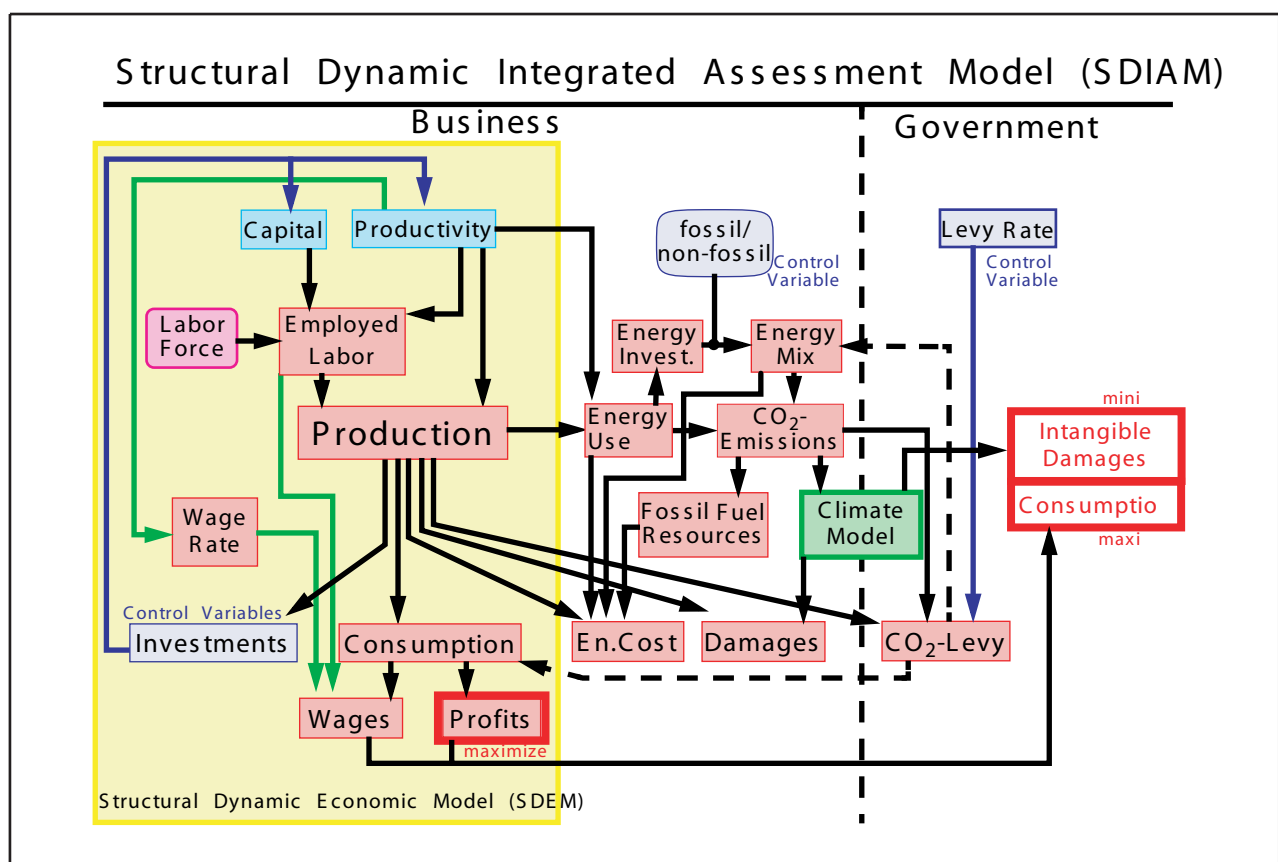




Examensarbeit Nr. 91



Integrated Assessment of Climate Change  
 Using Structural Dynamic Models

von

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Hamburg, Juni 2003

# Dissertation zur Erlangung des Doktorgrades

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# **Integrated Assessment of Climate Change Using Structural Dynamic Models**

Dissertation  
zur Erlangung des Doktorgrades  
der Naturwissenschaften im Fachbereich  
Geowissenschaften  
der Universität Hamburg

vorgelegt von

**Volker Barth**

aus  
München

Hamburg  
2003

Als Dissertation angenommen vom Fachbereich Geowissenschaften der  
Universität Hamburg

auf Grund der Gutachten von Herrn Prof. Dr. Klaus Hasselmann  
und Herrn Prof. Dr. Carlo Jaeger.

Hamburg, den 19.06.2003

Professor Dr. H. Schleicher  
(Dekan des Fachbereichs Geowissenschaften)

## Abstract

The interaction of society, economy and the climate system has been studied for the integrated assessment (IA) of climate change using a cost-benefit analysis (CBA). Structural dynamic models have been developed and applied to highlight two topics rarely addressed in current CBA/IA models: the different objectives that drive the actions of economic and societal actors, and the representation of economic growth. The models include an explicit description of the basic dynamic processes of the system, in contrast to the general equilibrium approach of standard economics.

The general Structural Dynamic Integrated Assessment Model (SDIAM) described in this thesis is developed from two basic models: the Structural Dynamic Economic Model (SDEM) and the impulse-response function climate model NICCS [Hooss *et al.* 2001]. SDEM focuses on the interaction of entrepreneurs and workers and describes growth as the consequence of profit-driven productivity increase. Entrepreneurs invest to enhance productivity, this generates profits that decay subsequently due to factor cost increases. Persistently positive profits require persistently growing productivity. Economic growth is achieved when part of the profits is also invested to enlarge the capital stock.

In SDIAM, society is introduced as an additional actor. Further extensions to SDEM are modules for energy costs, climate change (NICCS), and climate damage costs. Intangible climate damages enter directly in the societal welfare function, while tangible damages affect the entrepreneurial welfare function. To enhance its welfare, society influences economic decision-making by imposing levies on CO<sub>2</sub> emissions. Entrepreneurs optimize within the levy framework, which they take as given.

Numerical experiments with these models yield the following results. Without societal action, the transition from a fossil fuel based to a non-fossil driven energy system depends on the cost dynamics of fossil fuels and climate damage costs; it starts in the middle of the 21st century. Climate change damages are maximal around 2050 at 2.7% of GDP. When society imposes levies, these payments are included in the determination of the timing of the energy system transition. Levies lead to a reduction of emissions and climate damages, the magnitude depending on the way in which levy revenues are spent. However, if the same discount factors are applied uniformly to all costs, the net effect on societal welfare is negative, because the reduction of climate damages is overcompensated by losses in consumption. These losses result from entrepreneurial adaptations to higher costs related to energy use.

The contradiction of this result with recent societal efforts to curb greenhouse gas emissions (e.g. under the Kyoto protocol) is resolved if it is assumed that the valuation of climate damages in the societal welfare function increases with time relative to the value of consumption. This result confirms recent calls for differential discounting of climate change [Hasselmann *et al.* 1997, Hasselmann 1999].

Other features, like learning curves in the energy cost dynamics or unemployment in the societal welfare function, have been estimated but not yet been explicitly included in the present study.



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# Chapter 1

## Introduction

### 1.1 Problem Background

Over the past two decades, climate change and global warming have become an issue on the political agenda, both internationally and in many countries. People are concerned by the projected temperature increase, sea level rise and other consequences of climate change, which are considered, or at least expected, to be harmful to societies. Although nobody knows exactly what is going to happen, the general view is that “something has to be done”. The question is: What should be done? And when shall it be done?

These questions cannot readily be answered without clarifying another question beforehand: What is considered as a *good* solution? This is necessary because the particular difficulty of the climate change problem lies in its inherent linkage to the power source of the present economic system. The use of fossil fuels necessarily generates carbon dioxide, so that production of goods and services at the same time contributes to the so-called anthropogenic greenhouse effect. From a natural science perspective, the answer to the above questions is simple: “Reduce greenhouse gas emissions immediately!” However, this cannot easily be carried out without strongly affecting the functioning of current economies and thereby the way of life of today’s societies. Finding a “good solution” to handle climate change thus requires thorough consideration of the consequences and trade-offs of the measures to be taken. From an economic point of view, the questions above would be rephrased as: What can be done to reduce climate change as far as possible, while the effects on prosperity and wealth are still acceptable<sup>1</sup>? This is the core problem of the integrated assessment (IA) of climate change and is also the central question of the present thesis.

The classical tool of economics to solve this kind of problem is to conduct a cost-benefit-analysis (CBA). The basic strategy sounds simple: for each possible scenario, add all benefits (e.g. avoided climate damages), subtract all costs required to achieve these benefits (e.g. costs for more expensive energy sources), and compare the resulting net benefit (‘welfare’) for all scenarios. The scenario with the greatest welfare is the optimal solution. In the past decade this

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<sup>1</sup>The United Nations Framework Convention on Climate Change (UNFCCC) puts it as follows: “a stabilization of greenhouse gas concentrations [... to ...] prevent dangerous anthropogenic interference with the climate system [...] and to enable economic development to proceed in a sustainable manner.” [UNCED 1992]

approach has been widely used in integrated assessment models for policy optimization. Starting from the seminal models CETA [Peck and Teisberg 1992], DICE [Nordhaus 1993, 1994] and MERGE [Manne *et al.* 1993], today a great variety of models is in use (e.g. Weyant [1998]).

The simplicity of the CBA procedure makes it well suited for everyday problems in economic decision making. Its application to a long-term problem like climate change that affects more societal aspects than only its economic sphere, however, involves two fundamental problems, which have hitherto rarely been addressed. These are related to the construction of welfare functions in CBA models as well as to the representation of economic growth:

1. Neoclassical welfare theory (which is employed in virtually all CBA models so far) does not distinguish generally between the optimality criteria of economic decision makers and of society as a whole. In these models, the economic growth path is therefore determined by maximizing societal welfare through the control decisions of a hypothetical 'social planner'. However, economic and societal welfare should not be mixed, simply because economic decisions generally are *not* based on societal optimality criteria. Rather, different actor groups have different welfare criteria: entrepreneurs maximize profits, while employees maximize earnings and political decision makers of society are generally concerned about the well-being of all its members. Although economic prosperity is an important component of societal welfare, it is not all. Therefore, the assumption of **only one single decision maker and one single welfare function** for the whole socio-economic system is a misleading concept.
2. As climate change is a long-term process, the evolution and growth of the economy plays an important role in addressing future costs and benefits. The economic models employed in IA models are generally standard neoclassical growth models in which long run growth rates are based on the rate of **technological change, which is prescribed exogenously** by the modeller. Thus, feedbacks of climate change on the future evolution of the economy are poorly represented in these models and welfare effects due to changing growth rates cannot be addressed.

Besides these fundamental problems, the CBA method is subject to inherent limitations (cf. Tol [1996]):

- i. **Estimation** of climate impacts (which are used to assess costs and benefits) is made on the local and regional scale, which is still difficult to represent in state-of-the-art climate models, in particular global-scale general circulation models. Impact estimates are therefore unavoidably vague.
- ii. Cost and benefit assessments depend on the spatial and temporal **context** of the considered impacts. The most pronounced impacts of climate change may well occur in the second half of the 21st century. Nobody knows, the state of the affected future societies, so that the real impact is highly uncertain. This is generally solved by considering how climate change would affect a present-day society.
- iii. Another problem is the **aggregation** of regional costs and benefits to a global welfare function. This requires the assumption of relative weights between world regions which are difficult to estimate on a sound basis.

- iv. **Valuation** is a crucial issue in CBA, since costs and benefits have to be measured in a common metric in order to be traded against each other. Problems arise in the valuation of 'intangible' goods and services for which no markets exist (e.g. human health or landscape changes) so that their value has to be estimated. The usual concepts of 'willingness to pay' or 'willingness to accept compensation' are neither unanimous in themselves, nor is it clear which is appropriate to be employed to assess climate change. Related to that is the question whether all future assets should be devaluated using the same discount rate or if differentiations have to be applied [Hasselmann *et al.* 1997, Hasselmann 1999].

Despite these and other limitations, CBA is still considered to be a useful and relevant tool in the climate change context as it provides a method to objectify the assessment criteria. This objectivity requires that the criteria of valuation and assessment are clearly laid open in advance, so that the assessment process becomes transparent.<sup>2</sup>

## 1.2 Objectives of this Study

The present thesis primarily addresses the above-mentioned fundamental problems 1. and 2. by introducing a new approach in socio-economic modelling for the integrated assessment of climatic change based on *structural dynamic models*. These will be used to model the economic subsystem on the one hand and the interaction between society and the economic system on the other. The main objective of structural dynamic models is to find the right formulation for the system dynamics by modelling the basic dynamic interactions of the main actors in the system. Once this is achieved, such a model can easily be expanded to give a more complete picture through more detailed specific feedback mechanisms.

In combination with a fast but accurate impulse-response climate model (NICCS, [Hooss *et al.* 2001]) these models will be used to examine possible strategies of society to influence the economic evolution under climate change. The effect of these strategies on societal welfare, where material consumption is traded against intangible climate change impacts, is assessed by means of a cost-benefit analysis. The purpose of this study is to investigate under which circumstances societal measures to prevent climate change also have a beneficial effect on societal welfare. A secondary purpose is to find out whether structural dynamic models can be used to extend the scope of standard economic equilibrium models.

Two things should be noted in advance. First, uncertainties are an important issue in the context of climate change. However, it is not the purpose of this thesis to study the effect of uncertainty on decision making. The models used here are purely deterministic, and the consequences of uncertain parameters are investigated only through sensitivity studies. Second, since the technique of structural dynamic models is a new concept, only some fundamental effects will be considered here. Further refinement is left to future studies. In this sense, the present thesis is a conceptual study.

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<sup>2</sup>Note that this does not mean that the result is 'better' or 'more correct' than results from other methods. Alternative subjective assessment methods often arrive at more feasible recommendations or decisions, because the set of valuation and assessment criteria is implicitly clear to the involved persons but these generally find it hard to explicitly pin down their valuations with the exactness required for objectification. See Hisschemöller *et al.* [2001] for an overview.

### 1.3 Outline

The outline for the study is as follows. Chapter 2 introduces the Structural Dynamic Economic Model (SDEM), which represents the first-order economic subsystem of the full climate-socio-economic system. The central focus here is on the interaction between entrepreneurs and workers/employees, which is the source of economic growth. Additionally, it includes the determination of employment levels.

The third chapter explains the additional modules besides SDEM that form the Structural Integrated Assessment Model (SDIAM). The modules for energy conversion, energy and climate damage costs and society are specifically designed for SDIAM, while the nonlinear impulse-response climate model NICCS is taken from Hooss *et al.* [2001].

Chapter 4 focuses on the behaviour of SDIAM. Starting with SDEM from chapter 2, the constraints that follow from the 'world model' character of SDIAM are introduced: limited fossil resources and climate damages. The adverse effects that these constraints have on economic evolution are overcome by introducing the possibility to switch to a non-fossil energy source, however, at higher energy costs. A Constrained World Baseline (CWB) scenario is introduced as the reference point for further studies. Some sensitivity studies conclude this chapter.

The impact of societal regulation policies are the focal point of chapter 5. First, the societal welfare function is introduced. Next, the consequences of imposed emission levies are considered for the economic system, for societal welfare, and for employment. This is investigated for two levy recycling schemes. The results are first derived for equally discounted damages. Subsequently, the consequences of differential discounting in the societal welfare function are considered. Finally, the effect of reduced non-fossil energy costs is examined.

The final chapter 6 summarizes the approach and the main findings of the study and gives an outlook.



# Chapter 2

## The Structural Dynamic Economic Model (SDEM)

### 2.1 Introduction

Understanding economic growth has been a challenge for economic theory since its very beginnings. The difficulty was to incorporate the dynamic behaviour of optimizing economic agents over time into the theory of general equilibrium, which is essentially static. The approach taken by the standard neoclassical theory is to assume that the economy is already in general equilibrium and then ask how the investments should be chosen in order to find the so-called 'steady state' solution, where all variables grow at constant rates. The appropriate investment rate is either set exogenously in the Solow-Swan model [Solow 1956, Swan 1956], or derived endogenously by maximizing the inter-temporal utility of consumption [Ramsey 1928, Cass 1965, Koopmans 1965].

All of these models have the same shortcoming: economic growth exhibits declining rates and ceases eventually unless some explicitly growing factor (usually labelled 'technological progress') is introduced exogenously. The growth of this factor remained unexplained in the models mentioned above. More recently, 'endogenous growth theory' has attempted to explain technological progress by incorporating knowledge creation and dissemination, inspired by the seminal work of Romer [1986, 1987]. Despite a large variety of interesting approaches have been developed, a concise answer is still missing; observed growth rates can be explained by several mechanisms and the relevant one or the appropriate mix has not yet been determined.

The Structural Dynamic Economic Model (SDEM) presented in the following is a conceptual model that offers a new explanation by explicitly modelling the dynamic interactions of two major social groups: entrepreneurs and workers. The dynamic formulation explicitly allows for a system out of (general) equilibrium, which finally turns out to provide a mechanism for economic growth. The complexity of the system is reduced by focusing only on the most prominent interactions. A second non-conventional approach is the explicit modelling of the growth of (labour) productivity, although only in a very conceptual manner. Productivity is regarded as a proxy for technology, which determines the employed amounts of capital and labour. Changing the input shares of capital and labour requires an explicit change of technology, which, in

contrast to the neoclassical assumption, is neither costless nor instantaneous. Capital and labour are thus not substitutable at a given productivity, but productivity may change with time. These two features, non-equilibrium and non-substitutability, allow for profits as a growth incentive on the one hand and for unemployment on the other.

In order to focus on the dynamic core, SDEM is highly aggregated and consists of only one region with one sector producing only one uniform good. Two kinds of investment are distinguished: usual capital investments and investments that increase productivity. Both are controlled by entrepreneurs who have the opportunity to obtain positive profits by investing in productivity, which results in revenues exceeding (wage) costs. This pushes the system away from its 'equilibrium', which is understood in a natural sciences' perspective as a stationary, time-invariant state<sup>1</sup> where profits are zero. Market forces including the pressure of wage earners to increase wages tend to erode the profits and to restore the stationary state, but this process requires finite time. Since entrepreneurs aim at maximizing profits over time, the model economy is constantly pushed away from its stationary state and leads to a growth solution. This entrepreneurial striving for profits is thus the source for economic growth.

The following section 2.2 describes the basic model and the employed concepts. The behaviour of this model is analyzed analytically for two special cases in section 2.3. Although a general analytical solution could not be obtained, numerical experiments have been carried out and are presented in section 2.4. Both analytical and numerical calculations indicate that SDEM is too sensitive to small changes in dynamic parameters to reproduce the empirical data. The introduction of an unemployment feedback factor in section 2.5 offers a way to overcome this problem.

## 2.2 Model Setup

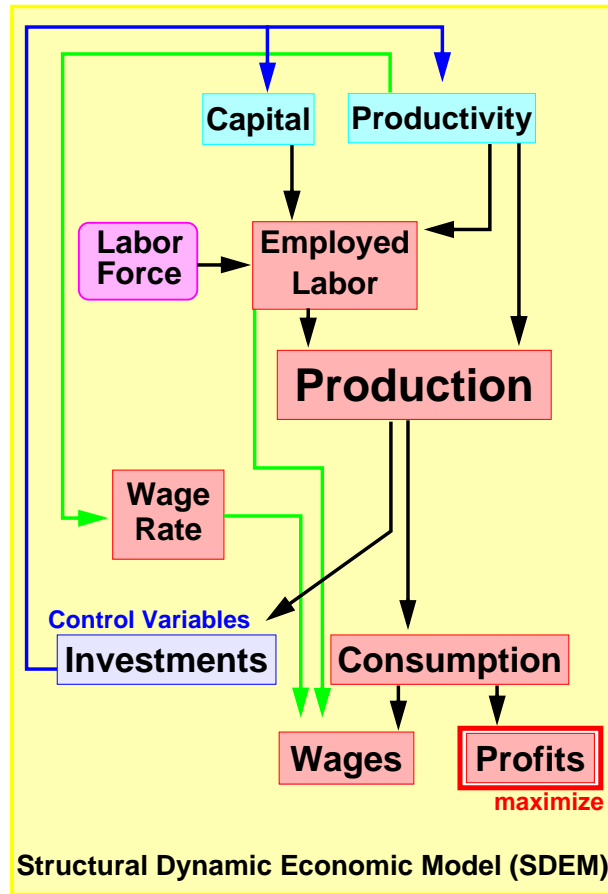
SDEM consists of a simple economy producing output which is used either for consumption or for investments. The allocation of output to consumption or investments goods occurs instantaneously and without friction so that supply and demand always balance<sup>2</sup>. In such an economy, the absolute price level is arbitrary and costs can be expressed in units of output. This economy is assumed to be closed, i.e. there is no interaction with foreign economies. Furthermore, there is no way to save money without simultaneously investing it.

SDEM simulates the dynamic behaviour of the basic actor groups in the economy: entrepreneurs and workers. Entrepreneurs run firms to produce output. They own the required capital but have to hire workers to utilize it. This assumption allows us to abstract from a capital market. Their intention is to maximize the profits or, rather, the net present value of the utility of these profits. They do this by investing part of the output in order to increase future profits. Workers on the other hand earn wages and use their social power (e.g. in trade unions) to keep the wage rate for each worker as high as possible. It is assumed that both actor groups can be aggregated to

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<sup>1</sup>Note that this definition is different from the 'general equilibrium' in economics, which is defined as the state where supply equals demand on all markets, with no explicit reference to time. It is also different from 'steady-state growth' in neoclassical growth theory, which is defined as a state where all quantities grow at constant rates.

<sup>2</sup>In economic terms this is a one-sector economy producing a uniform good that can be used simultaneously for consumption or investments. Supply and demand for that uniform good are assumed to be balanced.



**Figure 2.1:** Schematic dependencies in SDEM. Production output can be invested or consumed by workers (wages) or entrepreneurs (profits). The wage rate is determined dynamically by productivity, while entrepreneurs control the investments in capital and productivity in order to maximize the net present value of utility from profits. Productivity also determines the amount of employed labour. The labour force is prescribed exogenously. See sections 2.2.1 - 2.2.4 for details.

act as one representative entrepreneur or worker, respectively. A general sketch of the model is given in figure 2.1.

In the following, constants are denoted by greek letters, while latin letters indicate time-dependent quantities. Variables divided by the number of employed workers are marked by a hat ( $\hat{\cdot}$ ).

### 2.2.1 Time Evolution

We assume that an economy is completely described by three state variables: its stock of physical capital,  $k$ , its average labour productivity,  $\hat{p}$ , and the average wage per worker,  $\hat{w}$ . All other variables including employed labour,  $l$ , can be derived from those three. Their time evolution is controlled by the entrepreneurs via two forms of investments,  $i_k, i_{\hat{p}}$ , in capital and productivity, respectively, through the prognostic equations

$$\dot{k} = i_k - \lambda_k k, \quad (2.1)$$

$$\dot{\hat{p}} = \mu \frac{i_{\hat{p}}}{l}, \quad (2.2)$$

$$\dot{\hat{w}} = \lambda_w (\hat{w}^0(\hat{p}) - \hat{w}). \quad (2.3)$$

Equation (2.1) is the usual growth equation for capital that grows by investments in capital,  $i_k$ , and depreciates at the rate  $\lambda_k$ .

Investments in productivity,  $i_{\hat{p}}$ , cover all money that is spent to increase the productivity of workers. Since these expenses are assumed to be controlled by the entrepreneurs, we refer to them as investments, although in usual national accounts they will show up mainly as wages (of education and research staff). These investments increase  $\hat{p}$  according to (2.2), where  $\mu$  is an exogenous scaling factor that allows for a different effectiveness of one unit of investment in  $k$  and  $\hat{p}$ , respectively. Since productivity covers the technological state as well as the education and training of workers, its growth is likely to depend on many more variables besides invested money, and on some of these in a highly nonlinear way. However, in the overall effect, we assume that the scaling factor  $\mu$  is a fairly good approximation to these other dependencies. The factor  $l^{-1}$  appears because productivity is a per capita variable. Therefore, a given amount of investment  $i_{\hat{p}}$  will cause a greater increase of the productivity of each worker if that investment is distributed over less persons. We abstract from individual differences and consider only the average productivity.

The evolution (2.3) of the wage rate,  $\hat{w}$ , is driven by the difference between  $\hat{w}$  and some maximum wage rate,  $\hat{w}^0$ , which depends on productivity. The exogenous time constant  $\lambda_w$  describes the relaxation rate of this process, which is described in greater detail in section 2.2.3 below.

## 2.2.2 Production and Labour

To determine the amount of output, SDEM follows the pre-neoclassical approach of Leontief [1941] and assumes a well-defined relation between capital and employed labour. In contrast to Leontief [1941] it is assumed that this ratio is not a constant but is determined by productivity

$$\hat{p} = \nu \frac{k}{l}, \quad (2.4)$$

where  $\nu$  is a scaling constant. This assumption is supported by empirical findings from long time series of industrialized countries (Maddison [1982, 1995], Kummel [1998]) which show a relatively linear relation of production per worker (i.e. productivity) and capital per worker, at least in the long run. From those data sets it follows that the constant  $\nu$  is country-specific but generally in the order of  $\frac{1}{2}$  to  $\frac{1}{3}$ .

Production is simply derived from the definition of productivity

$$p = \hat{p} \cdot l. \quad (2.5)$$

Rearranging (2.4) gives the amount of employed labour

$$l = \nu \frac{k}{\hat{p}} \leq l_{max}, \quad (2.6)$$

where  $l_{max}$  denotes the total workforce which is prescribed exogenously. Thus, the employment rate is given as

$$q = \frac{l}{l_{max}}. \quad (2.7)$$

When (2.6) is inserted in (2.5) we obtain a second equation for production,

$$p = \nu k. \quad (2.8)$$

It is convenient to rewrite equations (2.5), (2.8) in the following limitational form

$$p = \min(\nu k, \hat{p}l), \quad (2.9)$$

where the minimum needs to be considered only when  $l = l_{max}$ , otherwise (2.5) and (2.8) are equivalent. Equation (2.9) tells us that at a given productivity level, production is restricted by the available amounts of either capital or labour. When there is not sufficient capital to employ the entire workforce,  $l < l_{max}$  and unemployment occurs. Idle capital is not explicitly excluded, but does not occur because the investments to build that idle capital will not generate production (revenues) and are thus avoided by profit optimizing entrepreneurs (see section 2.2.4).

Note that the introduction of productivity changes the capital-labour ratio of (2.9) compared to the standard limitational production function [Leontief 1941]. As in the standard function, equation (2.4) defines a fixed ratio of capital and labour at any given point in time. But as  $\hat{p}$  is not a constant and may increase over time, labour can be replaced by capital in the course of time. Capital deepening is thus an observable feature of SDEM, which was not present in previous applications of Leontief's [1941] production function (e.g. Harrod [1939], Domar [1946]).

