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A Method for Computing the Fraction of Attributable Risk Related to Climate Damages

Carlo C. Jaeger,[1]* Jette Krause,[1] Armin Haas,[1] Rupert Klein,[2] and Klaus Hasselmann[3]

The recent decision of the U.S. Supreme Court on the regulation of CO_2 emissions from new motor vehicles[(1)] shows the need for a robust methodology to evaluate the fraction of attributable risk from such emissions. The methodology must enable decisionmakers to reach practically relevant conclusions on the basis of expert assessments the decisionmakers see as an expression of research in progress, rather than as knowledge consolidated beyond any reasonable doubt.[(2,3,4)] This article presents such a methodology and demonstrates its use for the Alpine heat wave of 2003. In a Bayesian setting, different expert assessments on temperature trends and volatility can be formalized as probability distributions, with initial weights (priors) attached to them. By Bayesian learning, these weights can be adjusted in the light of data. The fraction of heat wave risk attributable to anthropogenic climate change can then be computed from the posterior distribution. We show that very different priors consistently lead to the result that anthropogenic climate change has contributed more than 90% to the probability of the Alpine summer heat wave in 2003. The present method can be extended to a wide range of applications where conclusions must be drawn from divergent assessments under uncertainty.

KEY WORDS: Bayesian learning; climate change; climate damages; heat wave; fraction of attributable risk

1. INTRODUCTION

With its decision of April 2, 2007, on *Massachusetts et al. v. Environmental Protection Agency et al.*, the U.S. Supreme Court has given new relevance to the task of assessing the fraction of climate-related risks that can be attributed to human actions. A group including state and local governments as well as private organizations had challenged the EPA's denial to regulate the emissions of greenhouse gases under the Clean Air Act. In its opinion, the Supreme Court states that "the rise in sea levels associated with global warming has already harmed and will continue to harm Massachusetts. The risk of catastrophic harm, though remote, is nevertheless real. That risk would be reduced to some extent if petitioners received the relief they seek. We therefore hold that petitioners have standing to challenge the EPA's denial of their rulemaking petition."[(1,p.23)] This reinforces the claim that "[t]here is an urgent need to develop improved methods and tools of climate impact assessment (such as the use of probabilities and Bayesian analysis)."[(5,p.1415)] A key challenge for such a methodology is the need to reach conclusions on the basis of conflicting expert judgments, and to do so with limited resources.

2. BAYESIAN ASSESSMENT OF THE FAR

A suitable methodology can be developed with the help of Bayesian decision theory (see Refs.

1 Potsdam Institute for Climate Impact Research, Potsdam, Germany.
2 Freie Universität Berlin, Department of Mathematics and Computer Science, Berlin, Germany.
3 Max Planck Institute for Meteorology, Hamburg, Germany.
* Address correspondence to Carlo Jaeger, Potsdam Institute for Climate Impact Research, Telegrafenberg A31, 14473 Potsdam, Germany; tel: +49-331-288-2601; carlo.jaeger@pik-potsdam.de.

6–8 for the use of Bayesian techniques in climate research, Ref. 9 for a discussion of decision theory related to climate change, and Refs. 10 and 11 for a Bayesian approach to decision theory). Different expert claims about a sequence of risks are then regarded as hypotheses that may guide action dealing with those risks. By assigning mixture weights to those hypotheses one obtains a convex set of further hypotheses that may guide action as well.

Let X_{t+1} be some damaging event the decisionmaker is interested in. In this case, this will be an event that may be influenced by climate change in ways that are not fully understood. The probability of that event can be assessed with the hypotheses the decisionmaker is willing to consider as well as with mixtures of these. On the basis of available new evidence, the decisionmaker can update the initial weights she assigns to the hypotheses. Using these updated weights, she can then assign revised probabilities to the event X_{t+1}. Call $P_N(X_{t+1} \mid e_t)$ the probability she assigns to that event on the basis of the evidence e_t available at time t, but taking into account only hypotheses without anthropogenic climate change. Call the probability she assigns to the same event on the basis of the same evidence but including also the hypotheses with anthropogenic climate change $P_A(X_{t+1} \mid e_t)$. As explained in the Appendix, the FAR, the fraction F_t of attributable risk, can then be computed as:

$$F_t = \frac{P_A(X_{t+1} \mid e_t) - P_N(X_{t+1} \mid e_t)}{P_A(X_{t+1} \mid e_t)}. \tag{1}$$

