On Compton scattering of energetic photons by light atoms in the presence of a low-frequency electromagnetic field

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Abstract

We consider the influence of a weak low-frequency electromagnetic field on Compton scattering of a high-energy photon by an electron which is initially bound in the ground state of a light atomic target. It is shown that this influence can be very substantial for the Compton scattering with a large (on the target scale) momentum transfer to the target when the electron, as a result of the scattering, makes a transition into a high-energy continuum state. It follows from our consideration that a weak low-frequency field can pronouncedly modify both spectra of emitted electrons and those of outgoing high-frequency photons. The modification of the latter spectra means that in the bound-free Compton scattering on a light target a high-energy photon can be indirectly but rather effectively coupled to a weak low-frequency field using the target electron as a mediator.

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I. INTRODUCTION

One of the prominent processes occuring when an electromagnetic field interacts with matter is represented by Compton scattering [1] of a high-energy photon on free or bound electrons. For nonrelativistic photon energies the cross section for Compton scattering is proportional to that of classical Thomson scattering and, thus, is quite small compared to typical atomic cross sections which are of order $\sim 10^{-16}$ cm^2. Despite this, it was pointed out already in the early days of quantum physics that the study of Compton scattering on atoms and in solids can yield unique information about electron momentum distributions in bound electronic states (see e.g. [2], [3] and references therein). Moreover, such a study can also provide important information about electron correlations in atomic bound states (see e.g. [4], [5] and references therein).

In the process of Compton scattering a momentum $\Delta\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the initial and final photon momenta, respectively, is transferred to the scatterer. If the Compton scattering takes place on an electron which is bound in an atomic target then for high energies of the incident photon and for not too small photon scattering angles this momentum transfer can be quite large compared with a typical electron momentum in the initial target state, especially if the target is a light atom. The most probable result of such scattering for the atomic target is target ionization, where a target electron undergoes a transition into continuum states, because only in such states the electron may possess considerable momentum which can compensate the large momentum transfer $\Delta\mathbf{k}$ in the atomic inelastic form-factor and, thus, prevent this form-factor from taking on negligible values. In other words, in case of Compton scattering with high momentum transfer to the target nearly the whole part of this momentum is absorbed by the target electron which finally finds itself in a high-energy atomic continuum state. Both spectra of the emitted electron and of the scattered photon may yield useful information about the properties of the initial atomic state since the Compton scattering probes the whole range of the electron momentum distribution in this state.

When an atom interacts with a high-energy electromagnetic field atom ionization can also proceed via photoeffect. However, for sufficiently large photon energies the Compton scattering dominates the process of target ionization. The reason for this is that the cross section for the photoeffect rapidly decreases [1] when the photon frequency ω increases whereas the Thomson cross section is frequency-independent. As an example, one could mention that for helium ionization Compton scattering begins to dominate over the photoeffect at incident photon energies of a few keV.

With the advent of sources of coherent electromagnetic radiation a great deal of efforts has been devoted to the study of elementary scattering/collision processes in the presence of a laser radiation. Such studies are of interest because of two main reasons. One reason is that some of the laser-assisted/modified processes can find important applications. The other reason is that such processes are quite interesting from the point of view of fundamental collision physics since the presence of a laser field by introducing new degrees of freedom can substantially influence the collision dynamics.

[1] In the nonrelativistic case this cross section decreases as $\omega^{-3.5}$.

In the present paper we attempt to consider modifications of Compton scattering of a high-energy photon on a light target atom which can appear in the presence of a low-frequency laser field. By low frequency we here mean a frequency which is much smaller than a typical transition frequency of the target electron in the target ground state. We shall assume that the laser field is weak enough in order that a free target atom in the ground state cannot be noticeably distorted by the field. Such a field, however, can substantially influence target continuum states. It will be shown that spectra of both emitted electrons and scattered photons can be remarkably modified by the interaction with a weak low-frequency laser field. This will be demonstrated by considering Compton scattering of a high-energy photon on a light hydrogen-like target, which is initially bound in the ground state and is embedded in a background laser field of circular polarization.

One should note that the present paper is not the first attempt to consider Compton scattering of a high-energy photon on a light hydrogen-like atom in the presence of a low-frequency and relatively weak laser field. After the results of the present paper had been formulated it came to light that much earlier considerations of such a problem were given in [6]- [7] and [8]. However, it turned out that the approaches of [6]- [7], [8] and the present paper are quite different and, therefore, lead to different results. In addition, in the present paper some important points (e.g. spectra of emitted electrons), which were not touched upon in [6]- [7] and [8], are considered. The approaches of the above mentioned papers to describe the laser field-assisted Compton scattering will be discussed in some detail in the last subsection of section III.

The paper is organized as follows. The general approach to the problem of laser-assisted Compton scattering is given in Section II. In section III we discuss modifications, induced by the presence of a laser field, in spectra of scattered photons and emitted electrons.

Atomic units are used throughout except where otherwise stated.

II. GENERAL CONSIDERATION

Let us consider inelastic scattering of a high-energy photon on a light hydrogen-like target. The scattering occurs in the presence of a weak low-frequency electromagnetic field with circular polarization. We shall assume that the interactions between the target and both electromagnetic fields are adiabatically switched on and off at $t \to -\infty$ and $t \to +\infty$, respectively. The high-energy photon initially has a momentum $\mathbf{k}_1$ and an energy $\omega_1 = ck_1$, where $c = 137$ is the speed of light. At the infinitely remote past $t \to -\infty$ the hydrogen-like target rests in the ground state $\varphi_0(\mathbf{r})$, where $\mathbf{r}$ is the coordinate of the target electron with respect to the target nucleus. The initial target state has an energy ε_0. As a result of the collision between the target and a high-energy incoming photon, which takes place in the presence of a laser background field, the incident photon is absorbed, the target makes a transition and a high-energy photon with momentum $\mathbf{k}_2$ and an energy $\omega_2 = ck_2$ is emitted [2]. At the remote future ($t \to +\infty$) the target is in a continuum state $\varphi_{\mathbf{p}}(\mathbf{r})$ where the

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[2]This division of the Compton scattering into the three parts and the order of enumeration of these parts, of course, does not imply that emission of the photon $\mathbf{k}_2$ must always be strictly

electron has a momentum $\mathbf{p}$ with respect to the target nucleus.

To ensure that a high-energy photon collides with the target when the latter is still in the ground state, the low-frequency background field is assumed to have a very weak effect on the free target. This will be the case if the following conditions are fulfilled: (i) $F_{0j} \ll F_{at}$, where $F_{0j} = F_0$ $(j = 1, 2)$ are the amplitudes of the electric component of the electromagnetic field and $F_{at} \sim Z_t^3$ is the typical atomic field in the target ground state; (ii) the field frequency ω_0 is small compared with the minimum excitation energy of the electron bound in the target ground state; (iii) there are no multiphoton resonances between the ground and excited states of the target in the presence of the low-frequency field. Although an electromagnetic field with the above conditions may have only very weak impact on the target ground state such a field can still very substantially modulate target continuum states, especially those with high energies. Note that such a situation can occur for a rather wide range of intensities of low-frequency fields.