### 2.2.3 Dynamic Wage Formation

An economy in which the produced output is completely used for consumption will not grow, since there is no way to build additional capital that would allow for more production in the future. Actually, it would feature shrinking production, because the capital stock depreciates at the rate  $\lambda_k$ . A stationary, time-invariant economy thus requires investments to maintain  $k$  at its current level, which set an upper limit to consumption. The maximum wage rate is obtained when profits are zero and given as

$$\hat{w}^0(\hat{p}) = \hat{p} - \lambda_k \hat{k} =: \hat{p}\alpha_w, \quad (2.10)$$

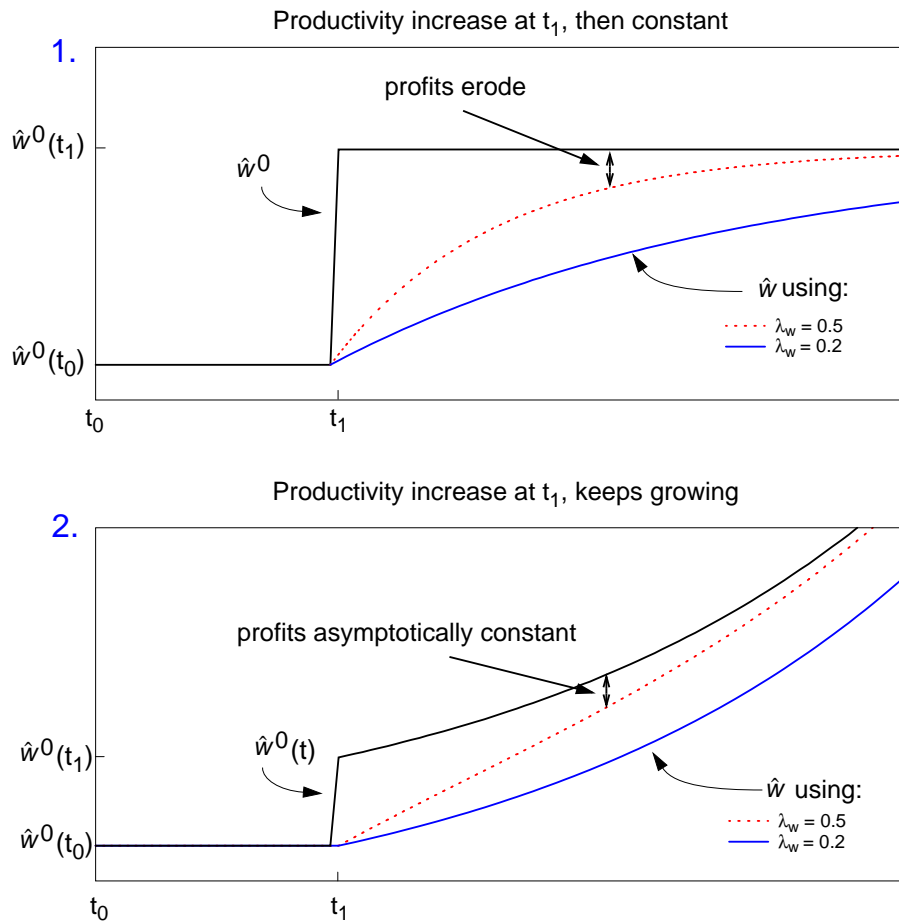
$$\text{where } \alpha_w = 1 - \frac{\lambda_k}{\nu} = \text{const.}, \quad (2.11)$$

which follows from equation (2.4). Of course, the maximum wage rate, i.e. the maximum possible wage per worker, depends on productivity, i.e. output per worker. An increase in productivity instantaneously generates higher output per worker, which also causes  $\hat{w}^0$  to rise. The actual wage rate, however, is sticky and follows this increase with a time lag characterized by the time constant  $\lambda_w$

$$\dot{\hat{w}} = \lambda_w (\hat{w}^0 - \hat{w}),$$

as has already been stated in (2.3). Growth in  $\hat{p}$  will thereby lead to wages below the maximum wage,  $\hat{w} < \hat{w}^0$ . This in turn offers a possibility to earn positive profits, which are defined as revenues less costs and investments

$$b = p - w - i_k - i_{\hat{p}}. \quad (2.12)$$



**Figure 2.2:** Dynamic wage rate adaptation mechanism. Panel 1 (top): A single increase in productivity raises the maximum wage rate  $\hat{w}^0$  once. The actual wage rate,  $\hat{w}$ , follows this increase with a time lag, characterized by the time constant  $\lambda_w$ . Greater  $\lambda_w$  leads to faster growth of  $\hat{w}$ . The difference between  $\hat{w}^0$  and  $\hat{w}$  goes to the entrepreneur and can be used as profits or investments. The difference erodes over time as  $\hat{w}$  approaches  $\hat{w}^0$  asymptotically. Panel 2 (bottom): When productivity (and  $\hat{w}^0$  with it) keeps growing, the difference  $\hat{w}^0 - \hat{w}$  approaches a constant but positive value. Persistently positive profits emerge.

Without productivity growth,  $\hat{w} \rightarrow \hat{w}^0$ , and profits vanish.

The idea behind this is the following (see also figure 2.2). Starting from a stationary state where profits are zero, the entrepreneur will act in order to obtain some profit. The capital replacement costs described by  $\alpha_w$  cannot be reduced, since otherwise the economy would decline and the emerging profits with it. Reducing actual wages is one option, but is difficult to assert. The way out is to invest in productivity leading instantaneously to higher output, while actual wages remain unchanged at first. Since workers now earn less than they could, this situation is not satisfactory for them and they will struggle to push the wage rate up again, e.g. by wage negotiations or strike. This process requires time, which is characterized by the time constant  $\lambda_w$ , and will sooner or later drive the profits down to zero again. Competition between entrepreneurs reduces product prices and plays a significant role in the real-world process of eroding profits. Due to the simplified model setup with only one aggregate representative producer and one

good it cannot be represented explicitly in SDEM. Competition enters implicitly as reduced production revenues, which is equivalent to a wage cost increase in the present price-independent formalism.

Note that an increase of productivity in SDEM has two consequences. One is to decrease the relative wage rate compared to productivity according to (2.3), which gives rise to profits. The other consequence is that the number of employed workers falls according to (2.6), when the productivity increase is not accompanied by a sufficient increase of physical capital. This is exactly what happens when rationalization takes place.

### 2.2.4 Optimization

In order to determine how the entrepreneur chooses the investments over time, SDEM requires a rule that the entrepreneur will follow. This rule is given by assuming that the entrepreneur maximizes the net present value of the utility of profits, discounted at the constant rate  $\lambda_d$ :

$$\max. U = \int_0^{\infty} u(b)e^{-\lambda_d t} dt. \quad (2.13)$$

Two different utility functions are examined more closely in section 2.3. (1) In analogy to the standard economic approach, a logarithmic utility function  $u = \ln(\tilde{b})$  translates profits nonlinearly into utility.  $\tilde{b}$  indicates that  $b$  is normalized by the constant  $p(t_0)$  in order to be dimensionless. This approach requires a discount rate for utility at the 'pure rate of time preference', which describes how today's actors weigh their well-being relative to that of future generations. (2) A linear utility function,  $u = \tilde{b}$ , simply accounts for the absolute level of profits. Here, the monetary value of profits is involved directly instead of its (somewhat arbitrary) transformation to 'utility'. Thus, the appropriate discount rate is the interest rate, which exceeds the pure rate of time preference essentially by the economic growth rate (see e.g. Nordhaus [1993]). This setting is supposed to come closer to the actual decision making of entrepreneurs.

## 2.3 Model Behaviour

Before we start to solve the model numerically in chapter 2.4 we provide in this section an analytic solution for the special case in which the investment rates

$$x_1 = \frac{i_k - \lambda_k k}{p} \quad (2.14)$$

$$x_2 = \frac{i_{\hat{p}}}{p} \quad (2.15)$$

are constant<sup>3</sup>. The purpose of this analysis is to give an impression of the behaviour of SDEM under various parameter settings and to gain some general insights on the consequences of the

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<sup>3</sup>In the following analysis we define  $x_1$  to cover only those investments that actually cause the capital stock to grow. The replacement investments  $\lambda_k k$  are assumed to be always undertaken and treated like costs. See appendix A for details.

assumptions made in section 2.2. We summarize the main results here, while the full calculations can be found in appendix A.

Summing the equations in section 2.2, the profit equation (2.12) may be rewritten as

$$b = \nu k \left( 1 - \frac{\hat{w}}{\hat{p}} \right) - i_k - i_{\hat{p}} \quad (2.16)$$

$$= \nu k \left( 1 - \frac{\hat{w}}{\hat{p}} - \frac{\lambda_k}{\nu} - x_1 - x_2 \right) \quad (2.17)$$

We see that entrepreneurs can increase their profits  $b$  in two ways: firstly, through rationalization by productivity growth, and secondly by capital expansion. Rationalization reduces the fraction  $\hat{w}/\hat{p}$ , that is, the relative share of wages from revenues, due to the delayed wage rate adaptation (2.3). Since rationalization also causes workers to be laid off, the production level is unchanged (2.5), (2.6). Capital expansion is possible as long as enough unemployed workers are available. This increases production and revenues, while the wage share is unchanged, so that only the absolute amount of profits increases.

Note that a strategy that invests either in capital alone or in productivity alone will generally lead to suboptimal results. To see this, consider first the case that all investments go to capital and  $i_{\hat{p}} = 0$ . Assume that  $\hat{w} < \hat{w}^0$  and that  $l < l_{max}$ , so that capital growth is not restricted by full employment. Although this will cause capital to grow, the wage rate  $\hat{w}$  will approach  $\hat{w}^0$  according to (2.3) because productivity remains constant, so that the term in brackets in (2.16) approaches  $\lambda_k k/p$ . In that limit, capital investments,  $i_k$ , can no longer exceed the replacement investments,  $\lambda_k k$ . The economy has reached the stationary state where capital growth as well as profits are zero.

Alternatively, consider the case in which all investments beyond those necessary for capital replacement go to productivity. Then, the amount of employed labour will decrease according to (2.6) because productivity grows while capital (and therefore also production) remains constant<sup>4</sup> at its initial value  $k(0)$ . Since production remains constant, profits depend only on the expenditures for wages and investments in productivity (see (2.16)),

$$w + i_{\hat{p}} = p \left( \frac{\hat{w}}{\hat{p}} + \frac{i_{\hat{p}}}{p} \right). \quad (2.18)$$

When  $p$  is constant and  $w$  grows according to (2.3), the sum (2.18) has one defined minimum with respect to  $x_2$  (see appendix A). When the minimum is reached (depending on the adaptation of  $w$ ), profits remain constant over time.

In short, investing only in capital or productivity leads at best to constant profits (in the ' $i_{\hat{p}}$  only' case). A better strategy is obvious: the entrepreneur can invest in productivity to reach the  $w + i_{\hat{p}}$  minimum, and then take some part of the occurring profits to invest in capital growth. This causes production to grow exponentially, and the constant profits from the ' $i_{\hat{p}}$  only' case above are transformed to exponentially growing profits. The decision how much to invest in capital, however, depends on the applied utility function and discount rate, and is determined by optimization in appendix A.

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<sup>4</sup>If entrepreneurs would not care for capital replacement and invested everything in productivity instead, the labour decrease would be even more rapid, since now in (2.6) the denominator  $\hat{p}$  would grow faster, while the nominator  $k$  (and therefore also production) would decline.



The calculations in appendix A show that the distinction between logarithmic and linear utility yield different results only for the optimal value of the investment rate in capital,  $x_1$ . The optimal value for the productivity investment rate,  $x_2$ , in both cases is equivalent to finding the minimum of the sum  $w + i_{\hat{p}}$  at an arbitrary level of  $p$ , which yields

$$\frac{(w + i_{\hat{p}})|_{min}}{p} = \frac{\hat{w}}{\hat{p}} + x_{2,opt} = 2\sqrt{\alpha_w \frac{\lambda_w}{\mu}} - \frac{\lambda_w}{\mu}, \quad (2.19)$$

$$\text{at } x_{2,opt} = \sqrt{\alpha_w \frac{\lambda_w}{\mu}} - \frac{\lambda_w}{\mu}. \quad (2.20)$$

independent from the employed utility function. The calculations from appendix A to determine of the optimal capital investment rate are summarized below.

### 2.3.1 Logarithmic Utility Function

In this subsection, equation (2.13) is maximized with respect to  $x_1$  for a logarithmic utility function (see section 2.2.4):

$$u = \ln(\tilde{b}). \quad (2.21)$$

The optimization separates in that for  $x_2$  with the result given by (2.19), (2.20), and that for  $x_1$ , which yields a distinct optimum at

$$x_{1,opt} = \frac{1}{\alpha_w} \left( \alpha_w - \sqrt{\alpha_w \frac{\lambda_w}{\mu}} \right)^2 - \frac{\lambda_d}{\nu} \quad (2.22)$$

$$\text{and } U = \frac{1}{\lambda_d} \left( \ln \left[ \frac{\lambda_d}{\nu} \right] + \frac{\left( \alpha_w - \sqrt{\alpha_w \frac{\lambda_w}{\mu}} \right)^2}{\alpha_w \frac{\lambda_d}{\nu}} - 1 \right), \quad (2.23)$$

where, of course,  $x_{1,opt}$  cannot be smaller than zero. Note that the growth rate of capital is now not restricted by  $\lambda_d$  because the logarithmic utility function transforms any exponential growth of capital or profits to only linear growth of utility, so that the integral  $U$  will converge for any discount rate  $\lambda_d > 0$  independent on the value of  $x_1$ .

The optimal solutions for the investment rates vary smoothly when the parameters describing the economy change. and policy measures that act on any of the model parameters now have visible effects, even when they are small. Namely the time evolution of employed labour can be influenced, as this directly depends on the choice of  $x_{1,opt}$ ,  $x_{2,opt}$  and the resulting growth rates of capital and productivity. Among the model parameters, only  $\lambda_w$  and  $\mu$  are easily accessible to the policy maker. All others are either given by initial or technical conditions ( $\nu$ ,  $\lambda_k$ ), or depend on moral and/or ethical values that are not easily changed by policy ( $\lambda_d$ ). In section 2.4 the behaviour of SDEM will be examined for empirically derived parameter settings.

### 2.3.2 Linear Utility Function

The above analysis is repeated here for the linear utility function,  $u = \tilde{b}$ . Again,  $x_2$  is determined according to (2.19), (2.20), while now the possible range for  $x_1$  is constrained: as long as full

employment is not reached, the growth rate of capital cannot exceed the discount rate, otherwise the integral in (2.13) will not converge. When full employment is reached,  $x_1$  is limited by the restriction that capital cannot grow faster than the product of productivity and labour force.

The optimal choice for  $x_1$  under the linear utility function is bistable:  $x_{1,opt}$  is either zero or at the appropriate upper boundary. This behaviour is based on the fact that profits are discounted at the rate  $\lambda_d$ . This effectively sets the time horizon for significant contributions to the integral  $U$ . Investing in capital leads to capital growth at the rate  $\lambda_a$ , which reduces the effective discount rate to  $\lambda_d - \lambda_a$  and thereby enlarges the effective time horizon for  $U$ , although the amplitude of profits (the term in brackets in (2.17)) is reduced. The question is now: Does the effect of the longer effective time horizon over-compensate the reduction in the profit amplitude? If so,  $x_{1,opt}$  is at its upper boundary, if not, it is zero. This depends only on the settings of the model parameters  $\lambda_w, \mu, \nu, \alpha_w$ , and the discount rate  $\lambda_d$ . As long as these parameters guarantee for capital investments and full employment has not been reached, the discount rate determines the maximum growth rate, since otherwise  $U$  would not converge.

Since the parameters in question are exogenous to the model, they may well change in real world economies. The above result essentially means that even slight parameter variations can cause a strong capital growth to shut down immediately, or to rise from zero to the maximum possible level, when economies are near the verge of the transition edge. On the other hand, when the economies are deep within the parameter range of one of the stable solutions, they would not respond at all to relatively strong parameter variations.

This behaviour does not match with empirically data of growing real-world economies, where neither abrupt changes in behaviour are commonly observed, nor do economies not respond at all to policy. The reason for these discrepancies appears to lie in the missing connection between the (endogenous) growth rate of production and the exogenous interest/discount rate. These discrepancies could presumably be overcome by some sophisticated feedback. However, as the logarithmic utility function already provides useful results, this refinement for the linear utility is left to future studies. For the rest of this thesis, the logarithmic utility function is used for entrepreneurs' optimization.

## 2.4 Numerical Experiments with SDEM

This sections describes results of model runs of the numerical model built from the equations in section 2.2 that have been done in comparison to the analytical solutions obtained in chapter 2.3. This numeric SDEM version does not have the limitations of the analytical analysis in section 2.3, i.e. investment rates are no longer required to be constant. However, the numerical formulation imposes two other constraints: a finite upper limit of the optimization integral (2.13) and a lower limit of the wage rate relaxation rate  $\lambda_w$ , which is set by the finite integration time step  $\Delta t$ . Additionally, the initial wage rate mismatch is an exogenous parameter, therefore  $c_1^*$  (A.10) can only be approximately set to zero (see equation (A.10) in appendix A). These features affect the optimal investment rates mostly at spin-up and at the end of the time horizon of the integration. The general behaviour of the analytical solutions is not changed by these terminal effects.

The numerical model is written in FORTRAN 77. The adjoint model that is used to calculate the gradient of the welfare function is generated automatically from the model source code by the Tangent and Adjoint Model Compiler (TAMC) [Giering and Kaminski, 1998]. The optimization itself is performed by the L-BFGS-B package [Zhu et al., 1994; Byrd et al., 1995], which has turned out to be very reliable and persevering in finding the optimal solution. The integration is run over 600 years with 5 year time steps. This captures most of the climate change signal for a not too large number of control variables (3 variables at 120 time steps each).

### 2.4.1 Baseline Settings

In order to check whether SDEM can reproduce long term economic growth, the US economy is chosen as a reference case. Data by Maddison [1982, 1995] are used to calibrate the model, who provides time series of numerous quantities for the period 1820-1992. In particular SDEM should reproduce the following characteristics:

1. The growth rate of production is in the order of 3% p.a.
2. The investment rate in capital (including replacement) is in the order of 20% p.a.
3. The growth rate of total hours worked (which is proportional to the labour force  $l$  in the model) is in the order of 1.5% p.a.
4. The employment rate has been (more or less) constant above 90%.

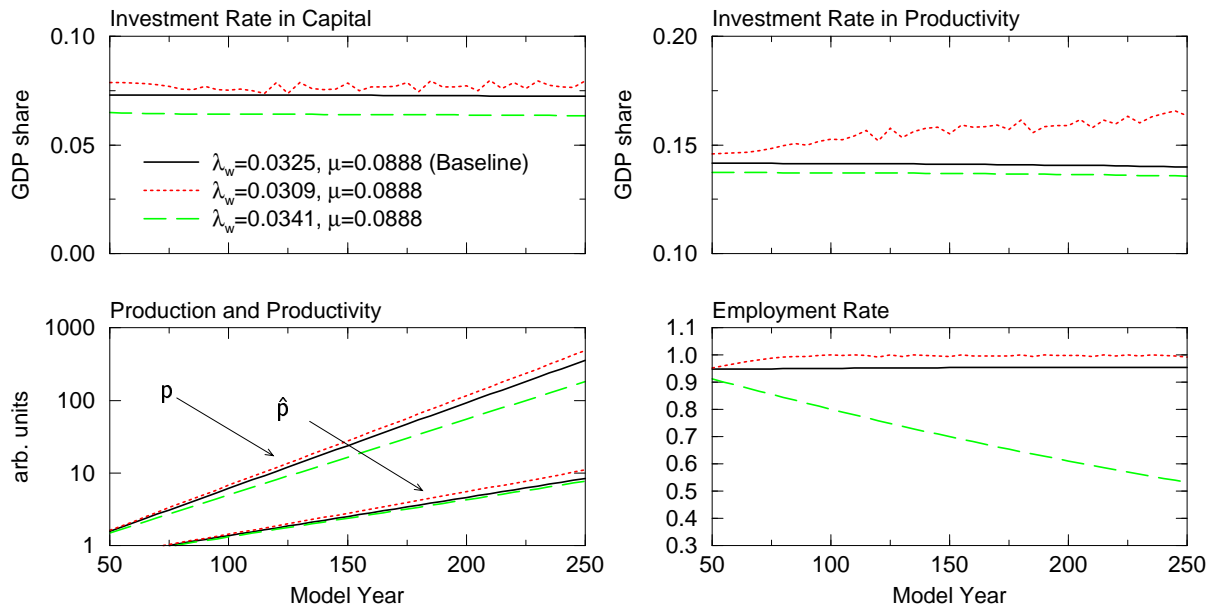
To derive the parameter settings that are required for an optimal solution with these characteristics, we assume a capital depreciation rate of  $\lambda_k = 0.05$  and a moderate discount rate  $\lambda_d = 0.01$ . The proportionality constant between productivity and capital per labour can be derived from Maddison [1982] and is found to be  $\nu = 0.4$  for the US. For other western economies that value ranges from 0.3 to 0.5.

From (2.10) and (2.11) it then follows that 12.5% of annual production have to be used to replace capital depreciation. In order to match the first two characteristics,  $x_{1,opt}$  must be in the order of 7.5% of annual production. Using this and inverting (2.22) gives the ratio  $\lambda_w/\mu$ , when  $\lambda_d$  is known. Equation (A.31) in the appendix can then be used to derive the absolute value of  $\mu$ . Using the above procedure yields  $\lambda_w = 0.0298 \text{ year}^{-1}$ ,  $\mu = 0.076$ . Empirically accounting for minor disturbances arising from terminal effects, we arrive at the baseline setting that meets the three conditions:

$$\lambda_w = 0.0325 \text{ year}^{-1} \quad (2.24)$$

$$\mu = 0.0888 \quad (2.25)$$

The optimization result achieved with these parameters is shown in figure 2.3 as solid curves. These comply well with the above criteria. The investment rates in capital and productivity are approximately 0.073 and 0.14, respectively. This results in growth rates of 2.7% p.a. for production and 1.2% p.a. for productivity, which is slightly smaller than the algebraically growth rates derived from (A.6), (A.7) in the appendix. The reason lies in the discrete formulation of



**Figure 2.3:** Sensitivities of SDEM to changes in the rate of wage adaptation  $\lambda_w$  at constant productivity investment efficiency  $\mu$ . Investment rates are given as fraction of current production. Production and productivity are normalized to initial capital (see appendix C). In the scenario with reduced  $\lambda_w$  (dotted), full employment is reached. This imposes an additional constraint to the numerical optimization and leads to increased 'noise' in the investment rates.

the prognostic equations (2.1), (2.2) combined with the large 5 year time step, which yields systematically too small growth rates from given investment rates. Considering this, both growth rates agree well with empirical data and the above three criteria<sup>5</sup>.

The investment rate in productivity appears large at first glance. Recall, however, from chapter 2.2.1 that  $i_{\hat{p}}$  also includes expenditures to train employees or salaries of teachers and thus covers more than only expenditures in research and development listed in usual national accounts. The value of  $i_{\hat{p}}$  can therefore not easily be verified by comparison with data, because these data are not available and providing them is beyond the scope of this study. However, as SDEM is a conceptual model, the purpose of  $i_{\hat{p}}$  is to provide a simple mechanism that relates productivity growth to explicit entrepreneurial decision making, while the exact mechanism is unresolved. As long as the exact mechanism of productivity growth is unknown the actual value of  $i_{\hat{p}}$  should be considered as an initial guess of the order of magnitude rather than an accurate value.

Despite these promising results, one shortcoming is obvious: wage adaptation is too slow in the baseline setting. It takes  $1/\lambda_w \approx 31$  years until the profits resulting from a single step-like productivity increase are reduced to  $1/e \approx 0.37$  by the wage rate adaptation process (2.3). Compared to real economic systems where wage contracts last only one to two years, values of  $\lambda_w$  in the order of 5 years appear more realistic. Since  $\lambda_w$  is determined by the empirical values

<sup>5</sup>In the literature, productivity is often defined as GDP per man-hour, while SDEM uses GDP per worker. Since the annual hours worked per worker has continually declined by some 0.6% p.a. over the last century in all industrialized countries [Maddison, 1982], the growth rate of GDP per man-hour is significantly greater than of GDP per worker.

of  $\nu$ ,  $a_w$ , and the growth rate of population, this contradiction with the empirical data cannot be overcome within the current model setup. A way out of this problem is offered in section 2.5.1.

## 2.4.2 Sensitivity to Variations in Dynamic Variables

To explore the dynamic behaviour of the current model in more detail, we examine the effect of slight variations of  $\lambda_w$  and  $\mu$  on that baseline (solid lines in figure 2.3). First we consider a 5% variation in  $\lambda_w$  at unchanged  $\mu$ , using  $\lambda_w = 0.0285$  (dotted) and  $\lambda_w = 0.0315$  (dashed). We start with the explanation for the reduced  $\lambda_w$  (dotted curves). The top panels in fig. 2.3 show the effects on the optimal investment rates  $x_1, x_2$ . Decreasing the rate of wage adaptation,  $\lambda_w$ , allows for larger investment rates because at a given time after a given productivity increase the ratio  $\hat{w}/\hat{p}$  is reduced. A small part of the saved wage cost increases profits, while the major part is used to increase investment rates. This leads to faster growth of both productivity and capital and thus production (lower left panel in fig. 2.3). Furthermore, because the wage per worker is cheaper, the entrepreneur has an incentive to hire more workers than at baseline conditions, so that the employment rate grows faster than in the baseline scenario (lower right panel in fig. 2.3).

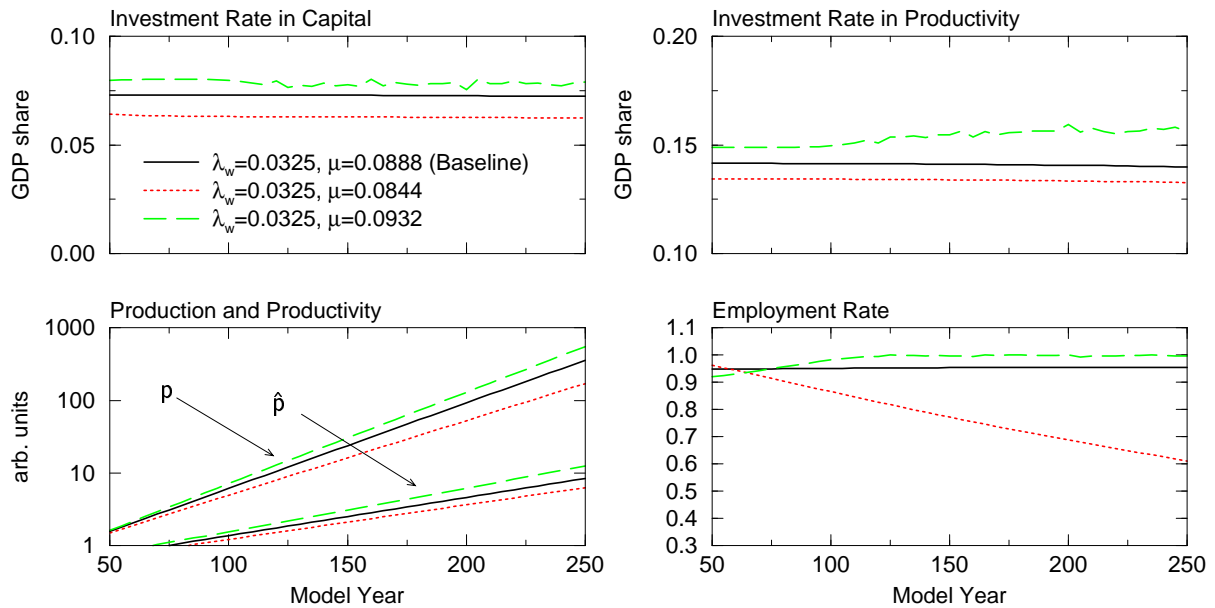
For the settings with enhanced  $\lambda_w$  (dashed lines), the argument is simply inverted: The wage rate adaptation is faster, this leads to a higher relative wage rate. Therefore, overall investments are reduced (top panels), productivity and production grow slower (third panel), and, since wages appear more expensive to the entrepreneur, less workers are hired, leading to a continually decreasing employment rate (last panel).