To take advantage of Bayesian learning when assessing the FAR of some critical event with regard to climate change, the following steps are necessary. First, identify a sequence of data that can be used to test a set of models, and define a critical event the FAR for which is to be estimated. Second, define a set of model hypotheses including both hypotheses under the assumption that there is and that there is no anthropogenic climate change. Third, define initial weights for the different model hypotheses and update these weights on the basis of the sequence of evidence. And fourth, compute the probability of the critical event for the different hypotheses and the FAR.

3. AN APPLICATION TO CLIMATE CHANGE AND THE SWISS SUMMER HEAT WAVE

It seems promising to analyze the Alpine summer heat wave with this methodology. From 2002 to 2003, Swiss summer temperatures jumped by 3.6°C from 18.7°C to 22.3°C (see Fig. 1). The 2003 heat wave increased mortality in Switzerland by 7%. The statistics of the 1,000 additional deaths show that by no means all can be attributed to people already being in bad health.[12] Throughout Europe, the 2003 heat wave caused about 35,000 people to die, many in an

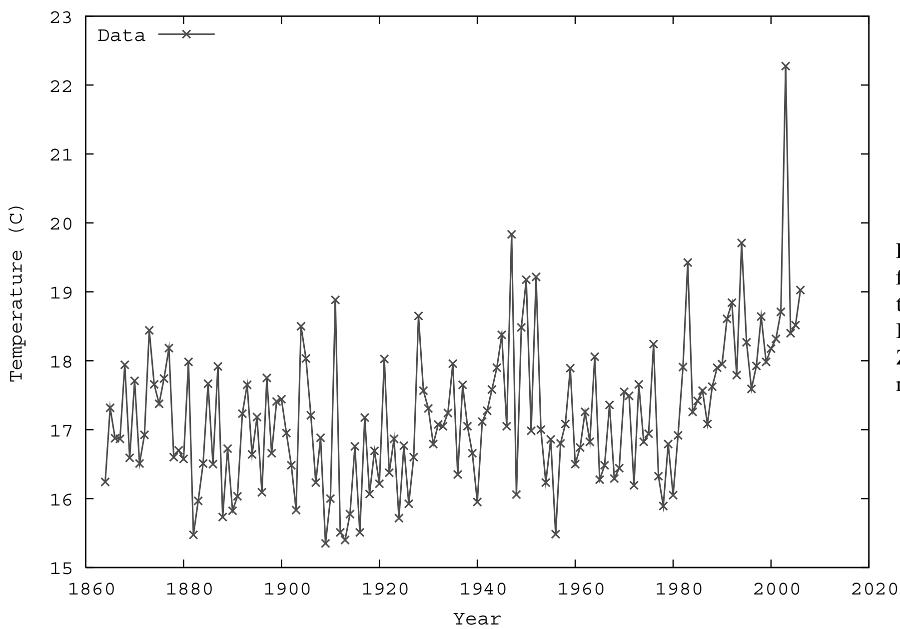

Fig. 1. Swiss mean summer temperature from 1864 through 2006, averaged over the four stations Basel-Binningen, Bern-Liebefeld, Genève-Cointrin, and Zurich and over the three summer months June, July, and August.

undignified manner.[13] The question arises whether the extreme 2003 summer temperatures and thus these deaths can be attributed to climate change.

Different authors have shown that the 2003 summer heat wave exhibited characteristics resembling those projected to occur more frequently by the end of the 21st century under scenarios of anthropogenic climate change.[14–18] Multimodel multiscenario simulations predict that in the absence of effective mitigation measures the frequency of occurrence of extremely warm seasons will rise remarkably in many parts of the world by the end of the 21st century.[19] Investigations of temperature records and model simulations suggest that there is an anthropogenic influence not only on recent warming,[20–23] but also on temperature extremes.[24] Applying time series analysis to German summer surface air temperature, Schönwiese *et al.*[25] find that the probability of summer temperature anomalies in the range of those experienced in 2003 has increased by a factor of 20 since 1760 due to a progressive warming trend since 1870.