In the presence of a laser field the interaction between the target atom [3] and a high-frequency incident photon field reads

$$W = \frac{1}{c}\mathbf{A}_V \cdot \left(\hat{\mathbf{p}} + \frac{1}{c}\mathbf{A}\right) + \frac{1}{2c^2}\mathbf{A}_V^2 \tag{1}$$

Here $\mathbf{A}_V$ is the vector potential of the high-frequency electromagnetic field which will be treated as a quantized field

$$\mathbf{A}_V = \sum_{\mathbf{k}\lambda} \sqrt{\frac{2\pi c^2}{V\omega_k}} \mathbf{e}_{\mathbf{k}\lambda} \left(\hat{a}_{\mathbf{k}\lambda} \exp(i\mathbf{k}\cdot\mathbf{r}) + \hat{a^+}_{\mathbf{k}\lambda} \exp(-i\mathbf{k}\cdot\mathbf{r})\right), \tag{2}$$

where $\hat{a^+}_{\mathbf{k}\lambda}$ and $\hat{a}_{\mathbf{k}\lambda}$ are the photon creation and destruction operators, respectively, $\mathbf{e}_{\mathbf{k}\lambda}$ is photon polarization vector and V is the normalization volume for the quantized field. The sum in (2) runs over all possible $\mathbf{k}$ and λ. Further, in Eq.(1) $\hat{\mathbf{p}}$ is the electron momentum operator and $\mathbf{A}$ is the vector potential of the background low-frequency laser field. This field will be considered as a classical one and is described in the dipole approximation by

$$\mathbf{A}(t) = \frac{c\mathbf{F}_{01}}{\omega_0}\cos(\omega_0 t) + \frac{c\mathbf{F}_{02}}{\omega_0}\sin(\omega_0 t), \tag{3}$$

where $\mathbf{F}_{0j}$ $(j = 1, 2)$ are the amplitudes of the electric components of the laser field.

The interaction W is the sum of two terms. The first term on the right-hand side of (1) proportional to A_V is "responsible" for the description of atomic photoeffect due to absorption of a high-frequency incident photon, where this absorption is modified by the presence of the background laser field. Concerning Compton scattering, the first term can lead to a non-zero contribution to this process only starting with second order of the perturbative expansion in W. In contrast, the second term on the right-hand side of (2),

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preceded in time by absorption of the incident photon $\mathbf{k}_1$.

[3]The interaction between the target nucleus and electromagnetic fields can be neglected because of the large nuclear mass.

which is proportional to A_V^2, yields a non-zero contribution to Compton scattering already in the first order of the perturbative expansion in W. It is known that, provided the energy transfer to the target is much larger than the typical target energy in the ground state $\varepsilon_0 \sim Z_t^2$, the first order contribution from the term $\mathbf{A}_V^2/2c^2$ is much more important for Compton scattering than the second order contribution from the first term in (1). A similar relation between the contributions holds also when Compton scattering is assisted by a weak low-frequency laser field. Therefore, assuming that the energy transfer to the target is high enough the transition amplitude for the process of Compton scattering assisted by a laser field can be written as

$$S_{fi} = -\frac{i}{2c^2} \int_{-\infty}^{+\infty} dt \langle \Psi_f^{(-)}(t) | \mathbf{A}_V^2 | \Psi_i(t) \rangle. \tag{4}$$

The initial and final states $|\Psi_i\rangle$ and $|\Psi_f^{(-)}\rangle$ are taken as an external product of a target (electron) state dressed by the low-frequency field and a high-energy photon state $|\mathbf{k}, \lambda\rangle \exp(-i\omega_k t)$

$$\begin{aligned} |\Psi_i\rangle &= \psi_i(\mathbf{r}, t) |\mathbf{k}_1, \lambda_1\rangle \exp(-i\omega_1 t) \\ |\Psi_f\rangle &= \psi_f(\mathbf{r}, t) |\mathbf{k}_2, \lambda_2\rangle \exp(-i\omega_2 t). \end{aligned} \tag{5}$$

The electron states, dressed by the laser field, are solutions of the Schrödinger equation

$$i\frac{\partial}{\partial t}\psi_{i,f} = \left(H_{at} + V_{int}(t)\right)\psi_{i,f}, \tag{6}$$

where H_{at} is the Hamiltonian of a free target and the target interaction with the low-frequency classical field, taken in the velocity gauge in the dipole approximation, reads

$$V_{int}(t) = \frac{1}{c}\mathbf{A}(t) \cdot \hat{\mathbf{p}} + \frac{1}{2c^2}\mathbf{A}^2(t). \tag{7}$$

As an approximate solution of Eq.(6) we will take that obtained in [9] where it was shown that a good approximation for the target ground state, dressed by a weak low-frequency field, is given by

$$\psi_i(\mathbf{r}, t) = \varphi_0(\mathbf{r}) \exp(-i\varepsilon_0 t) \exp\left(-i\mathbf{A}(t) \cdot \mathbf{r}/c\right) \left(1 - \frac{1}{\omega_{eff}}\mathbf{F}(t) \cdot \mathbf{r}\right). \tag{8}$$

In the above expression $\mathbf{F}(t) = -\frac{1}{c}\partial \mathbf{A}(t)/\partial t$ is the electric field strength, $\varphi_0(\mathbf{r})$ is the target ground state in the field-free case and $\omega_{eff} \sim Z_t^2$ is the mean target transition frequency [9]. Since $r \sim 1/Z_t$ it is easily seen that the approximate solution (8) represents the zero and first order terms of a perturbative expansion in $\eta = F_0/F_{at}$. In the case under consideration, where $\eta \ll 1$ [4], already the zero-order term in this expansion yields quite a good approximation for the state ψ_i and, therefore, in the following consideration the first order correction in η shall be neglected.

[4] In examples considered in the present paper $\eta \leq 10^{-2}$.

In addition to the term in the last brackets in (8), the field parameters enter the state (8) via the term $\exp(-i\mathbf{A}(t)\cdot\mathbf{r}/c)$. The fact, that the latter term represents a necessary "ingredient" of the wavefunction (8) in the velocity gauge, follows from the derivation given for this wavefunction in [9] [5]. The importance of this term can be stressed e.g. by noting that it is necessary in order to ensure the correct value of the kinetic electron momentum in the initial dressed state: $\langle\psi_i(t)|\hat{\mathbf{p}}+\frac{1}{c}\mathbf{A}(t)|\psi_i(t)\rangle=0$, $\langle\psi_i(t)|\left(\hat{\mathbf{p}}+\frac{1}{c}\mathbf{A}(t)\right)^2|\psi_i(t)\rangle\sim Z_t^2$. Since one has $\mathbf{A}(t)\cdot\mathbf{r}/c\sim\mathbf{F}_0\cdot\mathbf{r}/\omega_0$, it is clear that the condition of the weakness of the laser field compared to the atomic field is certainly not sufficient to approximate the term $\exp(-i\mathbf{A}(t)\cdot\mathbf{r}/c)$ by 1. Instead, in the case of low frequencies the term $\mathbf{F}_0\cdot\mathbf{r}/\omega_0$ may not be small compared to 1 even for quite weak laser fields and, as we shall see below, the factor $\exp(-i\mathbf{A}(t)\cdot\mathbf{r}/c)$ may have an important impact on calculated results.