In fig. 2.4 we show the effect of a variation of the factor  $\mu$  that describes how efficiently an investment  $i_{\hat{p}}$  increases productivity. We start from the same baseline as before, but now keep the rate of wage adaptation fixed at  $\lambda_w = 0.03$  and vary  $\mu$  from 0.0945 (dotted curves) to 0.1044 (dashed curves), again corresponding to 5% deviation from the baseline. We start our discussion again with the dotted curves indicating a decreased investment efficiency  $\mu$ . Therefore, a given investment  $i_{\hat{p}}$  results in a smaller increase of productivity and  $\hat{w}^0$ . Because  $\lambda_w$  is unchanged this leads to relatively higher wage rates and less money available for investments. Thus, both  $x_1$  and  $x_2$  are reduced (upper panels in fig. 2.4), production (or capital) and productivity grow slower (lower left panel in fig. 2.4), and, since wages are more expensive, the employment rate goes down (lower right panel in fig. 2.4).

For the dashed lines indicating increased  $\mu$ , the argumentation again simply needs to be inverted to explain the observed behaviour.

Summing up the findings of this sensitivity analysis, we see that moderate changes in the dynamic parameters  $\lambda_w$  and  $\mu$  transform linearly into variations of the corresponding growth rates of the prognostic variables productivity and capital as well as annual production, due to the assumed linear relation (2.8) between capital and production. This was also expected from the theoretical considerations in section 2.3.

What had also to be expected from theory and became even more obvious in the numerical experiments was the strong sensitivity of the employment rate to parameter variations, that turned from growing to decreasing employment. In the baseline scenario, the constant employment rate is the result of the equality of the growth rates of capital and production. The dynamic param-



**Figure 2.4:** Sensitivities of SDEM to changes in productivity investment efficiency  $\mu$  at constant rate of wage adaptation  $\lambda_w$ . Investment rates are given as fraction of current production. Production and productivity are normalized to initial capital (see appendix C). In the scenario with increased  $\mu$  (dashed), full employment is reached. This imposes an additional constraint to the numerical optimization and leads to increased 'noise' in the investment rates.

eters were explicitly chosen to achieve this equality. Any slight deviation from these parameter values results in an imbalance of capital and productivity growth, causing the employment rate to either grow or decline.

The policy implications of this finding are quite substantial: If the model is assumed to describe empirical data correctly, this means that the observed, more or less constant employment rates in western economies in the past were due to a permanent 'correct' setting of the dynamic parameters of these economies. If this would not have been the case, the 'luckier' ones of these economies would have run into a state of eternal full employment, while the 'unhappier' ones would suffer from ever increasing unemployment. This inherent instability of the current model setup can be overcome by the introduction of additional feedbacks. Section 2.5 below describes a feedback that effectively stabilizes employment rates.

## 2.5 Extended SDEM with Unemployment Feedback

The check of SDEM against empirical data in sections 2.4.1 and 2.4.2 revealed two shortcomings. First, realistic growth rates can only be modelled when the rate of wage adaptation is assumed to have an obviously too small value. Second, the modelled unemployment reacts too sensitively to small variations in the model dynamics. Both effects are the result of the model focusing only on entrepreneurs and employed workers. Although this is a good choice when attempting to understand the functional principles of the economy, it is too narrow when real

systems are considered.

### 2.5.1 Unemployment Feedback

Real-world economies face a large variety of influences, feedback mechanisms and so on. The most important of these have to be incorporated in a model to give realistic results. In the basic SDEM, feedbacks arising from unemployment were ignored to make the wage rate adaptation process and its role in the economy more pronounced. For example, it was ignored that in the basic SDEM unemployed persons have no income and would thus be bound to starve. In order to get at least some income, unemployed will be likely to work for less than the average wage rate of the already employed. This competition between workplace owners and unemployed reduces the achievable maximum wage rate  $\hat{w}^0$  and thereby the wage rate  $\hat{w}$ . However, this reduction will be bounded from below for two reasons. First, unemployed have to cover at least their basic needs and if the wage rate is below that level, they will seek some occupation outside the system in the informal sector, e.g. subsistence farming or micro-enterprises. Second, in most real societies there exist social support mechanisms, which effectively introduce a minimum wage rate in the economy and cannot be ignored when real wages are to be determined.

In order to bring the model behaviour closer to the observed data, SDEM has been extended to include this feedback of unemployment on the actual wage rate. Note that this is only a first approach and several other feedbacks would have to be included in order to arrive at a more accurate employment model, e.g. payroll taxes. In order to keep the model simple it is useful to introduce only as many feedbacks as necessary until the empirical data are reproduced sufficiently well.

To model the competition between unemployed and employed it is assumed that  $\hat{w}^0$  is reduced by multiplying it with the simple power function

$$F = F_{min} + (1 - F_{min}) q^{\alpha_q} \quad (2.26)$$

where  $\alpha_q, F_{min}$  are positive, exogenous constants.  $F$  is designed to equal unity at full employment and to fall to the minimum value  $F_{min}$  as the employment rate,  $q$ , goes to zero. The functional form of  $F$  is of no importance, what matters is that  $F$  declines sharply when the employment rate falls below unity. This expresses that the competition effect on the wage rate is strongest when there are only relatively few unemployed.

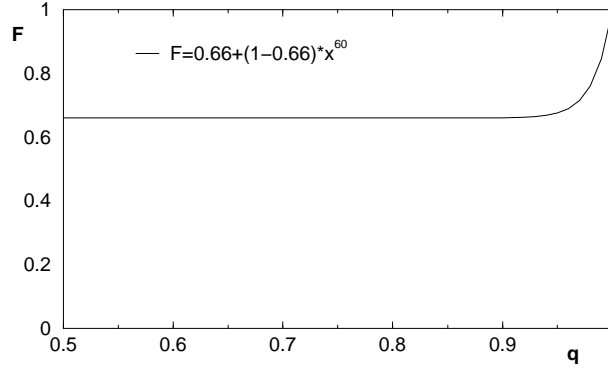
Extending SDEM by inserting (2.26) in the differential equation (2.3) for the wage rate adaptation yields

$$\dot{\hat{w}}^\dagger = \lambda_w (\hat{w}^0 F - \hat{w}^\dagger). \quad (2.27)$$

The wage rate resulting from this equation is denoted by  $\hat{w}^\dagger$  and turns out to be

$$\hat{w}^\dagger = \hat{w} F \quad (2.28)$$

when the integration of (2.27) is initialized at  $\hat{w}^\dagger(0)$ . Henceforth, we will refer to SDEM including equations (2.26)-(2.27) as 'extended SDEM', while the model depicted in section 2.2 will be referred to as 'basic SDEM'.



**Figure 2.5:** Schematic principle of the feedback factor  $F$  as a function of the employment rate  $q$ . The values  $F_{min} = 0.66$  and  $\alpha_q = 60$  are used later in section 2.5.2.

The introduction of  $F$  results in nonzero and constant unemployment rates for two reasons. First, unemployment occurs because the relative wage rate  $\hat{w}^\dagger/\hat{p}$  is smaller than the respective value  $\hat{w}/\hat{p}$  in the basic SDEM only when  $q < 1$ , so the entrepreneur has an incentive to keep the employment rate below full employment. The saved wage costs are either taken as instantaneous profits or are spent to increase capital growth, which is the most efficient way to increase time-integrated profits (see section 2.3). Second, employment rates are constant because increasing  $q$  would cancel the effect of  $F$  on wages while reducing  $q$  below a certain value becomes ineffective. To see this recall that reducing  $q$  requires devoting some capital growth in order to increase productivity. Since the absolute value of  $\partial F/\partial q$  decreases when  $F$  approaches  $F_{min}$ , the employment rate will not go to zero but rather remain where the profit gains from reduced  $F$  no longer outweigh the losses from decreased capital growth. It is therefore optimal to choose a constant employment rate at less than full employment at the largest possible value of  $x_1$ . The adjustment of the level of the employment rate takes place at the very beginning of the integration.

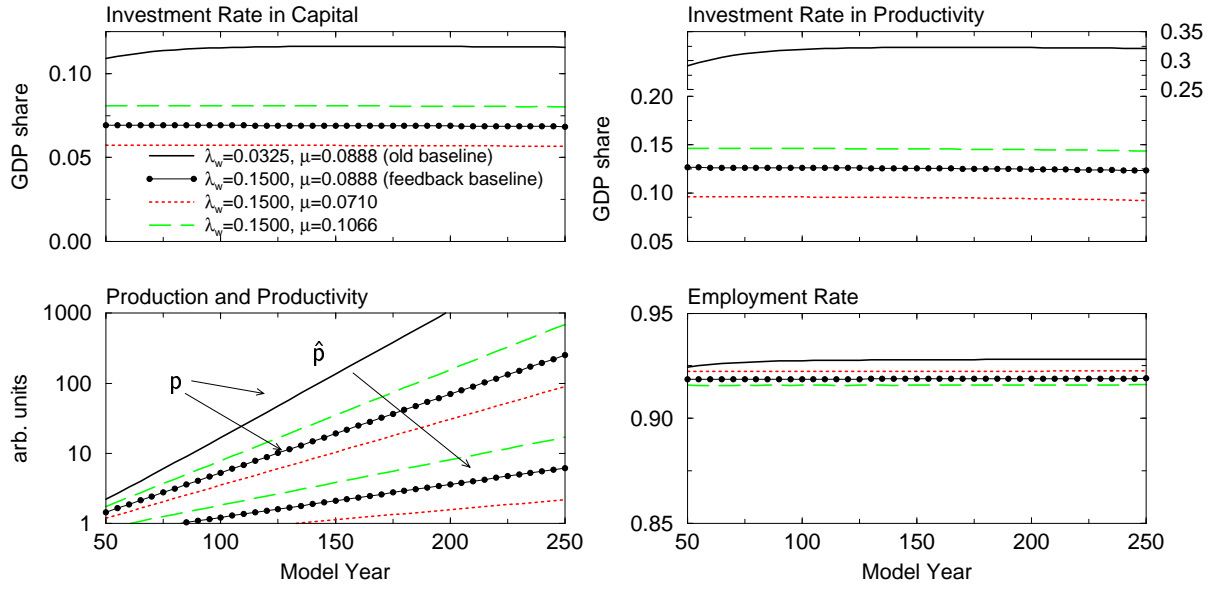
This changed optimality condition, however, overrides some of the findings from section 2.3. The constant employment rate couples the investment rates according to equation (A.32) in the appendix, which can be rewritten as

$$x_1 = \frac{1}{\nu} \left( \mu x_2 + \frac{d}{dt} \left( \frac{l_{max}}{l_{max}} \right) \right). \quad (2.29)$$

A strategy that maximizes  $x_1$  will do so according to (2.29) and as long as profits (2.16) remain positive. Note that the minimization of the sum  $w + i_{\hat{p}}$  is no longer important, since that  $x_1$  that corresponds to  $x_{2,opt}$  at the minimum of (2.18) will in general either be too large, so that negative profits would result, or too small to give maximum capital growth. Since now  $x_1$  and  $x_2$  are no longer independent of each other, the model loses one degree of freedom and is thus overdetermined. This increases the freedom in choosing  $\lambda_w$  and we may hope to find a solution that enables us to satisfy the three empirical growth criteria from section 2.4.1 at a more realistic rate of wage adaptation.

Apparently, the introduction of  $F$  allows to overcome both shortcomings from sections 2.4.1, 2.4.2 as it stabilizes the growth rate of employment to zero and allows for arbitrary  $\lambda_w$ . Instead of proving these findings analytically we demonstrate in the following only that the numerical model behaves as expected.





**Figure 2.6:** Sensitivities of extended SDEM to changes in the rate of wage adaptation  $\lambda_w$  and the productivity investment efficiency  $\mu$ . Results are given for  $F_{min} = 0.66$  and  $\alpha_q = 60$ . Solid black curves have been generated with the 'old' baseline settings from figs. 2.3, 2.4. The solid curves with bullets show the result of increased  $\lambda_w$  compared to that 'old' baseline. The dotted and dashed curves show the variation of  $\mu$  at that higher  $\lambda_w$ . Note that in the top right panel the vertical axis has been compressed. In the lower left panel, the productivity curve for the 'old' baseline scenario has been omitted since it overlaps the production curve of the 'feedback' baseline.

## 2.5.2 Parameter Variation in Extended SDEM

We start our discussion of the  $F$  feedback by applying it to the 'old' baseline parameter setting from section 2.4.1.  $F$  is specified by  $F_{min} = 0.66$  and  $\alpha_q = 60$  which correspond to a  $\hat{w}^0$  reduction to 66% when the employment rate is below approximately 0.9 (see fig. 2.5). The optimization result is again plotted as solid line in fig. 2.6. For comparison with the previous results all other parameter settings are the same as in fig. 2.3. Since the then used initial value for the wage rate mismatch,  $c_1^*$ , is not appropriate under feedback conditions, this results in an extended spin-up time until  $t = 100$  in fig. 2.6. This has been accounted for in subsequent calculations. The application of  $F$  reduces  $q$  slightly from 0.95 (see fig. 2.3) to 0.93. The reduction of the wage rate caused by  $F$  allows for very large investment rates, of  $x_1 \approx 0.118$  and  $x_2 \approx 0.33$ .

Obviously, the parameter settings need to be changed for the extended SDEM in order to come closer to empirical data. We therefore examine the response to a faster wage rate adaptation by enhancing  $\lambda_w$  up to 0.15, corresponding to a more realistic wage relaxation time of  $1/\lambda_w = 6.7$  years<sup>6</sup>. As this increases the wage rate relative to productivity, investment rates are expected to decline. The result of this optimization is plotted as solid line with bullets in fig. 2.6, denoted

<sup>6</sup>The discrete numeric formulation of the wage rate adaptation requires that  $1/\lambda_w$  is greater than the integration time step (currently 5 years). If wage rate adaptation is faster than the time step, the results are indistinguishable from immediate adaptation and might lead to numeric instabilities.

as the 'feedback' baseline. Investment rates are in fact drastically reduced compared to the 'old' baseline settings (solid lines) and amount to  $x_1 = 0.075$  and  $x_2 \approx 0.148$ , which again reproduces the empirical criteria from section 2.4.1.

Despite the strong increase of  $\lambda_w$  the employment rate remains constant due to the  $F$  feedback, and its level remains at  $q = 0.918$ , which is only slightly below the level of the 'old baseline' employment rate. This small but visible effect from increased  $\lambda_w$  on the employment rate has the same tendency to reduce  $q$  as in the basic model, but now for a different reason: a greater relative wage rate due to increased  $\lambda_w$  reduces (capital) investments and thereby the profit growth rate. Part of that wage rate increase can be compensated when  $F$  is reduced, which is done by lowering the employment rate.

Finally we examine the effect of variations in the productivity investment efficiency  $\mu$ . Starting from the 'feedback' baseline (solid line with bullets), we keep  $\lambda_w = 0.15$  fixed and vary  $\mu$  from 0.0710 (dotted lines) to 0.1066 (dashed lines), corresponding to  $\pm 20\%$  deviation. As in the basic model, the investment rates become greater when  $\mu$  increases because the enhanced productivity growth allows for stronger reduction of wages and thus higher investment rates.

Additionally, employment levels are reduced as  $\mu$  increases (lower right panel in fig. 2.6), which differs from the behaviour found in section 2.4.2 and is connected to  $F$ . Recall from section 2.5.1 that the  $q$  level is determined by the equilibrium point where profit gains from further decrease of  $q$  are compensated by losses from the capital growth reduction necessary to decrease  $q$ . When productivity grows faster due to greater  $\mu$ , relatively less capital growth reduction is necessary and therefore the equilibrium is reached at smaller  $q$ .

In less technical terms, this means that the feedback mechanism also changes the way in which changes in the productivity growth dynamics affect employment. In the basic model, facilitating productivity growth by increased  $\mu$  reduces relative wages and thereby makes the hiring of workers more attractive so that the employment rate grows faster (or decreases slower). In the extended SDEM, increased  $\mu$  still facilitates productivity growth and decreases relative wages, but this does not stimulate the entrepreneur to hire more workers, because this would lead to less competition for workplaces and wages would increase again. Instead, the creation of unemployment requires less strong cuts in capital growth, therefore the economy ends up at a reduced employment level.

Although the functional form of the  $F$  feedback is rather a sketch of the involved processes, it overcomes the major shortcomings of the basic model. Still, some processes are likely to be missing that would be necessary to model unemployment comprehensively. This is in particular the case since SDEM in its present formulation does not resolve the composition of actual labour costs, like e.g. payroll taxes, which do not change the general dynamics, but affect the detailed behaviour. Adding this or other feedbacks will presumably overcome the impression that the introduction of  $F$  has now made the unemployment rate too insensitive to parameter variations. However, setting up a more detailed model of unemployment is beyond the scope of this conceptual study. Since the model results already agree relatively well with empirical data, this topic is not elaborated further.

## Chapter 3

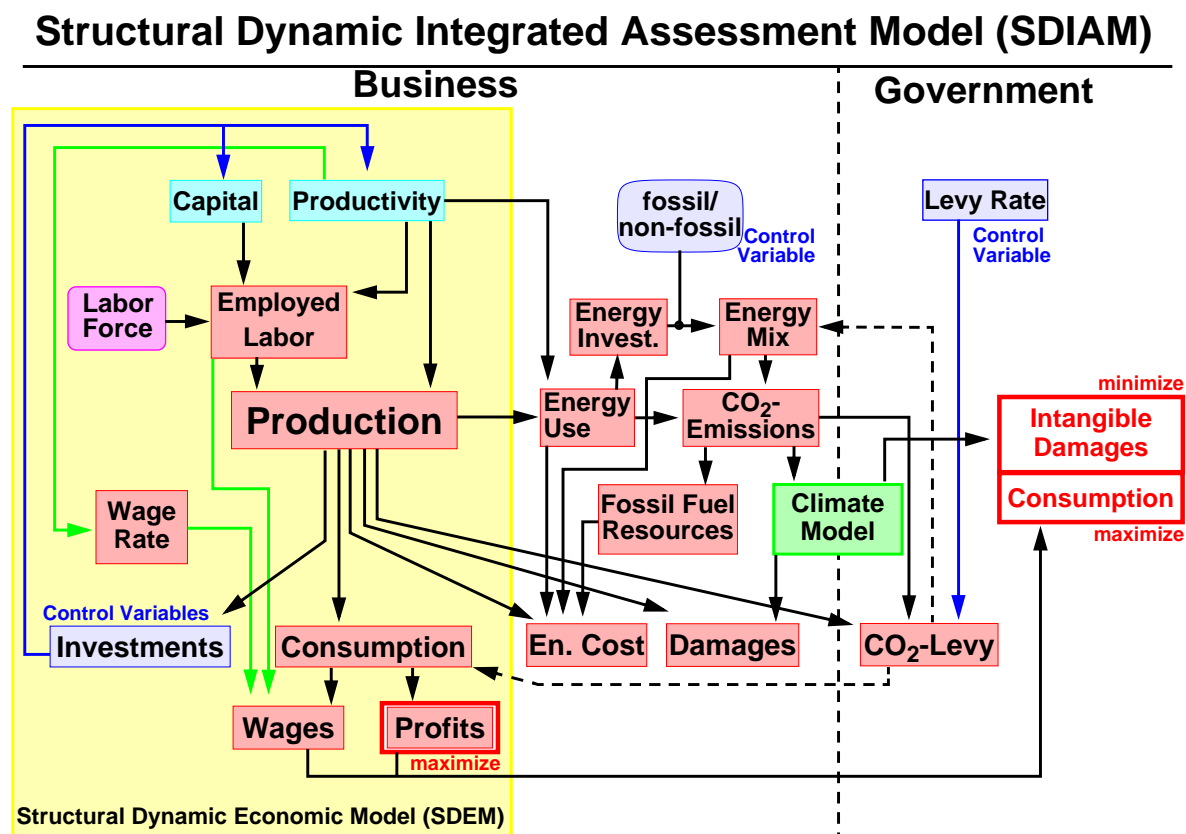
# The Structural Dynamic Integrated Assessment Model (SDIAM)

The following chapter embeds SDEM in a framework for the integrated assessment of climate change. This framework, the Structural Dynamic Integrated Assessment Model (SDIAM), is designed to capture the basic interactions of three major global subsystems: economy, climate and society. SDIAM couples these three subsystems dynamically, so that their interactions can be analyzed and assessed.

The first link between economic activity and climate is the emission of greenhouse gases like carbon dioxide (CO<sub>2</sub>). SDIAM determines the CO<sub>2</sub> emissions arising from energy consumption as a function of production modelled by SDEM, which is initialized to represent the world economy. The climate model NICCS [Hooss *et al.* 2001] is then used to calculate the resulting climate change. The second link between social systems and climate is provided by the damage module of SDIAM. It translates the impacts of climate change into costs that affect economy and society. A monetary representation is chosen to make impacts comparable to monetary efforts to reduce CO<sub>2</sub> emissions. This allows to determine an optimal time evolution path using cost-benefit analysis.

SDIAM introduces society as a separate subsystem in which government is the representative actor with separate controls to increase its own welfare function, which in general differs from the economic welfare function. This permits to endogenize conflicts between society as a whole and the economy, which are rooted in the different welfare definitions and are lost in the averaged definitions of economic welfare in usual economic models.

The outline for this chapter is the following. After a brief specification of the model initialization, we describe in section 3.2 how production determines greenhouse gas emissions and how the emissions per unit production can be changed by the entrepreneur. The incentive to change that ratio is given by changing energy prices, as specified in section 3.3. The climate model NICCS [Hooss *et al.* 2001] is briefly sketched in section 3.4. The following section 3.5 describes the translation of climate change into damages. In section 3.6 the way in which society is implemented in the model is presented. The concluding section 3.7 indicates how some SDEM equations have to be adapted to the SDIAM concepts. A general sketch of SDIAM is presented in figure 3.1.

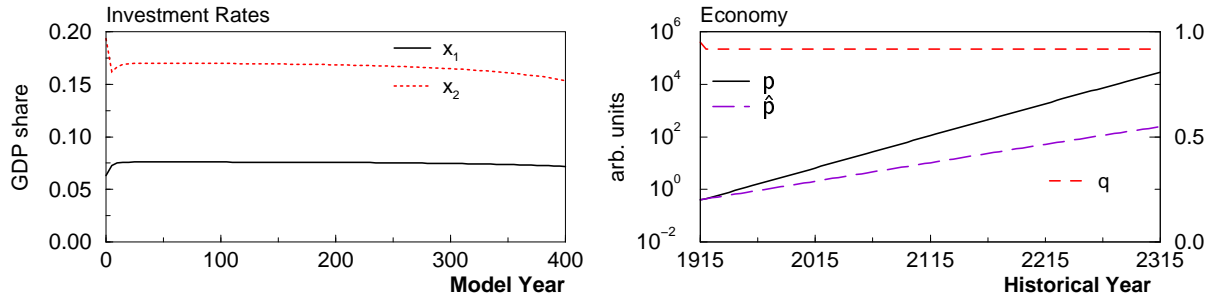


**Figure 3.1:** Schematic sketch of SDIAM with SDEM as the economic module (box on the left, see fig. 2.1). The decision to build fossil or non-fossil energy systems is a new control variable for the entrepreneur. Government/society is introduced as a new actor that controls the emission levy rate and optimizes a combination of consumption less climate damages. Levy revenues can be used to either change the energy supply mix, or to enhance consumption, or both.

### 3.1 Model Initialization

Anthropogenic climate change is predominantly driven by global anthropogenic CO<sub>2</sub> emissions, which are therefore taken as a proxy variable for the total greenhouse gas emissions. The economic module of an IA model of climate change needs to represent all economy-borne CO<sub>2</sub> emissions. Since SDEM is not regionalized yet, it is calibrated to match the characteristics of the aggregate world economy during the 20th century, in analogy to its calibration to the US economy in section 2.4.1. Although it is difficult to obtain economic data on a global scale for such long periods, Maddison [1995] provides some time series which have been used to re-calibrate SDEM for the use in SDIAM. The following characteristics are assumed:

1. Production grows at 2.8% p.a. [Maddison 1995]
2. Population grows at 1.2% p.a. [United Nations 1966, 1999]. The same growth rate is assumed for available workforce,  $l_{max}$ . It follows that  $\hat{p}$  grows at 1.6% p.a.
3. Employment rates  $q \geq 0.9$ .



**Figure 3.2:** Optimization results for the world baseline settings from equations (3.1)-(3.4). Left panel: investment rates; right panel: production (left ordinate) and unemployment (right ordinate).

4. Start year of optimization set to  $t_0 = 1915$ .
5. Initial (1915) emissions  $e(t_0) = 0.9 \text{ GtC/yr}$  [Marland *et al.* 2001].

The optimization is initialized in 1915 for which the global emissions/GDP ratio was maximal [Maddison 1995, Marland *et al.* 2001]. This setting avoids disturbing effects from the increasing fossil fuel share in world energy supply when calibrating the energy efficiency function (see section 3.2.3). Additionally it can be used to sort out those parameter settings that cannot reproduce the historic evolution from 1915-2000. As in chapter 2.4, the integration is performed over 600 years with a 5 years time step, i.e. it ends in 2515.

Calibrating SDEM to a world economy with the above specifications requires the following settings ('world baseline'):

$$\lambda_w = 0.15 \text{ year}^{-1} \quad (3.1)$$

$$\mu = 0.10 \quad (3.2)$$

$$F_{min} = 0.66 \quad (3.3)$$

$$\alpha_q = 60.0 \quad (3.4)$$

For comparison with previous results, these settings have been chosen to deviate only slightly from the 'feedback' baseline from chapter 2.5.2. In particular, only  $\mu$  has been changed to adapt the model to the world economy, while the wage rate relaxation speed and the unemployment feedback are not changed. The optimization result derived from these settings is shown in fig. 3.2.

## 3.2 Energy and Emissions

Production of goods (and also services) requires energy as an input to obtain work in the physical sense. Though this is an almost trivial statement, its implications for economic growth as well as for environmental impacts are enormous. While for millenia mechanical work was generated predominantly by chemical processes within the muscles of humans and animals, work provision switched almost completely to heat engines since the industrial revolution. This

switch allowed for extended use of physical work and thereby increased production and wealth, mostly in the 'industrialized countries', but also required the increased use of fossil fuels as energy source. Environmental side effects of intensive fossil fuel use first became apparent on a local scale as pollution of air, soils and water, while the threat of global climate change has only recently become an issue.

Heat generation from fossil fuels inevitably generates carbon dioxide (CO<sub>2</sub>), which is usually simply emitted to the atmosphere. By affecting the radiative properties of earth's atmosphere, the so-called 'anthropogenic greenhouse effect' has long been known to change the climate system [Arrhenius 1896, IPCC 2001a]. Due to this close coupling of economic growth and CO<sub>2</sub> emissions<sup>1</sup> we restrict the following studies to that gas, although there are many other gases known to have similar impacts, like methane or nitrous oxide. This restriction is also justified by the fact that carbon dioxide is the largest single contributor to the anthropogenic greenhouse effect and its relative share is expected to increase in the future [IPCC 2001a]. This restriction of the present study to only one greenhouse gas will, however, underestimate the extent of future climate change.