With regard to the 2003 heat wave in Europe, Stott *et al.*[14] have used climate model simulations to quantify the fraction of risk attributable to human influence. Using a temperature threshold that was exceeded in 2003, but in no previous year since the beginning of instrumental records, they offer a lower bound of 0.5 as their assessment of the fraction of attributable risk that European summer temperature would exceed that threshold in 2003. As regards the Alpine region, however, their study misses a key feature of the empirical record, namely, the increasing variability of summer temperatures around their long-term mean—perhaps because the area selected was too large and the data grid too coarse in this respect.[26] In the case of global mean temperature, the anthropogenic signal has clearly been demonstrated to lie well above the natural variability noise—independently of recent controversies over the precise level of the natural variability background.[27–29] However, on regional scales, the anthropogenic influence is more difficult to detect with statistical confidence, particularly with respect to extreme events. In view of the unavoidable scientific uncertainties, any attempt to assess the risks of anthropogenic climate change must in this case depend on subjective judgments and is thus amenable to a Bayesian approach.

3.1. Hypotheses on Temperature Development

"Summer 2003 was the hottest in Europe since 1500, very likely due in part to anthropogenic climate change."[30,p.1483] Our goal is to specify the phrases "very likely" and "in part" in quantitative terms. For this purpose, we consider the following four model hypotheses as well as mixtures of these. Each one of the four hypotheses assumes normally distributed random variables with no interannual correlation; their mixtures, however, include the possibility of nonnormal distributions and autocorrelated values. Each hypothesis corresponds to one of four patterns that may be used in the description of regional climate change.

H1: There is no climate change, just short-term random fluctuations. The model consists of a random variable distributed around the mean of the years 1864–2002 with a standard deviation (*SD*) equal to the *SD* of the same period.

$$\tau_1 = 17.15 + \varepsilon, \quad \varepsilon \sim \mathrm{N}(0, 0.945)$$

H2: There is a modest constant warming trend that may be explained by natural causes; interannual random fluctuations as in H1 are superimposed. The model consists of a random variable distributed around a linear trend leading to a temperature increase of about 0.3°C over the past 150 years. This corresponds to the assessment that "solar forcing may have contributed about half of the observed 0.55°C surface warming since 1860."[31,p.3195] As both changes in solar forcing and changes in mean summer temperatures have been relatively small, linearization appears justified to describe their relationship. As the initial value for the trend we use the average of the first 30 years, corrected for the model slope. (Let u be the average temperature of the first 30 years and a the slope according to the hypothesis under consideration. Then the initial value is set to $u - 15a$. We will use the same procedure for the initial values of H3 and H4 as well as for the initial value for the standard deviation in H3.)

$$\tau_2 = 16.952 + 0.002t + \varepsilon, \quad \varepsilon \sim \mathrm{N}(0, 0.924)$$

H3: Anthropogenic climate change leads to a significantly stronger warming trend that steadily increases Alpine summer temperatures; interannual random fluctuations increase linearly with time. The model consists again of a random variable distributed around a linear trend, starting with the average temperature of the first 30 years, but now we let the trend end with the average temperature of the last 30 years.

