begin{aligned} \chi^{(0)} &\equiv \xi^{(0)}, \\ \chi^{(\pm n)} &\equiv \frac{1}{\sqrt{2}}\left(\xi^{(n)} \pm \eta^{(n)}\right), \end{aligned} \tag{6.56}$$

we can write

$$\begin{aligned} -\mathcal{L}_{\text{mass}}^{\text{KK}} =& \frac{1}{2} m_\nu \chi^{(0)} \chi^{(0)} \\ &+ \frac{1}{2}\sum_{n=1}^{\infty} \left(\chi^{(+n)}, \chi^{(-n)}\right) \begin{pmatrix} m_\nu - n/R & 0 \\ 0 & m_\nu + n/R \end{pmatrix} \begin{pmatrix} \chi^{(+n)} \\ \chi^{(-n)} \end{pmatrix} + \text{H.c.} \end{aligned} \tag{6.57}$$

Rearranging the spinors into a vector as follows

$$\psi = \left(\chi^{(0)}, \chi^{(+1)}, \chi^{(-1)}, \chi^{(+2)}, \chi^{(-2)}, \dots\right)^T, \tag{6.58}$$

we may rewrite the mass part of the Lagrangian as

$$\mathcal{L}_{\text{mass}}^{\text{KK}} = -\frac{1}{2}\psi^T \mathcal{M}_\nu^{\text{KK}} \psi + \text{H.c.} \tag{6.59}$$

The resulting neutrino mass matrix then is

$$\mathcal{M}_\nu^{\text{KK}} = \begin{pmatrix} m_\nu & 0 & 0 & 0 & 0 & \cdots \\ 0 & m_\nu - \frac{1}{R} & 0 & 0 & 0 & \cdots \\ 0 & 0 & m_\nu + \frac{1}{R} & 0 & 0 & \cdots \\ 0 & 0 & 0 & m_\nu - \frac{2}{R} & 0 & \cdots \\ 0 & 0 & 0 & 0 & m_\nu + \frac{2}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{6.60}$$

6.2.3 Adding a Higgs Triplet

So far in our toy model, it is unexplained where the Majorana mass term m_ν comes from. We may add, as is usually done, a Higgs triplet to generate this term. As we work in the model of universal extra dimensions, the Higgs triplet will propagate in the extra dimension, too. The Lagrangian of the Higgs sector is the same as the one in the Higgs triplet model, which was given in equation (5.15). The only difference is that all the fields experience the extra spatial dimension, which means that they depend on the additional coordinate y.

Vertex	vertex factor
$\overline{(e_L)^c} e_L \delta^{++}$	iY
$\delta^{++}\delta^{--}$	$-iM_\Delta^2$
eeA_μ^a	$ig\gamma_\mu t^a$
eeA_5	$-igt^a$
$W_\mu ee$	$-\frac{1}{\sqrt{2}} ig\gamma_\mu$
$W_\mu W_\nu \phi\phi$	$2ig^2 g^{\mu\nu}$
$\phi^+\delta^{--}\phi^+$	$i\alpha$
$W_\mu W_\nu \delta^{--}$	$-\sqrt{2} i v_\Delta g^2 g^{\mu\nu}$

Table 6.1: Vertices for the Higgs triplet model in universal extra dimensions.

Writing out the fields in components, it is easy to give the Feynman rules. The vertices we are interested in are given in table 6.1.

For illustration, let us calculate one vertex in the language of KK modes

$$\mathcal{L} \supseteq \int dy \overline{e_L}(x,y)\delta^{--}(x,y)(e_L)^c(x,y). \tag{6.61}$$

We can expand the electrons into a sum over KK modes

$$e_L(x,y) = \frac{1}{\sqrt{2\pi R}} e_L^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left(e_L^{(n)}(x) \cos\frac{ny}{R} + e_R^{(n)}(x) \sin\frac{ny}{R} \right). \tag{6.62}$$

The Higgs triplet must be even to have a zero mode (which is necessary to create the Majorana neutrino mass). So we may expand

$$\delta^{--}(x,y) = \frac{1}{\sqrt{2\pi R}} \delta^{--}_{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \delta^{--}_{(n)}(x) \cos\frac{ny}{R}. \tag{6.63}$$

Plugging these expansions into equation (6.61) and using the integrals given in appendix B.1, we obtain

$$\begin{aligned}\mathcal{L} \supseteq & \frac{1}{\sqrt{2\pi R}} \overline{e_L^{(0)}}(x) \delta_{(0)}^{--}(x) (e_L^{(0)})^c(x) \\ & + \frac{1}{\sqrt{2\pi R}} \overline{e_L^{(0)}}(x) \sum_{n=1}^{\infty} \delta_{(n)}^{--}(x) (e_L^{(n)})^c(x) \\ & + \frac{1}{\sqrt{2\pi R}} \delta_{(0)}^{--}(x) \sum_{n=1}^{\infty} \left(\overline{e_L^{(n)}}(x) (e_L^{(n)})^c(x) + \overline{e_R^{(n)}}(x) (e_R^{(n)})^c(x) \right) \\ & + \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \overline{e_L^{(n)}}(x) \delta_{(n)}^{--}(x) (e_L^{(0)})^c(x) \\ & + \frac{1}{2\sqrt{\pi R}} \sum_{n,m,l=1}^{\infty} \overline{e_L^{(n)}}(x) \delta_{(m)}^{--}(x) (e_L^{(l)})^c(x) (\delta_{l,m+n} + \delta_{m,l+n} + \delta_{n,l+m}) \\ & + \frac{1}{2\sqrt{\pi R}} \sum_{n,m,l=1}^{\infty} \overline{e_R^{(n)}}(x) \delta_{(m)}^{--}(x) (e_R^{(l)})^c(x) (-\delta_{l,m+n} + \delta_{m,l+n} + \delta_{n,l+m}).\end{aligned} \tag{6.64}$$

We see that KK number conservation restricts the possible interactions.

6.3 Neutrinoless Double Beta Decay in Universal Extra Dimensions

6.3.1 KK Number Violating Couplings

Our aim is to discuss contributions to neutrinoless double beta decay from KK tower particles. As the involved external quarks and leptons are the SM particles (that is, the zero KK mode particles), diagrams that contain higher KK modes as virtual particles necessarily involve at least two KK number violating vertices.

First we need to note that, *at tree-level*, all KK number violating couplings vanish. As an example, let us discuss the $A_\mu^{(2n)}$-$\psi^{(0)}$-$\psi^{(0)}$ vertex: The relevant term in the Lagrangian is

$$\begin{aligned} & \propto \psi_L(x)\psi_L(x)A_\mu^{(2n)}(x,y) \\ & \propto \psi_L(x)\psi_L(x)A_\mu^{(2n)}(x)\cos\frac{2ny}{R}. \end{aligned} \tag{6.65}$$

Integrating out the fifth dimension via $\int_0^{2\pi R} dy$, it is clear that this term vanishes, as

$$\int_0^{2\pi R} dy \cos\frac{ky}{R} = 0, \text{ for } k \in \mathbb{N}. \tag{6.66}$$

The same is true for other vertices allowed by KK parity.

Such vertices, however, can be created at loop-level. The resulting couplings were discussed by Cheng, Matchev, and Schmaltz [73]. We state their results and calculate the additional couplings needed in appendix B.

6.3.2 Neutrinoless Double Beta Decay Mediated by Heavy KK Neutrinos

Figure 6.2 (a) shows the most general diagram for neutrinoless double beta decay mediated by heavy KK neutrinos. As only even KK modes can violate KK number, we may reduce the number of possible diagrams. A corrected diagram is shown in figure 6.2 (b). We already stated before that KK number violating couplings are loop-suppressed. Therefore, we want to consider only diagrams which involve two KK number violating vertices. So we can reduce the number of possible diagrams even further. The two diagrams to consider are shown in figure 6.2 (c) and (d).

The first realization possible of neutrinoless double beta decay would be the vertex $\overline{\psi^{(2n)}}$-$\psi^{(0)}$-$A_\mu^{(0)}$ (resulting in the vertex $\overline{\psi^{(2n)}}$-$\psi^{(0)}$-$W_\mu^{(0)}$), so that only the virtual neutrino would be a higher KK mode, the rest of the involved particles were the usual SM fields. The corresponding diagram is shown in figure 6.2 (c). This vertex, however, vanishes (see appendix B.4.1 for more details).

Taking the gauge fields A_μ^a or the W bosons, respectively, to be higher KK modes, too, there are now two vertices involving KK modes [cf. figure 6.2 (d)]. The vertex $\overline{\psi^{(2n)}}$-$\psi^{(0)}$-$A_\mu^{(2n)}$, where neutrino, electron and W boson interact is allowed by KK parity and therefore not suppressed. We do not have to discuss this vertex further. The KK number violating coupling now involves the quarks and the W boson. As the quarks are necessarily the SM particles (that is, zero KK modes), we have to examine the vertex $\overline{\psi^{(0)}}$-$\psi^{(0)}$-$A_\mu^{(2n)}$. This vertex does not vanish and thus we will lead to a realization of neutrinoless double beta decay. In appendix B.4.1 we obtain for the $\overline{\psi^{(0)}}$-$\psi^{(0)}$-$A_\mu^{a(2n)}$ interaction vertex:

$$(-i\gamma^\mu g t^a P_L)\sqrt{2}\frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[\frac{23}{3}C_2(G)-\frac{1}{3}\sum_{\text{real scalars}}(C(r)_{\text{even}}-C(r)_{\text{odd}})-9C_2(r)\right]. \tag{6.67}$$

The KK number violating vertex $\overline{\psi^{(0)}}$-$\psi^{(0)}$-$W_\mu^{-(2n)}$ between the quarks and the W-boson can directly be deduced from this expression. Using the generators of $SU(2)$ given in appendix A.9, we can see that

$$\begin{aligned} t^1 A_\mu^{1(2n)} + t^2 A_\mu^{2(2n)} &= \frac{1}{2}\begin{pmatrix} 0 & A_\mu^{1(2n)} - iA_\mu^{2(2n)} \\ A_\mu^{1(2n)} + iA_\mu^{2(2n)} & 0 \end{pmatrix} \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^{+(2n)} \\ W_\mu^{-(2n)} & 0 \end{pmatrix}, \end{aligned} \tag{6.68}$$

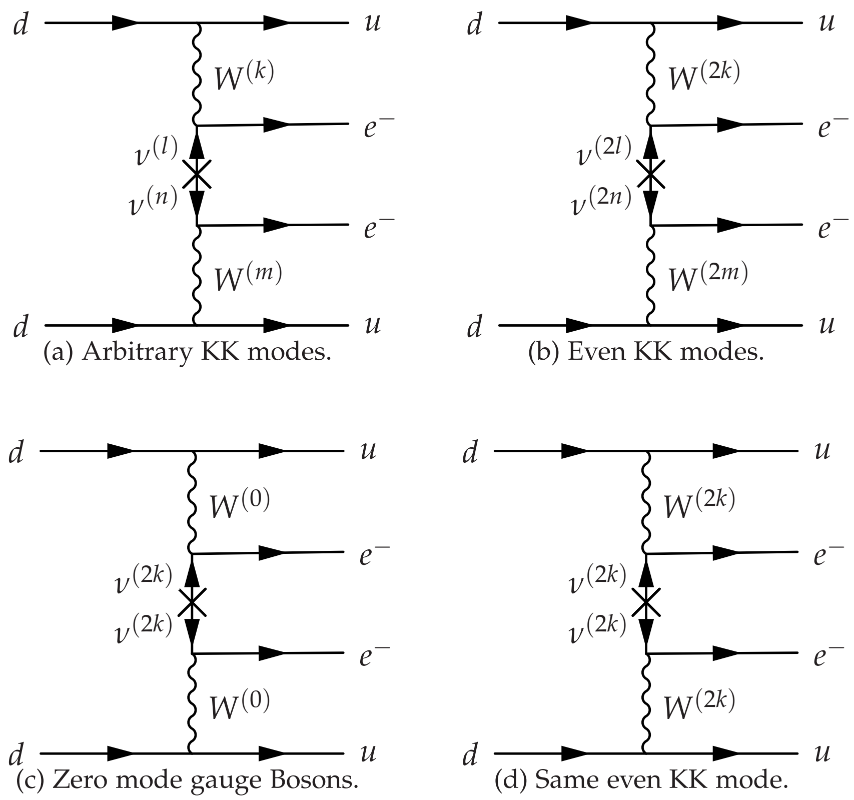


Figure 6.2: (a) Diagram for $0\nu\beta\beta$ decay mediated by the exchange of KK neutrino modes from universal extra dimensions. With the modes for the virtual particles being completely arbitrary, the diagram is either forbidden or strongly suppressed, as there are several KK number violating vertices involved. (b) Diagram for $0\nu\beta\beta$ decay mediated by the exchange of KK neutrino modes from universal extra dimensions with a reduced number of KK number violating vertices. By only including even KK numbers, diagrams that are forbidden by KK parity are already excluded. (c) Diagram for $0\nu\beta\beta$ decay mediated by the exchange of KK neutrino modes from universal extra dimensions with the minimal number of KK number violating vertices (two). Only the neutrinos are higher KK modes, the gauge bosons are zero modes. This diagram vanishes. (d) Diagram with two KK number violating vertices, which will be calculated in this section.

where

$$W_\mu^{\mp(2n)} = \frac{1}{\sqrt{2}}\left(A_\mu^{1(2n)} \pm i A_\mu^{2(2n)}\right). \tag{6.69}$$

So the desired vertex including the KK modes of the W-boson differs from the original vertex in equation (6.67) only by a factor of $1/\sqrt{2}$.