We shall restrict our attention to the Compton scattering where the emitted electron has momentum and energy which are much larger than the characteristic momentum and energy of the electron in the target ground state. Note that such a situation is rather typical for scattering of a high-frequency photon on a light target. If the electron momentum in a final state is large then quite a reasonable and simple approximation for the final electron state $\psi_f(t)$ can be obtained by using a nonrelativistic Coulomb-Volkov state which in the velocity gauge reads

$$\psi_f(t)=\varphi_{\mathbf{p}}(\mathbf{r})\exp\left(-i\frac{p^2}{2}t+\frac{i}{c}\int_\infty^t dt'\mathbf{A}(t')\cdot\mathbf{p}-\frac{i}{2c^2}\int_\infty^t dt'\mathbf{A}^2(t')\right), \tag{9}$$

where $\varphi_{\mathbf{p}}(\mathbf{r})$ is the target Coulomb continuum state. The approximation (9) is considered as quite a suitable one for describing an electron whose energy in the final continuum state is substantially larger than its binding energy in the initial state (see [10], [11]). Note that when the energy of the emitted electron increases, the difference between the Coulomb-Volkov and pure Volkov state, where $\varphi_{\mathbf{p}}(\mathbf{r})$ is replaced by a plane wave $\exp(i\mathbf{p}\cdot\mathbf{r})/(2\pi)^{3/2}$, becomes less and less important. [6]

Now the states (5) with the electronic parts approximated by Eqs.(8) and (9) and the representation (2) for the vector potential of the quantized high-frequency electromagnetic field can be inserted into the transition amplitude (4). The integration over time in (4) then can be performed by using the technique developed in [12]. The result is

$$S_{fi}=-i\frac{4\pi^2}{V}\frac{(\mathbf{e}_{\mathbf{k}_1\lambda_1}\cdot\mathbf{e}_{\mathbf{k}_2\lambda_2})}{\sqrt{\omega_1\omega_2}}\sum_{n=-\infty}^{+\infty}\langle\varphi_{\mathbf{p}}(\mathbf{r})|J_n(\alpha)\exp(i(\mathbf{k}_1-\mathbf{k}_2-\mathbf{q}_n)\cdot\mathbf{r})|\varphi_0(\mathbf{r})\rangle$$
$$\times\delta\left(\varepsilon_p+\omega_2-\varepsilon_0-\omega_1+n\omega_0+F_0^2/(2\omega_0^2)\right), \tag{10}$$

where $\varepsilon_p=p^2/2$, $J_n(\alpha)$ are the Bessel functions,

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[5] According to the derivation of [9] the term $\exp(-i\mathbf{A}(t)\cdot\mathbf{r}/c)$ can be interpreted as the gauge factor transforming the bound-state wavefunction ψ_i from the length gauge to the velocity gauge.

[6] In examples of the field-assisted Compton scattering, discussed in the next section, final electron energies are large enough and the difference between results of calculations, which use the Coulomb-Volkov or Volkov states to approximate the electron final states, is not very substantial.

$$\alpha = \alpha_0 - \boldsymbol{\alpha_1}\cdot\mathbf{r} = \frac{1}{\omega_0^2}\sqrt{(\mathbf{F}_{01}\cdot\mathbf{p})^2 + (\mathbf{F}_{02}\cdot\mathbf{p})^2} - \frac{(\mathbf{F}_{01}\cdot\mathbf{p})\,\mathbf{F}_{02} - (\mathbf{F}_{02}\cdot\mathbf{p})\,\mathbf{F}_{01}}{\omega_0\sqrt{(\mathbf{F}_{01}\cdot\mathbf{p})^2 + (\mathbf{F}_{02}\cdot\mathbf{p})^2}}\cdot\mathbf{r} \tag{11}$$

and

$$\mathbf{q}_n = n\frac{1}{\alpha_0^2\omega_0^3}\left((\mathbf{F}_{01}\cdot\mathbf{p})\,\mathbf{F}_{01} + (\mathbf{F}_{02}\cdot\mathbf{p})\,\mathbf{F}_{02}\right). \tag{12}$$

After standard manipulations with the transition amplitude (10) one obtains the following expression for the fully differential cross section for the laser-assisted Compton scattering

$$\frac{d\sigma}{d\omega_2 d\Omega_2 d^3\mathbf{p}} = r_e^2\frac{\omega_2}{\omega_1}\left(\mathbf{e}_{\mathbf{k}_1\lambda_1}\cdot\mathbf{e}_{\mathbf{k}_2\lambda_2}\right)^2\sum_{n=-\infty}^{+\infty}|\langle\varphi_{\mathbf{p}}(\mathbf{r})|J_n(\alpha)\exp(i(\mathbf{k}_1-\mathbf{k}_2-\mathbf{q}_n)\cdot\mathbf{r})|\varphi_0(\mathbf{r})\rangle|^2 \times\delta\left(\varepsilon_p+\omega_2-\varepsilon_0-\omega_1+n\omega_0+F_0^2/(2\omega_0^2)\right), \tag{13}$$

where $r_e = 1/c^2$ is the classical electron radius and $d\Omega_2$ is the infinitesimal solid angle for the scattered photon. Finally, assuming that the incoming photon is unpolarized, averaging over its polarization and summing over the polarization of the outgoing photon one has

$$\frac{d\sigma}{d\omega_2 d\Omega_2 d^3\mathbf{p}} = \left(\frac{d\sigma}{d\Omega_2}\right)_{Th}\frac{\omega_2}{\omega_1}\sum_{n=-\infty}^{+\infty}|\langle\varphi_{\mathbf{p}}(\mathbf{r})|J_n(\alpha)\exp(i(\mathbf{k}_1-\mathbf{k}_2-\mathbf{q}_n)\cdot\mathbf{r})|\varphi_0(\mathbf{r})\rangle|^2 \times\delta\left(\varepsilon_p+\omega_2-\varepsilon_0-\omega_1+n\omega_0+F_0^2/(2\omega_0^2)\right). \tag{14}$$

In the above expression $\left(\frac{d\sigma}{d\Omega_2}\right)_{Th} = 0.5r_e^2\left(1+\cos^2\vartheta_{12}\right)$ is the Thomson cross section and $\vartheta_{12} = \arccos(\mathbf{k}_1\cdot\mathbf{k}_2/k_1k_2)$ in the angle between the incident and scattered high-energy photons.

III. RESULTS AND DISCUSSION

A. Preliminary remarks

We start our discussion with a remark that the cross section (14) obtained by working in the velocity gauge would have the same form if the length gauge were used. The proof of this is similar to that given in [12] for the field-assisted binary-encounter emission.

In the absence of the laser field the cross section (14) reduces to its well known form for Compton scattering. The presence of a modulating electromagnetic field manifests itself in the cross section (14) by i) the Bessel functions $J_n(\alpha)$; ii) the terms $n\omega_0$ and $F_0^2/2\omega_0^2$ in the argument of the delta function, where the latter is the ponderomotive energy which describes the so-called AC-Stark shift of the energy of the final continuum state of the electron; iii) the quantities $\mathbf{q}_n$ which have the dimension of a momentum.

It is worth noting that the second term in Eq.(11), $-\boldsymbol{\alpha_1}\cdot\mathbf{r}$ [7], changes its sign when one of the polarization vectors of the laser field, $\mathbf{F}_{01}$ or $\mathbf{F}_{02}$, is inversed. In principle, due

[7] Since we assume that $p \gg Z_t$ and $\omega_0 \ll Z_t^2$, one effectively has that $|\boldsymbol{\alpha_1}\cdot\mathbf{r}| \ll \alpha_0$.

to the presence of this term, the cross section (14) could be different in cases of left and right polarizations of the laser field (the so called dichroism effect). We shall address this interesting point elsewhere and for the moment set $\alpha = \alpha_0$.

From the quantum point of view the terms with different n in the cross section (14) may be interpreted as arising due to the exchange of different numbers of low-frequency photons between the colliding system "high-energy photon + target" and the laser field. As we shall see such an exchange accompanying the scattering of a high-energy photon can, under certain conditions, considerably affect the characteristics of this process. How many laser photons can on average be exchanged with considerable probabilities and, thus, how substantially the different aspects of Compton scattering may be influenced by the laser field depends on the absolute value of the parameter α_0 (Eq.(11)). The latter describes the effective strength of the coupling between the system "high-energy photon + target" and the laser field. According to the properties of the Bessel functions J_n (see [13]) the maximum number of low-frequency photons, which can be exchanged between the colliding system and the laser field with a noticeable probability, is of the order α_0. Consequently, the energy exchange with the laser field may be characterized by the quantity $\pm\alpha_0\omega_0$ which is of order of the maximal possible exchanged energy. The value of α_0 depends on the angles between the electron momentum $\mathbf{p}$ and the field vectors $\mathbf{F}_{01}$ and $\mathbf{F}_{02}$ and, in general, increases when the momentum transfer to the target $\mathbf{k}_1 - \mathbf{k}_2 \approx \mathbf{p}$ or the ratio F_0/ω_0 increase.