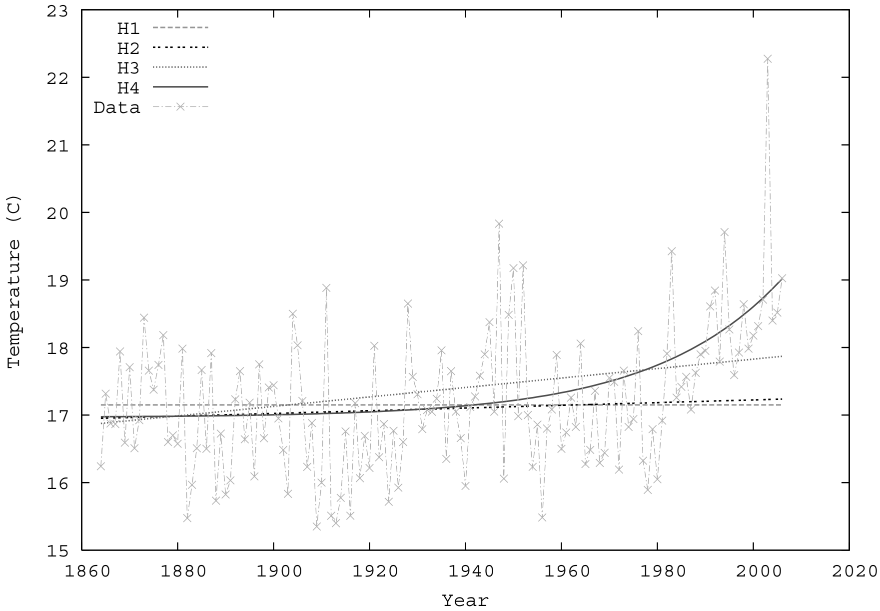

Fig. 2. Expected temperature trends from 1864 through 2006 according to H1 to H4 along with Swiss mean summer temperature data.

This corresponds to an average annual temperature increase of 0.007°C, roughly consistent with the IPCC third assessment: "The best estimate of global surface temperature change is a 0.6°C increase since the late 19th century."[22,p.101] Theoretical analyses predict a logarithmic increase of global mean temperatures with increasing greenhouse gas concentrations.[32]

As the greenhouse gas content of the atmosphere is observed to increase exponentially,[33] a linear global temperature response would accordingly be expected. H3 assumes that regional changes over Switzerland directly reflect this global trend. In accordance with the widespread view that anthropogenic climate change is accompanied with increasing climate variability,[34] we let the *SD* increase linearly from the *SD* of the residuals in the first 30 years to the *SD* of the residuals in the last 30 years.

$$\tau_3 = 16.876 + 0.007t + \varepsilon, \quad \varepsilon \sim \mathrm{N}(0, \sigma_3(t)),$$
$$\sigma_3(t) = 0.798 + 0.0006t.$$

H4: Anthropogenic climate change is raising Alpine summer temperatures at an accelerating pace; the variance of interannual random fluctuations remains constant. In this case a least-square exponential trend is fitted to the deviations from the mean temperature of the first 30 years, starting with a random fluctuation of 0.01°C. With this trend, the *SD* of the residuals from the exponential fit does not increase (it decreases slightly by $-3 * 10^{-4}$°C p.a.). We set it equal to the *SD* of the residuals. The model behind this hypothesis may be interpreted as follows: a deterministic linear instability is driving the regional mean summer temperatures away from their preindustrial levels, resulting in an exponential growth. Superimposed one finds fluctuations with time independent variance, which indicates that the mean trend and the fluctuations are likely driven by independent mechanisms.[4]

$$\tau_4 = 16.965 + 0.01 * \exp(0.0375t) + \varepsilon, \ \ \varepsilon \sim \mathrm{N}(0, 0.868)$$

For a representation of the four hypotheses, see Fig. 2.

3.2. Bayesian Updating of Hypotheses' Weights

Bayesian learning starts with a set of priors, and different decisionmakers—as well as different scientists—usually hold different priors with regard to a given problem. We, therefore, compare two sets of

[4] H4 was motivated by preliminary analyses that aimed at finding a transformation of the temperature data into a new time series of the transformed variable that would be approximately independent, identically distributed (iid). For such time series, De-Finetti's theorem provides a rigorous framework for the present Bayesian learning procedure.[35] We found that an autoregressive moving average process, fitted to the time series of the logarithms of annual temperature growth rates, $\mathrm{Log}[(T_{i+1} - T_i)/T_i]$, provided such a transformation with high accuracy. We will report on details of this study in a forthcoming publication. Moreover, a more detailed analysis of the interactions between an accelerating warming trend and the levels of interannual variability observed in the last few decades seems warranted.