We have to calculate the square brackets in our vertex formula, determined by group theory. We are working with particles in representations of $SU(2)$, so we have $C_2(G) = 2$ (with G being the adjoint representation of $SU(2)$) and $C_2(r) = 3/4$ (with r being the fundamental representation of $SU(2)$). To calculate the term $1/3 \sum_{\text{real scalars}} (C(r)_{\text{even}} - C(r)_{\text{odd}})$ we need to determine all scalar fields of our theory. The only scalars we have in the extra dimensional version of the SM are the Higgs scalars and the A_5. The Higgs boson is a complex scalar, which is even under the Z_2, whereas the A_5 is odd, to have a consistent assignment. So for this factor we have

$$\frac{1}{3}\sum_{\text{real scalars}} (C(r)_{\text{even}} - C(r)_{\text{odd}}) = \frac{1}{3}(2-1)\frac{1}{2} = \frac{1}{6}. \tag{6.70}$$

All in all we find

$$\left[\frac{23}{3}C_2(G) - \frac{1}{3}\sum_{\text{real scalars}} (C(r)_{\text{even}} - C(r)_{\text{odd}}) - 9C_2(r)\right] = \frac{101}{12}. \tag{6.71}$$

As higher KK modes are heavier than the light neutrino (the zero KK mode), the resulting amplitude for neutrinoless double beta decay can be calculated using the approximate propagator for heavy neutrino exchange. For one KK neutrino contributing, the propagator of the neutrino is proportional to $1/m_k \approx R/2k$, with $2k$ being the number of the corresponding KK mode. So for the amplitude we get

$$\begin{aligned}\mathcal{A}^{(2k)}_{\text{UEDs}} &= 2\left(\frac{g}{\sqrt{2}}\right)^2 \left(\frac{g^3\sqrt{2}}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\frac{101}{12}\frac{1}{\sqrt{2}}\right)^2 \frac{1}{\left(M_W^2 + \frac{(2k)^2}{R^2}\right)^2}\frac{R}{2k} \\ &= \frac{g^{10}}{64^2\pi^4}\left(\ln\frac{\Lambda^2}{\mu^2}\right)^2\left(\frac{101}{12}\right)^2 \frac{R^5}{2k(M_W^4R^4 + 4k^2M_W^2R^2 + 16k^4)}.\end{aligned} \tag{6.72}$$

The factor 2 on the r.h.s. of the equation comes from the fact that in our toy model (see section 6.2.2) the neutrinos with mass matrix entries $m_\nu - 2k/R$ and $m_\nu + 2k/R$ will contribute equally to the amplitude. Their physical mass is $m_k \equiv 2k/R \approx |m_\nu - 2k/R| \approx |m_\nu + 2k/R|$, for m_ν being the small neutrino mass.

Let us calculate the value for $k = 1$, that is, the contribution of the second KK mode only. We obtain

$$\mathcal{A}^{(2)}_{\text{UEDs}} = \frac{g^{10}}{64^2\pi^4}\left(\ln\frac{\Lambda^2}{\mu^2}\right)^2\left(\frac{101}{12}\right)^2 \frac{R^5}{2(M_W^4R^4 + 4M_W^2R^2 + 16)}. \tag{6.73}$$

We may now plug this result into formula (3.17) for the decay rate $\Gamma^{0\nu}$. However, for this purpose we need to know the effective coupling constant ϵ_{NP}. Recalling that the amplitude for light neutrino exchange is

$$\mathcal{A}_\nu = \left(\frac{g}{\sqrt{2}}\right)^4 \frac{1}{M_W^4} \frac{m_{ee}}{\langle p^2 \rangle}, \tag{6.74}$$

which leads to the decay rate

$$\frac{\Gamma^{0\nu}}{\ln 2} = \frac{1}{T_{1/2}^{0\nu}} = |m_{ee}|^2 \left|\mathcal{M}^{0\nu}\right|^2 G^{0\nu}, \tag{6.75}$$

we can see that

$$\epsilon_{\mathrm{UEDs}}^{(2)} = \frac{\langle p^2 \rangle M_W^4}{(g/\sqrt{2})^4} \frac{g^{10}}{64^2 \pi^4} \left(\ln \frac{\Lambda^2}{\mu^2}\right)^2 \left(\frac{101}{12}\right)^2 \frac{R^5}{2(M_W^4 R^4 + 4 M_W^2 R^2 + 16)}. \tag{6.76}$$

Thus we arrive at

$$\begin{aligned} \frac{\Gamma_{(2)}^{0\nu}}{\ln 2} =& \frac{1}{T_{1/2}^{0\nu}} = \left|\mathcal{M}^{0\nu}\right|^2 G^{0\nu} \\ & \times \left[\frac{\langle p^2 \rangle M_W^4}{(g/\sqrt{2})^4} \frac{g^{10}}{64^2 \pi^4} \left(\ln \frac{\Lambda^2}{\mu^2}\right)^2 \left(\frac{101}{12}\right)^2 \frac{R^5}{2(M_W^4 R^4 + 4 M_W^2 R^2 + 16)}\right]^2. \end{aligned} \tag{6.77}$$

For simplicity, we will use the same NMEs as in light neutrino exchange. There should be no problem with that, as the nuclear physics involved in our problem is practically the same as in light neutrino exchange and NMEs are of order 1 anyway. The phase space factor $G^{0\nu}$ is determined by the final states and is therefore the same as the one used in light neutrino exchange. We can now derive an equation of degree 5 in R, the radius of the extra dimension,

$$A R^5 - \frac{1}{\sqrt{T_{1/2}^{0\nu}}} R^4 - \frac{8}{\sqrt{T_{1/2}^{0\nu} M_W^2}} R^2 - \frac{16}{\sqrt{T_{1/2}^{0\nu} M_W^4}} = 0. \tag{6.78}$$

The factor A is given by

$$A = \frac{2\langle p^2 \rangle}{g} \frac{g^{10}}{64^2 \pi^4} \left(\ln \frac{\Lambda^2}{\mu^2}\right)^2 \left(\frac{101}{12}\right)^2 \left|\mathcal{M}^{0\nu}\right| G^{0\nu\,1/2}. \tag{6.79}$$

Using $\langle p^2 \rangle \approx (100\,\mathrm{MeV})^2$, $g = 0.652$, $\Lambda = 15\,\mathrm{TeV}$,[3] $\mu = 100\,\mathrm{MeV}$,[4] a matrix element $\mathcal{M}^{0\nu} = 4$ [42], and the half-life from the Heidelberg-Moscow experiment given before,

[3] Usually, about 50 KK modes are taken to contribute. This limit is calculated via the QCD coupling g_3 in a $4d$ effective theory obtained from the $5d$ universal extra dimensions model (see [74] for a more detailed discussion). With an approximate size of 300 GeV each, we obtain $\Lambda = 15\,\mathrm{TeV}$.

[4] We consider a nuclear process, so the renormalization scale μ should be taken to be near the nuclear momentum scale.

we obtain as a limit

$$\frac{1}{R} \geq 155\,\text{GeV}. \tag{6.80}$$

We will discuss the relevance of this limit after we have considered the contribution of all KK modes in the next step. As more modes will contribute, the limit on the radius of the extra dimension gets stronger. Remember that we had for one KK mode contributing the amplitude given in equation (6.72). To get the total amplitude, we need to sum up all contributing KK modes. We obtain

$$\mathcal{A}_{\text{tot}} \propto \sum_{k=1}^{\infty} \frac{R^5}{2kM_W^4R^4 + 16k^3M_W^2R^2 + 32k^5}. \tag{6.81}$$

The sum on the r.h.s. may be summed up analytically, and we find

$$\sum_{k=1}^{\infty} \frac{R^5}{2kM_W^4R^4 + 16k^3M_W^2R^2 + 32k^5} = \frac{R}{16M_W^4}\left[8\gamma + 4\Psi\left(1 - \frac{iM_WR}{2}\right)\right. \tag{6.82}$$

$$\left. +4\Psi\left(1 + \frac{iM_WR}{2}\right) + iM_WR\Psi^{(1)}\left(1 - \frac{iM_WR}{2}\right) - iM_WR\Psi^{(1)}\left(1 + \frac{iM_WR}{2}\right)\right]. \tag{6.83}$$

In this expression, γ is Euler's constant, which has a numerical value

$$\gamma \approx 0.577216. \tag{6.84}$$

$\Psi(z)$ is the digamma function (see Abramowitz and Stegun [75] for a detailed discussion) with z as a complex variable, which is defined as

$$\Psi(z) = \frac{d}{dz}\ln\Gamma(z), \tag{6.85}$$

where $\Gamma(z)$ is the usual gamma function. For $|z| < 1$, $\Psi(z)$ may be expanded as

$$\Psi(1+z) = -\gamma + \sum_{n=2}^{\infty}(-1)^n\zeta(n)z^{n-1}. \tag{6.86}$$

Here, $\zeta(n)$ is the Riemann zeta function. $\Psi^{(1)}(z)$ is the so-called trigamma function, defined as

$$\Psi^{(1)}(z) = \frac{d}{dz}\Psi(z). \tag{6.87}$$

This function is a special case of the polygamma function, given by

$$\Psi^{(n)}(z) = \frac{d^n}{dz^n}\Psi(z). \tag{6.88}$$

For $|z| < 1$, $\Psi^{(n)}(z)$ may be expanded as

$$\Psi^{(n)}(1+z) = (-1)^{n+1}\left[n!\zeta(n+1) - \frac{(n+1)!}{1!}\zeta(n+2)z + \frac{(n+2)!}{2!}\zeta(n+3)z^2 - \dots\right]. \tag{6.89}$$

In our case it is

$$\left|\frac{iM_W R}{2}\right| < 1, \tag{6.90}$$

as follows directly from the well-motivated assumption

$$M_W < \frac{1}{R}. \tag{6.91}$$

Expanding the gamma functions to fourth order in $\frac{iM_W R}{2}$ and collecting terms, one obtains

$$\sum_{k=1}^{\infty} \frac{R^5}{2kM_W^4 R^4 + 16k^3 M_W^2 R^2 + 32k^5} \approx \frac{\zeta(5)}{32} R^5. \tag{6.92}$$

We could have found this result more easily, as for $1/R > M_W$ the most important contribution in the denominator comes from the term $32k^5$. We thus have

$$\sum_{k=1}^{\infty} \frac{R^5}{2kM_W^4 R^4 + 16k^3 M_W^2 R^2 + 32k^5} \approx R^5 \sum_{k=1}^{\infty} \frac{1}{32k^5}. \tag{6.93}$$

As $\sum_{k=1}^{\infty} \frac{1}{k^5}$ converges, we do not have to think about a cutoff (usually done after 50 KK modes), so that we finally obtain

$$\sum_{k=1}^{\infty} \frac{R^5}{2kM_W^4 R^4 + 16k^3 M_W^2 R^2 + 32k^5} \approx \frac{R^5}{32}\zeta(5), \tag{6.94}$$

which is the same as in the more detailed analysis above. The numerical value of the Riemann zeta function is

$$\zeta(5) = 1.03693. \tag{6.95}$$

We can now plug this value into the formula for the decay rate and get

$$\frac{\Gamma_{0\nu}}{\ln 2} = \frac{1}{T_{1/2}^{0\nu}} = \left[\frac{\langle p^2\rangle M_W^4}{(g/\sqrt{2})^4}\right]^2 \mathcal{A}_{\mathrm{tot}}^2 \left|\mathcal{M}^{0\nu}\right|^2 G^{0\nu}. \tag{6.96}$$

Calculating the radius R, we obtain

$$\frac{1}{R} = \left[\left(2AM_W^4\right)^2 \left(\frac{\zeta(5)}{32}\right)^2 T_{1/2}^{0\nu}\right]^{\frac{1}{10}}. \tag{6.97}$$

With the values used before, we finally conclude

$$\frac{1}{R} \geq 160\,\text{GeV}. \tag{6.98}$$

This bound is just slightly stronger than the limit obtained from the contribution of the second KK mode. This result should be intuitively clear, as the higher KK modes are strongly suppressed and the biggest contribution indeed comes from the lowest modes contributing.

It is now interesting to see what happens if we additionally take into account the decay mediated by the light neutrinos. The amplitude must be added to the amplitude of the decay mediated by heavy KK modes calculated before. We obtain

$$\mathcal{A}_{\text{tot}} = \frac{g^{10}}{64^2\pi^4}\left(\ln\frac{\Lambda^2}{\mu^2}\right)^2\left(\frac{101}{12}\right)^2\frac{\zeta(5)}{32}R^5 + \left(\frac{g}{\sqrt{2}}\right)^4\frac{1}{M_W^4}\frac{m_{ee}}{\langle p^2\rangle}. \tag{6.99}$$

Plugging this expression into the formula for the decay rate and solving for $1/R$, we arrive at

$$\frac{1}{R} = \left[(2AM_W^4)\frac{\zeta(5)}{32}\frac{(T_{1/2}^{0\nu})^{\frac{1}{2}}}{1 - |m_{ee}|\,|\mathcal{M}^{0\nu}|\,(G^{0\nu})^{\frac{1}{2}}(T_{1/2}^{0\nu})^{\frac{1}{2}}}\right]^{\frac{1}{5}}. \tag{6.100}$$

This function is plotted for different values of $|m_{ee}|$ in figure 6.3. It can easily be seen that depending on the mass assumed, the limits on the radius R get stronger.