It is interesting to note that by considering the classical work performed by the laser field on the emitted electron one obtains that the maximal absolute change in energy of a fast moving electron ($v_e > F_0/\omega_0$) due to the action of the laser field is of order of $\alpha_0\omega_0$ and, depending on the field phase in the instant of time when the fast electron is released from the target, the energy change can be both positive and negative. It is also worth noting that, in this classical consideration, the time τ_L of the interaction between the electron and laser field which is necessary for such an energy exchange to occur is of order of ω_0^{-1}, i.e. this time is much larger than the typical orbiting time τ_{at} of the electron in the target ground state ($\tau_{at} \sim |\varepsilon_0|^{-1}$).

The origin of the momenta $\mathbf{q}_n$ can be traced back to be in the factor $\exp(-i\mathbf{A}(t)\cdot\mathbf{r}/c)$. As follows from (11) and (12) the maximum absolute value of these momenta is of the order F_0/ω_0. If the laser frequency is sufficiently low [8] then even for quite weak laser fields the quantity F_0/ω_0 can be of order or exceed the typical electron momentum ($\sim Z_t$) in the target ground state. Therefore one can expect that the quantities $\mathbf{q}_n$ may have an important impact on the cross section (14) provided F_0/ω_0 is not much smaller than Z_t.

Since the target nucleus is very heavy compared to an electron and, thus, the interaction between the nucleus and the electromagnetic fields was neglected, then the target electron is the only particle which directly couples to the laser field. We will see, however, that not only the electron spectra can be dramatically modified by the coupling with a relatively weak laser field but also that this field may noticeably influence the properties of the outgoing high-energy photon.

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[8] In the present consideration the frequency ω_0 cannot be too low. In particular, the limit $\omega_0 \to 0$ lies beyond the validity scope of the present treatment (for more detail, see discussions in [9] and [12]).

The latter point would be completely unexpected if one would go too far with the semiclassical time analysis of Compton scattering. Indeed, in such a time analysis it is usually assumed (see e.g. [14]) that effectively the scattering takes place on the time scale $\tau_S \sim 1/(\omega_1 - \omega_2) = 2/(p^2 + Z_t^2)$ which, in the case of emission of an energetic electron, is much shorter than the typical target atomic time $\tau_{at} \sim 1/Z_t^2$. In addition, as we have mentioned already, according to the semiclassical consideration, the time scale for the interaction with a low-frequency field is $\tau_L \sim \omega_0^{-1}$ and, thus, considerably exceeds even the target time scale. Therefore, from the point of view of such a time analysis there would be practically no opportunity for influencing the properties of the outgoing high-energy photon by the laser field since the photon is not coupled directly to the field.

B. Modification of electron energy spectrum

It follows from the cross section (14) that for fixed energy and angles of the scattered photon the energy spectrum of the emitted electron consists of discrete equidistant lines centered at energies $\varepsilon_p^{(n)} = \omega_1 - \omega_2 + \varepsilon_0 - n\omega_0 - F_0^2/(2\omega_0^2)$. The number of these lines is determined by the value of the parameter α_0 and is roughly given by $2\alpha_0 + 1$. An example of the electron energy spectrum is displayed in figure 1.

If the outgoing photon is not detected the electron spectrum is obtained by integrating over all possible energies and emission angles of the scattered high-energy photon

$$\frac{d\sigma}{d^3\mathbf{p}} = 0.5 r_e^2 \int d\omega_2 \int d\Omega_2 \left(1 + \cos^2\vartheta_{12}\right) \sum_{n=-\infty}^{+\infty} \frac{\omega_2}{\omega_1} J_n^2(\alpha_0) |\langle \varphi_{\mathbf{p}}(\mathbf{r})| \exp(i(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}_n)\cdot\mathbf{r}) |\varphi_0(\mathbf{r})\rangle|^2$$
$$\times \delta\left(\varepsilon_p + \omega_2 - \varepsilon_0 - \omega_1 + n\omega_0 + F_0^2/(2\omega_0^2)\right). \quad (15)$$

The electron energy spectrum which follows from (15) does not possess a discrete structure. Rather this spectrum is now continuous. An example of such a spectrum is displayed in figure 2. Calculations show that the laser field starts to substantially influence the electron energy spectrum (15) when the typical energy exchange with the field, roughly given by $\alpha_0\omega_0$, becomes of the order of the width $\delta\varepsilon_p$ of the energy distribution of the electron emitted in the field-free Compton effect, which is approximately given by $\delta\varepsilon_p \sim pZ_t \sim \sqrt{2(\omega_1 - \omega_2)}Z_t$. In such a case the double-peak structure can replace the usual single-peak one in the energy spectrum of the emitted electron. Note that the shape of the spectrum is rather similar to that found in [9] and [12] for electron emission in the field-assisted binary-encounter collisions with fast ions. Such a similarity does not look very surprising since in the absence of the background laser field the processes of electron emission due to Compton scattering and due to binary-encounter collisions with charged projectiles are known to have important common points (see e.g. [5] and references therein).

It turns out that the ponderomotive energy $F_0^2/2\omega_0^2$ influences very weakly the shape of the electron energy spectrum. Calculations for $F_0 = 0.007$ a.u., where the term $F_0^2/2\omega_0^2$ was neglected, resulted in a curve coinciding, within drawing accuracy, with the dot-dot curve of figure 2.

In figure 2 shown are also results of calculations for $F_0 = 0.007$ a.u. where the terms $\exp(i(\mathbf{q}_n \cdot \mathbf{r}))$ were neglected (dash-dot curve). In contrast to the ponderomotive energy, the terms $\exp(i(\mathbf{q}_n \cdot \mathbf{r}))$ have a very important impact on the calculated spectrum shape.

The latter is not unexpected since the condition $\alpha_0\omega_0 \gtrsim \delta\varepsilon_p$ can be also formulated as $max\{|\mathbf{q}_n|\} \gtrsim Z_t$ and, therefore, the factor $\exp(-i\mathbf{A}(t)\cdot\mathbf{r})$ plays an important role. When this factor is omitted, only a very small effect of the laser field is seen in the calculated energy spectrum.

We mentioned already that in the study of the binary-encounter emission assisted by a weak low-frequency laser field [9], [12] we found changes in the shape of the electron emission spectrum which were rather similar to those obtained for the Compton scattering. Note, however, that although the binary-encounter collisions with fast ions and Compton scattering have much in common these processes also have important differences. In particular, for these two processes the neglect of the terms $\exp(i(\mathbf{q}_n\cdot\mathbf{r}))$ leads to quite different results. In the case of the binary-encounter emission such a neglect results in the complete disappearance of any peak structure in calculated electron spectra (see [12]). In contrast, for the Compton scattering, this neglect practically restores in calculations the laser field-free electron spectrum. This strict difference in the role of the factor $\exp(-i\mathbf{A}(t)\cdot\mathbf{r})$ should be attributed to rather different kinematical conditions for collisions with a fast heavy projectile, whose energy and momentum are huge on the atomic target scale, and with a high-frequency photon, the momentum of which takes on relatively modest values.