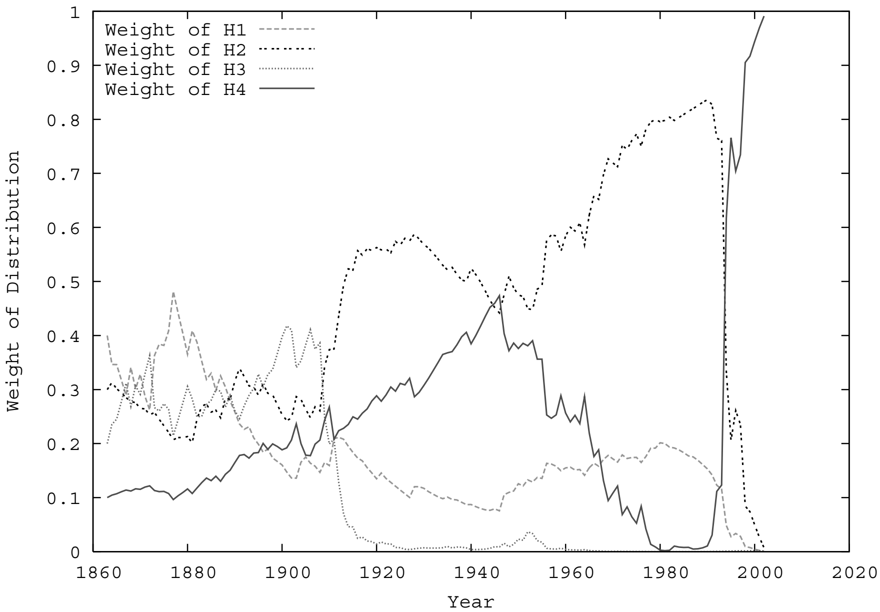

Fig. 3. Results from updating with nonanthropogenic oriented priors, starting from initial weights of 0.4 (H1), 0.3 (H2), 0.2 (H3), and 0.1 (H4) in 1863. Updating is carried out according to Bayes's rule (see the Appendix) using Swiss summer temperature data as displayed in Fig. 1.

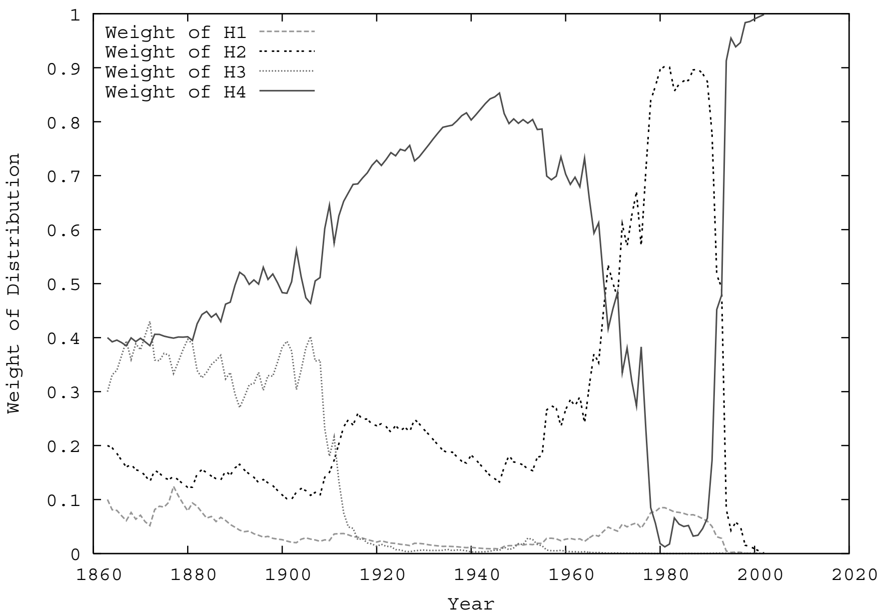

Fig. 4. Results from updating with anthropogenic climate change oriented priors, starting from initial weights of 0.1 (H1), 0.2 (H2), 0.3 (H3), and 0.4 (H4) in 1863.

initial priors representing two very different points of view on climate change. The first point of view, called "nonanthropogenic oriented priors," assigns weight 0.4 to H1, 0.3 to H2, 0.2 to H3, and 0.1 to H4. The second point of view uses the reverse order, we, therefore, call those priors "anthropogenic climate change oriented priors." Both priors can now be adjusted according to a process of Bayesian updating (see the Appendix). The resulting shift in the approximated weights is represented in Figs. 3 and 4.