There exist limits on the size of extra dimensions from experiments testing the inverse-square law of gravity. The most stringent bound comes from Kapner et al. [76], who conclude that an extra dimension must have a size $R \leq 44\,\mu$m. Using

$$(2\,\text{fm})^{-1} \approx 100\,\text{MeV}, \tag{6.101}$$

or alternatively

$$\frac{1}{\text{eV}} \approx 0.2\,\mu\text{m}, \tag{6.102}$$

we can see that the radius given above corresponds to

$$\frac{1}{R} \geq 4.55 \times 10^{-12}\,\text{GeV}. \tag{6.103}$$

This limit is much weaker than the limit we obtain from double beta decay. There are, however, more stringent bounds: At the moment, best constraints come from electroweak precision tests [77, 78] and give for a Higgs mass $m_H = 115\,\text{GeV}$ (and a top quark mass of $m_t = 173\,\text{GeV}$)

$$\frac{1}{R} \gtrsim 600\,\text{GeV}. \tag{6.104}$$

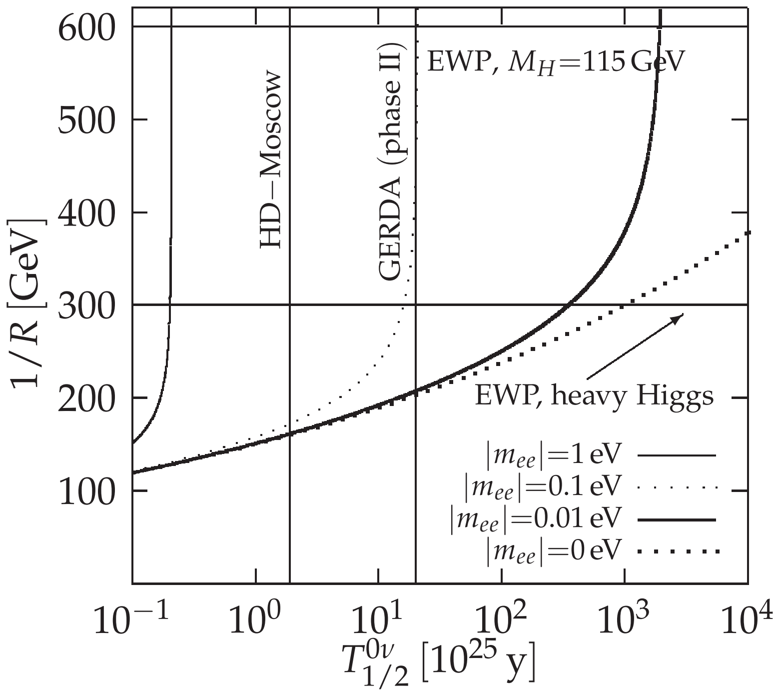


Figure 6.3: Lower limit on $1/R$ as a function of the half-life of $0\nu\beta\beta$ for different values of $|m_{ee}|$.

For a heavier Higgs, the constraints could be as low as

$$\frac{1}{R} \gtrsim 250 - 300\,\text{GeV}. \tag{6.105}$$

Both limits are marked in figure 6.3.

However, we would like to point out that the constraints we obtain from neutrinoless double beta decay are complementary to the ones from electroweak precision data, as they are obtained from a different class of experiments. Furthermore, with the next generation of experiments, limits on extra dimensions will improve considerably. With the GERDA experiment [13] it is planned to reach sensitivities of order $T^{0\nu}_{1/2} > 2 \times 10^{26}$ y (phase II). This value would directly translate into a limit on the size of one universal extra dimension,

$$\frac{1}{R} \geq 203\,\text{GeV}. \tag{6.106}$$

There are more $0\nu\beta\beta$ experiments on the way, which as a side effect will improve the bounds given. With better limits for the half-lives of $0\nu\beta\beta$, we expect the bounds for the size of the extra dimension to become competitive in the future. This fact can be seen from the plot in figure 6.3. The situation will improve, if we find evidence

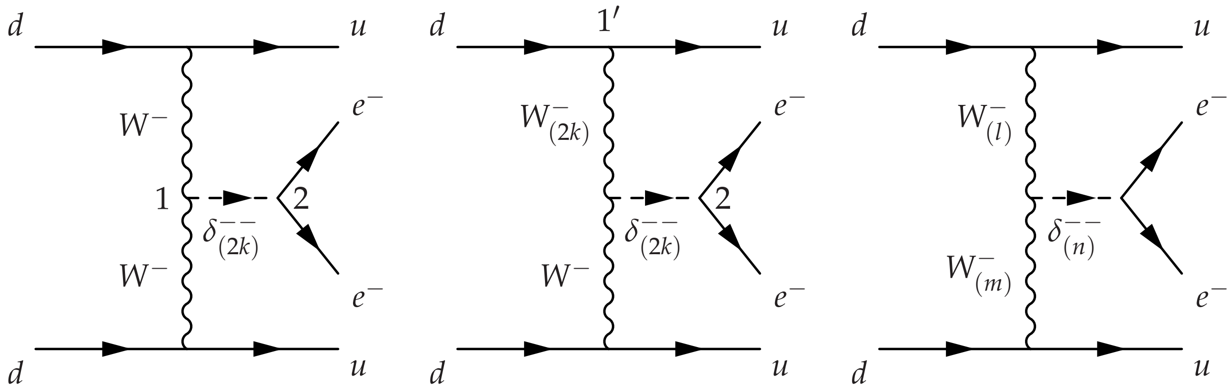


Figure 6.4: Diagrams for $0\nu\beta\beta$ decay in the Higgs triplet model in universal extra dimensions. The vertices 1, 1′, and 2 are KK number violating. We want to consider the least suppressed contributions. The diagrams on the left and in the center contain two KK number violating vertices. The diagram on the right contains three or more KK number violating vertices, depending on the choice of the numbers l, m, n. As KK number violating couplings are loop suppressed, we will consider the first two diagrams only.

for a Majorana neutrino mass different from zero. Figure 6.3 nicely illustrates that in this case bounds on the size of the extra dimension will be much stronger.

Of course, new bounds on the size of extra dimensions are to be expected from LHC in the next years, too.

6.3.3 Neutrinoless Double Beta Decay Mediated by Higgs Triplets

Putting a Higgs triplet into the extra dimension gives a tower of KK Higgs triplets. These may mediate neutrinoless double beta decay, in analogy to the Higgs triplet model. We want to concentrate on the least suppressed realizations, which only involve two KK number violating vertices. Figure 6.4 gives more details. We can see that the vertices 1, 1′, and 2 are KK number violating.

We can now perform the same analysis as we did in the last subsection for the heavy KK neutrinos. For the amplitude of the left diagram in figure 6.4 (where only the triplet is of higher KK modes) we find

$$\mathcal{A}_1 = \left(\frac{g}{\sqrt{2}M_W^2}\right)^2 V_1 V_2 \sum_{k=1}^{\infty} \frac{R^2}{R^2 M_\Delta^2 + 4k^2}. \tag{6.107}$$

Here, the vertex factors were calculated in the appendices B.4.2 and B.4.3 to be

$$V_1 = \frac{g^2\alpha}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}[4+6C(r)], \tag{6.108}$$

$$V_2 = \frac{g^2\Upsilon}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}6C(r). \tag{6.109}$$

For the fundamental representation, we have $C(r) = \frac{1}{2}$. The sum on the r.h.s. of equation (6.107) may be summed up analytically, and we obtain

$$\sum_{k=1}^{\infty}\frac{R^2}{R^2M_\Delta^2+4k^2} = \frac{-2+M_\Delta\pi R\coth\left(\frac{M_\Delta\pi R}{2}\right)}{4M_\Delta^2}. \tag{6.110}$$

For any sensible value of M_Δ, it is

$$\frac{M_\Delta\pi R}{2} \geq 2. \tag{6.111}$$

For this argument, however, we have

$$\coth\left(\frac{M_\Delta\pi R}{2}\right) \approx 1. \tag{6.112}$$

Thus we find

$$\mathcal{A}_1 \approx \left(\frac{g}{\sqrt{2}M_W^2}\right)^2 V_1V_2\frac{-2+M_\Delta\pi R}{4M_\Delta^2}. \tag{6.113}$$

For the diagram in the center of figure 6.4 (where one of the gauge bosons is a higher KK mode, too), we can write down the amplitude

$$\begin{aligned}\mathcal{A}_2 &= \left(\frac{g}{\sqrt{2}}\right)\sqrt{2}v_\Delta g^2V_1'V_2\frac{1}{M_W^2}\sum_{k=1}^{\infty}\frac{1}{M_W^2+\frac{(2k)^2}{R^2}}\frac{1}{M_\Delta^2+\frac{(2k)^2}{R^2}}\\ &= \frac{g^3v_\Delta}{M_W^2}V_1'V_2R^4\sum_{k=1}^{\infty}\frac{1}{R^4M_W^2M_\Delta^2+4k^2R^2(M_W^2+M_\Delta^2)+16k^4}.\end{aligned} \tag{6.114}$$

The vertex factor V_1' was already used in the case of the decay mediated by heavy KK neutrinos and is given by

$$V_1' = \frac{g^3}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[\frac{23}{3}C_2(G)-\frac{1}{3}\sum_{\text{real scalars}}(C(r)_{\text{even}}-C(r)_{\text{odd}})-9C_2(r)\right]. \tag{6.115}$$

We have to recalculate the group theory factor in square brackets, as we have a different scalar sector now. The complex Higgs triplet is in the adjoint representation of $SU(2)$, so we have $C(r) = 2$. All in all we find

$$\left[\frac{23}{3}C_2(G)-\frac{1}{3}\sum_{\text{real scalars}}(C(r)_{\text{even}}-C(r)_{\text{odd}})-9C_2(r)\right] = \frac{85}{12}. \tag{6.116}$$

As before, we can approximate the sum on the r.h.s. of equation (6.114), as the most important term in the denominator is the one proportional to k^4. Thus we find

$$\sum_{k=1}^{\infty} \frac{1}{R^4 M_W^2 M_\Delta^2 + 4k^2 R^2 (M_W^2 + M_\Delta^2) + 16k^4} \approx \sum_{k=1}^{\infty} \frac{1}{16k^4} = \frac{\zeta(4)}{16} = \frac{\pi^4}{1440}. \tag{6.117}$$

Summing up the amplitudes and including the amplitude

$$\mathcal{A}_0 = \left(\frac{g}{\sqrt{2} M_W^2} \right)^2 \frac{\sqrt{2} g^2 Y v_\Delta}{M_\Delta^2} \tag{6.118}$$

for the diagram containing only zero modes (the usual particles), we find for the total amplitude

$$\begin{aligned} \mathcal{A}_{\text{tot}} &= \mathcal{A}_0 + \mathcal{A}_1 + \mathcal{A}_2 \\ &= \left(-\frac{g^2 V_1 V_2}{4 M_W^4 M_\Delta^2} + \frac{g^4 Y v_\Delta}{\sqrt{2} M_W^4 M_\Delta^2} \right) + \frac{g^2 V_1 V_2 \pi}{8 M_W^4 M_\Delta} R + \frac{g^3 v_\Delta V_1' V_2 \pi^4}{1440 M_W^2} R^4. \end{aligned} \tag{6.119}$$

Plugging this expression into the formula for the half-life

$$\frac{1}{T_{1/2}^{0\nu}} = \left(\frac{\langle p^2 \rangle M_W^4}{(g/\sqrt{2})^4} \right)^2 |\mathcal{A}_{\text{tot}}|^2 \left| \mathcal{M}^{0\nu} \right|^2 G^{0\nu}, \tag{6.120}$$

we arrive at an equation of degree 4 in R:

$$\begin{aligned} &\left(-\frac{g^2 V_1 V_2}{4 M_W^4 M_\Delta^2} + \frac{g^4 Y v_\Delta}{\sqrt{2} M_W^4 M_\Delta^2} \right) + \frac{g^2 V_1 V_2 \pi}{8 M_W^4 M_\Delta} R + \frac{g^3 v_\Delta V_1' V_2 \pi^4}{1440 M_W^2} R^4 \\ &\qquad = \frac{g^4}{4 \langle p^2 \rangle M_W^4 (T_{1/2}^{0\nu})^{1/2} \, |\mathcal{M}^{0\nu}| \, (G^{0\nu})^{1/2}}. \end{aligned} \tag{6.121}$$

This equation may be solved for $1/R$. However, the parameters of the Higgs triplet model are less constrained than for other models. Let us choose the parameters such that $\sqrt{2} Y v_\Delta = 0.1\,\text{eV}$, which is of the order of the light neutrino mass. For the Higgs triplet mass we chose a value $M_\Delta = 1\,\text{TeV}$, and a numerical analysis shows that the solution does not depend too much on the triplet mass, as long as it is of the order $100\,\text{GeV}$ to TeV. Plugging in the values given before [see equation (6.80)], we obtain using the limit on the half-life of the Heidelberg-Moscow experiment (1.9×10^{25} y):

$$\frac{1}{R} \geq 0.4\,\text{GeV}. \tag{6.122}$$

We can see that the constraints we obtain here are weaker than the ones from KK neutrino mediation calculated before.

Chapter 7

Summary and Outlook

We have investigated various topics related to neutrinoless double beta decay, a field which remained a little bit static during the past years. And indeed, there are some news to report.

We saw that the Schechter-Valle (or Black Box) theorem is not as strong as it may seem. It is of merely academic interest, and one has to be a little bit careful when using its assertion to explain the Majorana nature of neutrinos. If a mass is generated via the the well-known diagram in figure 4.1, it is far too small to account for the neutrino masses we expect to have. Moreover, we found an operator mediating $0\nu\beta\beta$, but giving zero contribution to the neutrino mass via this diagram. Of course, other diagrams (which, however, will be more strongly suppressed) may give some non-zero contribution for the same operator. But this mass will be many orders of magnitude smaller than the expected one, too. Note that our finding does not invalidate the Schechter-Valle theorem, as the proof presented in section 4.1 excludes that a Majorana mass term is forbidden by symmetry and still neutrinoless double beta decay takes place. We, however, found that such a mass term from the well-known diagram is not forbidden but may vanish for some operators giving neutrinoless double beta decay. Therefore, we can say that we indeed need to know more about the underlying mechanism triggering neutrinoless double beta decay before we may safely interpret the results from up-coming experiments and extract a neutrino mass.