C. Modulation of photon energy spectrum

According to the cross section (14), for fixed energy and emission angles of the emitted electron the energy spectrum of the scattered photon consists of discrete equidistant narrow lines which are centered at $\omega_2 = \omega_1 + \varepsilon_0 - \varepsilon_p - n\omega_0 - F_0^2/2\omega_0^2$ (see for an illustration figure 3). The number of these lines is roughly given by $2\alpha_0 + 1$. Although the absolute change in the photon energy ω_2 due to the presence of the weak low-frequency field can reach rather substantial (on the target scale) values (even for the case considered in the figure 3, where $F_0/\omega_0 \ll 1$, one has $\Delta\omega_2 \approx \pm 2a.u. \approx \pm 54$ eV), the relative change in the photon energy $\Delta\omega_2/\omega_2$ remains quite modest. Note, however, that even this relatively modest effect cannot be understood within the (semi)classical picture, briefly discussed in the last paragraph of the subsection A, where the effective scattering time of the high-energy photon $\tau_S \sim 1/(\omega_1 - \omega_2)$ is much smaller than the electron orbiting time $\tau_{At} \sim 1/Z_t^2$ and the effective time for the interaction with the laser field $\tau_L \sim \omega_0^{-1}$ is even much larger than the orbiting time. This shows natural limits for the semiclassical analysis of the Compton effect. [9]

If in the photon-target collisions the emitted electrons are not detected, one has to perform the integration over the final electron momentum $\mathbf{p}$. Then, the corresponding photon spectrum follows from the cross section

[9]It is useful to note that there is a striking similarity, with respect to the breakdown of an analysis which operates with different time scales, between the field-assisted Compton scattering and such a well known phenomenon as the Mössbauer effect. In the latter the "characteristic time" of emission of a high-energy photon by the nucleus of an atom in a crystal lattice is much shorter than the "typical lattice times" but still the crystal absorbs the recoil arising due to the photon emission as a single body.

$$\frac{d\sigma}{d\omega_2 d\Omega_2} = 0.5r_e^2 \left(1+\cos^2\vartheta_{12}\right) \frac{\omega_2}{\omega_1} \int d^3\mathbf{p} \sum_{n=-\infty}^{+\infty} J_n^2(\alpha_0) |\langle\varphi_{\mathbf{p}}(\mathbf{r})| \exp(i(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}_n)\cdot\mathbf{r})|\varphi_0(\mathbf{r})\rangle|^2$$
$$\times\delta\left(\varepsilon_p + \omega_2 - \varepsilon_0 - \omega_1 + n\omega_0 + F_0^2/(2\omega_0^2)\right). \quad (16)$$

Like in the case with the electron spectrum, considered in the previous subsection, the photon energy spectrum resulting from Eq.(16) loses the discreteness and becomes continuous. An example of the photon spectrum following from (16) is shown in figure 4.

There are important differences between the changes arising in the electron and photon spectra in the presence of a low-frequency laser field. First, we do not observe any double peak structure in the photon energy spectrum. Instead, it follows from calculations that the spectrum shape in the presence of the laser field remains quite similar to that when the field is absent. The main effect of the laser field is the shift of the maximum in the photon energy spectrum towards lower frequencies. In contrast to the electron spectrum, the ponderomotive energy plays quite an important role in forming the photon spectrum. It is seen in figure 4 where the dot curve presents the calculated photon spectrum where the ponderomotive energy is neglected. In such a case the calculated spectrum is shifted to higher frequencies. Note that the calculated spectra with and without the ponderomotive energy are situated nearly symmetrically with respect to the photon spectrum in the absence of a laser field. Further, for the photon spectrum we observe that the effect of the quantized energy exchanges with the laser field, described by the terms $n\omega_0$, and the effect of the ponderomotive energy tend to considerably reduce each other. Another important difference between the effects of the laser field on the electron and photon spectra is that for the latter spectra the factor $\exp(-i\mathbf{A}(t)\cdot\mathbf{r})$ seems to play quite a different role. This can be seen in figure 4 where we also show results for the photon spectrum which were calculated by omitting this factor. The omission completely removes from the calculations the characteristic Compton shape of the photon spectrum: in the energy range under discussion the spectrum becomes nearly a constant and is strongly reduced in intensity, the latter is connected with the redistribution of the spectrum to the regions of lower and higher frequencies. The neglect of the gauge factor means that the initial electron state (8) is obtained in the length gauge whereas the final state (9) is taken in the velocity gauge or vice versa. Therefore the need to keep this factor can be formulated as the need to give a gauge consistent treatment for the field-assisted Compton scattering.

As it is suggested by figure 4, the total cross section for the field-assisted Compton scattering is practically not influenced by the laser field.

D. Some remarks on the earlier considerations of Compton effect in the presence of a weak low-frequency field

As we noted in the introduction, the first considerations of the laser-assisted Compton scattering on light atoms were given many years ago in [6], [7] and [8]. Later on, some rather technical points of how to evaluate the series appearing in the treatment of [8] were considered in [15] and [16]. The latter two papers will not be discussed here.

In [6] and [7] the Rayleigh and Compton scattering were considered. The main difference between the considerations of [7] and the present paper is in a choice of the initial target dressed state. We shall briefly comment on the approximation, which was used in [7] (and [6]) to describe the atomic ground state in the presence of a low-frequency weak laser field. In this approximation the state of a tightly bound ($F_{at} \gg F_0$) electron is represented as an integral over Volkov states convoluted with the momentum distribution inherent to the electron in the ground state of a free target. According to this approximation a tightly bound electron responds to a weak low-frequency field as a basically free electron. Such an approximation, in particular, means that a tightly bound electron oscillates in the velocity and coordinate spaces with amplitudes roughly given by $v_0 \sim F_0/\omega_0$ and $a_0 \sim F_0/\omega_0^2$, respectively. It is obvious, however, that even for quite weak but low-frequency fields the amplitudes v_0 and a_0 can reach values which are very large on the atomic scale. In particular, one can easily assume a situation where a_0 would exceed by orders of magnitude the size of the electron orbit in the bound state. It is very unlikely, however, that a tightly bound electron can participate in such oscillations. While the approximation used in [6]- [7] can be quite reasonable when dealing with super-strong laser fields ($F_0 \gtrsim F_{at}$ or $F_0 \gg F_{at}$) [10], it is clearly not appropriate for considering weak low-frequency fields. In particular, it is predicted in [6] that the Rayleigh scattering of an X-ray, where the atomic target remains in its initial state, can be very substantially modified by a weak low-frequency field independently of the value of the ratio $\eta = F_0/F_{at}$ [11]. However, a rather straightforward consideration, which uses dressed bound target states obtained by means of the perturbation expansion in F_0/F_{at}, shows that the modulation of the Rayleigh scattering is determined by the ratio F_0/F_{at} and, therefore, is very small for weak laser fields.

The consideration of the field-assisted Compton scattering on atoms, given in [8], seems to be heavily based on the derivation of the cross section for the field-free Compton effect presented in [14]. From the analysis of the latter derivation, given in the Appendix of the present paper, it follows that this derivation is rather questionable. Even if we disregard the latter point, there still remain important differences between the consideration of [8] and that of the present paper. Here we mention two of them. First, in [8] the field-assisted Compton scattering was considered in such a way that the initial and final electron states, modulated by the laser field, were actually obtained by tacitly using different gauges. That resulted in the missing of the gauge factor $\exp(-i\mathbf{A}(t) \cdot \mathbf{r}/c)$ whose importance was discussed in the previous subsections. Second, in the treatment of [8] the term $F_0^2/2\omega_0^2$ appeared not only in continuum states but also as the energy shift of the tightly bound ground state (as a result, the term $F_0^2/2\omega_0^2$ cancelled out in the energy balance). However, since the origin of the term $F_0^2/2\omega_0^2$ lies in the laser field-governed oscillations of a free or at least unbound electron with the corresponding amplitudes $v_0 \sim F_0/\omega_0$ and $a_0 \sim F_0/\omega_0^2$, it is very unlikely that, for the case of a weak and low-frequency laser field, such a term can appear in the

[10] Namely for such super-strong fields the approximation, in which the initial atomic state is represented as the convolution of the Volkov states describing a free electron in the field of a laser, was initially introduced in [17] and [18].