Reduction of the amount of CO<sub>2</sub> emitted per unit of production can be established in two ways: reduction of the energy use per unit production, or reduction of the CO<sub>2</sub> emissions per unit energy generated. While the first way is closely related to technological progress and thus occurs 'automatically', the second generally requires substantial and deliberate changes to the energy supply system. We therefore model these two efficiencies separately and describe them in the following.

### 3.2.1 Carbon Efficiency

In modern industrialized economies, Production  $p$  requires carbon dioxide emissions,  $e$ . The carbon efficiency of production,  $z$ , relates both quantities:

$$e = \frac{p}{z}. \quad (3.5)$$

The carbon efficiency is determined by two more specific efficiencies:  $f_e$  describes how much primary energy is generated per unit emissions, while  $f_p$  gives the amount of production output per unit primary energy. The carbon efficiency of production,  $z$ , is thus defined as

$$z = z(t_0) \frac{f_e f_p}{f_e(t_0) f_p(t_0)}. \quad (3.6)$$

The growth rates of  $f_e, f_p$  are normalized to the initial carbon efficiencies at  $t = t_0$ , while  $z(t_0)$  can be derived from available data using (3.5). Before we examine both steps in more detail note that we do not treat energy as an explicit input factor to production. Rather, energy use is one component of overall labour productivity, so that productivity can be used to determine the work-energy efficiency  $f_p$  in section 3.2.3 below.

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<sup>1</sup>A good example for this linkage is the german phrase „*Die Schlote rauchen.*” – “The chimneys are smoking.” to express economic prosperity.

### 3.2.2 Energy-Carbon Ratio

The ratio of primary energy per unit CO<sub>2</sub> emission,  $f_e$ , is determined mainly by the share of non-fossil power generation units in the energy sector, which is controlled by the entrepreneur. In order to assess that share it is assumed that the energy produced,  $E$ , is proportional to the capital,  $k_e$ , required to produce it

$$k_e = \kappa E := E(t_0) \frac{p/p(t_0)}{f_p/f_p(t_0)}, \quad (3.7)$$

$$k_{e,n} = \kappa E_n. \quad (3.8)$$

Only the growth rate of  $k_e$  is of interest here. Thus, the proportionality constant  $\kappa$  between  $k_e$  and  $E$  is arbitrary and set to unity, while energy  $E$  is derived from the growth rates of  $p$ ,  $f_p$  (see appendix C). The first equation describes the whole energy sector, the second is the analogue for the non-fossil sector, denoted by the index  $n$ . The initial amount of non-fossil energy is given as

$$E_n(t_0) = \epsilon E(t_0) \quad (3.9)$$

where  $\epsilon$  is a small constant in the order of few percent. We will usually employ

$$\epsilon = 0.01 \quad (3.10)$$

As the evolution of  $E$  determines  $\dot{k}_e = \dot{E}$ , the necessary investments  $i_e$  cannot be independently chosen by the entrepreneur, but are diagnostically given as

$$i_e = \dot{k}_e + \lambda_k k_e \left[ -\sigma_s \frac{s_e}{\beta} \right]. \quad (3.11)$$

In fact,  $i_e$  is already contained in the general capital investments  $i_k$ , because  $k_e$  is itself only part of the capital stock  $k$  and decays at the same rate  $\lambda_k$ . The last term in brackets indicates that  $i_e$  can in part be substituted by emission levies,  $s_e$ , when society decides to do that by setting  $\sigma_s > 0$  (see section 3.6 and equations (3.14), (3.15) at the end of this section).

Although they cannot control the size of  $k_e$ , entrepreneurs can very well decide which part of these investments shall be invested in the fossil or in non-fossil energy sector. The control variable  $x_3$  defines the share of  $i_e$  that goes to the non-fossil sector which then grows according to

$$\dot{k}_{e,n} = x_3 i_e - \lambda_k k_{e,n} \left[ +\sigma_s \frac{s_e}{\beta} \right]. \quad (3.12)$$

The control variable may range from  $0 \leq x_3 \leq 1$ . When emission levies are redistributed ( $\sigma_s > 0$ ) they are used exclusively for the build-up of  $k_{e,n}$ .

In (3.12) it is tacitly assumed that the specific investment to build a given amount of installed power is independent of the energy source. This simplifying assumption is used here as a starting point. The actually observed differences in the investment costs can be partly accounted for by appropriate setting of the unit costs of energy generation (see section 3.3). In subsequent studies, more elaborate concepts, like e.g. learning curves, should be employed.

Finally, the energy-carbon ratio is defined as

$$f_e = \frac{E}{e} = 1 + \frac{k_{e,n}}{k_e - k_{e,n}} \quad (3.13)$$

where the second formula is derived from normalizing the (constant but unknown) ratio of fossil energy per unit emission to unity, so that a completely fossil-driven energy system is characterized by  $f_e = 1$ . As the energy system becomes less and less 'decarbonized',  $f_e$  grows indefinitely. This normalization has no effect on the growth rate of  $f_p$  which is used in (3.6).

We postponed the explanation of the factor  $\nu\beta$  in the denominator within the brackets of equations (3.11), (3.12). The constant  $\beta$  is required to scale levies and energy investments appropriately, because only the growth rate of  $k_e$  is exactly defined by (3.7)-(3.12), while the absolute size was left arbitrary. This is sufficient to determine  $f_e$ , but when levies are included in (3.11), (3.12), the absolute amount matters.  $\beta$  is the fraction

$$\beta = \frac{k_e(t_0)}{k(t_0)} \quad (3.14)$$

of energy capital in the overall capital stock at  $t_0$ . Due to lack of data we estimate  $\beta$  from the share of energy expenditures from total GDP [DoE/EIA 2001] as

$$\beta = 0.1. \quad (3.15)$$

### 3.2.3 Energy Efficiency

Once energy is generated from a fossil or non-fossil energy source, it needs to be transformed into some kind of physical work that can be used in the production process. The incentive to reduce the energy cost share gave rise to technological improvements that made that transformation increasingly more efficient over the past century. For example the work per energy ratio in the USA has quintupled over the 20th century [Ayres and Warr 2001].

Improvements in energy utilization are usually accompanied by improvements in labour efficiency, i.e. increased productivity. It is therefore assumed that the production per energy unit,  $f_p$ , is a function of productivity,  $\hat{p}$ , rather than an independent variable. Additionally it is assumed that production is proportional to the employed physical work, so that  $f_p$  becomes the efficiency of energy conversion. As there are thermodynamic bounds to the transformation of energy into physical work,  $f_p$  is limited as well. These considerations result in a logistic dependency of  $f_p$  from  $\hat{p}$ ,

$$f_p = f_{p,max} \left( 1 - \frac{\phi_1}{\phi_2 + \hat{p}} \right), \quad (3.16)$$

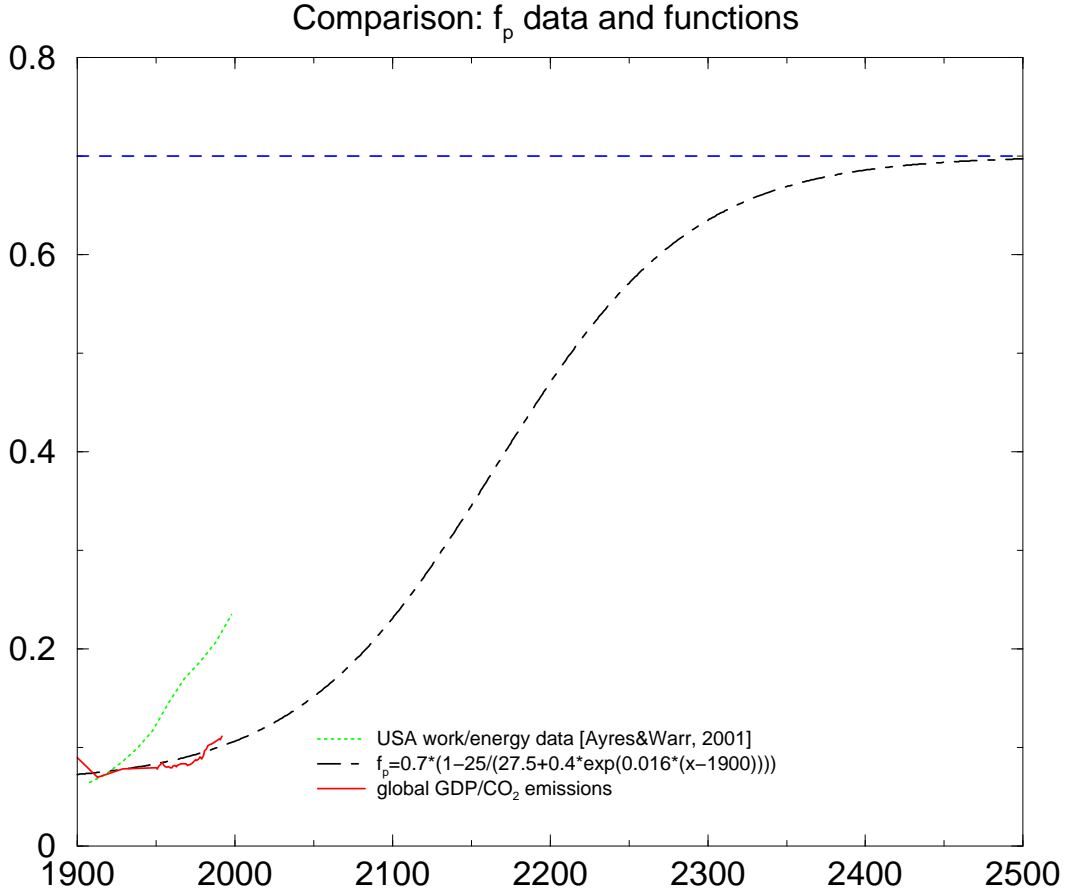
where the parameters  $f_{p,max}$ ,  $\phi_1$ ,  $\phi_2$  have to be determined empirically. The asymptotic maximum level  $f_{p,max}$  will be smaller than unity for thermodynamic reasons, but we also expect it to be far greater than 0.5. Modern gas power plants already have efficiencies in that order of magnitude [Kümmel 1998], and technical progress will continue to increase the economy-wide average efficiency so that a value of

$$f_{p,max} = 0.7 \quad (3.17)$$

is assumed. The initial value is derived from Ayres and Warr [2001] as  $f_p(t_0) = 0.07$  and set by adjusting  $\phi_1$ ,  $\phi_2$  which also determine the growth behaviour of  $f_p$ .

As data on  $f_p$  on the global scale were not available, these have been derived using a somewhat simplified procedure. First, the value of GDP per emission (i.e.  $z$ ) has been determined using





**Figure 3.3:** Comparison of historical data of  $z$  (GDP/emissions, solid line) with the  $f_p(\hat{p})$  function specified by equations (3.16)-(3.19) (dash-dotted line). Both curves are scaled to match the work/energy data (dotted line) from Ayres and Warr [2001] at  $t_0 = 1915$  and to approach  $f_{p,max} = 0.7$  for large  $t$  (dashed line).

global data on emissions [Marland *et al.* 2001] and GDP [Maddison 1995]. We then assume that  $f_e$  is constant in the period 1915-1992 so that  $f_p = z$  can be used to scale  $\phi_1, \phi_2$  to

$$\phi_1 = 25 \text{ year}^{-1}, \quad (3.18)$$

$$\phi_2 = 27.5 \text{ year}^{-1} \quad (3.19)$$

when we use a global productivity growth rate of 1.6% p.a. as suggested by the benchmark in section 3.1. The fit result is shown in fig. 3.3.

The strongest assumption used in the derivation of these settings is that  $f_e$  has been constant and therefore  $f_p = z$  during the last century. In fact, national US data ([Ayres and Warr 2001], dotted line in fig. 3.3) show that  $f_{p,US}$  has grown much faster than global  $z_{global}$ , and it can be assumed that  $f_{p,global}$  grew almost as fast as  $f_{p,US}$ , so that the equality  $f_p = z$  did not hold globally. The explanation is that  $f_e$  has really declined because industrialization and 'fossilization' of energy systems in many parts of the world had not even started in 1915. What we therefore capture in our estimation of  $f_p$  is the superposition of this decrease of  $f_e$  due to industrialization and the parallel increase of 'pure'  $f_p$  due to technological progress that happened during the 20th century. Considering that our definition in section 3.2.2 allows only for growing  $f_e$ , this choice is the only possibility to arrive at a correct representation of historical emissions scenar-

ios. Furthermore, since 'fossilization' of the world energy systems is getting more and more complete, the growth rate of global  $f_p$  will approach that of  $f_p$  of already fossilized countries. Our estimation of  $f_p$  in fig. 3.3 assumes that this happens by the end of the 21st century, when productivity grows at the same rate as in the past. This appears not to be unrealistic.

### 3.3 Energy Costs

The costs for energy use in SDIAM are modelled conceptually rather than in a very detailed manner, therefore only a simple dynamic scheme is employed. In particular, the evolution of unit costs for fossil and non-fossil energy is considered. These together with the current composition of the energy supply system determine the cost per unit of average energy. Although unit costs generally are not the same as market prices, the term 'price' is sometimes used as a synonym for 'unit cost' in the following.

In SDIAM, primary energy<sup>2</sup> can be generated from two kinds of energy sources: a fossil fuel composite whose unit costs are denoted by  $w_f$  and exhibit some dynamics as fossil resources<sup>3</sup> decline, and a non-fossil energy composite at unit cost  $w_n$ , which is for simplicity assumed to be constant and initially much greater than the fossil unit cost:

$$w_f(t_0) \ll w_n = \text{const.} \quad (3.20)$$

The fossil fuel unit cost is assumed to increase when the available resources,  $c_{res}$ , decline, since resources become increasingly more difficult to exploit and more and more expensive technology is required to withdraw them. This is expressed by an inverse power function of available resources:

$$w_f = w_f(t_0) \left( \frac{c_{res}(t_0)}{c_{res}} \right)^{\alpha_e}, \quad (3.21)$$

which depends on the assumed initial resources,  $c_{res}(t_0)$ , as well as on the constant exponent,  $\alpha_e$ . The resource base is assumed to be

$$c_{res}(t_0) = 10^4 \text{ GtC}, \quad (3.22)$$

which is twice as large as the estimates given by the IPCC [2001c] or Rogner [1997]. When gas hydrates and other more uncertain occurrences are included, however, these estimates increase by another 12,000 – 24,000 GtC [IPCC 2001c, Rogner 1997]. Our estimate is somewhere in between, assuming that some but not all of those uncertain resources can be utilized. Rogner [1997] also provides an estimate for the extraction cost increase at declining resources, which is reasonably well reproduced by (3.21) when

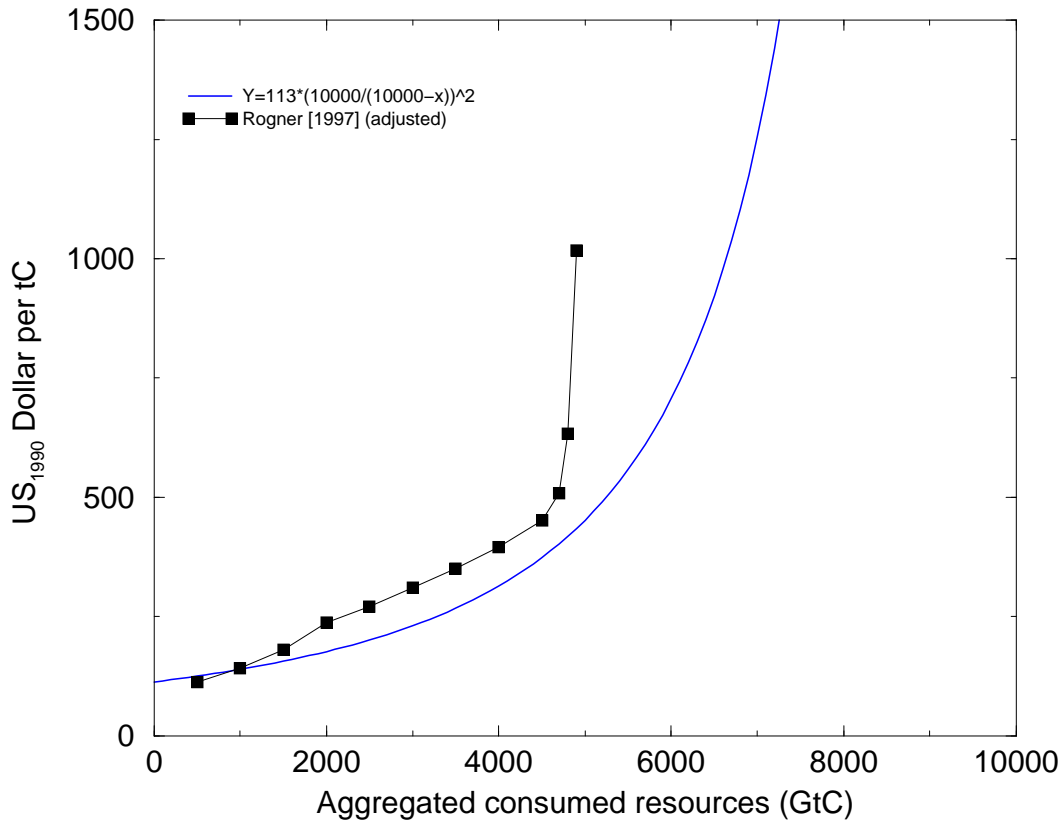
$$\alpha_e = 2 \quad (3.23)$$

is used (see fig. 3.4). For simplicity we neglect uses of fossil fuels that do not emit CO<sub>2</sub> and

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<sup>2</sup>Processing and distribution of primary energy is the same for fossil and non-fossil energy and therefore are not considered here.

<sup>3</sup>Resources and reserves are not distinguished in the context of this study. The term 'resources' implicitly denotes both resources and reserves in the following.



**Figure 3.4:** Fossil carbon extraction costs at declining resources following Rogner [1997] and equation (3.21). Both curves are scaled to current costs of 113 \$/tC.

describe the decline of fossil resources by

$$\dot{c}_{res} = -e. \quad (3.24)$$

Having determined  $w_f$  and  $w_n$ , the overall energy unit cost is then the average of these two values, weighted with their share in the energy mix:

$$w_e = w_f \frac{k_e - k_{e,n}}{k_e} + w_n \frac{k_{e,n}}{k_e}, \quad (3.25)$$

and total energy cost is that average unit cost times total energy use:

$$c = w_e E. \quad (3.26)$$

Late 20th century's unit costs for the fossil fuel average are some 10 US-\$ per barrel oil equivalent (\$/bboe) [DoE/EIA (2001)] which is hardly more expensive than in 1915. When expressed as current total energy costs per unit production and then normalized to the initial capital stock and energy use (see appendix C), these 10 \$/bboe correspond to a default unit cost value of

$$w_f = 0.007. \quad (3.27)$$

The normalized unit cost for non-fossil energies is assumed to

$$w_n = 0.1, \quad (3.28)$$

corresponding to about 140 \$/bboe. This value is rather high compared with current prices for available non-fossil energies, like wind. However, these cheap non-fossil energy sources are not enough to cover all energy demand, so that other, more expensive and in part not even developed energy sources have to be used. The high value for  $w_n$  is supposed to account for these effects and to cover the investment costs that exceed the investments from section 3.2.2. As  $w_n$  was assumed to be constant, learning curves (i.e. the reduction of specific investments depending on the already installed capital) are currently not represented in the model.

### 3.4 The Climate Module

To calculate responses of the climate system to changes in the atmospheric CO<sub>2</sub> content in an accurate and efficient way, the NICCS model [Hooss *et al.* 2001] is used in SDIAM. This model mimics the behaviour of a complex climate model using the technique of impulse-response functions (IRFs). For that technique, the 'big' model is disturbed by a small and short variation (the 'impulse') in one input variable. The reaction of the model is a changed behaviour of the output variables which can be described by one function per variable (the 'response' to the 'impulse'). Using that response function and convolving it with the time-dependent function of the input variable gives the same result that would have been obtained with the 'big' model using the same function of the input variable. The response function is also known as the GREENS function of the model, and the IRF method generally is accurate for sufficiently small perturbations, as long as the response can be approximated by linear extrapolation. The NICCS model extends this range of accuracy into the nonlinear domain by explicitly accounting for major nonlinearities in the system.

NICCS consists of two submodules: the carbon cycle module (CarC) transforms CO<sub>2</sub> emissions to changes in atmospheric CO<sub>2</sub> concentrations, while the climate change module (CliC) transforms CO<sub>2</sub> concentration changes into oceanic and atmospheric responses. Both modules are IRF representations of state-of-the-art three-dimensional models: CarC is based on the HAMOCC model [Maier-Reimer and Hasselmann 1987], while CliC is derived from the ECHAM3/LSG model<sup>4</sup> ([Cubasch *et al.* 1992], [Voss *et al.* 1998], [Voss and Mikolajewicz 2001]). In these models, the linear IRF approximation is only accurate for CO<sub>2</sub> concentrations less than twice the preindustrial value  $2 \times 280 \text{ ppm} = 560 \text{ ppm}$  [Maier-Reimer and Hasselmann 1987]. NICCS explicitly accounts for the major nonlinearities and thus allows for credible results at even higher CO<sub>2</sub> concentrations. However, other effects and feedbacks may occur at higher concentrations that "cannot be adequately reproduced by an IRF model" [Hooss *et al.* 2001]. In particular, abrupt changes in the climate system, like e.g. a shutdown of the thermohaline circulation, are beyond the model's scope. Therefore, the climate predictions of NICCS should be considered as a conservative approximation, in particular when the CO<sub>2</sub> concentration exceeds 560 ppm.

NICCS uses the annual CO<sub>2</sub> emissions  $e$  as input. Historical emissions before the initial time  $t_0$  of SDIAM are approximated by an exponential growth function for the period 1850 to  $t_0$ .

Output variables of NICCS are the time-dependent change in the annual and global mean of

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<sup>4</sup>Updating NICCS to imitate more recent models is easy, as long as the necessary IRFs are provided.

near-surface temperature,  $\Delta T$ , column-integrated cloud coverage, precipitation, and sea level. For each of these four variables the model supplies a time-independent spatial scaling pattern of the relative regional changes on the subcontinental scale. For the purposes of SDIAM only the globally averaged temperature change  $\Delta T$  is used as a proxy variable for climate change. The regional information is currently discarded, as SDIAM is not yet regionalized.

### 3.5 Climate Damage Costs

The anticipated effects of a changed climate are manifold, and many will affect humans and societies: shifts in vegetation zones that require changes in agriculture, increased numbers of heat and/or cold waves that affect health and/or productivity of workers, or a rising sea level that threatens coastal zones. Some of these changed conditions may also be beneficial: increased precipitation might lead to either flooding or better conditions for agriculture. A more comprehensive picture is given in IPCC [2001b].

Assessing these effects in a manner suitable for cost-benefit-analysis requires a formula that returns monetary impacts as a function of climate change over time. To achieve this it is necessary first to identify all possible impacts of a changed climate, next to quantify these impacts at various climate change levels, and finally to transform these impacts into benefits and losses in a way that makes them comparable across different world regions and time. None of these three points are easy to tackle. The last one, in particular, is highly controversial, as the valuation of non-monetary impacts depends on the underlying ethics.

We do not go into the details of this debate and follow the advice of the IPCC that “aggregate damage functions [...] should be regarded as ‘placeholders’” [IPCC 2001b:p.944] with mostly illustrative purposes. Nevertheless, the order of magnitude as well as the principal impact mechanisms should be represented adequately. Following Hasselmann *et al.* [1997], we define the damage function as

$$d = \delta_b p \cdot \frac{1}{2} \left( \left( \frac{\Delta T}{\tau_b} \right)^2 + \left( \frac{\frac{d}{dt}(\Delta T)}{\frac{d}{dt}\tau_b} \right)^2 \right). \quad (3.29)$$

Damages,  $d$ , depend both on the absolute level of climate change, approximated by the global temperature increase,  $\Delta T$ , and the rate of climate change, approximated by  $d(\Delta T)/dt$ , since the adjustment to climate change is more difficult when climate changes faster. This dependency is assumed to be quadratic to express that damages increase nonlinearly at higher levels of climate change. Both  $\Delta T$  and  $d(\Delta T)/dt$  are scaled to a benchmark climate change, characterized by the temperature level  $\tau_b$  and the corresponding rate of temperature increase  $d(\tau_b)/dt$ . The damage at that benchmark climate change is expressed as a fraction  $\delta_b$  of production.

The benchmark climate change is set as a  $2^\circ C$  temperature increase by the end of the 21st century, or

$$\tau_b = 2^\circ C, \quad (3.30)$$

$$\frac{d}{dt}\tau_b = 0.02 \frac{^\circ C}{yr}. \quad (3.31)$$

This is consistent with the climate change predicted by various climate models when  $CO_2$  concentrations increase by 1% per year [Meehl *et al.* 2000, IPCC 2001a].

Estimates for the magnitude of damages are usually given for a changed but static climate and do not consider climate change rates. Assessed climate changes range from 1 °C warming [Tol 2002] to 2.5 °C warming [Nordhaus and Boyer 2000, Mendelsohn *et al.* 2000], others refer to a doubling of (preindustrial) CO<sub>2</sub> levels [Berz 2001, Pearce *et al.* 1996]. The resulting damage estimates vary from costs of 1.9% of GDP [Tol 1995, Nordhaus and Boyer 2000] to benefits of 0.2% of GDP [Tol 2002]. This results from differently assumed climate change, but also from different valuation schemes or different cost estimates for sectoral damages. The approach taken in SDIAM to escape from this unclear situation is to start with a rather large damage estimate of 2% of GDP,

$$\delta_b = 0.02, \quad (3.32)$$

and to carry out sensitivity studies in chapter 4 in order to address the response of SDIAM to different damage estimates.

Despite the very coarse approximations made so far to derive a damage function we will, nevertheless, distinguish between tangible damages,  $d_t$ , that affect production directly, and intangible damages,  $d_i$ , that affect the well-being of individuals and/or society rather than GDP. We assume a fixed ratio

$$\begin{aligned} d_t &= 0.8 d, \\ d_i &= 0.2 d. \end{aligned} \quad (3.33)$$

The purpose of this splitting is to enable the separate assessment of intangible damages in the societal welfare function (see section 3.6). Although this is not very useful in the current model setup with its 'hind-cast' determination of intangible damages, it might lead to a better accounting and assessment of damages when a more sophisticated damage function is used.

### 3.6 Emission Levies and the Role of Society

Society (represented by government) in SDIAM is an additional actor which is different from the economic actors already represented in SDEM. The dynamic structure of SDIAM that focuses on the interaction of actors with diverging goals does not have a 'central planner' to integrate over all societal actors as in standard economic welfare. Instead, each actor is modelled with its individual properties.