Concerning our second topic, neutrinoless double beta decay in universal extra dimensions, we have found limits on the size of one extra dimension from the experimental bounds on the half-life of neutrinoless double beta decay. It is well known that bounds on universal extra dimensions are generally low due to the approximate KK number conservation. However, the limits we obtained from neutrinoless double beta decay were slightly weaker than these obtained from electroweak precision data. Nevertheless, we would like to point out that constraints from $0\nu\beta\beta$ are complementary to these ones, as they come from a different class of experiments. Moreover, next generation experiments are expected to improve the bounds we found.

So all in all, we think that neutrinoless double beta decay is a research area with interesting perspectives, from the theoretical side as well as from the experimental point of view. Hopefully, during the next years there will be more news to report.

Appendix A

Notations and Conventions

The notations and conventions mainly follow these of the textbook written by Peskin and Schroeder [34]. To be clear, however, we will list the used ones in this appendix.

A.1 Tensors

The metric tensor is given by

$$(g_{\mu\nu}) = (g^{\mu\nu}) = \mathrm{diag}(+1, -1, -1, -1). \tag{A.1}$$

This convention is the same as used by Bjorken and Drell [79, 80], and Peskin and Schroeder [34], but differs frome the one used by Weinberg [81]. Greek indices run over 0, 1, 2, 3, corresponding to t, x, y, z. Roman indices usually run only over the three spatial components. Einstein summation convention is assumed, meaning that repeated indices are summed over in all cases.
The totally antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$ is defined by

$$\epsilon^{0123} = +1. \tag{A.2}$$

A.2 Units

Throughout the whole thesis, we work in natural units, setting

$$\hbar \equiv c \equiv 1. \tag{A.3}$$

Therefore, dimensions are as follows:

$$[\mathrm{length}] = [\mathrm{time}] = [\mathrm{energy}]^{-1} = [\mathrm{mass}]^{-1}. \tag{A.4}$$

A.3 Delta Distribution

The four-dimensional delta distribution may be represented in the following form:

$$\int d^4x\, e^{ik\cdot x} = (2\pi)^4 \delta^{(4)}(k). \tag{A.5}$$

A.4 Pauli Matrices

The Pauli sigma matrices are traceless and Hermitian. They are given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \;\text{ and }\; \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.6}$$

Products of the Pauli matrices satisfy the identity

$$\sigma_i \sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k. \tag{A.7}$$

A.5 Gamma Matrices

The four gamma matrices satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{A.8}$$

and

$$\gamma_\mu^\dagger = \gamma_0\gamma_\mu\gamma_0. \tag{A.9}$$

We may introduce an additional gamma matrix

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma. \tag{A.10}$$

The following properties can be verified easily:

$$\left(\gamma^5\right)^\dagger = \gamma^5, \tag{A.11}$$

$$\left(\gamma^5\right)^2 = \mathbb{1}, \tag{A.12}$$

$$\left\{\gamma^5, \gamma^\mu\right\} = 0. \tag{A.13}$$

Contractions of gamma matrices in d dimensions are given by

$$\begin{aligned} \gamma^\mu\gamma_\mu &= d, \\ \gamma^\mu\gamma^\nu\gamma_\mu &= -(d-2)\gamma^\nu, \\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4g^{\nu\rho} - (4-d)\gamma^\nu\gamma^\rho, \\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu + (4-d)\gamma^\nu\gamma^\rho\gamma^\sigma. \end{aligned} \tag{A.14}$$

In four dimensions, these reduce to the well-known formulae

$$\begin{aligned} \gamma^\mu\gamma_\mu &= 4, \\ \gamma^\mu\gamma^\nu\gamma_\mu &= -2\gamma^\nu, \\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4g^{\nu\rho}, \\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\sigma\gamma^\rho\gamma^\nu. \end{aligned} \tag{A.15}$$

A.6 Trace Formulae

It is straight forward but tedious to show the following trace formulae:

$$\mathrm{Tr}(\mathbb{1}) = 4, \tag{A.16}$$

$$\mathrm{Tr}(\text{any odd number of } \gamma^\mu\text{'s}) = 0, \tag{A.17}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}, \tag{A.18}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right), \tag{A.19}$$

$$\mathrm{Tr}(\gamma^5) = 0, \tag{A.20}$$

$$\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu) = 0, \tag{A.21}$$

$$\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = -4i\epsilon^{\mu\nu\rho\sigma}. \tag{A.22}$$

A.7 Feynman Parameterization

In loop calculations in quantum field theory, we often have to calculate integrals over fractions with a product of polynomials A_i in the denominator. These may be simplified by a trick invented by Feynman:

$$\frac{1}{A_1A_2\ldots A_n} = (n-1)!\int_0^1 dx_1\int_0^1 dx_2\ldots\int_0^1 dx_n \frac{\delta(1-x_1-\ldots-x_n)}{[A_1x_1+A_2x_2+\ldots+A_nx_n]^n}. \tag{A.23}$$

Using the delta distribution, we may directly perform one of the integrations. However, we have to be careful, as not only the variables inside the integral have to be replaced, but also integration limits change. In the case of three Feynman parameters, which is used often in this thesis, we obtain

$$\begin{aligned} &\int_0^1 dx_1\int_0^1 dx_2\int_0^1 dx_3 \frac{\delta(1-x_1-x_2-x_3)}{[A_1x_1+A_2x_2+A_3x_3]^3} \\ &= \int_0^1 dx_1\int_0^{1-x_1} dx_2 \frac{1}{[A_1x_1+A_2x_2+A_3(1-x_1-x_2)]^3}. \end{aligned} \tag{A.24}$$

A.8 Dimensional Regularization

In dimensional regularization (as opposed to regularization via cutoff), we compute the Feynman diagram of a physical process as an analytic function of the dimensionality d of spacetime. Every observable quantity should have a well-defined limit for $d \to 4$.
In this thesis, we often need the following general integrals

$$\begin{aligned}\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2-\Delta)^n} &= \frac{(-1)^n i}{(4\pi)^{d/2}} \frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}}, \\ \int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2-\Delta)^n} &= \frac{(-1)^{n-1} i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1}.\end{aligned} \tag{A.25}$$

We may expand the combination

$$\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}} \left(\frac{1}{\Delta}\right)^{2-d/2} = \frac{1}{(4\pi)^2} \left(\frac{2}{\epsilon} - \gamma + \log(4\pi) - \log\Delta\right). \tag{A.26}$$

Here, γ is Euler's constant and $\epsilon = 4-d$. We can now renormalize this expression by the so-called modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme and obtain

$$\frac{1}{(4\pi)^2} \left(\frac{2}{\epsilon} - \gamma + \log(4\pi) - \log(\Delta)\right) \to \frac{1}{(4\pi)^2} \left(-\log\frac{\Delta}{M^2}\right), \tag{A.27}$$

where M is an arbitrary mass parameter (the so-called renormalization scale).

A.9 Group Theory

Let the matrices t^a form a representation of a Lie algebra G. That means, they obey the relation

$$[t^a, t^b] = i f^{abc} t^c, \tag{A.28}$$

where f^{abc} are called the structure constants. They are totally antisymmetric, and form a representation of the Lie algebra, called the adjoint representation, which is denoted by the symbol G, too. There exist two invariants of a representation r, which are defined as

$$\mathrm{tr}[t^a t^b] = C(r)\delta^{ab}, \qquad t^a_{ik} t^a_{kj} = C_2(r)\delta_{ij}. \tag{A.29}$$

Here, $C_2(r)$ is called quadratic Casimir operator and $C(r)$ is called Dynkin index. The following relations hold:

$$\begin{aligned} t^a t^b t^a &= [C_2(r) - \frac{1}{2} C_2(G)] t^b, \\ f^{acd} f^{bcd} &= C_2(G)\delta^{ab}, \\ f^{abc} t^b t^c &= \frac{i}{2} C_2(G) t^a. \end{aligned} \tag{A.30}$$

In $SU(N)$ groups, the fundamental representation is denoted by N, and the invariants are given by

$$\begin{aligned} C(N) &= \frac{1}{2}, \\ C_2(N) &= \frac{N^2-1}{2N}, \\ C(G) &= C_2(G) = N. \end{aligned} \tag{A.31}$$

In this thesis, we often consider the group $SU(2)$, which has two widely used irreducible representations. The spin-$\frac{1}{2}$ representation is given by

$$\begin{gathered} t_1^{1/2} = \frac{\sigma_1}{2} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad t_2^{1/2} = \frac{\sigma_2}{2} = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\ \text{and } t_3^{1/2} = \frac{\sigma_3}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{gathered} \tag{A.32}$$

The spin-1 representation is given by

$$t_1^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t_2^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \text{ and } t_3^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{A.33}$$

Appendix B

Extra Dimensions

B.1 Integrals

The following integrals repeatedly arise in calculations in extra dimensions:

$$\int_0^{2\pi R} dy \cos\frac{ny}{R} = 0, \tag{B.1}$$

$$\int_0^{2\pi R} dy \sin\frac{ny}{R} = 0, \tag{B.2}$$

$$\int_0^{2\pi R} dy \cos\frac{ny}{R}\cos\frac{my}{R} = \pi R\delta_{n,m}, \tag{B.3}$$

$$\int_0^{2\pi R} dy \sin\frac{ny}{R}\sin\frac{my}{R} = \pi R\delta_{n,m}, \tag{B.4}$$

$$\int_0^{2\pi R} dy \sin\frac{ny}{R}\cos\frac{my}{R} = 0, \tag{B.5}$$

$$\int_0^{2\pi R} dy \cos\frac{ny}{R}\cos\frac{my}{R}\cos\frac{ly}{R} = \frac{\pi R}{2}\left(\delta_{l,m+n}+\delta_{m,l+n}+\delta_{n,l+m}\right), \tag{B.6}$$

$$\int_0^{2\pi R} dy \sin\frac{ny}{R}\sin\frac{my}{R}\sin\frac{ly}{R} = 0, \tag{B.7}$$

$$\int_0^{2\pi R} dy \sin\frac{ny}{R}\sin\frac{my}{R}\cos\frac{ly}{R} = \frac{\pi R}{2}\left(-\delta_{l,m+n}+\delta_{m,l+n}+\delta_{n,l+m}\right), \tag{B.8}$$

$$\int_0^{2\pi R} dy \sin\frac{ny}{R}\cos\frac{my}{R}\cos\frac{ly}{R} = 0. \tag{B.9}$$

B.2 Lagrangian of the Toy Model

In this appendix, we will give the calculations for the Lagrangian of the toy model from section 6.2.2. Let us rewrite the Lagrangian as

$$\mathcal{L}_\nu = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}}, \tag{B.10}$$

where we define (with A running over $0, 1, 2, 3, 5$)

$$\mathcal{L}_{\text{kin}} = \int_0^{2\pi R} dy\, \overline{\nu} i \Gamma^A \partial_A \nu \quad \text{and} \quad \mathcal{L}_{\text{mass}} = -\int_0^{2\pi R} dy\, \frac{1}{2} m_\nu \left(\nu^T C_5^{-1} \nu + \text{H.c.} \right). \tag{B.11}$$

We want to calculate $\mathcal{L}_{\text{kin}}$ first. We can write

$$\begin{aligned}
\mathcal{L}_{\text{kin}} &= \int_0^{2\pi R} dy\, \overline{\nu} \left(i\gamma^\mu \partial_\mu + \gamma_5 \partial_y \right) \nu \\
&= \int_0^{2\pi R} dy \left(\xi^\dagger, \overline{\eta}^\dagger \right) \gamma_0 \begin{pmatrix} -\partial_y \mathbb{1} & i\sigma^\mu \partial_\mu \\ i\overline{\sigma}^\mu \partial_\mu & \partial_y \mathbb{1} \end{pmatrix} \begin{pmatrix} \xi \\ \overline{\eta} \end{pmatrix} \\
&= \int_0^{2\pi R} dy \left(\overline{\eta}^\dagger, \xi^\dagger \right) \begin{pmatrix} i\sigma^\mu \partial_\mu \overline{\eta} - \partial_y \xi \\ i\overline{\sigma}^\mu \partial_\mu \xi + \partial_y \overline{\eta} \end{pmatrix} \\
&= \int_0^{2\pi R} dy \left(\overline{\eta}^\dagger i\sigma^\mu \partial_\mu \overline{\eta} + \xi^\dagger i\overline{\sigma}^\mu \partial_\mu \xi - \overline{\eta}^\dagger \partial_y \xi + \xi^\dagger \partial_y \overline{\eta} \right).
\end{aligned} \tag{B.12}$$

The second term in the last line already has the desired form. The first term can be rewritten in the same form

$$\begin{aligned}
\overline{\eta}^\dagger i\sigma^\mu \partial_\mu \overline{\eta} &= \eta^T (-i\sigma_2) i\sigma^\mu i\sigma_2 \partial_\mu \eta^* = \eta^\alpha \left[\sigma_2 i\sigma^\mu \sigma_2 \right]_{\alpha\beta} \left(\partial_\mu \eta^* \right)^\beta \\
&\overset{(*)}{=} -\left(\partial_\mu \eta^* \right)^\beta \left[\left(\sigma_2 i\sigma^\mu \sigma_2 \right)^T \right]_{\beta\alpha} \eta^\alpha = -\left(\partial_\mu \eta^* \right)^\beta \left[\sigma_2 i\sigma^{\mu T} \sigma_2 \right]_{\beta\alpha} \eta^\alpha \\
&= -\partial_\mu \left\{ (\eta^*)^\beta \left[\sigma_2 i\sigma^{\mu T} \sigma_2 \right]_{\beta\alpha} \eta^\alpha \right\} + (\eta^*)^\beta \left[\sigma_2 i\sigma^{\mu T} \sigma_2 \right]_{\beta\alpha} \partial_\mu \eta^\alpha.
\end{aligned} \tag{B.13}$$