[11] Such an η-independence is possible for the bound-free Compton scattering since the typical atomic field in the initial (ground) state, F_{at}, is an irrelevant parameter for high-energy continuum states.

energy of a tightly bound state where such oscillations are, in general, not possible at all.

CONCLUSION

We have considered few points of the question of how the different aspects of Compton scattering of a high-frequency photon on a light atomic target can be modified by the presence of a relatively weak low-frequency field. In our consideration the parameters of the latter field were chosen in such a way that the field effect on a free target atom in the ground state would be negligible. It turns out, however, that such a weak field can quite effectively influence the process of Compton scattering provided large momenta and energies are transferred in the scattering to the target leading to the emission of the electrons with energies which considerably exceed that of the electron in the target ground state.

If the momentum $\mathbf{k}_2$ of the scattered photon is fixed the energy spectrum of the emitted electron consists of discrete equidistant lines centered at energies $\varepsilon_p^{(n)} = \omega_1 + \varepsilon_0 - \omega_2 - n\omega_0 - F_0^2/(2\omega_0^2)$. A similar discrete spectrum with lines centered at $\omega_2^{(n)} = \omega_1 + \varepsilon_0 - \varepsilon_p - n\omega_0 - F_0^2/(2\omega_0^2)$ is predicted also for the scattered photon provided the momentum of the emitted electron is fixed. The number of lines in both spectra is roughly given by $2\alpha_0 + 1$, where the parameter α_0 describes the effective strength of the coupling between the colliding "high-energy photon + target" system and the laser field. Both electron and photon spectra show, on the basic level of the fully differential cross section, the quantized character of the energy exchange with the low-frequency field despite the latter is treated classically.

In contrast to the high-energy photon field, the target electron is directly coupled to the background low-frequency field. Therefore, if one of the outgoing particles (the emitted electron or scattered photon) is not observed and, thus, one has to integrate over its final states, then, as we have seen, the "integrated" effect of a laser field can be much more pronounced in electron emission spectra. Still, since the electron interacts with both high- and low-frequency fields, the low-frequency laser field can indirectly but rather noticeably influence also the spectra of the scattered high-frequency photons integrated over the final electron states.

APPENDIX

Starting with the basic expression for the Compton cross section

$$\frac{d\sigma}{d\omega d\Omega_2} = \left(\frac{d\sigma}{d\Omega_2}\right)_{Th} \frac{\omega_2}{\omega_1} \sum_f |\langle f|\exp(i\mathbf{k}\cdot\mathbf{r})|i\rangle|^2 \delta(E_f - E_i - \omega), \tag{17}$$

where $\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$, $\omega = \omega_1 - \omega_2$ and E_i and E_f are the energies of the initial and final target states, the authors of [14] attempted to derive the so called impulse approximation for the Compton scattering. By using the identities

$$\delta(E_f - E_i - \omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp(i(\omega - E_f + E_i)t) \tag{18}$$

and

$$\exp(iHt)|i, f\rangle = \exp(iE_{i,f}t)|i, f\rangle, \tag{19}$$

where H is the target Hamiltonian, and by summing in (17) over the final states $\{|f\rangle\}$ the cross section (17) was transformed into

$$\frac{d\sigma}{d\omega d\Omega_2} = \left(\frac{d\sigma}{d\Omega_2}\right)_{Th} \frac{\omega_2}{\omega_1}\frac{1}{2\pi}\int_{-\infty}^{+\infty} dt \exp(i\omega t)$$
$$\times\langle i|\exp(iHt)\exp(-i\mathbf{k}\cdot\mathbf{r})\exp(-iHt)\exp(i\mathbf{k}\cdot\mathbf{r})|i\rangle. \tag{20}$$

The authors of [14] formulate the key point of their derivation of the impulse approximation as follows (p. 417, after Eq.(16)). "... If we reexamine Eq.(20) we immediately see that those times which are of importance in the integration (over time) are of order ω^{-1}. When ω is much larger than all of the other characteristic energies associated with the ground state of the electron, one can..." keep only the first term

$$\exp(iHt) \approx \exp(iH_0t)\exp(iVt) \tag{21}$$

in the operator expansion

$$\exp(iHt) = \exp(iH_0t)\exp(iVt)\exp(-[H_0, V]t^2/2)...., \tag{22}$$

where H_0 and V are the operators of kinetic and potential energies, respectively. Then the authors of [14] insert the right-hand side of (21) (and its conjugate) into (20), the potential V cancels out and the impulse approximation for the Compton cross section follows (for more detail see [14]).

In fact, it is rather obvious that *the whole time interval* $-\infty < t < +\infty$ and not only $\Delta t \sim \omega^{-1}$ is of importance in (20). This is because the time integration in this expression originates from the representation for $\delta(\omega - E_f + E_i)$ and the delta-function by no means can be obtained by performing integration over any finite interval of time. A simple analysis shows that the term $\langle i|....|i\rangle$ in (20) actually involves, due to the presence of $\exp(-i\mathbf{k}\cdot\mathbf{r})\exp(-iHt)\exp(i\mathbf{k}\cdot\mathbf{r})$, the oscillating term $\exp(-i\Omega t)$ with $\Omega \approx k^2/2$. The frequency Ω is much higher than the frequency $|\varepsilon_0|$ associated with the bound state and it is the term $\exp(-i\Omega t)$ which "compensates" the rapidly oscillating term $\exp(i\omega t)$ in the time integral in (20).

By discussing the questionable points of the derivation of the impulse approximation given in [14] we, of course, do not intend to put in doubt the validity of the impulse approximation itself. It is well known that this approximation yields rather good results provided the momentum transfer $\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$ is large on the target scale. In such a case, however, the impulse approximation for the cross section (17) can be obtained by replacing the final atomic states $|f\rangle$ by plane waves (and neglecting in the arguments of the delta-function terms associated with ground state frequencies) without artificial assumptions about how much time does the Compton scattering need to occur.