By 1980, the differences between the two sets of priors have largely faded out. In 2002, for the nonanthropogenic oriented priors the (rounded) weights for the four hypotheses are $5 * 10^{-4}, 7 * 10^{-3}, 1 * 10^{-3}$, and 0.991. For the anthropogenic climate change oriented priors the weights are $3 * 10^{-5}$, $1 * 10^{-3}$, $4 * 10^{-4}$, and 0.998.

3.3. The Critical Event and its FAR

In order to compute the probability of the critical event under each hypothesis, we need to define that event in statistically meaningful terms. The summer temperature of 2003 exceeded the mean of the preceding 100 years by nearly 5 *SD* of those 100 years. We define the critical event as a summer temperature in 2003 exceeding the mean of the preceding 100 years by at least 4 *SD*, that is, a temperature of at least 21.26°C. Under the different hypotheses, the probabilities for such an event are $7 * 10^{-6}$, $6 * 10^{-6}$, $5 * 10^{-5}$, and $3 * 10^{-3}$. Using the weights from Bayesian learning, the overall probability for the critical event is the same (rounded to four digits) for both perspectives, namely, 0.0029. Under the assumption that there is no anthropogenic climate change, the resulting probability is $6 * 10^{-6}$ for both perspectives. This leads to a FAR of 99.8% for both the nonanthropogenic and the anthropogenic climate change oriented priors.

4. CONCLUSIONS

Three main conclusions can be drawn. First, the present analysis supports the claim by Stott *et al.*[14] that the FAR of the 2003 heat wave is larger than 0.5—and it supports the much stronger claim of a FAR of 0.9 and more. Second, the analysis shows that this finding represents a consensus that can be reached from widely diverging starting points. Third, the method of Bayesian learning can be used to track the evolution of probability assessments based on the accumulation of additional evidence and to compute a resulting FAR for possible climate damages.

The FAR is a parameter of considerable practical importance. Insurance contracts, for example, may specify payment of a fraction of damages depending on FAR estimates. Moreover, it is standard practice in liability trials to ground decisions on compensation payments on some assessment of what proportion of damage can be attributed to a specific cause. This leads to one important advantage of the proposed Bayesian learning method compared with simple model fitting. The latter can identify a best model, and the fit can be improved once additional data become available. But simple model fitting cannot assess the FAR, and it is not applicable to small data sets because it cannot use priors.

Of course, Bayesian learning can only be as good as the hypotheses available to start with. In this case, Figs. 3 and 4 display a rapid shift in the weights of H3. In the history of climate research, there has been a related shift from a debate about the dangers of global cooling due to natural cycles[36] or even due to human influence[37] to the current debate about global warming. In climate science, this shift was based on a new model of climate change that combined a long-term warming trend related to greenhouse gases with a temporary cooling caused by aerosols, discarding the possibility of a rapid end of the current interglacial.[38] This yields a pattern that cannot be produced by a mixture of the hypotheses that were formulated before 1970. We restrict ourselves to those hypotheses. However, including the more complex pattern postulated by Mitchell[38] would strengthen, not weaken, our assessment of the FAR for the 2003 Alpine heat wave.

Volatile patterns of Bayesian learning can show decisionmakers that the set of proposed hypotheses may neglect some important unknown mechanism. But developing additional hypotheses is a business for experts, and decisions may well need to be taken before experts have developed a satisfactory set of hypotheses. For practical applications of the proposed method, then, it is important to distinguish situations where the available hypotheses yield a robust assessment and those where this is not the case. Robust here means stable through time and across different priors. As long as this is not the case, risk-averse decisionmakers may be well advised to focus on the worst scenario, following the course of action that minimizes maximum conceivable damage. When a robust assessment is possible, standard risk management practices that balance risks and opportunities are more appropriate.