The minus in step $(*)$ arises because the two fermions η^α and $\left(\partial_\mu \eta^*\right)^\beta$ are exchanged. The total derivative which arises in the last step can be omitted, as the Lagrangian will be integrated over all space. To rewrite the matrix $\left[\sigma_2 i\sigma^{\mu T} \sigma_2\right]$, first note that

$$(\sigma_0)^T = \sigma_0, \quad (\sigma_1)^T = \sigma_1, \quad (\sigma_2)^T = -\sigma_2, \quad \text{and} \quad (\sigma_3)^T = \sigma_3. \tag{B.14}$$

Furthermore,

$$[\sigma_i, \sigma_0] = 0 \quad \text{and} \quad \left\{ \sigma_i, \sigma_j \right\} = 2\delta_{ij}. \tag{B.15}$$

Thus we obtain (using $\sigma_2\sigma_2 = \mathbb{1}$)

$$\sigma_2 i\sigma^{\mu T}\sigma_2 = \left\{\begin{array}{lll} \text{for} \quad \mu = 0: & \sigma_2 i\sigma_0\sigma_2 & = i\sigma_0 \\ \text{for} \quad \mu = 1: & \sigma_2 i\sigma_1\sigma_2 & = -i\sigma_1 \\ \text{for} \quad \mu = 2: & \sigma_2(-i\sigma_2)\sigma_2 & = -i\sigma_2 \\ \text{for} \quad \mu = 3: & \sigma_2 i\sigma_3\sigma_2 & = -i\sigma_3 \end{array}\right\} = i\overline{\sigma}^\mu, \tag{B.16}$$

Finally we have

$$(\eta^*)^\beta \left[\sigma_2 i\sigma^{\mu T}\sigma_2\right]_{\beta\alpha} \partial_\mu\eta^\alpha = (\eta^*)^\beta i(\overline{\sigma}^\mu)_{\beta\alpha}\partial_\mu\eta^\alpha = \eta^\dagger i\overline{\sigma}^\mu\partial_\mu\eta. \tag{B.17}$$

So the kinetic part of the Lagrangian now has the form

$$\mathcal{L}_{\text{kin}} = \int_0^{2\pi R} dy \left(\eta^\dagger i\overline{\sigma}^\mu\partial_\mu\eta + \xi^\dagger i\overline{\sigma}^\mu\partial_\mu\xi - \overline{\eta}^\dagger\partial_y\xi + \xi^\dagger\partial_y\overline{\eta}\right). \tag{B.18}$$

We can plug in the KK expansions for the even and odd fields given in equation (6.53) and calculating term by term, we find for the first term

$$\begin{aligned}
\int_0^{2\pi R} dy\, \eta^\dagger i\overline{\sigma}^\mu\partial_\mu\eta &= \int_0^{2\pi R} dy\, \frac{1}{\pi R}\sum_{n,m=1}^{\infty} \eta^{(n)^\dagger} \sin\frac{ny}{R} i\overline{\sigma}^\mu\partial_\mu\eta^{(m)} \sin\frac{my}{R} \\
&= \int_0^{2\pi R} dy\, \frac{1}{\pi R}\sum_{n,m=1}^{\infty} \eta^{(n)^\dagger} i\overline{\sigma}^\mu\partial_\mu\eta^{(m)} \frac{e^{i\frac{ny}{R}} - e^{-i\frac{ny}{R}}}{2i}\frac{e^{i\frac{my}{R}} - e^{-i\frac{my}{R}}}{2i} \\
&= -\frac{1}{4\pi R}\sum_{n,m=1}^{\infty} \eta^{(n)^\dagger} i\overline{\sigma}^\mu\partial_\mu\eta^{(m)} \int_0^{2\pi R} dy\, \Big(e^{i(n+m)y/R} - e^{i(n-m)y/R} \\
&\qquad\qquad -e^{i(m-n)y/R} + e^{-i(n+m)y/R}\Big) \\
&= -\frac{1}{2}\sum_{n,m=1}^{\infty} \eta^{(n)^\dagger} i\overline{\sigma}^\mu\partial_\mu\eta^{(m)} \left(\delta_{n,-m} - \delta_{n,m} - \delta_{n,m} + \delta_{m,-n}\right) \\
&= \sum_{n=1}^{\infty} \eta^{(n)^\dagger} i\overline{\sigma}^\mu\partial_\mu\eta^{(m)}.
\end{aligned} \tag{B.19}$$

For the second term from equation (B.18) we have

$$\begin{aligned}
\int_0^{2\pi R} dy\, \xi^\dagger i\overline{\sigma}^\mu\partial_\mu\xi &= \int_0^{2\pi R} dy\, \Bigg(\frac{1}{2\pi R}\xi^{(0)^\dagger} i\overline{\sigma}^\mu\partial_\mu\xi^{(0)} + \frac{1}{\sqrt{2}\pi R}\xi^{(0)^\dagger} i\overline{\sigma}^\mu\partial_\mu\sum_{n=1}^{\infty}\xi^{(n)}\cos\frac{ny}{R} \\
&+ \frac{1}{\sqrt{2}\pi R}\sum_{n=1}^{\infty}\xi^{(n)^\dagger}\cos\frac{ny}{R} i\overline{\sigma}^\mu\partial_\mu\xi^{(0)} + \frac{1}{\pi R}\sum_{n,m=1}^{\infty}\xi^{(n)^\dagger}\cos\frac{ny}{R}\xi^{(m)}\cos\frac{my}{R}\Bigg).
\end{aligned} \tag{B.20}$$

The second and third term on the r.h.s. vanish, as $\int_0^{2\pi R} dy \cos\frac{ny}{R} \equiv 0$, so we find

$$
\begin{aligned}
&\int_0^{2\pi R} dy\, \xi^\dagger i\overline{\sigma}^\mu \partial_\mu \xi \\
&= \int_0^{2\pi R} dy \left(\frac{1}{2\pi R} \xi^{(0)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(0)} + \frac{1}{\pi R} \sum_{n,m=1}^{\infty} \xi^{(n)\dagger} \cos\frac{ny}{R} \xi^{(m)} \cos\frac{my}{R} \right) \\
&= \xi^{(0)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(0)} + \int_0^{2\pi R} dy\, \frac{1}{\pi R} \sum_{n,m=1}^{\infty} \xi^{(n)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(m)} \frac{e^{i\frac{ny}{R}} + e^{-i\frac{ny}{R}}}{2} \frac{e^{i\frac{my}{R}} + e^{-i\frac{my}{R}}}{2} \\
&= \xi^{(0)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(0)} + \frac{1}{4\pi R} \int_0^{2\pi R} dy \sum_{n,m=1}^{\infty} \xi^{(n)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(m)} \Big(e^{i(n+m)y/R} + e^{i(n-m)y/R} \\
&\qquad\qquad + e^{i(m-n)y/R} + e^{-i(n+m)y/R} \Big) \\
&= \xi^{(0)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(0)} + \frac{1}{2} \sum_{n,m=1}^{\infty} \xi^{(n)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(m)} \left(\delta_{n,-m} + \delta_{n,m} + \delta_{n,m} + \delta_{m,-n} \right) \\
&= \xi^{(0)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(0)} + \sum_{n=1}^{\infty} \xi^{(n)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(n)}.
\end{aligned}
\tag{B.21}
$$

Finally, for the second last term from equation (B.18) we obtain

$$
\begin{aligned}
&- \int_0^{2\pi R} dy\, \overline{\eta}^\dagger \partial_y \xi \\
&= \int_0^{2\pi R} dy \left(-\frac{1}{\sqrt{2}\pi R} \sum_{n=1}^{\infty} \overline{\eta^{(n)}}^\dagger \sin\frac{ny}{R} \partial_y \xi^{(0)} + \frac{1}{\pi R} \sum_{n,m=1}^{\infty} \overline{\eta^{(n)}}^\dagger \sin\frac{ny}{R} \xi^{(m)} \frac{m}{R} \sin\frac{my}{R} \right) \\
&\overset{(*)}{=} \frac{1}{\pi R} \int_0^{2\pi R} dy \sum_{n,m=1}^{\infty} \overline{\eta^{(n)}}^\dagger \xi^{(m)} \sin\frac{ny}{R} \sin\frac{my}{R} \overset{(**)}{=} \sum_{n=1}^{\infty} \frac{n}{R} \overline{\eta^{(n)}}^\dagger \xi^{(n)}.
\end{aligned}
\tag{B.22}
$$

Step $(*)$ is possible, as $\int_0^{2\pi R} dy \sin\frac{ny}{R} \equiv 0$. Step $(**)$ comes from equation (B.19). An analogous calculation gives

$$
\int_0^{2\pi R} dy\, \xi^\dagger \partial_y \overline{\eta} = \sum_{n=1}^{\infty} \frac{n}{R} \xi^{(n)\dagger} \overline{\eta^{(n)}}. \tag{B.23}
$$

Thus we find

$$
\begin{aligned}
\mathcal{L}_{\text{kin}} =& \xi^{(0)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(0)} \\
&+ \sum_{n=1}^{\infty} \left[\xi^{(n)\dagger} i\overline{\sigma}^\mu \partial_\mu \xi^{(n)} + \eta^{(n)\dagger} i\overline{\sigma}^\mu \partial_\mu \eta^{(m)} + \frac{n}{R} \left(\overline{\eta^{(n)}}^\dagger \xi^{(n)} + \xi^{(n)\dagger} \overline{\eta^{(n)}} \right) \right].
\end{aligned}
\tag{B.24}
$$

Often this expression is rewritten in supersymmetry (SUSY) language, where for two Weyl spinors conventionally is defined

$$\xi\eta \equiv \xi^{\alpha}\eta_{\alpha} = \xi^{\alpha}\epsilon_{\alpha\beta}\eta^{\beta}, \tag{B.25}$$

with

$$(\epsilon_{\alpha\beta}) = -(\epsilon^{\alpha\beta}) = -i\sigma_2. \tag{B.26}$$

We then have

$$\begin{aligned}\overline{\eta^{(n)}}^{\dagger}\xi^{(n)} + \xi^{(n)\dagger}\overline{\eta^{(n)}} &= \eta^{(n)^{T}}\left(-i\sigma_2\right)\xi^{(n)} + \xi^{(n)^{\dagger}}(-i\sigma_2)i\sigma_2\overline{\eta^{(n)}} \\ &= \xi^{(n)}\eta^{(n)} + \overline{\xi^{(n)}}\,\overline{\eta^{(n)}}.\end{aligned} \tag{B.27}$$

So we may finally write

$$\begin{aligned}\mathcal{L}_{\text{kin}} =&\xi^{(0)^{\dagger}}i\overline{\sigma}^{\mu}\partial_{\mu}\xi^{(0)} \\ &+ \sum_{n=1}^{\infty}\left[\xi^{(n)^{\dagger}}i\overline{\sigma}^{\mu}\partial_{\mu}\xi^{(n)} + \eta^{(n)^{\dagger}}i\overline{\sigma}^{\mu}\partial_{\mu}\eta^{(m)} + \frac{n}{R}\left(\xi^{(n)}\eta^{(n)} + \overline{\xi^{(n)}}\,\overline{\eta^{(n)}}\right)\right].\end{aligned} \tag{B.28}$$

For the mass term, we have

$$\begin{aligned}\mathcal{L}_{\text{mass}} &= -\frac{1}{2}m_{\nu}\int_{0}^{2\pi R}dy\left(\nu^{T}C_{5}^{-1}\nu + \text{H.c.}\right) \\ &= -\frac{1}{2}m_{\nu}\int_{0}^{2\pi R}dy\left[\left(\xi^{T},\overline{\eta}^{T}\right)\begin{pmatrix}-i\sigma_2 & 0\\ 0 & -i\sigma_2\end{pmatrix}\begin{pmatrix}\xi\\ \overline{\eta}\end{pmatrix} + \text{H.c.}\right] \\ &= \frac{1}{2}m_{\nu}\int_{0}^{2\pi R}dy\left(\xi^{T}\overline{\xi}^{*} + \eta^{\dagger}\overline{\eta} + \text{H.c.}\right).\end{aligned} \tag{B.29}$$

Plugging in the KK expansions, and rewriting the result in SUSY language, we obtain

$$\mathcal{L}_{\text{mass}} = -\sum_{n=1}^{\infty}\frac{1}{2}m_{\nu}\left(\xi^{(n)}\xi^{(n)} + \eta^{(n)}\eta^{(n)} + \text{H.c.}\right). \tag{B.30}$$

So all in all we find

$$\begin{aligned}\mathcal{L}_{\nu} = \xi^{(0)\dagger}i\overline{\sigma}^{\mu}\partial_{\mu}\xi^{(0)} - \frac{1}{2}m_{\nu}\left(\xi^{(0)}\xi^{(0)} + \text{H.c.}\right) + \sum_{n=1}^{\infty}\Big[\xi^{(n)\dagger}i\overline{\sigma}^{\mu}\partial_{\mu}\xi^{(n)} + \eta^{(n)\dagger}i\overline{\sigma}^{\mu}\partial_{\mu}\eta^{(n)} \\ + \frac{n}{R}\left(\xi^{(n)}\eta^{(n)} + \overline{\xi}^{(n)}\overline{\eta}^{(n)}\right) - \frac{1}{2}m_{\nu}\left(\xi^{(n)}\xi^{(n)} + \eta^{(n)}\eta^{(n)} + \text{H.c.}\right)\Big].\end{aligned} \tag{B.31}$$

Diagram	a_1	a_2	a_3
(a)	$\left[\frac{19}{6}-(\xi-1)\right]C_2(G)$	$\left[\frac{11}{3}-(\xi-1)\right]C_2(G)$	$\left[\frac{9}{2}+\frac{9}{4}(\xi-1)\right]C_2(G)$
(b)	0	0	$\left[3+\frac{3}{4}(\xi-1)\right]C_2(G)$
(c)	$\frac{1}{3}C_2(G)$	$\frac{1}{3}C_2(G)$	$[-1+(\xi-1)]\,C_2(G)$
(d)	$\frac{1}{6}C_2(G)$	$-\frac{1}{3}C_2(G)$	$-\frac{1}{2}C_2(G)$
(e)	$C_2(G)$	0	$\left[-3-\frac{3}{2}(\xi-1)\right]C_2(G)$
(f)	$C_2(G)$	0	$[1-(\xi-1)]\,C_2(G)$
(g)	0	0	0
(h)	$\mp\frac{1}{3}C(r_s)$	$\mp\frac{1}{3}C(r_s)$	$\pm C(r_s)$
(i)	0	0	$\mp C(r_s)$

Table B.1: Contributions of the diagrams in figure B.1 (Values taken from [73]). In this and the following tables, $C_2(r)$ is the quadratic Casimir operator, and $C(r)$ is the Dynkin index. Both are group theory constants for a given representation r. More details on group theory may be found in appendix A.9.