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FIGURE CAPTIONS

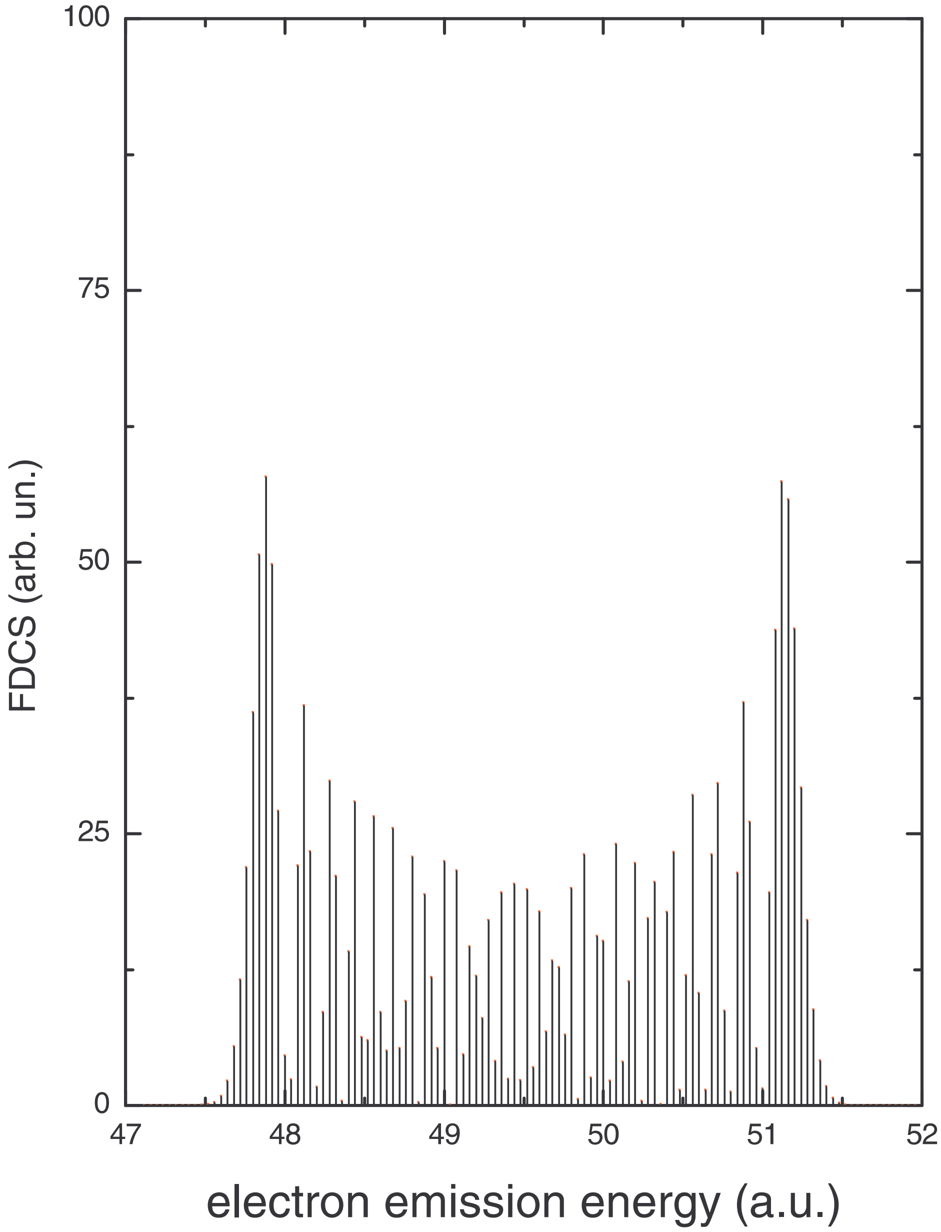

Figure 1

FIG. 1. Energy spectrum of electrons emitted from the hydrogen ground state in the process of Compton scattering. Momenta of the incident and scattered high-energy photons as well as the electron emission angles are fixed: $\omega_1 = 750$ a.u., $\vartheta_1 = 0^0$; $\omega_2 = 700$ a.u., $\vartheta_2 = 180^0$; and $\vartheta_e = 0^0$. The parameters of the background field are $F_0 = 0.004$ a.u. and $\omega_0 = 0.04$ a.u.. The circular laser field is polarized along the x and z axes.

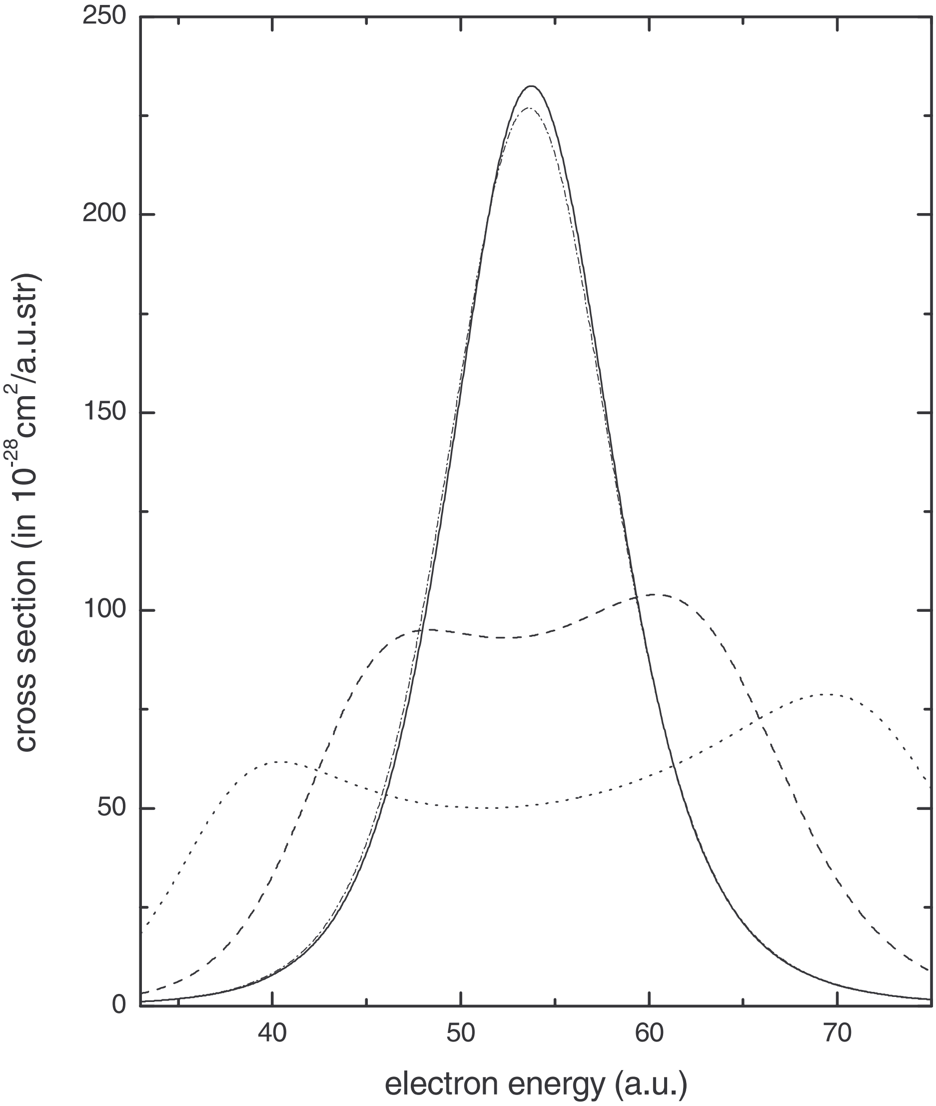

Figure 2

FIG. 2. Energy spectrum of electrons emitted from the hydrogen ground state in the process of Compton scattering. Electron emission angles are fixed: $\vartheta_e = 10^0$, $\phi_e = 0^0$. Integration over all possible states of the scattered photon $\mathbf{k}_2$ has been performed. The momentum of the incident photon is defined by $\omega_1 = 750$ a.u. and $\vartheta_1 = 0^0$. Polarization of the laser field is the same as in figure 1 but its frequency is ten times smaller: $\omega_0 = 0.004$ a.u.. Solid curve: field-free Compton scattering. Dash curve: Compton scattering assisted by a laser field with $F_0 = 0.004$ a.u.. Dot Curve: Compton scattering at $F_0 = 0.007$ a.u.. Dash-dot curve: Calculated spectrum for $F_0 = 0.007$ a.u. when the gauge factor was removed from calculations.