Society aims at a high level of consumption and few damages. Its welfare is thus defined as overall consumption less intangible damages

$$b_s = w + b + (1 - \sigma_s)s_e - d_i e^{\lambda_{d,d} t}. \quad (3.34)$$

The exponential term  $\exp(\lambda_{d,d} t)$  has been introduced to examine the possibility that the relative value of intangible damages increases more strongly than that of tangible damages. However, the parameter  $\lambda_{d,d}$  is initially set to zero.

To increase  $b_s$ , society controls a levy on carbon dioxide emissions<sup>5</sup>. More specifically, the levy per tonne carbon emitted can be expressed as a fraction  $y_e$  of production. This fraction  $y_e$  is the

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<sup>5</sup>The specific implementation of the levy is not the scope of the present study. Currently, any kind of monetary tool that imposes a cost on emissions can be covered by this notion.

control variable of society. Total levies,  $s_e$ , thus amount to

$$s_e = (y_e p) e. \quad (3.35)$$

These levies are assumed to phase in linearly according to

$$y_e = 0 \quad \text{for } t < t_1, \quad (3.36)$$

$$y_e = \frac{y_{e,2}}{t_2 - t_1} t \quad \text{for } t_1 \leq t \leq t_2, \quad (3.37)$$

$$y_e = y_{e,2} \quad \text{for } t > t_2, \quad (3.38)$$

where  $y_{e,2}$  determines the final levy rate, while  $t_1, t_2$  denote the last year without levies and the year where the final levy rate is reached, respectively.

After levies are collected, they have to be spent. This can be done in two ways. The appropriate way in a climate protectionist society is to use them specifically to reduce emissions. In SDIAM this is done by increasing the growth of the non-fossil energy sector using the fraction  $\sigma_s s_e$  with  $0 \leq \sigma_s \leq 1$  (see section 3.2.2). The other way is to use the remaining fraction  $(1 - \sigma_s) s_e$  unspecifically as public expenditures. Since SDIAM does not resolve an explicit government sector, this is treated like additional general consumption.

The net present value of societal welfare is defined as

$$U_s = \int_0^{\infty} \ln(\tilde{b}_s) e^{-\lambda_d t} dt \quad (3.39)$$

where again  $\tilde{b}_s$  indicates that  $b_s$  is normalized appropriately to be dimensionless for the use as an argument of the logarithm.  $U_s$  could be utilized to determine the optimal societal and economic welfare simultaneously using a nested optimization: the optimization of society iteratively adjusts  $y_e$  in order to find the optimal  $U_s$ . After each variation of  $y_e$ , a full SDEM optimization is performed to find the optimal time evolution of the economy under the current levy framework. When  $U_s$  is optimal, an optimal solution for  $U$  is given as well.

Generally, amplitude, timing, and spending of levies should completely be controlled by society, i.e.  $y(t)$  and  $\sigma(t)$  are intended as time-dependent social control variables. However, the implementation of a full society optimization turned out to be beyond what could be achieved within the time restrictions of this study. Instead, the behaviour of  $b_s$  and  $U_s$  is explored using various levy schemes and settings of  $\lambda_{d,d}$  as input parameters for scenario calculations in chapter 5.

### 3.7 Adaptations to SDEM Equations

Integrating the additional costs for energy, damages, and levies in the economy requires changes in some SDEM equations from chapter 2.2. The most obvious is the inclusion of  $c$ ,  $d_t$ , and  $s_e$  as additional cost terms in the profit function (2.12)

$$b = p - w - c - d_t - i_k - i_{\hat{p}} - s_e. \quad (3.40)$$

At the same time, damages, energy costs and levies add to the list of necessary costs that have to be covered even in a stationary economy. In analogy to section 2.2.3 the maximum wage rate is decreased by

$$\hat{w}^0 = \hat{p}a_w, \quad (3.41)$$

$$\text{where } a_w = \alpha_w - \frac{c}{p} - \frac{d_t}{p} - \frac{s_e}{p} = 1 - \frac{\lambda_k}{\nu} - \frac{c}{p} - \frac{d_t}{p} - \frac{s_e}{p}. \quad (3.42)$$

Note that the reduction factor  $a_w$  now is time dependent in contrast to equation (2.10), where  $\alpha_w$  was constant.

In this approach it is assumed that damages to the capital stock are immediately replaced from the current production, so that all damages appear as output loss while the production basis is unaffected. This avoids the separate handling of damages to capital and to output, which is hard to quantify. Furthermore, no changes to the capital growth equations are required, and the interpretation of  $x_1$  as that part of the capital investments that actually causes growth remains unchanged (see section 2.3).

At first glance, the inclusion of all these costs in  $a_w$  affects only workers. However, profits and investments are also affected, since these depend on the response of wages to a changes in productivity,  $\partial(w/p)/\partial\hat{p}$ , which decreases when  $w/p$  is reduced. Profits and/or investments are thus reduced similar to wages.



# Chapter 4

## From SDEM to SDIAM: Numerical Experiments

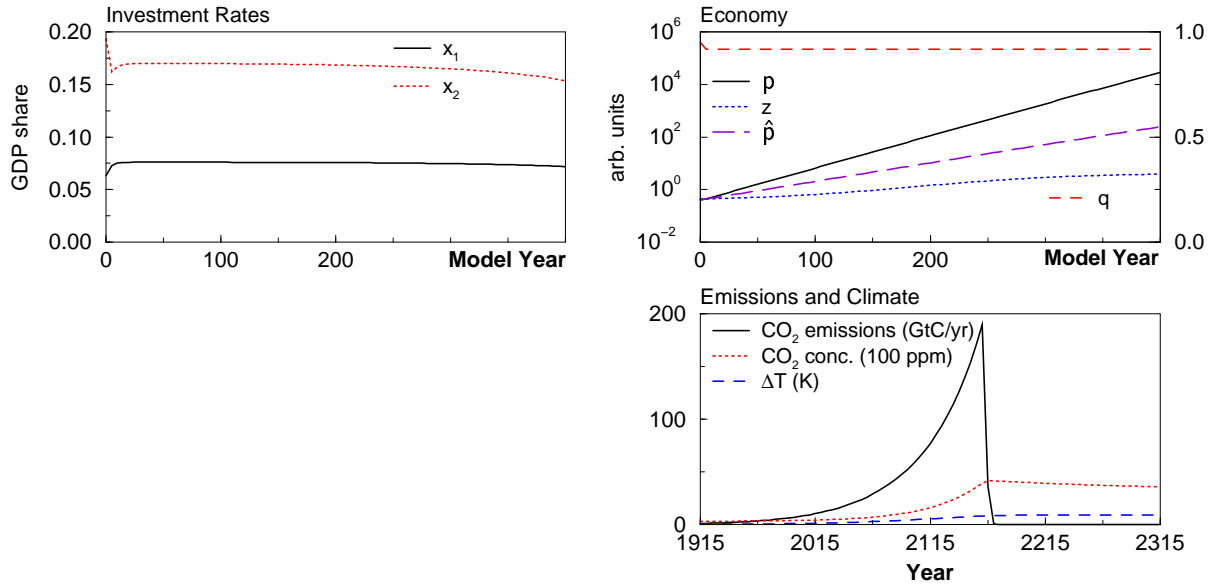
Having formally described the concepts and mechanisms at work in SDIAM in the previous chapters, the following sections show the results of numerical experiments that have been conducted in order to examine the behaviour of the model. Basically, SDIAM imposes additional limitations and constraints to the SDEM economy, in the form of limited fossil resources and climate damages. At the same time it offers new opportunities to avoid or circumvent these limitations, by increasing the energy and carbon efficiency of production. This chapter deals only with the reaction of the economic system to the additional constraints, while the role of society is examined more closely in chapter 5.

The starting point for the current chapter is the SDEM/SDIAM world baseline, already depicted in chapter 3.1. From there, we proceed by examining the reactions to fossil fuel resource limitation and climate impacts in section 4.2. The drastic consequences which result mainly from the decline in fossil fuels can, of course, be circumvented by the transition to a fossil-free energy supply system, which is introduced in section 4.3 and results in the Constrained World Baseline (CWB) scenario as the starting point for the subsequent studies. The final section 4.4 of this chapter is devoted to sensitivity studies which address the uncertainties regarding the availability of fossil resources and the magnitude of climate damages and their effects on the CWB scenario.

### 4.1 The World Baseline

The world baseline in chapter 3.1 has been chosen to reproduce the growth rates of the world economy in the 20th century. Growth rates have been the only meaningful results in SDEM; the long time horizon of the integration had nothing to do with prediction but was necessary to achieve results undisturbed from terminal effects. In this respect, SDEM has not been related to history or prediction, apart from the need for calibration.

In SDIAM the model years correspond to particular historic years starting with  $t_0 = 1915$ . Insofar, the calibration of the 'world baseline' to the dynamic parameters in equations (3.1)-



**Figure 4.1:** Optimization results for the world baseline settings from equations (3.1)-(3.4). Upper left panel: investment rates; upper right panel: production and carbon efficiency (left ordinate, logarithmic), and unemployment (right ordinate, linear). For illustration, the lower right panel shows the evolution of CO<sub>2</sub> emissions and climate when it is assumed that energy unit costs are zero and that fossil fuels can be replaced instantaneous and at no cost when all resources are used up.

(3.4) is only valid for the period 1915-2000, or model years 0-85. Assuming that the dynamic parameters remain the same for the future is mostly speculative but will nevertheless be assumed as there is no better guess available.

The time evolution of several economic variables in the world baseline setting is shown in figure 4.1. This model run still shows pure SDEM dynamics, since all additional feedbacks of SDIAM are not yet switched on.

The investment rates  $x_1, x_2$  (upper left panel) are comparable to those found in chapter 2 and cause production to grow at the historic growth rate. The initial behaviour in the first four time steps is required to adjust the employment rate. As model time approaches the end of the time horizon, investment rates decrease. The reason is that investments sacrifice profits today in order to build stocks that generate and increase profits in the future. Beyond the time horizon, there is no more future in the model, therefore investments become more and more useless and are thus reduced in favour of profits as the terminal time comes closer<sup>1</sup>. The analysis in the following is thus restricted to the first 400 years of each model run to keep the focus on the model dynamics, independent from terminal effects.

The decrease in investments at the end of the first 400 years is, however, too small to be clearly visible in the production and productivity curves (upper right panel). The growth of the carbon

<sup>1</sup>It is technically impossible to circumvent this 'end-of-the-world problem' in a numerical model, since 'infinity' as the upper limit of the optimization integral cannot be represented in the number space of a computer. The only possibility to reduce the unwanted effects is to push the terminal time far away from the time period under examination.

efficiency of production,  $z$ , is currently only achieved by technological progress, which is represented by  $f_p$ . Therefore, the growth rate of  $z$  decreases as  $\hat{p}$  increases because  $f_p$  is a logistic function of  $\hat{p}$ . The unemployment rate is found to be constant as a result of the  $F$  feedback, which causes the investment rates to be coupled even during the decrease as they approach the end of the time horizon.

Although there is yet no feedback of climate damages or energy costs to the model, CO<sub>2</sub> emissions and the resulting climate change are already calculated and their evolution is plotted in the lower right panel of 4.1. This plot is purely illustrative, since emissions drop to zero when there are no resources left, while production continues as if nothing had happened. Such a scenario would only be possible when a non-fossil energy supply could replace the fossil-driven supply instantaneously and at zero cost ('backstop technology'). Although this is an unrealistic assumption it illustrates a 'business as usual' strategy. The outcomes are impressive: the carbon dioxide concentration would rise to around 4000 ppm, and global mean temperature would increase by more than 8 °C within the next 150 years. In the following, more realistic assumptions are analyzed.

## 4.2 Adding Constraints

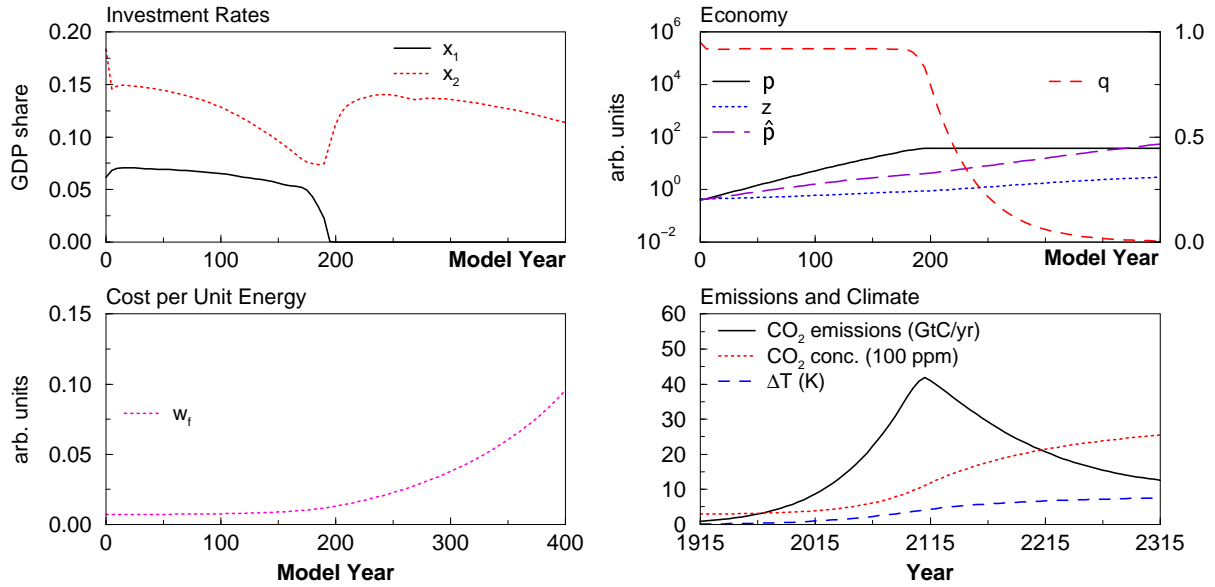
This section examines the response of the economy to changes in the non-economic environment, namely the finiteness of fossil resources and the impacts of climate change. These external changes lead to changed and generally increased costs, which reduce profits and are thus attempted to be avoided or mitigated by the entrepreneurs. Throughout this section it is assumed that the economy has no other energy source than fossil fuels, i.e. the transition to non-fossil energy sources is not yet allowed.

### 4.2.1 Decreasing Fossil Resources

The first constraint that is activated is the influence of increasing unit costs for fossil fuels, which go up as resources decline, according to equations (3.21)-(3.23) in chapter 3.3. As the model economy is assumed to depend on fossil fuels, the depletion of resources cannot be avoided when output is produced to obtain profits. Zero resources would thus result in infinite unit costs. To avoid these costs, a profit optimizing entrepreneur will therefore choose production over time in such a way that resources are not completely exhausted within the time horizon.

The resulting investment behaviour to achieve maximum profits while resources are not depleted within the time horizon is shown in fig. 4.2. To extend the availability of fossil resources over the full time horizon, production growth is stopped eventually and the then given output level is maintained until the end of the time horizon. This is achieved first by slightly reducing and finally by shutting down capital investments around the year 2100 (model year 185). Fossil unit costs grow increasingly faster within the first 400 years, but stay well below infinity even at the end of the time horizon (not shown).

Another imaginable alternative for the entrepreneur would have been to produce as long as resources are available and then to shut down production completely. This is not an alternative



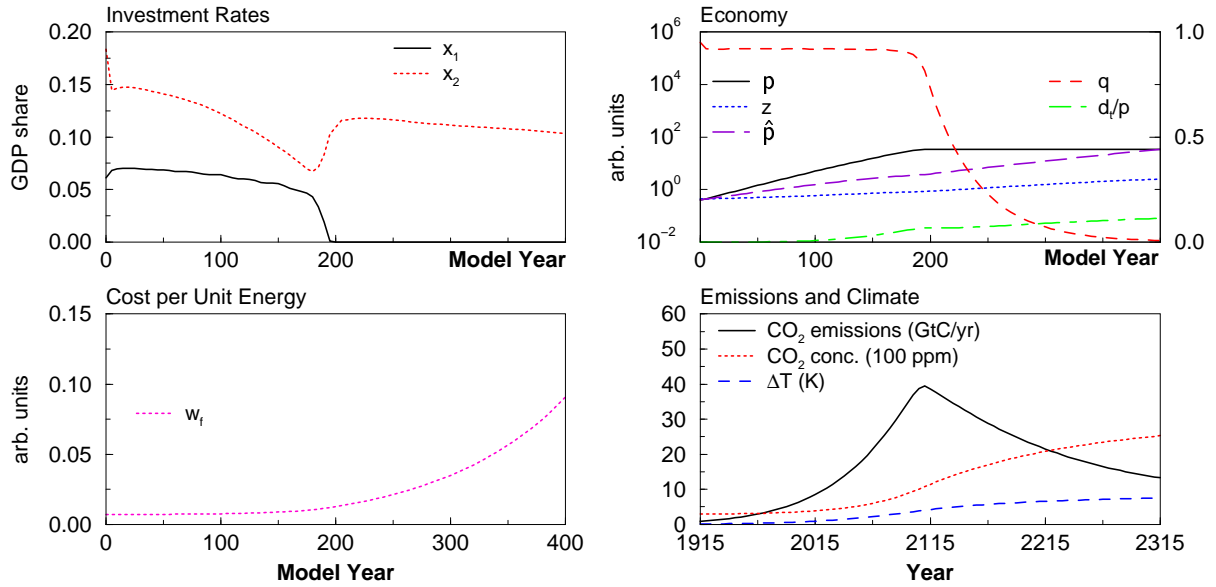
**Figure 4.2:** Optimization results for the world baseline settings with finite fossil resources and without non-fossil alternatives. Upper panels: see fig. 4.1. Lower left panel: fossil unit energy costs normalized to  $k(t_0)$ . Lower right panel: the climate change resulting from  $\text{CO}_2$  emissions has no economic impact in this scenario and is displayed only for illustration.

in the model for two reasons. Firstly, the model economy production function generates output as long as capital (or labour) is available; idle capital is excluded from the SDEM/SDIAM setup. Therefore, production could only be reduced at the depreciation rate of capital, when no more capital replacement investments would be undertaken, and never be exactly zero. Secondly, as SDEM assumes that capital is always replaced (see chapter 2.3), this is no alternative, either. If this assumption would be dropped and declining production thus become possible, the observed investment stop in fig. 4.2 would occur later, because the emissions saved at later times could be used earlier to allow longer production growth. However, as a decaying economy is not necessary when it is possible to overcome the fossil fuel dependency, this assumption turns out to be irrelevant in subsequent sections and is therefore not examined in more detail here.

The productivity investments,  $x_2$ , follow the decline of  $x_1$  until the year 2100 in order to keep the employment rate constant. After the  $x_1$ -shutdown,  $x_2$  increases again, because productivity growth also increases the energy efficiency  $f_p$ , so that available resources at shutdown time last longer. The shutdown time is thus chosen to achieve the highest possible output level that can be maintained over the rest of the time horizon. If the time horizon were longer, the shutdown would occur earlier, while with a greater resource base it would take place later.

A side-effect of this investment behaviour is that employment decays after the  $x_1$  shutdown, because increasing productivity at constant capital stock means laying off workers due to ongoing rationalization. Note that the unemployment feedback does not prevent this because profits are not reduced when employment is further reduced, so there is no disadvantage for the entrepreneur.

The resulting emissions peak at the same time as capital investments shut down, and decline



**Figure 4.3:** Optimization results for the world baseline settings with climate damages, finite fossil resources, and no alternative supply. The fraction of tangible damages to overall output,  $d_t/p$ , is shown as dash-dotted curve in the top right panel (right ordinate). For the other curves see fig. 4.2.

afterwards as a consequence of the increasing energy efficiency. However, the resulting climate change still amounts to  $\Delta T = 3.6^\circ C$  in the year 2100 and  $\Delta T = 7.5^\circ C$  after 400 years. The feedback of climate damages on the economy is not included in this scenario; it will be examined in the next section.

### 4.2.2 Climate Damages

The next constraint to be added are tangible climate damages set to the default values in (3.29)-(3.32). The resulting optimal economic time evolution is shown in fig. 4.3. Although climate change is comparable to that in section 4.2.1 (see fig. 4.2), so that damages are high and amount to more than 6% of GDP after the end of the 21st century, the overall picture is almost the same as in fig. 4.2. Again, production growth stops and capital investments shut down around the year 2100. However, investment levels are increasingly reduced due to the additional expenditures for damage payments, and productivity investment rates after the  $x_1$  shutdown are reduced by approximately 2 percent of production. The resulting slightly slower productivity growth leads to reduced levels of energy efficiency,  $z$ , and thus somewhat greater emissions after the peak. Similarly, employment levels decrease slightly slower because rationalization does not proceed as fast as in fig. 4.2. Both effects are, however, almost negligible.

The reason for this small effect of damages on the current model economy is twofold. Firstly, climate damages are a minor constraint compared to the limitations by finite resources and fossil unit costs growing according to a power law, which determine the overall investment behaviour. Secondly, climate damages reach high levels after the end of the 21st century and after the shutdown of capital investments. Therefore, initial production growth with its large

contribution to the  $U$  integral is hardly affected, whereas damages are large at later times when profits are already strongly discounted and the contributions to the net present value of profits are small.

This confirms the well-known characteristic of the economics of climate change using uniform discounting, that climate change hardly affects the economic growth path, because its adverse effects occur at times that, due to discounting, are of no relevance to present day economic decisions. The consequences of this observation for SDIAM are studied in more detail in chapter 5, where arguments for differential discounting are presented. In the following section the introduction of SDIAM features is completed by adding non-fossil energy sources.

### 4.3 The Non-fossil Alternative

The considerations in section 4.2 had something of a “Malthusian logic”: Imposing constraints on a growing system will eventually lead at least to a collapse of growth (if not a breakdown of the whole system). Although this kind of argumentation is justified when asking questions of the “what happens, if ...?”-type to outline limitations or the consequences of current behaviour, its application to predict future evolution has almost always failed (one much discussed example, besides Malthus himself and many others: “The Limits To Growth” [Meadows *et al.* 1973]). The general reason for this failure is that social systems can invent new solutions to overcome or circumvent the upcoming limitations once these are recognized. These new developments are not and usually cannot be foreseen by the authors of “Malthusian studies”<sup>2</sup>.

In order to escape the fallacies of this “Malthusian logic”, SDIAM explicitly accounts for non-fossil energy supply that may overcome the actual limitations in fossil resources. The build-up of a non-fossil energy supply system requires the decision of the entrepreneur to invest in non-fossil power plants rather than in fossil ones. Recall from chapter 3.2.2, that it is assumed that (1) investments in the energy system are part of the overall capital investments  $i_k$ , (2) overall investments in energy supply always match the energy needs of production, and (3) non-fossil energy supply has the same specific investment costs as fossil supply. Under these assumptions, investing in either form of energy supply cannot be distinguished from the other when only energy output is regarded. The control variable  $x_3$  simply adds an additional “attribute” (fossil/non-fossil) to the energy investments, but does not change the settings for production<sup>3</sup>.

#### 4.3.1 Non-fossil Alternative Without Damages

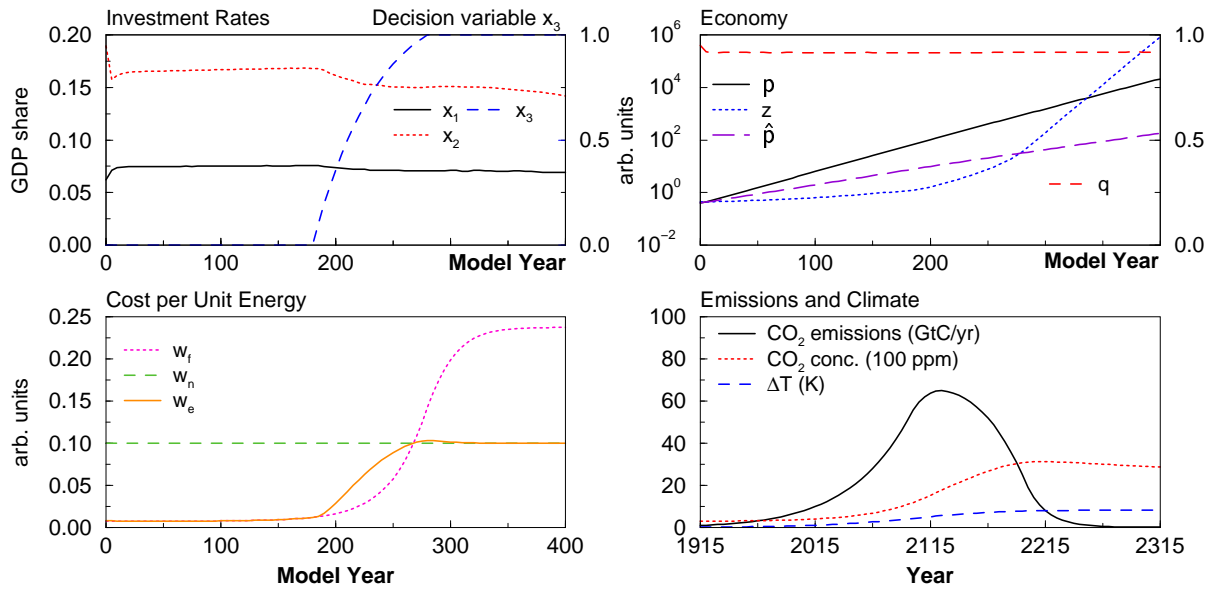
While the mere build-up of the energy supply system is modelled to be independent of the fossil/non-fossil “attribute”, the costs of energy use are not<sup>4</sup>. According to equation (3.25), average costs per unit energy reflect the difference between fossil and non-fossil energy unit

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<sup>2</sup>For example, Meadows *et al.* [1973] did not foresee the recent development in oil and gas exploration and drilling technologies, which made their estimates of available resources obsolete very quickly.

<sup>3</sup>This attribute could also be labelled ‘green’ and ‘black’, and denote the colour to paint the power plant in.

<sup>4</sup>In the picture of the coloured power plants, energy from ‘green’ plants is then more expensive than that from ‘black’ plants.

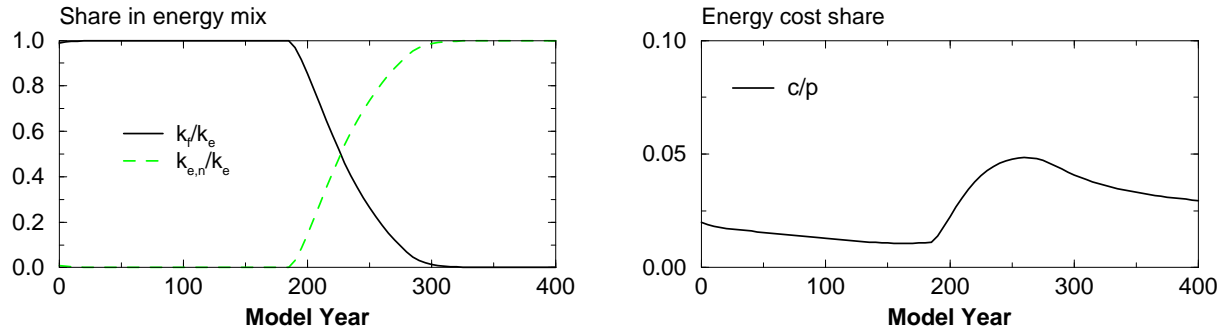


**Figure 4.4:** Optimization results for the world baseline settings with finite fossil resources, no damage impacts but *with* a non-fossil alternative. Upper left panel includes  $x_3$  on the right ordinate. Upper and lower right panels: see fig. 4.2. Lower left panel includes average unit costs,  $w_e$ , as well as fossil ( $w_f$ ) and non-fossil unit costs ( $w_n$ ), respectively.

costs as well as the actual share of fossil and non-fossil sources in the supply system. As the transition from a fossil to non-fossil energy system does not happen instantaneously, there will be a mixed energy system for a certain period of time. During that period, the average unit cost will be between the costs for a fossil and a non-fossil energy unit and thus greater than the cheapest available “price”. The start and duration of the transition period will be triggered by the attempt of the profit-maximizing entrepreneur to minimize the integrated energy costs.