B.3 One-Loop Diagrams

B.3.1 Diagrams for the Gauge Boson Self Energy

Figure B.1 shows the contributing diagrams. The logarithmically divergent contributions may be written as (for $p_5' = p_5 + \frac{2n}{R}$)

$$\overline{\Pi}_{\mu\nu} = \frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left\{g_{\mu\nu}p^2 a_1 - p_\mu p_\nu a_2 + g_{\mu\nu}\frac{p_5^2+p_5'^2}{2}a_3\right\}. \tag{B.32}$$

Here and in the following subsections, Λ is the cutoff scale and μ is the renormalization scale. Both scales arise in the calculation of divergent loop diagrams. Their role becomes more clear in subsection B.3.5, where we calculate the divergent contributions explicitly.

Table B.1 gives the values for a_1, a_2, and a_3. For completeness, these are given in general R_ξ-gauge, but in this thesis we will use Feynman gauge, which means that we set $\xi = 1$. Thus we do not have to worry about the fact that A_λ and A_5 do not decouple in R_ξ-gauge. In this case, the two additional diagrams shown in figure B.2 would exist.

Note that in table B.1 from fermion loops no contributions arise at one-loop level. This result is due to the cancellation between Z_2 even and Z_2 odd fermion compo-

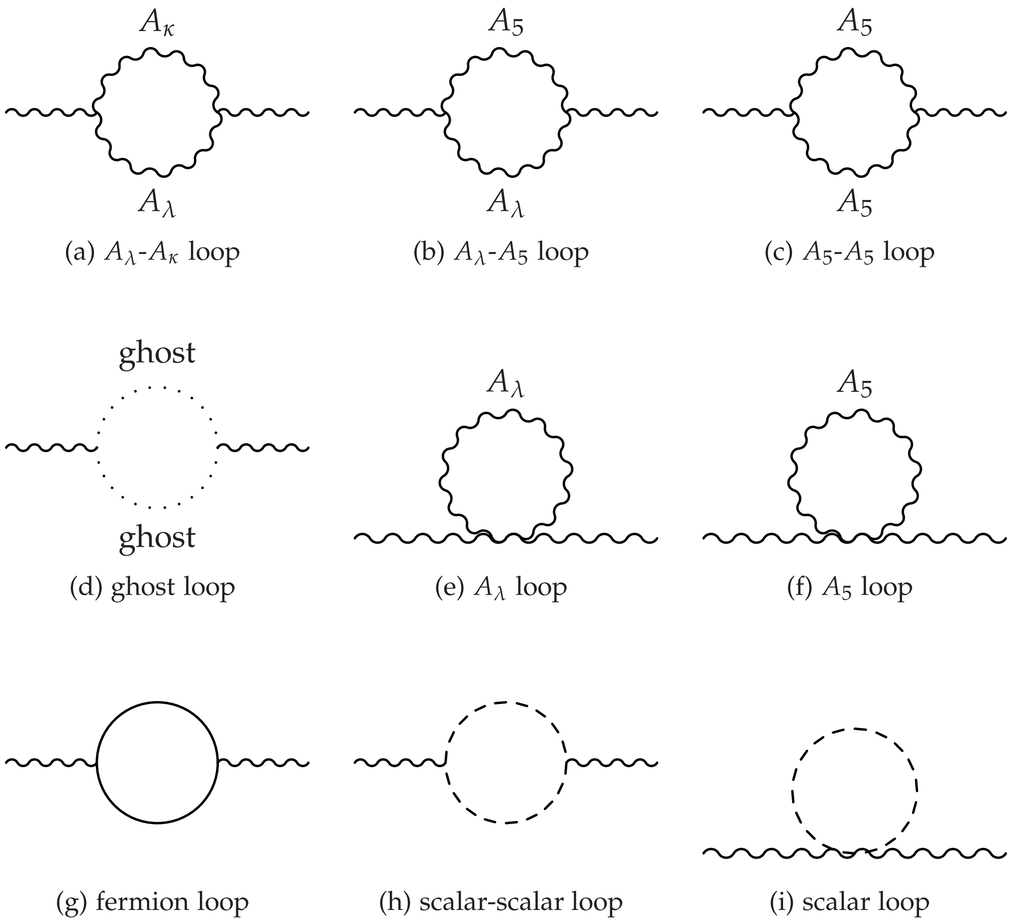


Figure B.1: One-loop diagrams for the gauge boson self energy (Figures as in [73]).

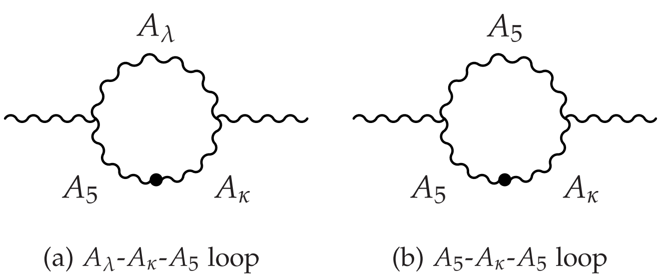
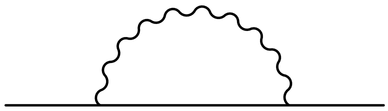
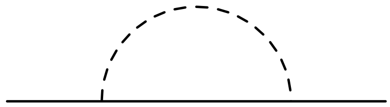


Figure B.2: Additional diagrams in R_ξ-gauge (Figures as in [73]). The coefficients are given as: (a) $a_1 = 0, a_2 = 0, a_3 = \frac{3}{2}(\xi - 1)C_2(G)$ and (b) $a_1 = 0, a_2 = 0, a_3 = -2(\xi - 1)C_2(G)$.



(a) gauge boson loop



(b) scalar boson loop

Figure B.3: One-loop diagrams for the fermion self energy (Figures as in [73]).

Diagram	b_1	b_2
(a)	$[-1-2(\xi-1)]\, g^2 C_2(r)$	$[5+(\xi-1)]\, g^2 C_2(r)$
(b)	$\mp h^2$	$\mp h^2$

Table B.2: Contributions of the diagrams in figure B.3 (Values taken from [73]).

nents. For the scalar loops in (h) and (i), the upper (lower) sign is for the Z_2 even (odd) components. All results are for the real components and must be multiplied by a factor of 2 for complex scalars.

Adding up all contributions, we obtain (for $p_5' = p_5 + \frac{2n}{R}$)

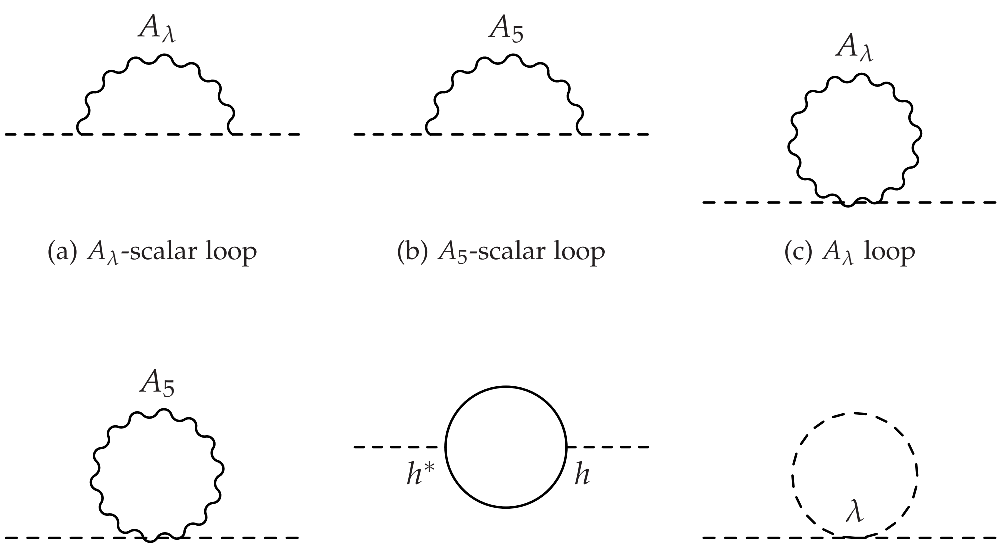
$$\overline{\Pi}_{\mu\nu} = \frac{g^2}{64\pi^2} \ln \frac{\Lambda^2}{\mu^2} \Bigg\{ \Big(g_{\mu\nu} p^2 - p_\mu p_\nu \Big) \Bigg[\left(\frac{11}{3} - (\xi - 1) \right) C_2(G) \\ - \frac{1}{3} \sum_{\text{real scalars}} \left(C(r)_{\text{even}} - C(r)_{\text{odd}} \right) \Bigg] + g_{\mu\nu} \frac{p_5^2 + p_5'^2}{2} \left[4 + (\xi - 1) \right] C_2(G) \Bigg\} . \tag{B.33}$$

B.3.2 Diagrams for the Fermion Self Energy

Figure B.3 shows the contributing diagrams. The logarithmically divergent contributions may be written as (for $p_5' = p_5 + \frac{2n}{R}$)

$$\overline{\Sigma} = \frac{1}{64\pi^2} \ln \frac{\Lambda^2}{\mu^2} \left[\not{p} \frac{1 \pm \gamma_5}{2} b_1 - \left(i p_5 \frac{1 \pm \gamma_5}{2} - i p_5' \frac{1 \mp \gamma_5}{2} \right) b_2 \right] . \tag{B.34}$$

Table B.2 gives the values for b_1 and b_2. As before, in (b) the upper (lower) sign is for the Z_2 even (odd) components.



(a) A_λ-scalar loop (b) A_5-scalar loop (c) A_λ loop

(d) A_5 loop (e) fermion loop (f) scalar loop

Figure B.4: One-loop diagrams for the scalar boson self energy (Figures as in [73]).

B.3.3 Diagrams for the Scalar Boson Self Energy

Figure B.4 shows the contributing diagrams. The logarithmically divergent contributions may be written as

$$\frac{1}{64\pi^2} \ln \frac{\Lambda^2}{\mu^2} \left[p^2 c_1 + \frac{p_5^2 + p_5'^2}{2} c_2 \right], \text{ for } p_5' = p_5 + \frac{2n}{R}. \tag{B.35}$$

Table B.3 gives the values for c_1 and c_2 (in Feynman gauge). As before, in (f) the upper (lower) sign is for the Z_2 even (odd) components. For an odd scalar, only boundary kinetic terms arise:

$$\frac{1}{64\pi^2} \ln \frac{\Lambda}{\mu^2} p_5 p_5' d_1, \text{ for } p_5' = p_5 + \frac{2n}{R}. \tag{B.36}$$

Table B.3 gives the values for d_1.

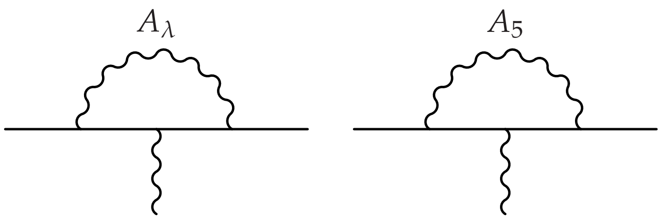
B.3.4 Diagrams for the Fermion–Gauge Boson Interaction

KK number violating couplings may only arise at one-loop level. In this subsection we give the contributions from the one-loop vertex corrections for the fermion–gauge boson interaction. The corresponding diagrams are shown in figure B.5. The logarithmically divergent contributions to the vertex can be written as

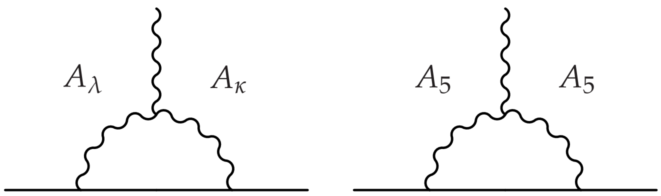
$$f_1 \sqrt{2} \frac{g^2}{64\pi^2} \ln \frac{\Lambda^2}{\mu^2} g \overline{\psi} \gamma^\mu t^a P_L \psi A_\mu^a. \tag{B.37}$$

Diagram	c_1	c_2	d_1
(a)	$4g^2C(r)$	$2g^2C(r)$	0
(b)	0	$3g^2C(r)$	$5g^2C(r)$
(c)	0	$-4g^2C(r)$	$4g^2C(r)$
(d)	0	$g^2C(r)$	$-g^2C(r)$
(e)	0	0	0
(f)	0	$\mp\frac{\lambda}{2}$	$\pm\frac{\lambda}{2}$

Table B.3: Contributions of the diagrams in figure B.4 for even scalars (c_1, c_2) and for odd scalars (d_1) (Values taken from [73]).