In the case of the 2003 Alpine heat wave, attributing a FAR of at least 90% to anthropogenic climate change is a robust assessment. An interesting case that may or may not lead to a similar assessment is the one of hurricane damages like those caused by Hurricane Katrina. Investigating such cases with the present method is a promising avenue for further research.

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Liebefeld to Zollikofen, which may lead to slight deviations in measurements. The authors declare that they have no competing financial interests.

APPENDIX ON METHODS

A.1. Bayesian Updating of Prior Probabilities

The method of updating subjective probabilities by means of available evidence is governed by Bayes's celebrated formula, based on the symmetrical relation

$$P(h \mid e) * P(e) = P(e, h) = P(e \mid h) * P(h) \quad \text{(A.1)}$$

for conditional probabilities. Here, evidence (e) and hypothesis (h) are both treated as possible propositions that may or may not turn out to be true. Equation (A.1) specifies the probability $P(e, h)$ that the evidence and hypothesis are both true. It implies that, given the probability $P(e \mid h)$ that the evidence e is true (i.e., that a prediction is verified) under the condition that the hypothesis h is true, one can infer the reciprocal $P(h \mid e)$ that the hypothesis h is true for the case that the evidence e is true. To update the *a priori* probability $P(h)$ (the "prior"), however, one needs to know in addition the total probability $P(e)$ that the evidence is true, independent of the truth of the hypothesis h.

Unfortunately, this information is not always available. In our application to the Swiss summer heat wave, for example, a given hypothesis h consists of a particular model of the evolution of Swiss summer temperatures, and the hypothesis test e is a predicted temperature measurement. Assuming that the model is correct, one can determine the probability $P(e \mid h)$ of verifying the prediction. However, the overall probability of verifying the prediction $P(e)$, independent of the particular model hypothesis, is not known, as the correct model, among the infinite set of all conceivable summer temperature models, is not known.

The usual application of Bayes's theorem is therefore not to update the absolute probability of a single hypothesis, but rather to update the relative probabilities of two (or more) competing hypotheses.[8] These relative probabilities can be looked at as mixing weights, approximating the correct model by a combination of the given ones. Applying Equation (A.1) to the ratio of the (relative) probabilities of two hypotheses h_1, h_2, one obtains:

$$\frac{P(h_1 \mid e)}{P(h_2 \mid e)} = \frac{P(e \mid h_1)}{P(e \mid h_2)} * \frac{P(h_1)}{P(h_2)}. \quad \text{(A.2)}$$

Thus, the posterior ratio of the probabilities of the two hypotheses is modified relative to the prior probability ratio by the Bayes factor $B = P(e \mid h_1)/P(e \mid h_2)$.[39]

To analyze the 2003 Alpine heat wave, we generalize Equation (A.2) to a set of n hypotheses $h_{1\le i\le n}$, which we subdivide into k hypotheses $h_{1\le i\le k}$ of climate variations without anthropogenic influence, and $n{-}k$ alternative hypotheses $h_{k<i\le n}$ including such influence. We consider, furthermore, a sequence of updated *a posteriori* relative probabilities $P(h_i \mid e_t)$ based on a sequence of evidences e_t that become available at times $t = 1, \ldots, T$. The relative probabilities, by definition, sum to unity, $\sum_{i=1}^{n} P(h_i \mid e_t) = 1$. Finally, as evidence we require that the model-predicted temperatures agree with the observed temperatures within a given infinitesimal increment $\Delta\tau$. This implies that the probability that the evidence is true is infinitesimal, but as Equation (A.2) involves only the ratio of probabilities, the formalism can be applied both to normal probabilities P and probability densities p.