The result is shown in fig. 4.4, where the settings from fig. 4.2 have again been employed, but now with the opportunity to switch from fossil to a non-fossil energy source by allowing for  $x_3$ . Without damage impact on the economy and given the difference between the unit energy costs  $w_f$  and  $w_n$ , the transition will not start before the end of the 21st century at model year 185. The share of energy investments that build non-fossil supply increases from zero to unity over the next 100 years. Fig. 4.5 (left panel) shows the resulting increase of non-fossil energy in the overall energy supply. Note that the share of fossil-driven energy capital is still greater than zero when  $x_3$  reaches unity, but continues to decline due to depreciation in the following decades and becomes increasingly insignificant. The evolution of energy costs as fraction of production,  $c/p$ , is shown in the right panel of fig. 4.5. These decline when the energy unit cost is constant, because the energy use per output unit,  $1/f_p$  declines. When unit energy costs increase, this leads to a less pronounced increase in overall costs. Note that the representation of costs relative to production does not require to apply a discount factor; this is only necessary for absolute (cost) values.

The effects of the transition to the economic and climate systems modelled by SDIAM are manifold. In the first place, the limitations to economic growth due to constrained fossil energy resources are no longer binding, thus the shutdown of capital investments is no longer needed. The small drop in the investment levels when  $x_3$  starts to grow (top left panel in fig. 4.4) is the



**Figure 4.5:** Further variables in the scenario from fig. 4.4. Left: relative shares of fossil (solid) and non-fossil (dashed) energy supply. Right: overall energy cost relative to production.

result of increased energy costs; this is explained below. Secondly, the growth rate of the carbon efficiency  $z$  increases rapidly after  $x_3 > 0$  as the non-fossil share in energy supply grows. As the  $z$  growth rate soon exceeds the growth rate of production, emissions decline after they peak at  $64.9 \text{ GtC/yr}$  in 2125 and approach zero by the middle of the 23rd century. Compared to the previous two scenarios this means higher emission levels over a shortened period of time, leading to greater  $\text{CO}_2$  levels and thus higher initial climate change ( $\Delta T = 4.2^\circ\text{C}$  in the year 2100), while further change proceeds less rapidly ( $\Delta T = 8.0^\circ\text{C}$  after 400 years).

The effect of the  $x_3$  transition on energy unit costs is depicted in the lower left panel of fig. 4.4, while the overall energy cost (the product of unit costs and energy use) is shown in fig. 4.5. As long as there are no investments in non-fossil energy ( $x_3 = 0$ ), the energy system is purely fossil driven and the average energy unit cost  $w_e$  (solid line in fig. 4.4) equals the fossil unit cost  $w_f$  (dotted line). Due to the decrease of fossil resources,  $w_f$  grows over time. When  $x_3 > 0$ , the energy mix contains an increasing share of non-fossil energy sources that come at a higher unit cost  $w_n$  (dashed line), therefore  $w_e$  exceeds  $w_f$ .

The further growth of  $w_e$  has two origins. Firstly, the increasing share of non-fossil energy in the energy mix drags the average unit cost towards  $w_n$ . Secondly, the unit cost for fossil energy continues to rise while resources decline, although the fossil share in the energy mix decreases. When  $w_f$  and  $w_n$  intersect, they equal their weighted average  $w_e$ . After the intersection,  $w_f$  exceeds  $w_e$  and, since the fossil share in the energy mix is still significantly positive,  $w_e$  exceeds  $w_n$  as well. However, as the fossil share approaches zero rather quickly,  $w_e$  soon approaches  $w_n$  from above. Note that the increase of  $w_f$  slows down as emissions are reduced, and reaches a constant value when emissions are zero because then fossil resources are no longer depleted. Note furthermore that  $w_e$  is the response to the anticipated increase of  $w_f$ : in order to avoid very high energy costs in the future, the optimizing entrepreneur accepts moderately higher costs in advance.

What finally matters for the optimization are the overall energy costs (fig. 4.5), which are calculated from  $w_e$  and the energy efficiency  $f_p$ . As these costs are necessary even for a stationary economy and are thus incorporated in  $a_w$  (see chapter 3.7), their evolution affects profits and thereby effectively limits the investment levels and thus economic growth. The increase in  $c/p$  is reflected by the drop in investment rates  $x_1, x_2$ , where the drop in  $x_2$  is more pronounced according to the characteristics of the unemployment coupling (2.29). Note, however, that the



drop in  $x_1 + x_2$  of approximately  $-0.023$  can only compensate for part of the  $c/p$  increase of  $0.038$ . The rest is mostly covered by a reduced wage rate  $\hat{w}/\hat{p}$  (not shown), which follows from equations (3.41), (3.42) when  $c/p$  increases. The response in investment rates is adjusted in such a way by the entrepreneur that  $c/p$  increase and the drops in investment and wage rates virtually cancel each other in the profit equation (3.40) so that profits remain unaffected by the changes in the cost structure.

The drop in investment rates affects the growth rate of production only slightly, it falls from  $0.029$  to  $0.028$ . Likewise, the employment rate shows negligible response to the transition in the energy system. Employment rates,  $q$ , drop from  $0.916$  to  $0.914$  between model years 180 and 220, because the increase in energy costs is compensated by a reduction of wage payments, partly by employing less workers and by the reduction of the relative wage rate  $\hat{w}/\hat{p}$  (not shown). Afterwards,  $q$  relaxes back to its initial value. The small amplitude of changes is a result of the rigidity of the  $F$  feedback.

### 4.3.2 Non-fossil Alternative With Damages: The Constrained World Baseline (CWB)

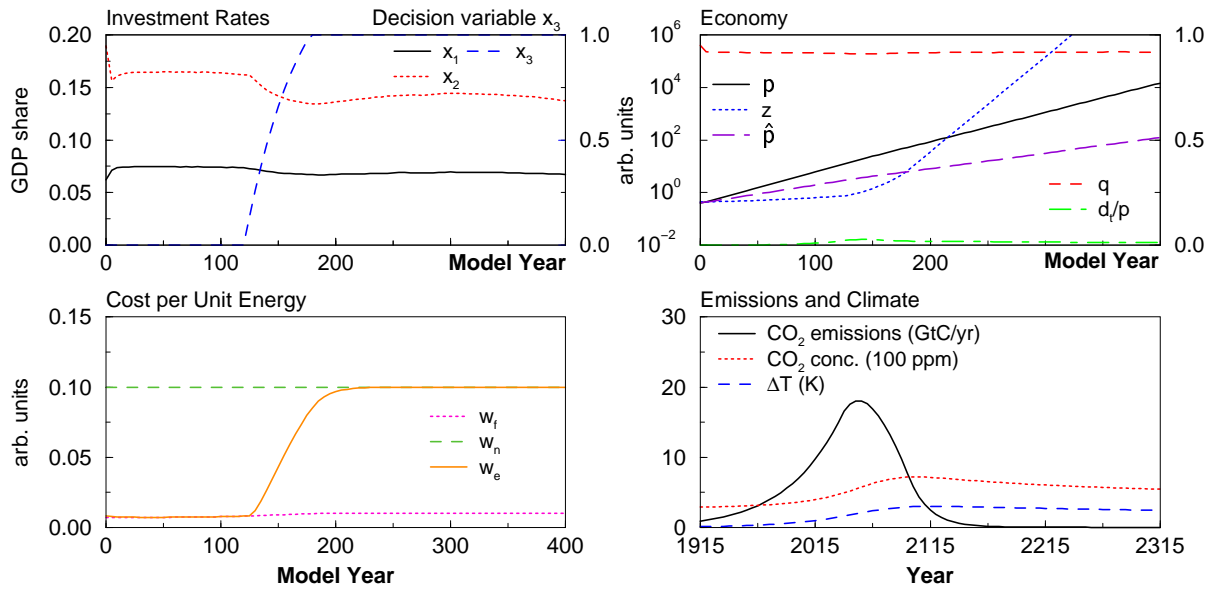
This section describes the behaviour of the model with finite resources, whose limitations can be overcome by switching to a non-fossil driven energy system, and where climate change damages are considered during the economic optimization. This completes the transition from SDEM to SDIAM. The settings employed are either taken from the world baseline in section 4.1 above, or are the default values given in chapter 3. Thus, the scenario described here is referred to as the Constrained World Baseline (CWB) scenario in the following.

The findings from section 4.3.1 change significantly when damages are included (see fig. 4.6). The most important change compared to fig. 4.3 is the timing of the  $x_3$  increase, which now starts 60 years earlier after the year 2035 (model year 120) and is finished in 2095 (model year 180). This early transformation of the energy system significantly reduces emissions, which now peak at  $18 \text{ GtC/yr}$  in 2055 and approach zero around the year 2150. This means that only  $1670 \text{ GtC}$  of fossil resources are consumed in total, whereas in previous scenarios  $8000 - 10000 \text{ GtC}$  have been used. Therefore, climate change is now greatly reduced compared to fig. 4.3, with a maximum level of  $\Delta T = 3^\circ\text{C}$  in 2120 and slowly declining afterwards. Consequently, maximum damages amount only to  $2.7\%$  of production.

The reduced depletion of fossil resources also affects the evolution of the fossil energy unit costs, which increase only slightly above their initial value and do not exceed the non-fossil unit costs. Nevertheless, the average unit costs,  $w_e$ , increase according to the transformation of the energy system and approach  $w_n$  from below.

Again, the reason for the acceptance of this price increase is the anticipation of the costs that would arise if the  $x_3$  transition would start later. While this would cause reduced energy costs, damage costs would increase. The optimal cost evolution is plotted in fig. 4.7. The optimizing entrepreneur aims at reducing the integral of energy and damage costs to the smallest value possible because this requires the smallest cuts in investments.

The required trade-offs in energy and damage costs can best be understood when turning again



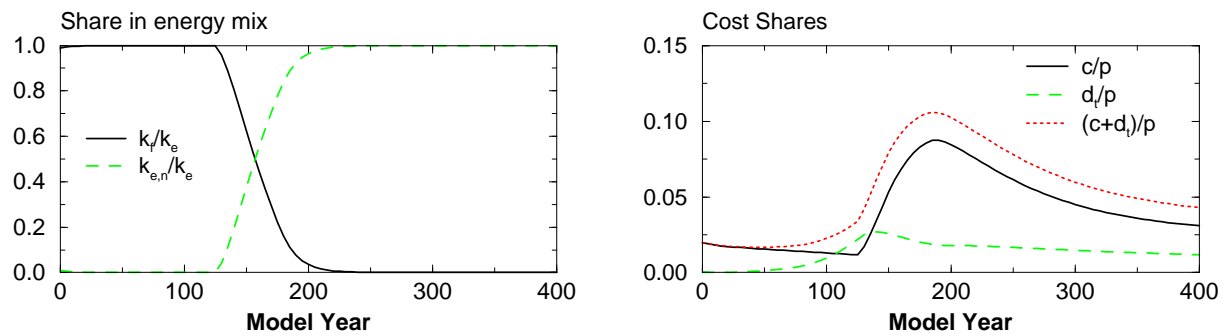
**Figure 4.6:** Optimization results for the Constrained World Baseline (CWB) with finite fossil resources, damages and a non-fossil alternative. Upper right panel includes  $d_t/p$  on the right ordinate. Other panels: see fig. 4.4.

to the scenario from section 4.3.1 (see fig. 4.4) where climate damages were not taken into account during the optimization. However, climate change was calculated for illustration, which can be used to derive the damages that would have occurred in such a scenario. These hypothetical damage costs are shown in fig. 4.8.

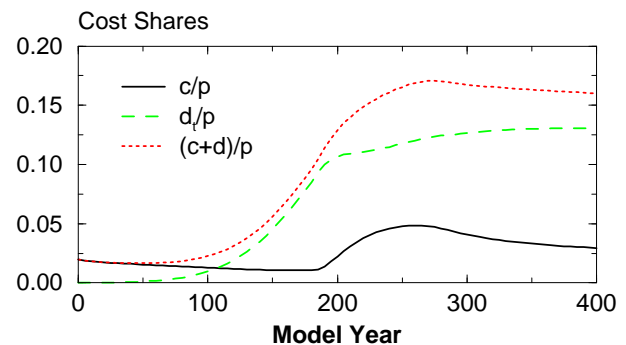
Comparing figures 4.7 and 4.8 shows that when only energy costs are considered in the optimization (fig. 4.8), the peak in  $c/p$  occurs later and is much smaller, although the increase of  $w_e$  is almost the same in both scenarios. This is the result of the logistic growth of the energy efficiency  $f_p$  with smaller growth rates of  $f_p$  at earlier times. An increase in  $w_e$  is thus much more strongly attenuated by the parallel  $f_p$  increase, and thereby energy costs are reduced more, when it happens later. The consequence of such a late transformation of the energy system would be heavy climate change and thus high damage costs of over 10.5% of production after model year 200. Overall costs reach their maximum level of 17% of GDP after model year 250, where some 75% of the overall costs are caused by damages.

When both damage and energy costs are considered during the optimization (fig. 4.7), the transition occurs earlier, thereby increasing the energy cost share (because the  $f_p$  attenuation is not as efficient), but at the same time greatly reducing damage costs. The resulting overall costs peak earlier in model year 185, but at a much smaller level of only 10.5% of GDP, and decline sharply afterwards. Damage costs amount to only 16% of overall costs at the time of the peak. This fraction increases when energy costs decline due to the ongoing growth of energy efficiency  $f_p$  and thus reduced demand for energy in production.

In short, what happens when damage costs are considered during the optimization is that action is taken to avoid part of these damages. This is done by avoiding emissions, which is achieved by making the transformation to a fossil free energy system earlier than would be necessary when no climate change were occurring so that damages could be disregarded and only the



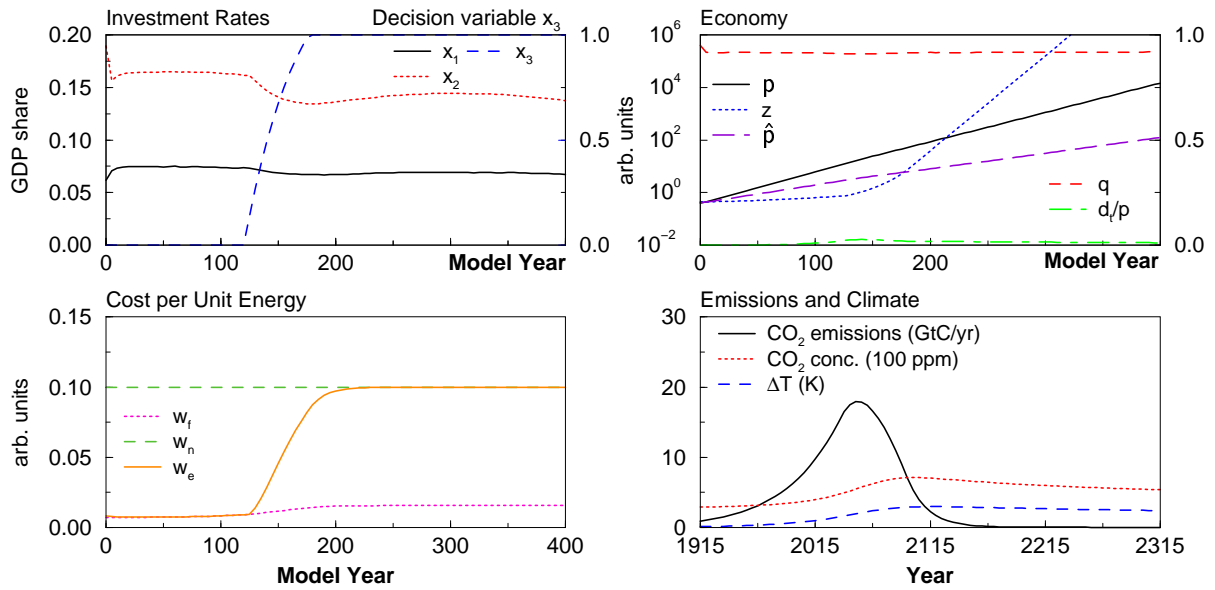
**Figure 4.7:** Further variables in the CWB scenario from fig. 4.6. Left: relative shares of fossil (solid) and non-fossil (dashed) energy supply. Right: overall energy costs,  $c$  (solid), tangible damage costs,  $d_t$  (dashed), and their sum  $c + d_t$  (dotted), each expressed as fraction of production.



**Figure 4.8:** Overall energy costs  $c$  (solid line) from fig. 4.5, and hypothetical damage costs that would arise from the corresponding scenario in section 4.3.1 but have been disregarded in the optimization there. Dashed line: tangible damage costs,  $d_t$ ; dotted line: the sum  $c + d_t$ . All cost terms are expressed as fraction of production.

finiteness of resources would matter. The earlier transition to the more expensive non-fossil energy sources exerts higher energy costs, but this is rewarded by strongly reduced damages. Thereby, overall costs are much smaller than they would have been if the transition timing were the same as in a world without climate change.

Nevertheless, the sum of energy and damage costs is greater in fig. 4.7 than the energy costs alone in the scenario without climate damages (figs. 4.5, 4.4) and thereby force greater reductions in the investment rates (fig. 4.6). Again, this drop in investment rates is much more pronounced in  $x_2$ . The employment rate drops from initial 0.916 to 0.909 in 2055 (model year 135), but relaxes back to its initial value afterwards.



**Figure 4.9:** The CWB with reduced resource base,  $c_{res} = 5,000 \text{ GtC}$ .

## 4.4 Sensitivity to Uncertainties

On the way from the world baseline of SDEM to the Constrained World Baseline (CWB) of SDIAM a number of assumptions and parameter guesses have been employed. This has been necessary because the exact values are highly uncertain, e.g. the magnitude of damages, or simply not known at the present time, e.g. the fossil resource base. These parameters are hardly affected by human activities, while others, like the unit cost for non-fossil energy, can more easily be influenced by policy measures.

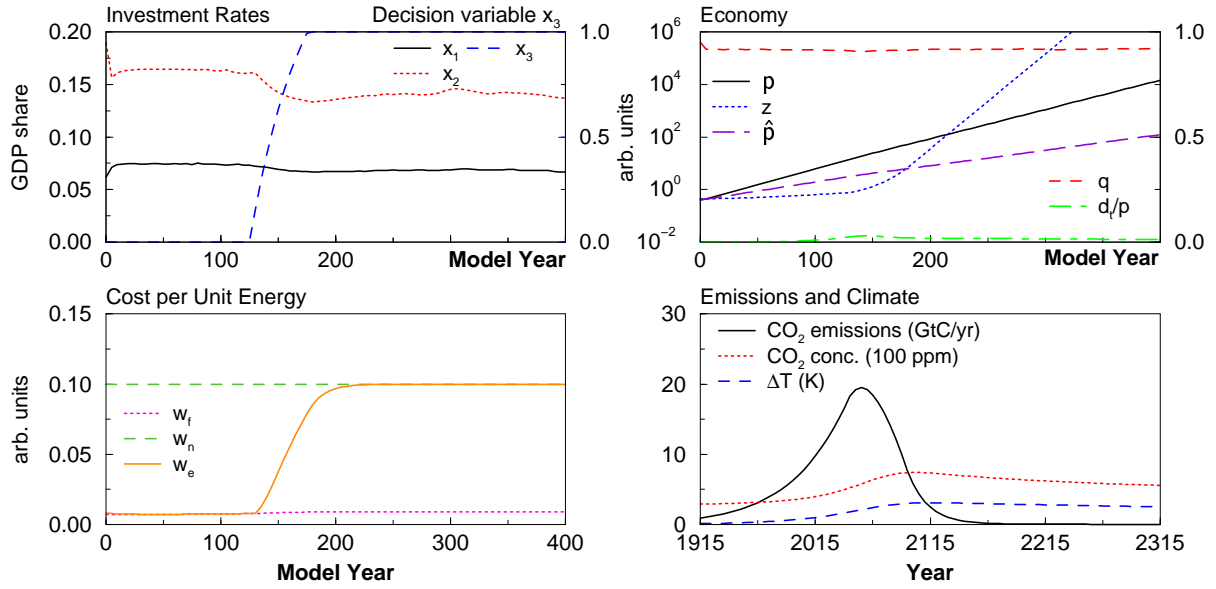
SDIAM in the version presented here does not account for uncertainties within the model. In order to address this issue at least in a descriptive manner, the sensitivity of the SDIAM-CWB to variations of the resource base and the climate impacts is examined in this section. The results will also set the stage for the effects of policy measures that will be introduced in chapter 5.

### 4.4.1 Uncertain Resource Base

As already mentioned in chapter 3.3, estimates about available resources differ between some  $5,000 \text{ GtC}$  as a relatively certain lower level, and  $17,000 - 29,000 \text{ GtC}$ , when gas hydrates (clathrates) are included [Rogner 1997, IPCC 2001c]. The total amount of clathrates and the degree to which these can be technically utilized at all are highly uncertain.

To assess this uncertainty, the CWB has been optimized using either  $c_{res} = 5,000 \text{ GtC}$  and  $c_{res} = 15,000 \text{ GtC}$ , while the default value in fig. 4.6 was  $c_{res} = 10,000 \text{ GtC}$ . The resulting scenarios are shown in figures 4.9 and 4.10.

The changes in the economic evolution in response to variations in the resource base are very small and hardly visible. Positive  $x_3$  values emerge earlier when  $c_{res}$  is smaller, because then



**Figure 4.10:** The CWB with expanded resource base,  $c_{res} = 15,000 \text{ GtC}$ .

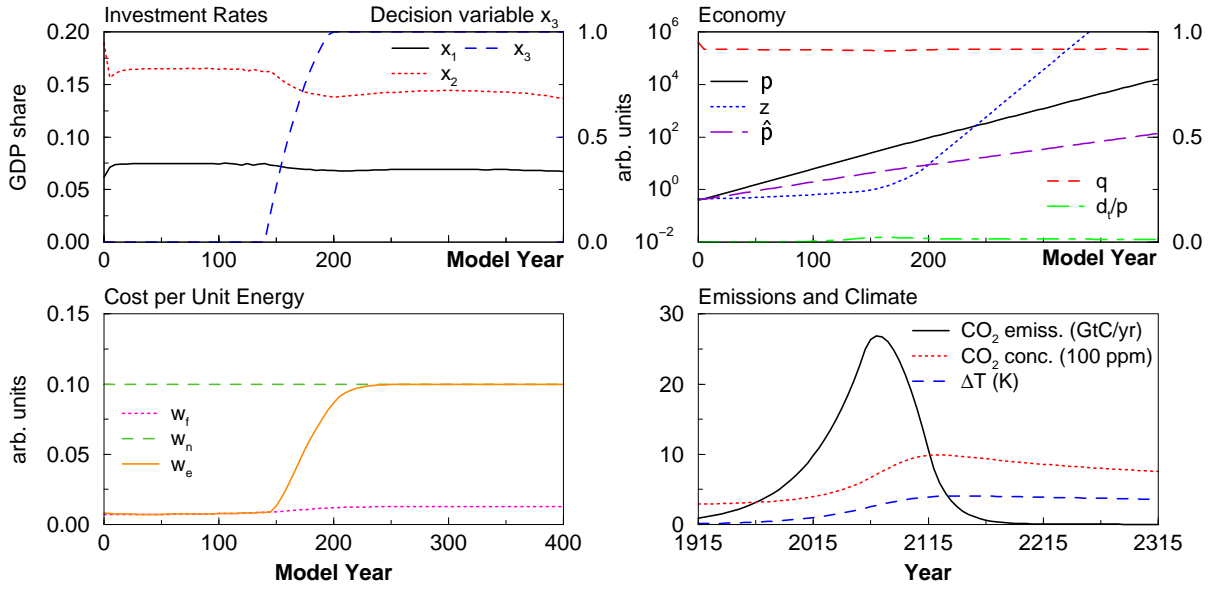
fossil unit costs increase faster and this is one of the triggering factors which induce the transformation of the energy system. However, the variation of  $c_{res}$  by  $\pm 50\%$  compared to the CWB shifts the starting point for positive  $x_3$  values only by one time step (5 years) more to the present or to the future. This earlier start of the  $x_3$  transition in fig. 4.9 leads to slightly reduced emissions that peak at  $17.7 \text{ GtC/yr}$  in 2050, while in fig. 4.10 the maximum value of  $19.4 \text{ GtC/yr}$  is reached in 2055. Peak damages amount to  $d_t/p = 0.027$  in fig. 4.9 and  $d_t/p = 0.03$  in fig. 4.10, respectively.

The reason for this minor response lies in the fact that the limitation of resources is no longer a binding constraint, as it can be overcome by changing the energy system. The general timing of the  $x_3$  transition around model year 125 is mostly triggered by the damages, which depend on emissions but not on the available resources. The fine tuning of the start of the  $x_3$  transition is, however, influenced by the evolution of fossil unit costs, which depends on the resource base. Smaller  $c_{res}$  leads to faster  $w_f$  growth and thus higher energy costs. Starting the  $x_3$  transition earlier reduces climate change and, thus, damage costs, so that the increase in energy costs can be compensated.

#### 4.4.2 Uncertain Climate Change Impacts

Present day estimates for benchmark climate damages vary from several per cent of GDP down to zero damages, and even slight benefits are derived by some authors [Tol 2002]. As already mentioned in chapter 3.5, these variations are based to some extent on differences in valuation criteria. It is therefore difficult (if not impossible) in principle to justify the selection of one criterion as the 'right' one, while other sources of divergence (e.g. aggregation schemes or the choice of included effects) might be overcome by future research.