(a) A_λ-fermion-fermion loop (b) A_5-fermion-fermion loop



(c) A_λ-A_κ-fermion loop (d) A_5-A_5-fermion loop

Figure B.5: One-loop diagrams for the fermion–gauge boson interaction (Figures as in [73]).

Diagram	f_1
(a)	$[2C_2(r) - C_2(G)][1 + (\xi - 1)]$
(b)	$-C_2(r) + \frac{1}{2}C_2(G)$
(c)	$C_2(G)[3 + \frac{3}{2}(\xi - 1)]$
(d)	$-\frac{1}{2}C_2(G)$

Table B.4: Contributions of the diagrams in figure B.5 (Values taken from [73]).

Diagram	g_1
(a)	$C_2(r)[8 - 2(1 - \xi)]$
(b)	$2C_2(r)$
(c)	neglected
(d)	0
(e)	0
(f)	0

Table B.5: Contributions of the diagrams in figure B.6.

Table B.4 gives the values for f_1. We will not give the calculations for these contributions. They may be found (in Feynman gauge) in the textbook by Peskin and Schroeder [34], p. 521 ff. We will do explicit calculations in the next subsections, which are very similar to the ones to be done here.

Summing over all values for f_1, we obtain (in R_ξ gauge)

$$f_1(\text{total}) = C_2(r)[1 + 2(\xi - 1)] + C_2(G)[2 + \frac{1}{2}(\xi - 1)]. \tag{B.38}$$

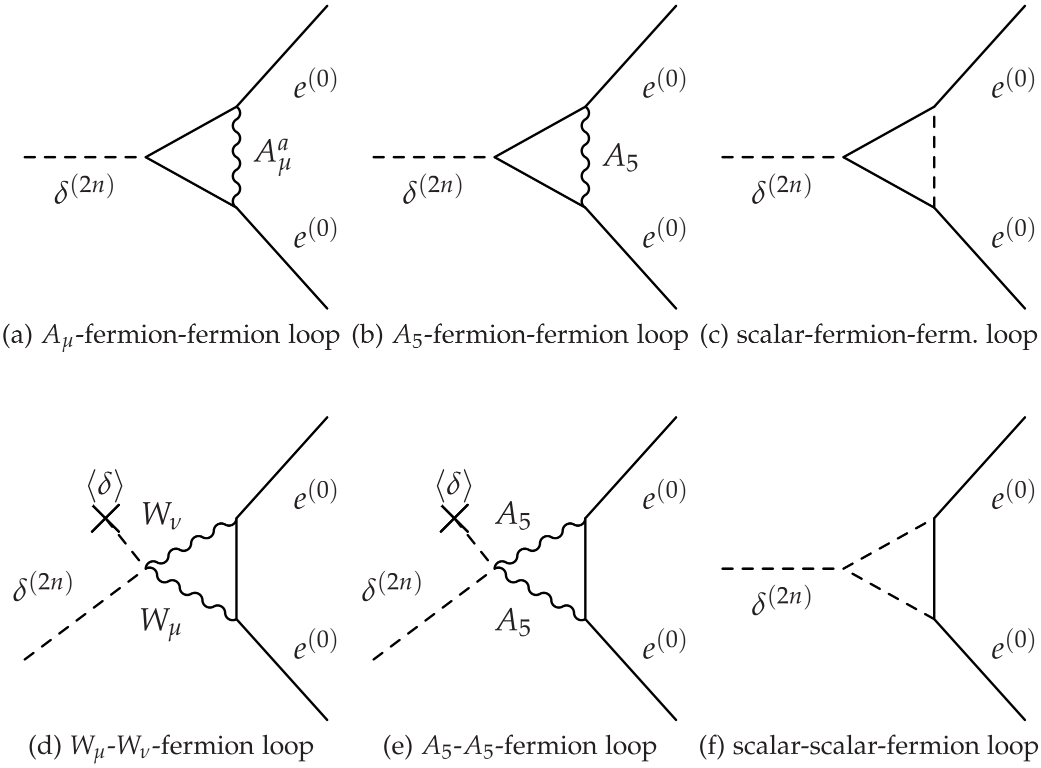
B.3.5 Diagrams for the Triplet Scalar–Fermion Interaction

In this subsection, we give the contributions of the one-loop vertex corrections for triplet scalar–fermion interaction. The corresponding diagrams are shown in figure B.6. The logarithmically divergent contributions to the vertex can be written

$$g_1 \frac{g^2 Y}{64\pi^2} \ln \frac{\Lambda^2}{\mu^2}. \tag{B.39}$$

Table B.5 gives the values for g_1, which we want to calculate now. Concerning the prefactors, we adopt the convention used by Cheng, Matchev, and Schmaltz [73], so that our results are consistent with their results given in the previous sections. Let us start with the fermion–fermion–gauge boson loops. Momentum assignments are given in figure B.7 (a). We have, for the gauge boson being A^a_μ,

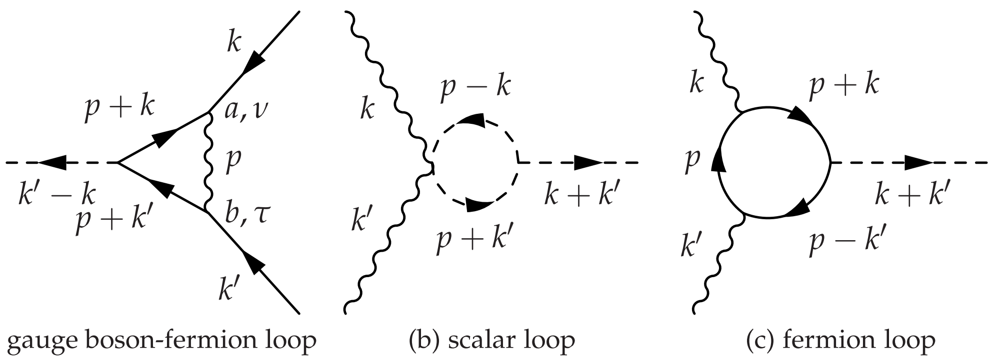
$$\begin{aligned}
&\frac{1}{2}\int \frac{d^4p}{(2\pi)^4} ig\gamma^\tau t^b \frac{i}{(\not{p} + \not{k}')} iY \frac{i}{(\not{p} + \not{k})} ig\gamma^\nu t^a \frac{(-i)}{p^2}\left(g_{\nu\tau} - \frac{p_\nu p_\tau}{p^2}(1 - \xi)\right)\delta^{ab} \\
&= \frac{g^2 Y}{2} t^a t^a \int \frac{d^4p}{(2\pi)^4} \frac{\gamma^\tau(\not{p} + \not{k}')(\not{p} + \not{k})\gamma^\nu}{(p + k')^2(p + k)^2 p^2}\left(g_{\nu\tau} - \frac{p_\nu p_\tau}{p^2}(1 - \xi)\right).
\end{aligned} \tag{B.40}$$



(a) A_μ-fermion-fermion loop (b) A_5-fermion-fermion loop (c) scalar-fermion-ferm. loop

(d) W_μ-W_ν-fermion loop (e) A_5-A_5-fermion loop (f) scalar-scalar-fermion loop

Figure B.6: One-loop diagrams for the triplet scalar–fermion interaction. In the calculations, we neglect diagram (c), as the scalar–fermion couplings are expected to be much weaker than gauge boson–fermion couplings. Diagram (f) vanishes anyway.



(a) gauge boson-fermion loop (b) scalar loop (c) fermion loop

Figure B.7: Momentum assignments for the diagrams to be calculated in sections B.3.5 and B.3.6.

We can now easily see by counting powers that this integral is logarithmically divergent. Thus it will be sufficient to consider the limit where the integration variable p is much bigger than the external momenta. Moreover, therefore it is justified that in the beginning we assumed all particles to be massless. We then find for the value of the diagram [with $t^a t^a = C_2(r)$]

$$\begin{aligned} &\frac{g^2 Y}{2} C_2(r) \int \frac{d^4p}{(2\pi)^4} \frac{\gamma^\tau \gamma^\nu}{(p^2)^2} \left(g_{\nu\tau} - \frac{p_\nu p_\tau}{p^2}(1-\xi) \right) \\ &= \frac{g^2 Y}{2} C_2(r)[4-(1-\xi)] \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2)^2} \end{aligned} \tag{B.41}$$

We now need to Wick rotate the momentum p in the integral in the last line to Euclidean momentum p_E, and obtain

$$\begin{aligned} \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2)^2} &= \frac{i}{(2\pi)^4} \int d^4p_E \frac{1}{(p_E^2)^2} = \frac{i}{(2\pi)^4} \int d\Omega_4 \int_0^\infty dp_E \frac{p_E^3}{(p_E^2)^2} \\ &= \frac{i}{16\pi^2} \int_0^\infty dp_E 2p_E \frac{1}{p_E^2} = \frac{i}{16\pi^2} \int_0^\infty d(p_E)^2 \frac{1}{p_E^2} = \frac{i}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2}, \end{aligned} \tag{B.42}$$

where Λ is the cutoff scale, and μ is the renormalization scale. We thus obtain for our vertex

$$\frac{i}{64\pi^2} g^2 Y C_2(r)[8-2(1-\xi)] \ln \frac{\Lambda^2}{\mu^2}. \tag{B.43}$$

With the same momentum assignment [cf. figure B.7 (a)], but now having A_5 in the loop, we obtain for the vertex

$$\begin{aligned} &\frac{1}{2} \int \frac{d^4p}{(2\pi)^4} (-igt^a) \frac{i}{(\not p + \not k')} iY \frac{i}{(\not p + \not k)} (-igt^a) \frac{(-i)}{p^2} \\ &= \frac{g^2 Y}{2} C_2(r) \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2)^2} = \frac{i}{64\pi^2} g^2 Y 2C_2(r) \ln \frac{\Lambda^2}{\mu^2}. \end{aligned} \tag{B.44}$$

In the same manner, we may calculate the rest of the diagrams, which are zero or may be neglected.

Summing over all values for g_1, we obtain

$$g_1(\text{total}) = C_2(r)[10-2(1-\xi)]. \tag{B.45}$$

B.3.6 Diagrams for the Triplet Scalar–Gauge Boson Interaction

In this subsection, we give the contributions from the one-loop vertex corrections for the triplet scalar–gauge boson interaction. The corresponding diagrams are shown in figure B.8. The logarithmically divergent contributions to the vertex can be written as

$$h_1 \frac{g^2 \alpha g^{\mu\nu}}{64\pi^2} \ln \frac{\Lambda^2}{\mu^2}. \tag{B.46}$$

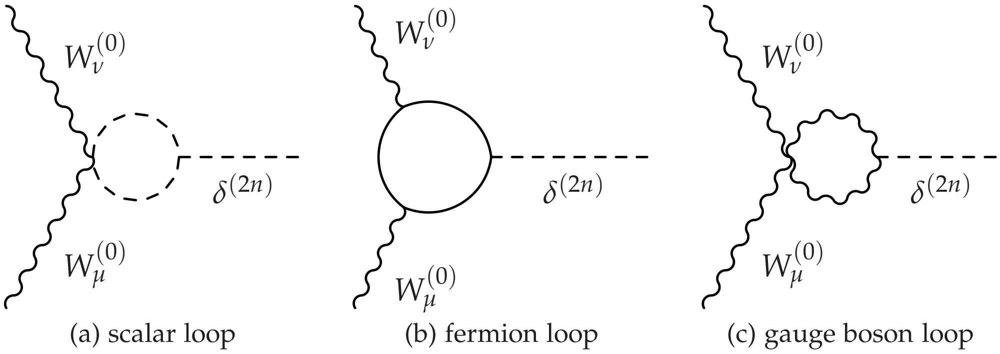


Figure B.8: One-loop diagrams for the triplet scalar–gauge boson interaction.