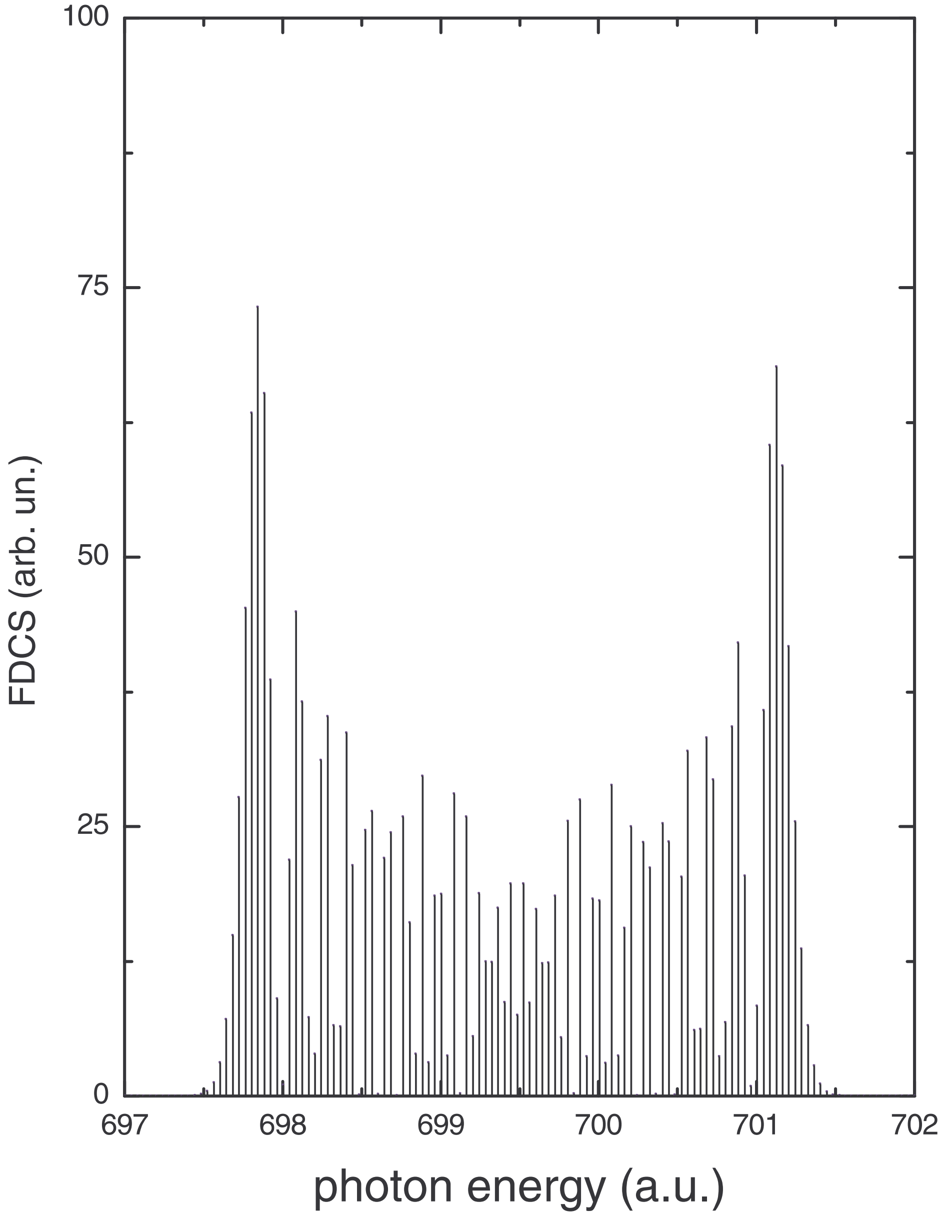

Figure 3

FIG. 3. Energy spectrum of photons scattered on a hydrogen target which is initially in the ground state. Momenta of the incident photon and emitted electron as well as angles of the scattered photon are fixed: $\omega_1 = 750$ a.u., $\vartheta_1 = 0^0$; $\vartheta_2 = 180^0$; $\varepsilon_p = 50$ a.u., $\vartheta_e = 0^0$. The parameters of the background field are $F_0 = 0.004$ a.u. and $\omega_0 = 0.04$ a.u.. The laser field is polarized along the x and z axes.

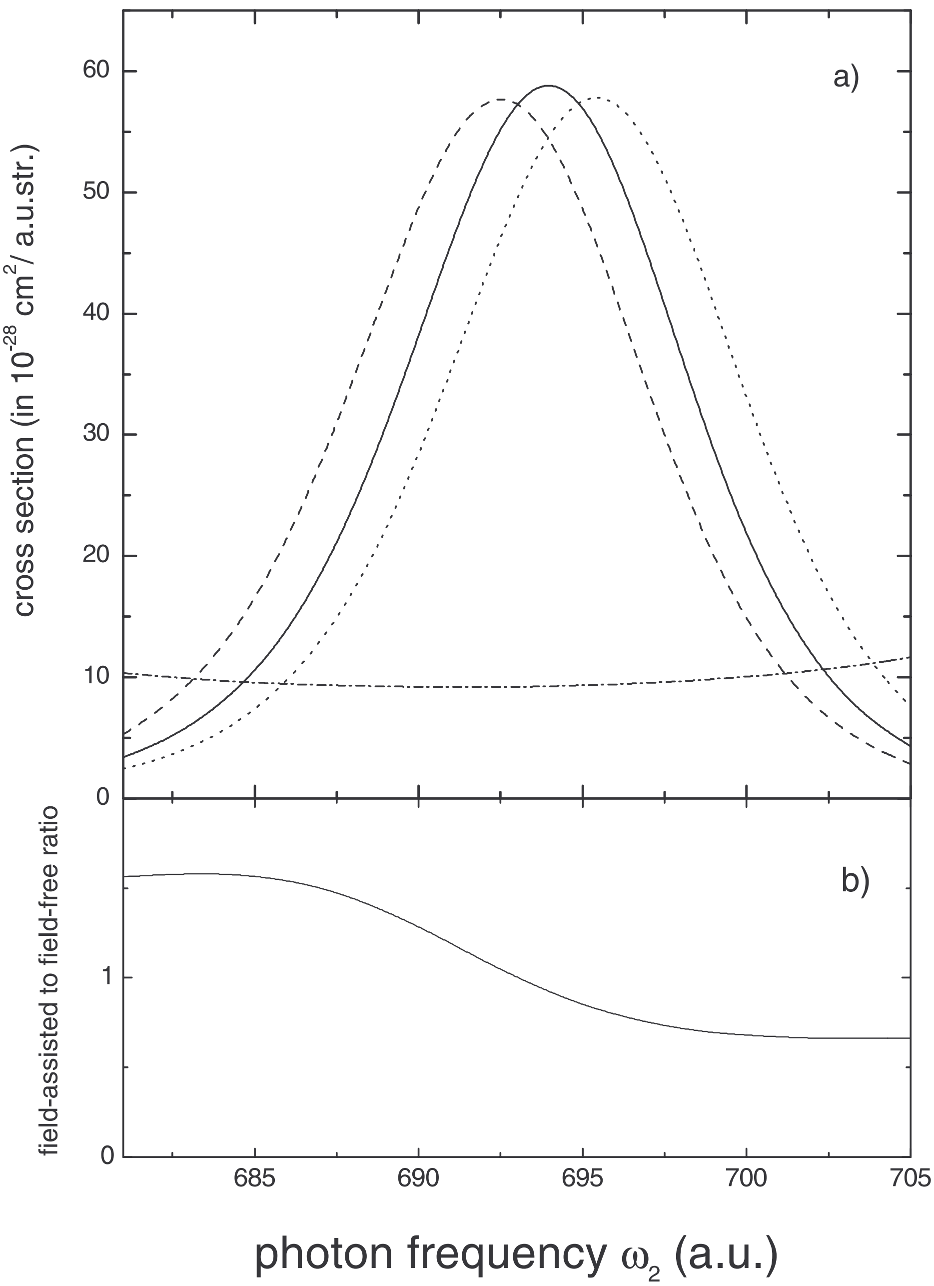

Figure 4

FIG. 4. a) Energy spectrum of photons scattered on hydrogen target which is initially in the ground state. Parameters of the incident photon are the same as in figure 3. Photon scattering angles are fixed: $\vartheta_2 = 180^0$. Integration over all possible states of the emitted electron has been performed. Polarization of the laser field and its frequency are the same as in figure 2. Solid curve: field-free Compton scattering. Dash curve: Compton scattering assisted by a laser field with $F_0 = 0.01$ a.u.. Dot curve: calculated spectrum for $F_0 = 0.01$ a.u. when the ponderomotive energy is neglected. Dash-dot curve: calculated spectrum for $F_0 = 0.01$ a.u. when the gauge factor is ignored. b) Ratio of the field-assisted to the field-free photon spectra (the ratio of the dash to solid curves in the part a) of the figure).