In order to address these uncertainties in climate change impacts and assess their effect on the



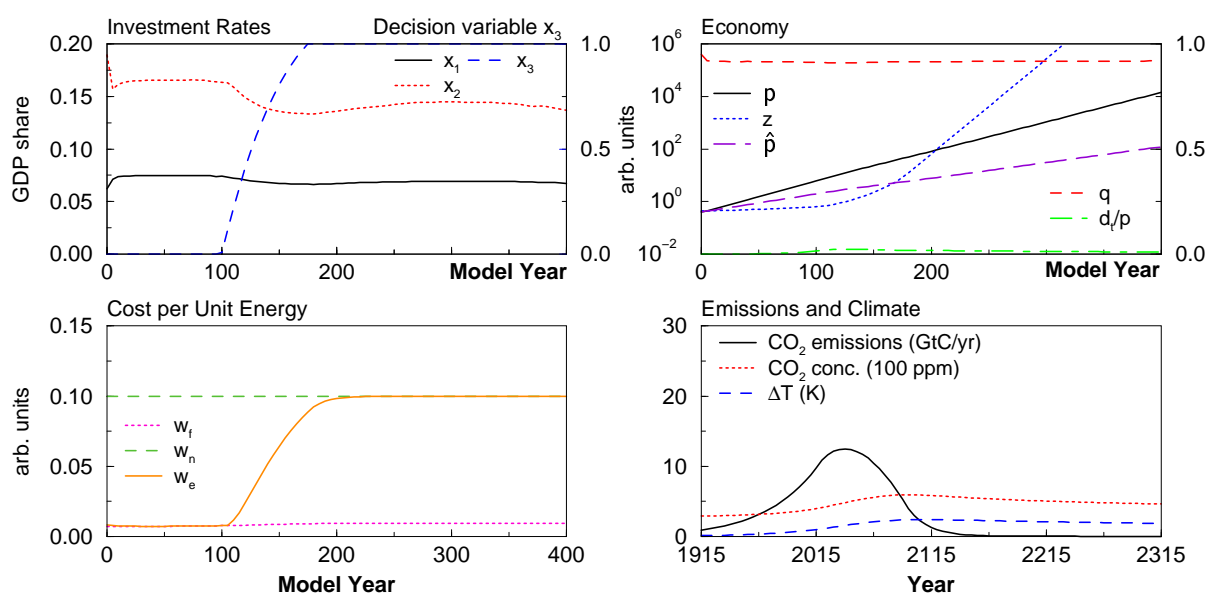
**Figure 4.11:** The CWB with reduced benchmark climate damage,  $\delta_b = 0.01$ .

model economy, sensitivity experiments have been carried out in analogy to those in section 4.4.1. This is done in the following by varying the magnitude of the benchmark climate change,  $\delta_b$ . Other candidates for variation are the scaling parameters  $\tau_b$  and  $\frac{d}{dt}\tau_b$  of the damage function. However, as  $\delta_b$  has been derived by already assuming a benchmark climate change which is described by equations (3.30), (3.31), these three parameters are not independent of each other. Examining only the variation of  $\delta_b$  thus covers the essential responses of the model to damage function variations, and a separate discussion of  $\tau_b$  and  $\frac{d}{dt}\tau_b$  can be omitted.

A basic experiment in that respect has tacitly been carried out already, since the exclusion of climate damages from the optimization in section 4.3.1 is nothing else than setting  $\delta_b = 0$ , while in the CWB  $\delta_b = 0.02$ . This is supplemented in the following by examining scenarios with  $\delta_b = 0.01$  and  $\delta_b = 0.03$  which cover the range of present day damage estimates (see chapter 3.5). The results are shown in figures 4.11 and 4.12.

Since the timing of the  $x_3$  transition mostly depends on climate damages, much greater effects than in section 4.4.1 are observed. As  $\delta_b$  scales the damage costs for a given climate change, the greater  $\delta_b$  is chosen, the earlier the transformation of the energy system starts in order to reduce climate change. For  $\delta_b = 0.01$  the transition starts in the year 2055, while for  $\delta_b = 0.03$  this happens already in 2015. Note that these dates are the result of purely profit optimizing considerations, where no societal/governmental action is included. The resulting damage costs all have peak values in the order of  $d_t/p = 0.023$ , while emissions are greatly reduced when  $\delta_b$  is set to higher values. Likewise, climate change is reduced from  $\Delta T = 4^\circ\text{C}$  in the year 2145 in fig. 4.11 to  $\Delta T = 2.4^\circ\text{C}$  in the year 2105 in fig. 4.12. Although climate damage costs remain almost unchanged throughout these scenarios, the earlier transition incurs relatively higher energy costs (see section 4.3.2). This leads to greater cuts in investments the higher the benchmark damage, but only to small reductions in the average growth rate of production, which is reduced by only 0.03% p.a. in fig. 4.12 compared to fig. 4.11.

Obviously, the magnitude of damages plays a significant role in the determination of the mit-



**Figure 4.12:** The CWB with increased benchmark climate damage,  $\delta_b = 0.03$ .

igation strategy that is taken voluntarily by the entrepreneur. As climate change can only be reduced by pre-emptive emissions reduction, the actual amount of damage is unknown when the action to prevent it is undertaken. Therefore, these actions are based on damage cost estimates rather than factual values. Any misestimation – or wrong perception – of the prospective damages will thus lead to adverse effects: underestimating damages leads to delayed energy system transformation and thus greater climate change and damages, while overestimating damages leads to premature mitigation and unnecessary reduction of economic growth. Delivery of sound estimates for climate damages is therefore an important research task. On the other hand, even with the best damage estimate, a deterministic model like SDIAM is principally limited in addressing the uncertainties that are connected to the nonlinearities of and simplifying assumptions of the systems under consideration. To address these principal uncertainties more thoroughly, SDIAM is currently being extended to account for stochastic parameter variations [Weber 2004].





# Chapter 5

## Society, Economy, and Climate

This chapter explores the role that society can play in the interaction between climate and the economy. In the previous chapters, profit optimization of entrepreneurs has been the only decision criterion in finding the optimal way to deal with climate change. The introduction of society as a new actor in the model offers additional possibilities of assessing and controlling climate change. This is achieved by allowing society to impose levies on the economy in order to maximize its own welfare. The societal welfare function generally differs from the entrepreneurial welfare function with respect to its components and their relative valuation. In the current setup of SDIAM the societal welfare function includes overall consumption and intangible climate damages. Note that this choice of constituents is based on subjective assumptions and not on empirical data, and that it is open to extensions.

The relative valuation of these two components reflects normative concepts, which have great influence on the obtained results and policy implications. This includes particularly the question whether climate damages should be discounted in the same way as usual economic costs. The standard, neoclassical answer is that they should, although this results in GDP losses when carbon levies are applied (see Hourcade *et al.* [2001] for an overview) and thus has difficulties to explain why many societies are actually willing to support emissions reduction measures. Other questions addressed in the following sections are: How are various definitions of societal welfare affected by climate change? How does societal welfare become more responsive to climate change? Are levies an adequate tool to increase social welfare, or to reduce climate change, or both? What could be done instead? Separated welfare functions for business and society enable the discussion of these questions for societal welfare independent from the entrepreneurial decision-making.

The existence of two actors in SDIAM generally leads to conflicting goals: reduction of intangible damages or the increase of workers' consumption does not necessarily correspond to maximum profits. The present setup makes these conflict lines visible and thus accessible for policy analysis. On the other hand, its incorporation in the model requires assumptions about how these conflicts are handled. Basically, the question is how to represent social power in the model, although this is not discussed in detail in this study. Instead, a simple and pragmatic approach is taken in which the entrepreneur accepts the levy framework imposed by society as a side condition to his own optimization and does not attempt to change it. Furthermore, only a limited set of levy schemes is examined, since a full society optimization had not been

implemented due to time constraints.

This chapter begins with an introduction of the concept of the societal welfare function in section 5.1. In the following sections two different methods of imposing and recycling emission levies are applied and examined with respect to climate change prevention, to societal welfare and to unemployment. When the standard assumption of uniform discounting is utilized in section 5.2 the result of societal welfare losses through climate protection measures is reproduced, which contradicts the observed perception and actual behaviour of governments, e.g. in the UNFCCC process. This discrepancy is explained and resolved by re-considering the relative valuation of the constituents of the social welfare function in section 5.3. Finally, in section 5.4 the potential effects of learning curves (which are currently not implemented in SDIAM) are discussed.

## 5.1 The Societal Welfare Function in the CWB

Recall from chapter 3.6 that the societal welfare function is defined according to (3.34) while its net present value is calculated according to (3.39) as

$$U_s = \int_0^{\infty} \ln[\tilde{w} + \tilde{b} + (1 - \sigma_s)\tilde{s}_e - \tilde{d}_i e^{\lambda_{d,d}t}] e^{-\lambda_{d,d}t} dt \quad (5.1)$$

$$=: \int_0^{\infty} u_s dt, \quad (5.2)$$

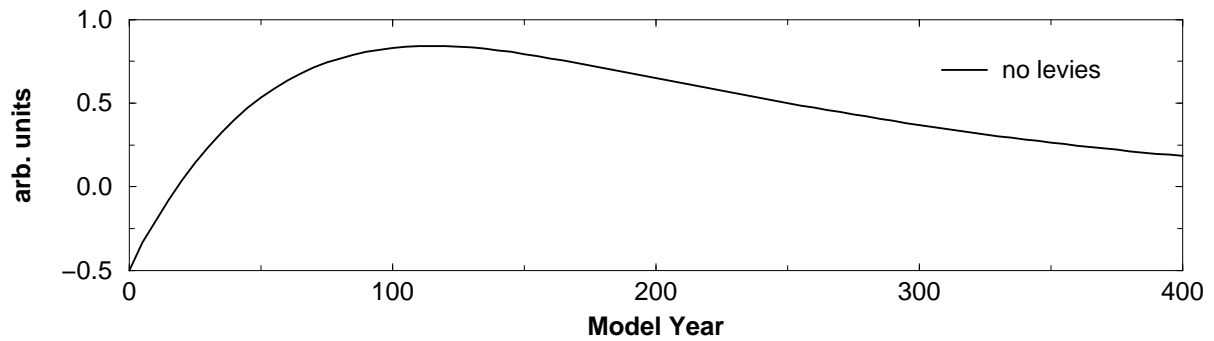
where the terms with a tilde have been divided by the constant  $p(t_0)$  to be dimensionless. The integrand  $u_s$  denotes the discounted and logarithmized societal welfare function, for which initially a uniform discount rate (because  $\lambda_{d,d} = 0$ ) is assumed.

In the previous chapters, the time evolution of the business welfare function was of no particular interest once the optimal solution was found; what mattered there were the investment decisions taken to yield that optimal solution. For the societal welfare function, no optimization is implemented, instead the local gradient of the welfare function is examined by manually varying the levy parameters. Therefore, the time evolution of  $u_s(t)$  and its integral,  $U_s$ , need to be examined to determine the way how changes in levy settings affect societal welfare.

Figure 5.1 shows  $u_s(t)$  for the Constrained World Baseline (CWB, see chapter 4.3.2). The choice of the normalization constant causes a constant vertical offset, therefore the absolute value of  $u_s(t)$  is arbitrary and only relative changes matter. Different scenarios remain comparable as long as the normalization constant is the same. Initially at small  $t$ , the discounting term  $\exp(-\lambda_{d,d}t)$  is close to unity so that  $u_s$  grows almost linear like the logarithm of an exponential function. Over time, the discounting term approaches zero and becomes the dominant factor in  $u_s$ , which thus goes through a maximum and finally approaches zero as well<sup>1</sup>. It is only due to the relatively modest discount rate  $\lambda_d = 0.01$  that  $u_s$  is still significantly positive at the end of the displayed time horizon. The integral  $U_s$  amounts to 214.6 for the full integration time horizon of 600 years.

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<sup>1</sup>Note that this asymptotic value of zero in the long run is not affected by the constant offset from the normalization constant. In other words, the 'distant end' of the  $u_s$  curve is fixed to zero by the discount factor, while the 'recent end' depends on the choice of the normalization constant.



**Figure 5.1:** Discounted, logarithmized societal welfare  $u_s(t)$  for the Constrained World Baseline. Levies have not been applied yet, likewise damages are valued like consumption ( $\lambda_{d,d} = 0$ ). Due to the normalization of  $b_s$  in the definition of  $u_s$ , the ordinate units are arbitrary.

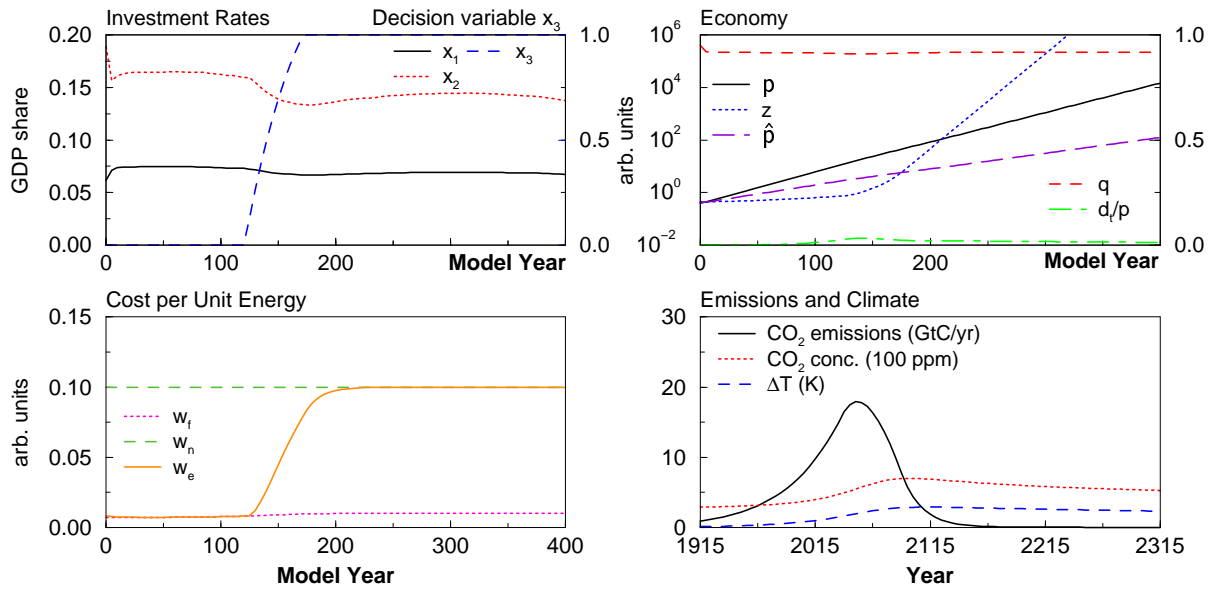
The CWB has been calculated without levies. The societal welfare function from fig. 5.1 serves as a reference for the following sections where the effect of various levy settings on  $u_s(t)$  and  $U_s$  as well as on the economic evolution is considered.

## 5.2 Uniform Discounting: The Effects of Levies

In order to address the effects of emission levies on the economy and the societal welfare this is first examined for the standard setting of a uniform discount rate (i.e.  $\lambda_{d,d} = 0$ ). In all following sections, levies start in the year 2000 (model year 85), increase linearly and reach their final value  $y_{e,2}$  in the year 2020 (model year 105). To start with, the final levy rate is set to 10 \$/tC, which is at the lower end of the range of recent economic model estimates for marginal abatement costs (i.e. emission certificate prices) under a global emissions trading scheme [Hourcade *et al.* 2001]. Such a scheme is most appropriate to represent the situation of a strictly global economy without regionalization like in SDIAM. In such an economy, the costs for reducing emissions can be comparatively small in the global average because the absolutely least costly CO<sub>2</sub> reduction measures can be executed first. When measures have to be taken on a national level, the levy estimates for industrialized countries to achieve a given emission reduction increase by one to two orders of magnitude because more expensive reduction potentials have to be exploited [Hourcade *et al.* 2001]. The two ways of spending the levy revenues – consumption or investments in non-fossil energy supply – are considered in detail in the next subsections.

### 5.2.1 Levy Revenues Spent for Consumption

Levies that are not used for investments can in SDIAM only be spent in a lump-sum manner for consumption, since there is no other way of spending due to the lack of intermediate sectors or trade. This possibility is expressed by setting  $\sigma_s = 0$  in equation (5.1). The result of the business optimization of the CWB for such a levies is shown in fig. 5.2.



**Figure 5.2:** Business optimization results for the Constrained World Baseline under a 10 \$/tC levy (linear increase from zero in model year 85 to 10 \$/tC in year 105, constant afterwards). The levy is used for general consumption ( $\sigma_s = 0$ ).

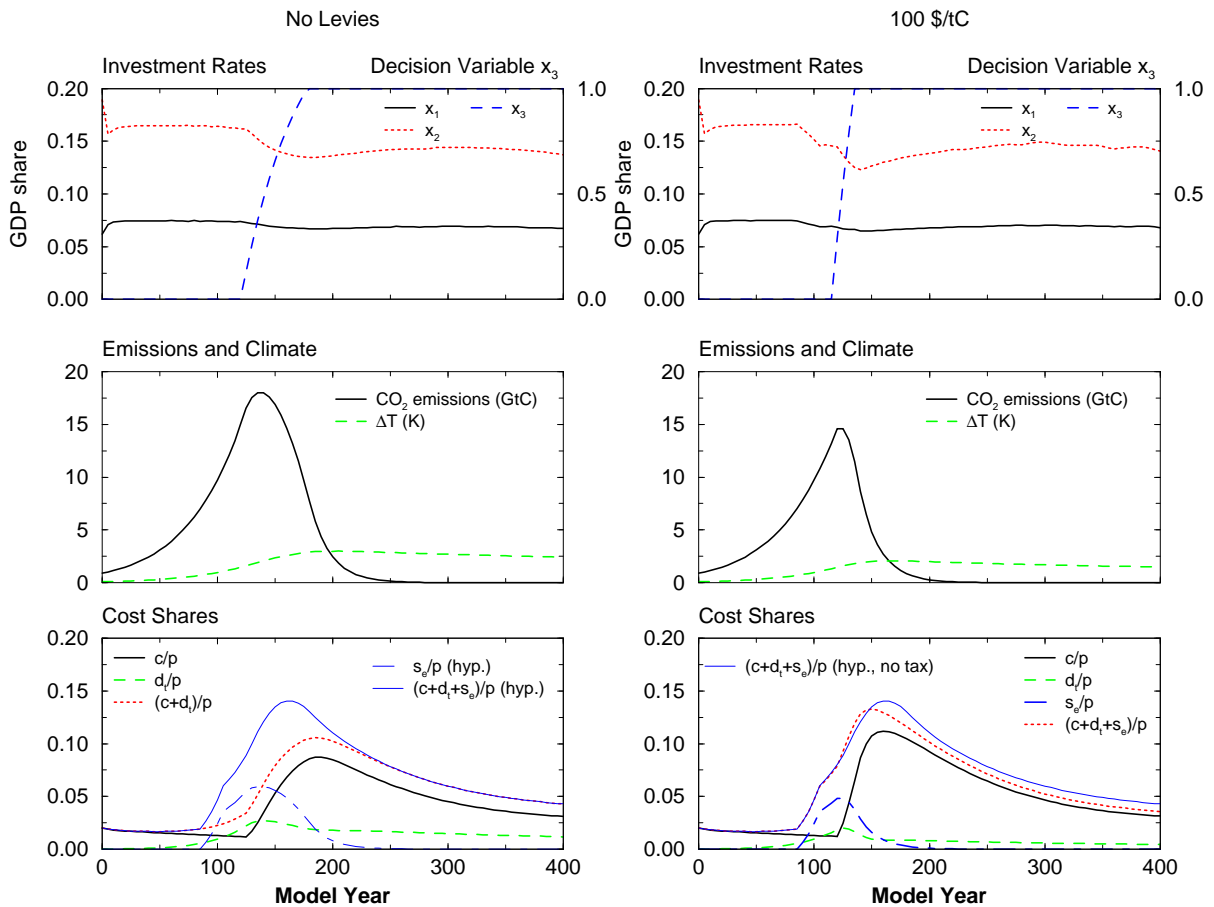
Comparing this result with the 'unlevied' CWB in fig. 4.6 yields almost no difference, only the transition proceeds slightly faster and finishes one time step earlier. Consequently, emissions decay somewhat faster, so that levy payments as well as damages are slightly reduced compared to the CWB. However, the small effect is surprising, since it is a wide-spread opinion that making  $\text{CO}_2$  emissions costly will foster the transition to non-fossil fuels. The reason is that the additional costs resulting from the given levy rate are still small compared to the non-fossil energy unit costs. Therefore, the timing of the transition is still almost completely driven by the trade-off between the avoided damage costs and the accepted increase in energy unit costs that results from the  $x_3$  transition. For the given  $y_e$  and  $w_n$ , levies are only a minor disturbance.

Despite this result, it is important to understand the business response to levies thoroughly, since it is central for the interaction between society and business. To make the effect of levies more pronounced, levies need to be increased relative to the energy unit cost difference. In the following, two examples with greater levy rates are considered: the first example in section 5.2.1.1 employs a levy of 100 \$/tC, which is at the upper end for the global trading scheme estimates, while the second example (section 5.2.1.2) features a levy of 20 \$/tC, which is well within the range of the currently estimated permit costs<sup>2</sup> [Hourcade *et al.* 2001].

### 5.2.1.1 Levy rate 100 \$/tC

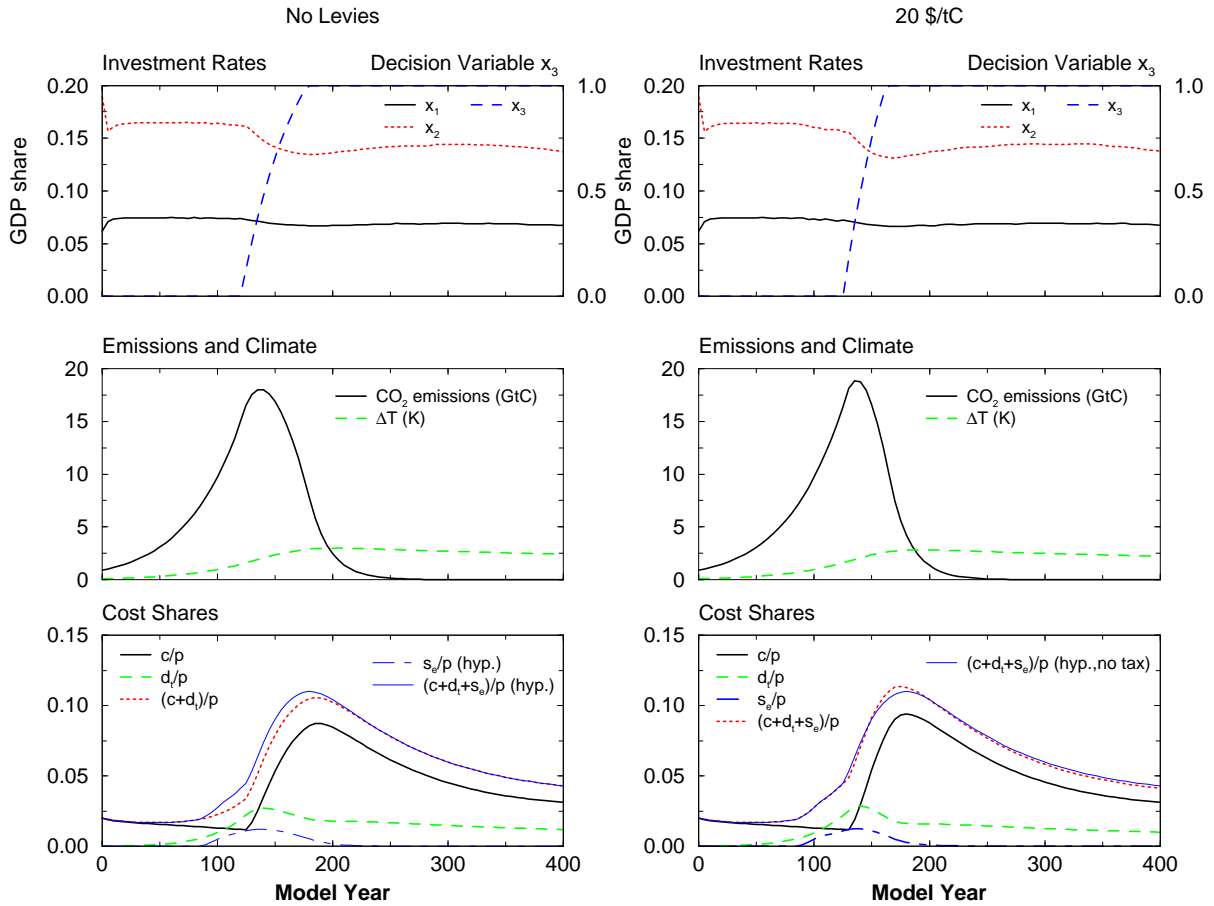
With the 100 \$/tC levy rate, the economy responds as commonly expected: the  $x_3$  transition starts earlier (though still 15 years after the levies have reached their final level) and proceeds much faster than for the CWB (top panels of fig 5.3). Emissions as well as climate change are

<sup>2</sup>Reducing  $w_n$  instead of increasing  $y_e$  yields analogous effects but changes the timing of the  $x_3$  transition even when no levies are applied and is thus not explained here.



**Figure 5.3:** Business optimization results for increased levy rate. Left column: no levies applied (same as fig. 4.6). Right column: under a 100 \$/tC levy using  $\sigma_s = 0$ . Thin lines in the lower left panel denote the hypothetical levy payments (dash-dotted), and the sum of energy costs, damages and hypothetical levies (solid). The thin solid line is repeated in the lower right panel. See text for more details.

significantly reduced (middle panels). The effect of levies on the cost structure is shown in the lower panels (thick lines). The dotted lines denote the total costs that accrue in both scenarios: in the 'no levy' scenario (lower left panel), these consist of energy costs and tangible damages; in the '100 \$/tC levy' scenario, energy, damage and levy payments add up to that sum. Obviously, levies lead to much greater costs, both because of the levy payments themselves and because non-fossil fuels are used earlier and faster, so that the unit cost increase is less dampened by the  $f_p$  increase (cf. chapter 4.3.1). This initial expenses, however, are rewarded by a relative reduction of payments later on, when emissions (and thus levies) are reduced, and also less damages have to be covered. This reduction can be seen most easily when comparing total costs in the levy scenario (dotted line in lower right panel) to the expenses that would arise, if the emissions from the 'no levy' scenario would have also been levied with 100 \$/tC. These hypothetical levies are plotted in the lower left panel as dash-dotted thin curve. The sum of (real) energy and damage costs and (hypothetical) levies is plotted as thin solid curve. This solid curve is also shown for comparison in the lower right panel, where the hypothetical costs after the cost peak are clearly greater than the costs that accrue when levies are considered in the optimization (thick dotted curve). Before the peak, both cost curves are nearly the same,



**Figure 5.4:** Business optimization results for increased levy rate. Left column: no levies applied (same as fig. 4.6). Right column: under a 20 \$/tC levy using  $\sigma_s = 0$ . Thin lines in the lower left panel denote the hypothetical levy payments (dash-dotted), and the sum of energy costs, damages and hypothetical levies (solid). The thin solid line is repeated in the lower right panel. See text for more details.

because emissions and damages are almost equal until the  $x_3$  transition starts.

The earlier and faster  $x_3$  transition when levies are applied is thus again a cost-reducing strategy. In contrast to previous findings, the trade-off is now not only between accepted higher energy costs and avoided damages and, but between initial levy payments and higher energy costs on the one hand and avoided damages and reduced levy payments in the long run on the other hand. This trade-off can also lead to counter-intuitive results as will be seen in the next subsection.

### 5.2.1.2 Levy rate 20 \$/tC

The results from a 20 \$/tC levy rate along with the CWB settings is shown in fig. 5.4. While the standard feature of an accelerated  $x_3$  transition is reproduced, the transition now starts *later* than without levy! As a result, the maximum emissions in the '20 \$/tC levy' scenario are greater than without levies, although emissions decline faster when levies are applied so that the integral emissions are nevertheless reduced.