We want to calculate the values for h_1 now. The momentum assignments are given in figure B.7. For the first diagram, (b) in figure B.7, we obtain

$$\begin{aligned}&\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}2ig^2g^{\mu\nu}\frac{i}{(k'+p)^2}i\alpha\frac{i}{p-k)^2}\\&=\alpha g^2g^{\mu\nu}\int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2)^2}=\frac{i}{64\pi^2}4\alpha g^2g^{\mu\nu}\ln\frac{\Lambda^2}{\mu^2}.\end{aligned}\tag{B.47}$$

For the second one, (c) in figure B.7, we can calculate

$$\begin{aligned}&(-1)\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\left(-i\frac{g}{\sqrt{2}}\gamma_\mu\right)\frac{i}{(\not{p}+\not{k})}iY\frac{i}{(\not{p}-\not{k}')}\left(-i\frac{g}{\sqrt{2}}\gamma_\nu\right)\frac{i}{\not{p}}\right]\\&=\frac{g^2}{4}Y\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\frac{\gamma_\mu(\not{p}+\not{k})(\not{p}-\not{k}')\gamma_\nu\not{p}}{(p+k)^2(p-k')^2p^2}\right].\end{aligned}\tag{B.48}$$

Neglecting, as before, all external momenta, we obtain

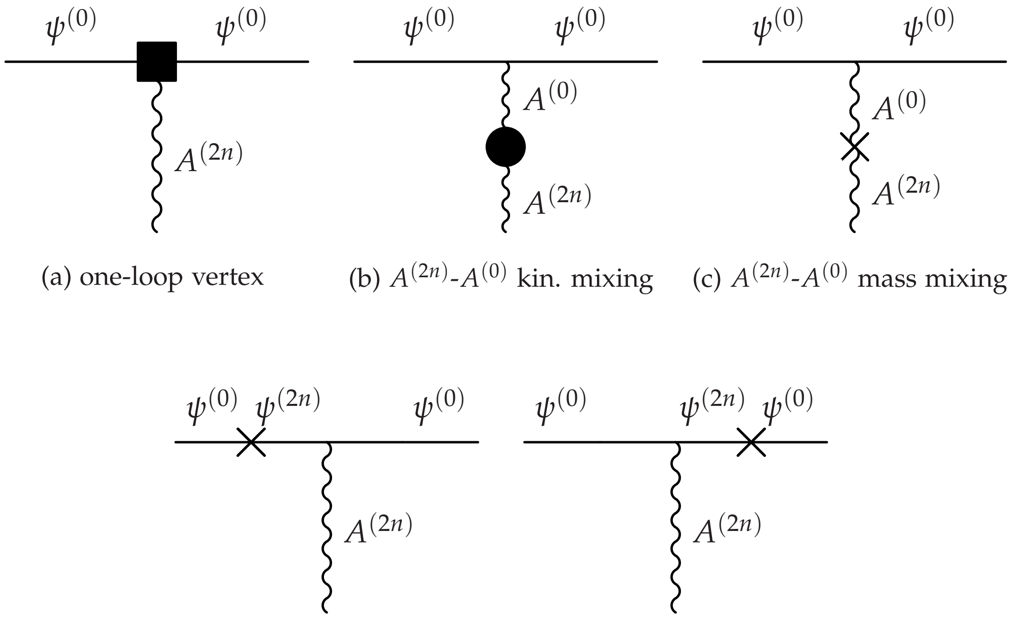
$$\begin{aligned}&\frac{g^2}{4}Y\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\frac{\gamma_\mu\not{p}\not{p}\gamma_\nu\not{p}}{(p^2)^3}\right]\\&=\frac{g^2}{4}Y\int\frac{d^4p}{(2\pi)^4}\frac{p^\alpha}{(p^2)^2}\mathrm{Tr}\left[\gamma_\mu\gamma_\nu\gamma_\alpha\right]\equiv 0.\end{aligned}\tag{B.49}$$

An analogous calculation for the gauge boson loop in diagram B.8 (c) gives

$$h_1=[4+\frac{1}{2}(\xi-1)]C_2(G).\tag{B.50}$$

So in total, we obtain

$$h_1(\text{total})=4+[4+\frac{1}{2}(\xi-1)]C_2(G).\tag{B.51}$$



(a) one-loop vertex (b) $A^{(2n)}$-$A^{(0)}$ kin. mixing (c) $A^{(2n)}$-$A^{(0)}$ mass mixing

(d) $\psi^{(0)}$-$\psi^{(2n)}$ mass mixing (e) $\psi^{(0)}$-$\psi^{(2n)}$ mass mixing

Figure B.9: The KK number violating coupling $\overline{\psi^{(0)}}\psi^{(0)}A_\mu^{a(2n)}$ (Figures as in [73]).

B.4 KK Number Violating Couplings

In this section, we will give the explicit formulae used for the KK number violating vertices in section 6.3.1. Parts of these were discussed by Cheng, Matchev, and Schmaltz [73].

B.4.1 Fermion–Gauge Boson Vertex

Here we consider the two vertices $\overline{\psi^{(0)}}\psi^{(0)}A_\mu^{a(2n)}$ and $\overline{\psi^{(2n)}}\psi^{(0)}A_\mu^{a(0)}$, which are given in figures B.9 and B.10, respectively. In addition to the one-loop vertex corrections, the KK number violating mass and kinetic mixing effects on the external legs, which are one-loop effects, too, have to be included to obtain the couplings between the physical mass eigenstates. For the first (figure B.9) of the two vertices, the contributions from the shown diagrams are

$$\sqrt{2}g\frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}k_1. \tag{B.52}$$

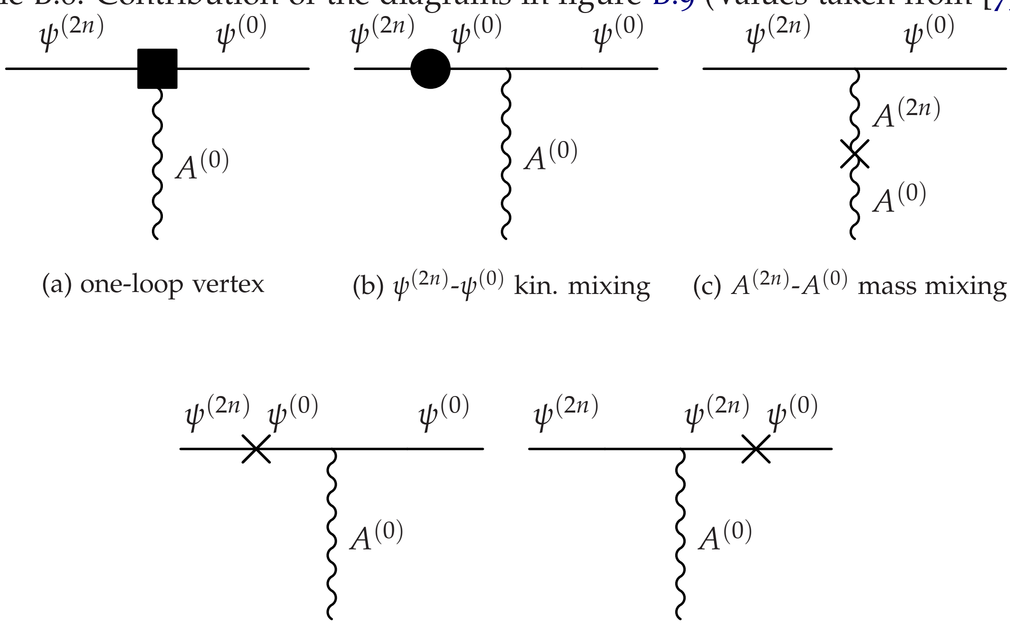
The values of k_1 are given in table B.6. Summing up all the contributions, we obtain

$$\sqrt{2}g\frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left\{\frac{23}{3}C_2(G)-\frac{1}{3}\sum_{\text{real scalars}}\left(C(r)_{\text{even}}-C(r)_{\text{odd}}\right)-9C_2(r)\right\}. \tag{B.53}$$

The additional factor $\sqrt{2}$ comes from the different normalization of the KK modes.

Diagram	k_1
(a)	$C_2(r)\,[1+2(\xi-1)] + C_2(G)\left[2+\frac{1}{2}(\xi-1)\right]$
(b)	$\left[\frac{11}{3}-(\xi-1)\right]C_2(G) - \frac{1}{3}\sum_{\text{real scalars}}(C(r_+)-C(r_-))$
(c)	$\left[2+\frac{1}{2}(\xi-1)\right]C_2(G)$
(d),(e)	$\{-\,[5+(\xi-1)]\,C_2(r)\}\times 2$

Table B.6: Contribution of the diagrams in figure B.9 (Values taken from [73]).



(a) one-loop vertex (b) $\psi^{(2n)}$-$\psi^{(0)}$ kin. mixing (c) $A^{(2n)}$-$A^{(0)}$ mass mixing

(d) $\psi^{(2n)}$-$\psi^{(0)}$ mass mixing (e) $\psi^{(0)}$-$\psi^{(2n)}$ mass mixing

Figure B.10: The KK number violating coupling $\overline{\psi^{(2n)}}\psi^{(0)}A_\mu^{a(0)}$ (Figures as in [73]).

Diagram	l_1
(a)	$C_2(r)\,[1+2(\xi-1)] + C_2(G)\left[2+\frac{1}{2}(\xi-1)\right]$
(b)	$-\,[1+2(\xi-1)]\,C_2(r)$
(c)	$-\left[2+\frac{1}{2}(\xi-1)\right]C_2(G)$
(d)	$[5+(\xi-1)]\,C_2(r)$
(e)	$-\,[5+(\xi-1)]\,C_2(r)$

Table B.7: Contribution of the diagrams in figure B.10 (Values taken from [73]).

For the second (figure B.10) of the two vertices, the contributions from the shown diagrams are

$$\sqrt{2}g\frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}l_1. \tag{B.54}$$

The values of l_1 are given in table B.7. Summing up all contributions, we see that the vertex indeed vanishes.

B.4.2 Scalar Triplet–Fermion Vertex

Here we give the vertex $e^{(0)}e^{(0)}\delta^{(2n)}$, which is shown in figure B.11. The contributions from the shown diagrams are

$$\frac{g^2 Y}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}m_1. \tag{B.55}$$

The values for m_1 are given in table B.8. Summing up all the contributions, we obtain

$$m_1(\text{total}) = 6C(r). \tag{B.56}$$

B.4.3 Scalar Triplet–Gauge Boson Vertex

Here we give the vertex $W^{(0)}W^{(0)}\delta^{(2n)}$, which is shown in figure B.12. The contributions from the shown diagrams are

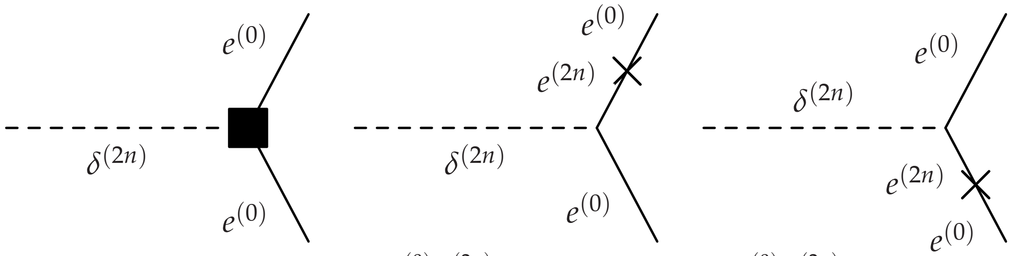
$$\frac{g^2\alpha}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}n_1. \tag{B.57}$$

The values for n_1 are given in table B.8. Summing up all the contributions, we obtain

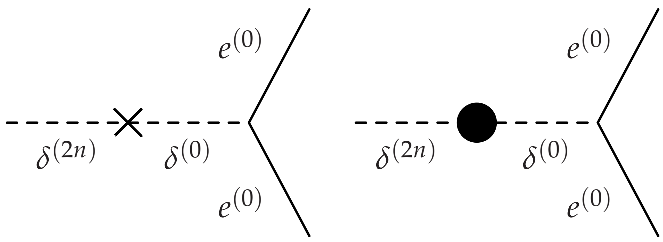
$$n_1(\text{total}) = 4 + 6C(r). \tag{B.58}$$

Diagram	m_1	n_1
(a)	$C_2(r)[10+2(\xi-1)]$	$4+[4+\frac{1}{2}(\xi-1)]C_2(G)$
(b),(c)	$\{-[5+(\xi-1)]C_2(r)\}\times 2$	$\{-[2+\frac{1}{2}(\xi-1)]C_2(G)\}\times 2$
(d)	$4C(r)$	$4C(r)$
(e)	$2C(r)$	$2C(r)$

Table B.8: Contributions m_1 of the diagrams in figure B.11 and contributions n_1 of the diagrams in figure B.12.

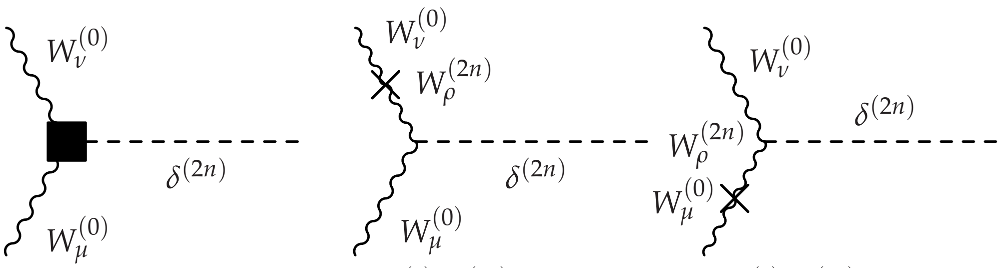


(a) one-loop vertex correction (b) $e^{(0)}$-$e^{(2n)}$ mass mixing (c) $e^{(0)}$-$e^{(2n)}$ mass mixing

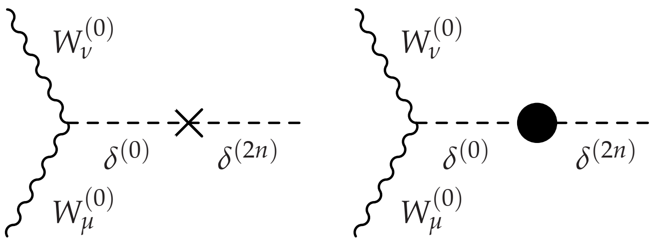


(d) $\delta^{(2n)}$-$\delta^{(0)}$ mass mixing (e) $\delta^{(2n)}$-$\delta^{(0)}$ kinetic mixing

Figure B.11: The KK number violating coupling $e^{(0)}e^{(0)}\delta^{(2n)}$.



(a) one-loop vertex correction (b) $W^{(0)}$-$W^{(2n)}$ mass mixing (c) $W^{(0)}$-$W^{(2n)}$ mass mixing



(d) $\delta^{(2n)}$-$\delta^{(0)}$ mass mixing (e) $\delta^{(2n)}$-$\delta^{(0)}$ kinetic mixing

Figure B.12: The KK number violating coupling $W^{(0)}W^{(0)}\delta^{(2n)}$.

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Erklärung

Ich, Michael Dürr, versichere, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den 27. September 2010.

Unterschrift