Forming the ratio of the posterior relative probability of the hypothesis h_i to the posterior sum of relative probabilities one then obtains the algorithm for updating these probabilities:

$$P(h_i \mid e_{t+1}) = \frac{p(e_{t+1} \mid h_i) * P(h_i \mid e_t)}{\sum_{j=1}^{n} p(e_{t+1} \mid h_j) * P(h_j \mid e_t)}. \quad \text{(A.3)}$$

This algorithm allows updating the weight of each hypothesis on the basis of new evidence. As an example, this is how it can be applied for updating the weight of H1 according to the first available summer temperature measurement of 1864. We start at 1963, that is, $t = 0$, define a set of hypotheses (H1 to H4, as described above), and assign initial weights to them, in this example the nonanthropogenic oriented priors given in Section 3.2:

$$P(h_1 \mid e_0) = 0.4; \quad P(h_2 \mid e_0) = 0.3; \quad P(h_3 \mid e_0) = 0.2; \quad P(h_4 \mid e_0) = 0.1$$

We then consider the first data point, Swiss summer temperature of the year 1864, $e_{t+1} = e_1 = 16.2417°C$. Now, the probability of this temperature event within each of the four hypotheses has to be calculated, that is, the probability density for the value e_1 has to be computed for each hypothesis. As the present hypotheses take the form of trends with normally distributed deviations, probability density for an event e_{t+1} within a hypothesis h_i is given as $p(e_{t+1} \mid h_i) = \frac{1}{\sigma_i \sqrt{2\pi}} * e^{-\frac{1}{2}(\frac{e_{t+1}-\mu_i}{\sigma_i})^2}$, where μ_i, σ_i are specified by hypothesis h_i. As H1 is $\tau_1 = 17.15 + \varepsilon,\ \ \varepsilon \sim \mathrm{N}(0, 0.945)$, $\mu_1 = 17.15$, and $\sigma_1 = 0.945$. (In fact, as for H1, neither the trend value nor deviations from the trend change over time, these values remain constant for all updating steps.)

Thus, $p(e_1 \mid h_1) = \frac{1}{0.945\sqrt{2\pi}} * e^{-\frac{1}{2}(\frac{16.2417-17.15}{0.945})^2} = 0.2660$. Analogously, in correspondence with the definitions of H2, H3, and H4, for $t = 1$

$$p(e_1 \mid h_2) = \frac{1}{0.924\sqrt{2\pi}} * e^{-\frac{1}{2}\left(\frac{16.2417-16.954}{0.924}\right)^2} = 0.3208,$$

$$p(e_1 \mid h_3) = \frac{1}{0.7986\sqrt{2\pi}} * e^{-\frac{1}{2}\left(\frac{16.2417-16.883}{0.7986}\right)^2} = 0.3619,$$

and

$$p(e_1 \mid h_4) = \frac{1}{0.868\sqrt{2\pi}} * e^{-\frac{1}{2}\left(\frac{16.2417-16.975}{0.868}\right)^2} = 0.3216.$$

Thus, the updating procedure yields as a posterior weight for H1 after the first updating step: $P(h_1 \mid e_1) = \frac{0.2660*0.4}{0.2660*0.4+0.3208*0.3+0.3619*0.2+0.3216*0.1} = 0.3464$.

The same procedure is carried out for all three hypotheses at each updating step. For more details, see the supplementary material provided.

A.2. Calculating the Fraction of Attributable Risk

Having updated the probabilities of the hypotheses based on the sequence of evidence, one can then compute the probabilities of observing a particular damaging event X_{t+1} under the assumption of either no anthropogenic influence (P_N) or including such influence as well (P_A):

$$P_{N/A}(X_{t+1} \mid e_t) = \frac{\sum_{i=1}^{k/n} P(X_{t+1} \mid h_i) * P(h_i \mid e_t)}{\sum_{i=1}^{k/n} P(h_i \mid e_t)}. \tag{A.4}$$

On the basis of these definitions, it is then natural to define the FAR, the fraction F_t of attributable risk, as in Equation (1). If climate change does indeed increase $P(X_{t+1})$, a number between 0 and 1 results that can be used for purposes of damage attribution. If climate change actually decreases $P(X_{t+1})$, no damage attribution problem arises and a negative number without lower bound results. If in this case one wants to attribute a fraction of the benefit resulting from a lower $P(X_{t+1})$, it is sufficient to consider the complementary event $\overline{X}_{t+1}$ that X_{t+1} does not occur, and compute F_t for the resulting probabilities.

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ADDITIONAL MATERIAL

Additional information accompanies the article, available at http://www.pik-potsdam.de/research/research-domains/transdisciplinary-concepts-and-methods/aspik/Heatwave.