Like in section 5.2.1.1 this surprising result follows from the changed trade-off scheme, whose cost structure is depicted in the lowermost panels of fig. 5.4. Recall that initially the energy cost share  $c/p$  decreases due to the increased energy efficiency  $f_p$  at (almost) constant fossil unit costs  $w_f$ . Postponing the  $x_3$  transition allows to compensate some of the levy payments by the reduced energy cost share<sup>3</sup>. This effect causes the reduction of total costs (thick dotted line in the lower right panel) around model year 130 when compared to the hypothetical total costs derived from the 'no levy' scenario (thin solid curve). After that, the further cost evolution is similar to that in section 5.2.1.1: the faster transition again causes higher peak energy costs but is rewarded by slightly smaller levy and damage payments after the cost peak, so that the integral overall costs in the levy scenario are smaller than those from the no levy scenario with added hypothetical damages.

Apparently, an emission levy does not necessarily lower emissions immediately and at all points in time. For a moderate levy rate, peak emissions can even be greater than in a scenario without levy. However, levies reduce climate change because the faster transition of the energy system drives emissions faster to zero so that the total integrated emissions are reduced. To business, the levies appear solely as costs, so that its response to levies is driven by cost minimization. Instead of immediately avoiding emissions (and levies) by transforming the energy system it can thus be a reasonable strategy to start the transition later and pay the levy when at the same time the energy price drops due to improved energy efficiency. Whether this is a reasonable strategy depends on the magnitude of the levy rate.

### 5.2.1.3 The Effect on Societal Welfare

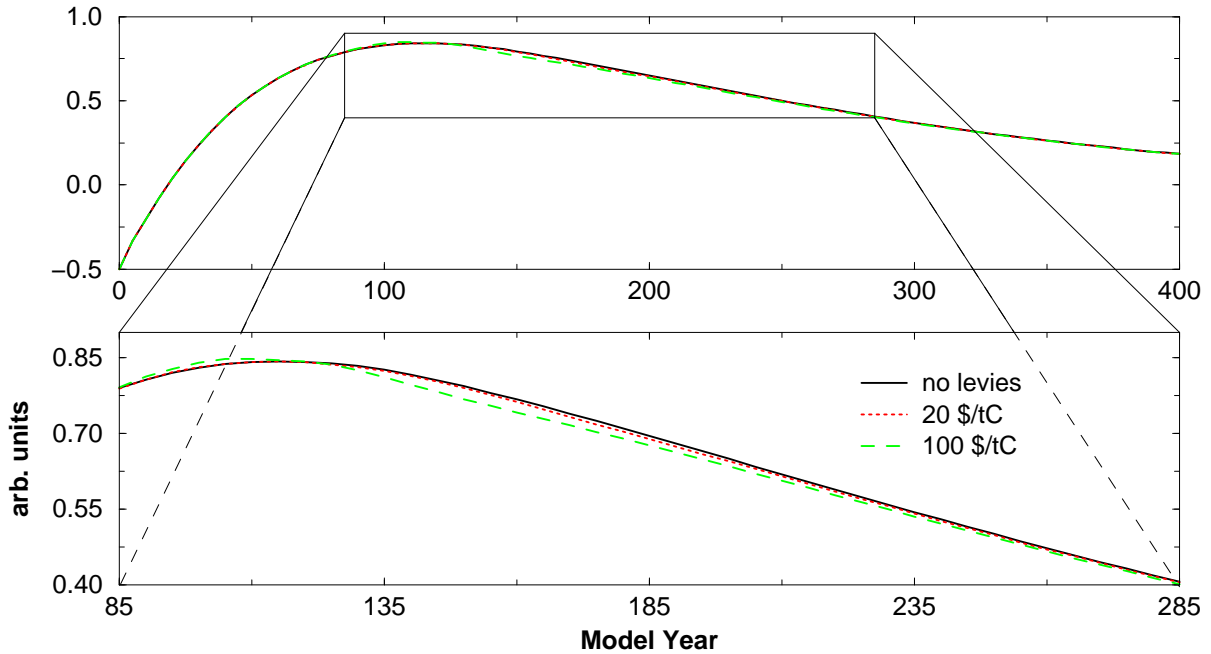
Societal welfare is also affected by the changes in the economic evolution that occur when levies are introduced. Fig. 5.5 shows the results of the societal welfare function in the scenarios with no levy (i.e. the CWB), with 20 \$/tC, and 100 \$/tC, respectively. The 10 \$/tC scenario would lie between the zero and 20 \$/tC levy scenario but is omitted here to enhance the legibility of the figure. It can be seen in fig. 5.8 below.

Until levies are imposed after model year 85, the three scenarios are identical. After this date, social welfare becomes greater the greater the levy rate is chosen. The reason is that emissions are still high in that period, so that the resulting high revenues from levies simply add to consumption – the higher the levy rate is, the more. When emissions start to decline, this source of societal welfare ceases, while the adverse effects of a faster transformation of the energy system become apparent. The rapid switch to a non-fossil energy system which is stimulated by higher levies results in relatively higher energy costs. Since energy costs are always covered, the wage rate is reduced (see chapters 4.3.1 and 3.7). Since wages are one major component of societal welfare (see equation (3.34), the  $u_s$  reduction increases with the levy rate. Proceeding further in time, the three scenarios converge, both because the growth path in a fossil-free economy is the same in all scenarios, and because the discount rate dampens small differences if they are far enough in the future.

The resulting net present values of societal welfare reflect this behaviour: still,  $U_s^{CWB} = 214.6$  for the 'no levy' scenario, while values of  $U_s^{20} = 213.8$  and  $U_s^{100} = 212.3$  for the '20 \$/tC levy'

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<sup>3</sup>For the 10 \$/tC levy the postponing effect did not appear due to the discrete time structure of the numerical model which allow no transition start 'in between' time steps.



**Figure 5.5:** Discounted, logarithmized societal welfare  $u_s(t)$  for the Constrained World Baseline without levies (solid line), with the 20 \$/tC levy rate (dotted line), and with the 100 \$/tC levy rate. The lower panel is a magnification of the time interval between model year 85 (where levying starts) and model year 285. Due to the normalization of  $b_s$  in the definition of  $u_s$ , ordinate units are arbitrary.

and the '100 \$/tC levy' scenario, resp., show that in those scenarios society is worse off than without levies<sup>4</sup>. Apparently, lump-sum levy recycling is not a beneficial option for society when (intangible) damages and consumption are discounted uniformly.

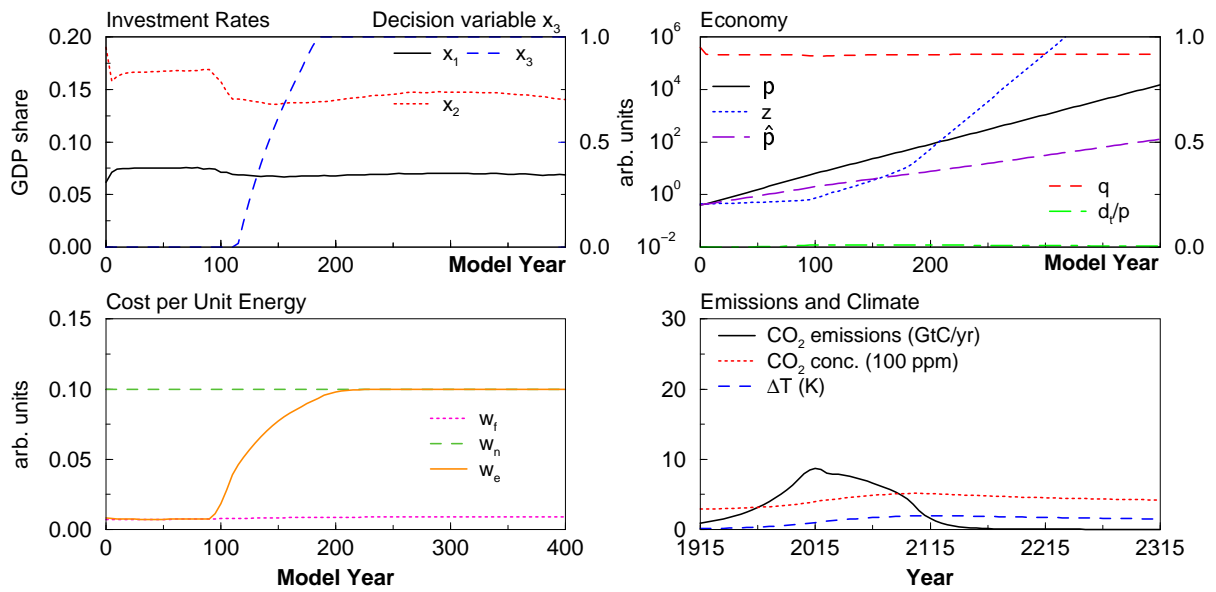
## 5.2.2 Levy Revenues Spent to Induce the Energy System Transformation

The other possibility for society to spend revenues from levies in SDIAM is to invest them specifically in non-fossil energy. This setup is chosen to reflect an environmentally concerned society, which does not rely only on the cost effect of levies<sup>5</sup> but wishes to play a more active part in the transition towards a fossil-free energy system. In that case, the revenues from levies do not add to consumption but add to the investments spent to increase the non-fossil driven capital stock in the energy system. Thereby, the transition of the energy system is no longer the sole result of entrepreneurial decision making. It now also depends on the levy settings by society as well. The calculation of societal welfare, however, is still performed using a uniform discount rate.

<sup>4</sup>Note that these figures for  $U_s$  are derived from by integrating over the limited time horizon of the numerical model. Integrating to an infinite upper time horizon would yield somewhat larger absolute values, while the absolute difference would remain the same and the relative difference would thus be slightly reduced.

<sup>5</sup>Simply making emissions more costly in section 5.2.1 turned out to have negative consequences for societal welfare although climate change is reduced by levies.



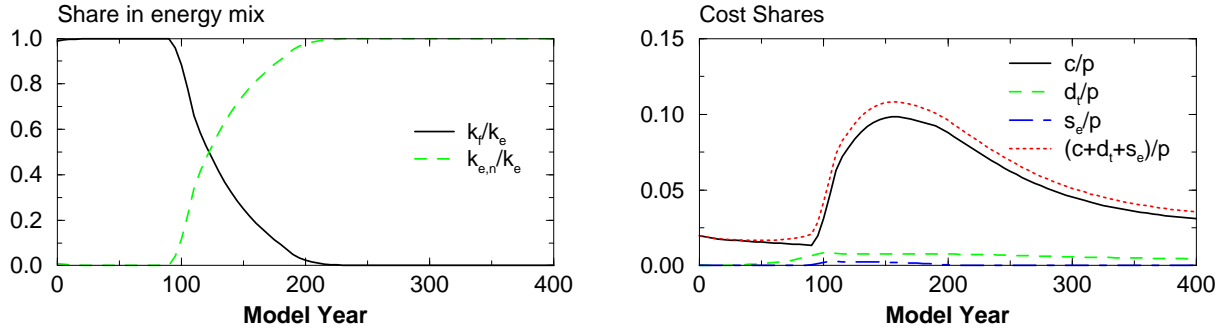


**Figure 5.6:** Business optimization results for the Constrained World Baseline under a 10\$/tC levy (linear increase from zero in model year 85 to 10\$/tC in year 105, constant afterwards). The levy revenues are used to increase the non-fossil capital stock in the energy system ( $\sigma_s = 1$ ).

In the following calculations it is assumed that levy revenues are completely spent for non-fossil energy investments ( $\sigma_s = 1$ ). The result of the usual CWB business optimization where a 10\$/tC emissions levy rate is used in that way is shown in fig. 5.6. In contrast to the case where the same levy rate was used for consumption ( $\sigma_s = 0$ , fig. 5.2), the utilization of levies for non-fossil energy investments results in pronounced differences from the 'no levy' CWB scenario (see fig. 4.6). The first thing to note is the decoupling of the energy system transition (indicated by the increase in energy unit costs  $w_e$ ), and the entrepreneur's decision to contribute to the transition (indicated by the increase in  $x_3$ ). The transition of the energy system now starts when the first levies are collected in model year 90, so that the first non-fossil plants are in operation (and thereby  $w_e > w_f$ ) one time step later in model year 95. In contrast to that,  $x_3$  starts to become positive in model year 115 and reaches unity in year 185, which is comparable but somewhat flatter than in the 'no levy' CWB case. Obviously, levy reinvestment reduces the efforts required from the entrepreneur to carry out the transformation of the energy system.

The positive effect of the early start of the transformation is that the emission efficiency  $f_e$  starts to increase as soon as levies are imposed, i.e. much earlier than in the 'no levy' case. This makes the  $z$  growth rate sufficiently large to overcompensate production growth, and this leads to slightly declining emissions; emissions peak level is 8.7 GtC/yr in 2015 (model year 100). The resulting maximum climate change with  $\Delta T = 1.9^\circ\text{C}$  in model year 2110 is comparable with the low estimates of the B1 scenario of the latest IPCC report [IPCC 2001a]. Accordingly, the maximum tangible damage costs amount to only 1% of production.

The initial investments in the transformation of the energy system are now only based on the current levy revenues and in no way optimized with regards to business costs. Rather, the initial costs of the transformation arise from societal decisions exogenous to the business optimization, so that entrepreneurs can only react by adapting the investment rates  $x_1, x_2$ . As emissions and,



**Figure 5.7:** More variables from the CWB scenario with a 10 \$/tC levy used for non-fossil energy investments in fig. 5.6. Left panel: relative shares of fossil (solid) and non-fossil (dashed) energy supply. Right panel: overall energy cost,  $c$  (solid), tangible damage costs,  $d_t$  (dashed), levy revenues,  $s_e$  (dot-dashed), and their sum  $c + d_t + s_e$  (dotted), each expressed as fraction of production.

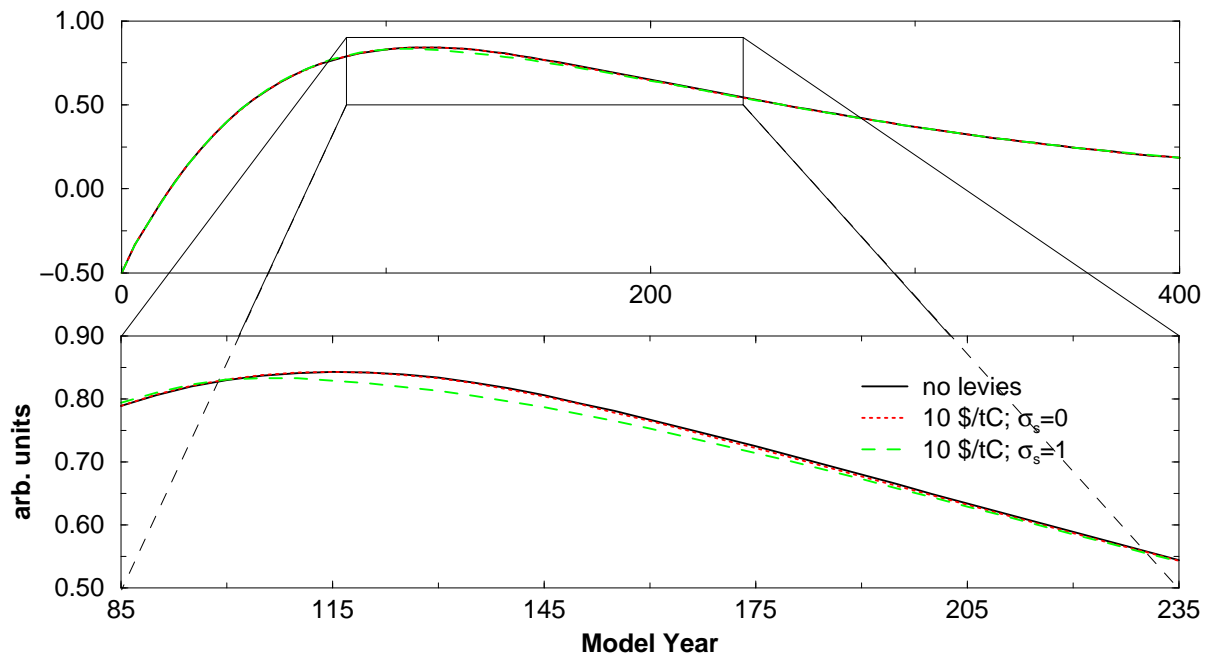
thus, levy revenues are largest at the time when levies are introduced, the investments in and the growth rate of non-fossil energy capital,  $k_{e,n}$ , are maximal at the start of the transition (see fig. 5.7, left panel). This fast and early start of the transformation leads to a very rapid increase in energy costs (fig. 5.7, right panel), which the entrepreneur compensates through abrupt cuts in the investment rates for capital,  $x_1$ , and productivity,  $x_2$ . The average growth rates of productivity and production drop from 1.6% p.a. to 1.4% p.a. and from 2.8% p.a. to 2.6% p.a., respectively. As a result of these investment cuts the employment rate drops by 0.6% around model year 100, but relaxes to its initial value when investment rates have stabilized at their new levels.

Note that the 'success' of levying emissions in reducing actual emissions also leads to a reduction of levy revenues, and thereby to a slowdown in societal investments in non-fossil energy. As the transition is also necessary from the entrepreneur's point of view (because of limited resources and to avoid climate damages), business eventually takes over these investments by setting  $x_3 > 0$  and completing the transformation of the energy system.

The consequences of the 10 \$/tC levy rate for the societal welfare are shown in fig. 5.8, where both the  $\sigma_s = 0$  case from section 5.2.1 and the current  $\sigma_s = 1$  scenario are plotted along with the 'no levy' CWB. Under uniform discounting, it turns out that for both ways of recycling levies positive levy rates reduce the societal welfare because the increased energy costs caused by levies reduce the (maximum) wage rate and thus overall consumption although this is hardly visible in the  $\sigma_s = 0$  scenario (see section 5.2.1.3). In contrast to the case  $\sigma_s = 0$  this reduction of consumption is not accompanied by a significant initial increase of  $u_s$  when  $\sigma_s = 1$ , because the use of levy revenues as investments has no direct beneficial effect on consumption. The reduction is also reflected in the net present values of integrated societal welfare. Recall that the 'no levy' CWB scenario from section 5.2.1.3 yielded

$$\begin{aligned}
 U_s^{CWB} &= 214.6, \\
 \text{while } U_s^{10,0} &= 214.2, \\
 \text{and } U_s^{10,1} &= 213.3
 \end{aligned}
 \tag{5.3}$$

for the scenarios with a 10 \$/tC levy rate and  $\sigma_s = 0$  or  $\sigma_s = 1$ , respectively. The reduction



**Figure 5.8:** Discounted, logarithmized societal welfare  $u_s(t)$  for the Constrained World Base-line without levies (solid line) and with 10 \$/tC levy rate used either for consumption ( $\sigma_s = 0$ , dotted line), or for investments in non-fossil energy capital ( $\sigma_s = 1$ , dashed line). The lower panel is a magnification of the time interval between model year 85 (where levies are imposed) and model year 235. Due to the normalization of  $b_s$  in the definition of  $u_s$ , ordinate units are arbitrary.

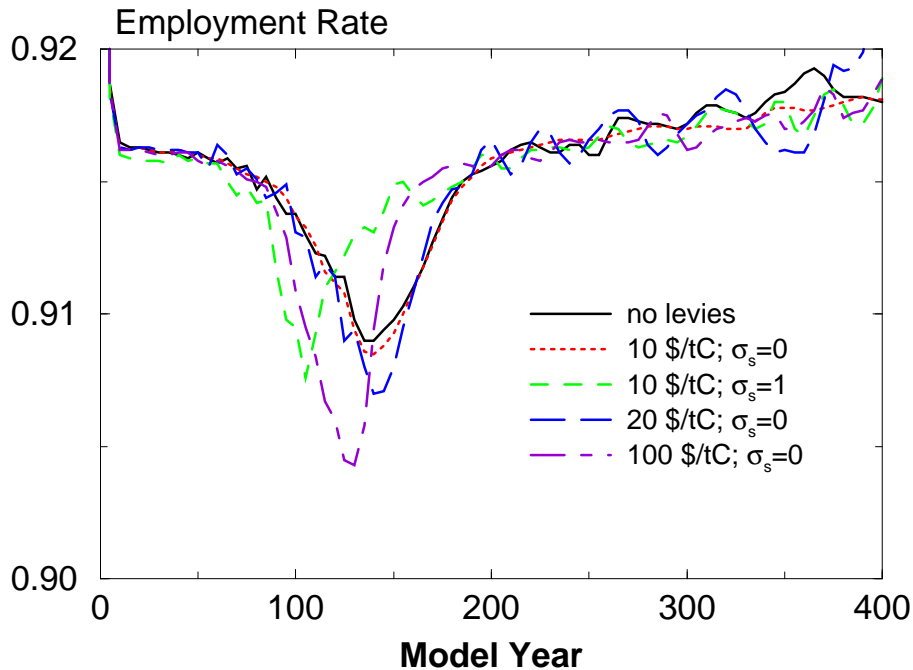
amounts to less than 1% of the net present value of societal welfare<sup>6</sup>.

### 5.2.3 The Effect on Employment

When intangible damages and consumption are discounted uniformly, the application of levies revealed no beneficial effects on societal welfare. Therefore, one might ask if there are at least beneficial effects on employment. Currently, unemployment has no effect on the societal welfare function to keep that function simple for the 'manual' assessment that is performed in the present study. The lack of individual sectors in SDIAM leads to a uniform productivity in the economy, in particular there is no difference between the fossil and the non-fossil energy sector. Therefore, the transition to a non-fossil energy system currently does not affect the model employment. Furthermore, ancillary benefits that may arise from a possible reduction of payroll taxes (see [Hourcade *et al.* 2001] for an overview) also cannot be addressed, since there are no such taxes in SDIAM. These limitations make the assessment of unemployment effects somewhat arbitrary, which is another reason why this is not included in the societal welfare function. What can nevertheless be examined is the reaction of the employment rate as a result of the overall cost increase when levies are introduced. The result is shown in fig. 5.9.

All scenarios show the already known feature of a drop in employment when the energy costs

<sup>6</sup>See footnote 4



**Figure 5.9:** Employment rate  $q$  for the CWB scenario ('no levies', solid line) and the other scenarios discussed in sections 5.2.

increase during the transition of the energy system. The reason is that the entrepreneur avoids a reduction of profits by three mechanisms. First, investment rates can be reduced. Second, the relative wage rate is reduced automatically via  $a_w$  (see equation (3.42)). Third, the wage rate can be reduced even further by lowering the feedback factor  $F(q)$ , which is achieved when the employment rate  $q$  is reduced. As the optimal value of  $F(q)$  is already close to  $F_{min}$  even when there is no distortion (see chapters 2.5.2, 3.1), the further reduction of  $q$  yields only a marginal reduction of the wage rate and is therefore limited.

What is expected from this argumentation is that the employment rate drops according to the timing of the energy cost transition, and that the magnitude of reduction follows the magnitude of additional costs that arise from the levies. This is exactly what is observed: the CWB without levies (solid line) exhibits the smallest reduction of the employment rate by 0.7 percentage points (from  $q = 91.6\%$  to  $q = 90.9\%$ ). Greater levy rates (dotted, long dashed and dash-dotted lines) lead to significantly stronger reductions of up to 1.2 percentage points for the 100 \$/tC levy. When levy revenues are invested in the energy transformation (the  $\sigma_s = 1$  case), this leads to an early and strong increase in energy costs, which is reflected by the early and strong  $q$  reduction (dashed line).

Apparently, the application of levies in any form also leads to greater unemployment during the transformation of the energy system. This would generally have a negative effect on societal welfare. When the energy system is eventually transformed and energy-related costs have returned to their initial levels, the employment rates also relax back to their initial levels. Recall, however, that the employment mechanism in SDIAM is only a first-order approximation based on an averaged productivity, which neglects intersectoral adaptation effects that may occur in real-world systems. In particular the transition from a highly automatized fossil energy sector

with high labour productivity to an initially relatively labour intensive non-fossil sector could also lead to an increase in employment. These effects are, however, not yet resolved in the present, aggregated model setup.

The findings from sections 5.2.1-5.2.3 indicate that even with an active re-investment of levy revenues by society to speed up the transformation of the energy system, the resulting societal welfare is reduced. Although climate change is smaller when emissions levies are applied, the adverse effects on the possible consumption outweigh the benefits from reduced intangible climate damages. This is due to the small share of intangible damage within the overall damages, which themselves are small compared to consumption. The entrepreneur's reactions to emission levies in order to maintain maximal profits leads to larger cuts in consumption than is gained by saving intangible damage losses. In the present setup, society will therefore be worse off if it chooses to apply levies in any form. The best choice would be to leave it to business alone to find the optimal response to climate change.

However, this result is derived from a welfare function where consumption and intangible damages are valued equally. What, if that relative valuation would be different or change over time? This question will be addressed in the next section.

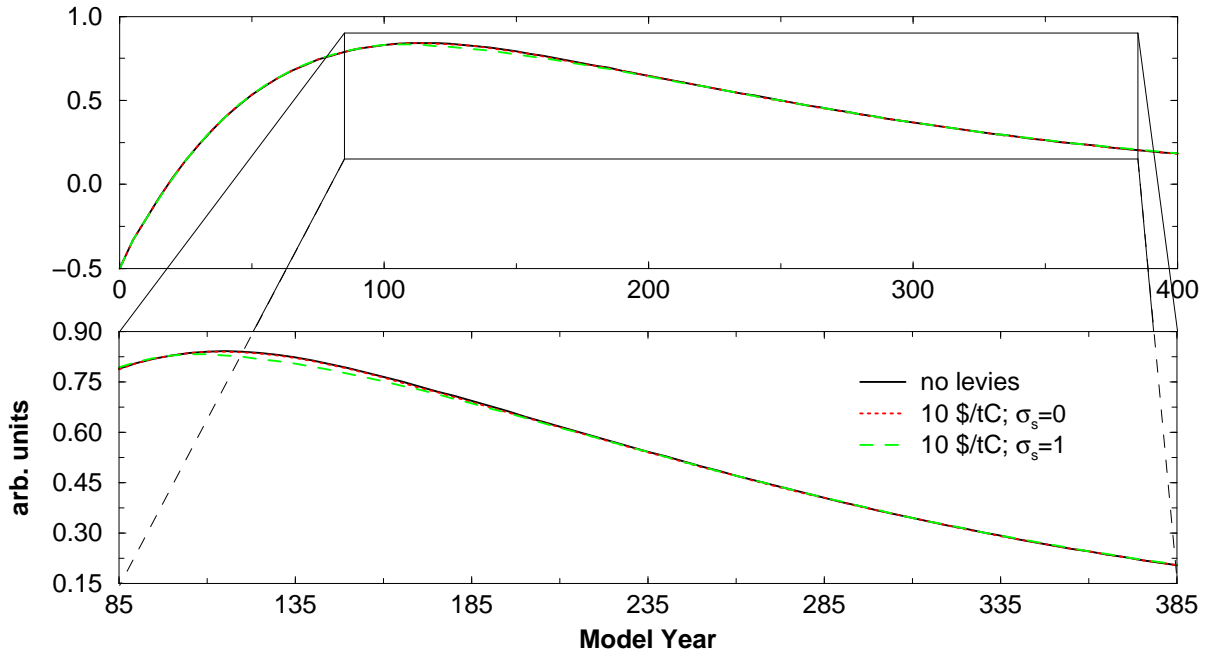
### 5.3 Differential Discounting: Changed Relative Valuation of Climate Damages

In the previous sections of this chapter it was assumed that future utility losses from intangible damages and future gains from consumption are discounted uniformly by society. In other words, a given monetarized intangible damage from climate change is compensated when consumption increases by the same amount. It turned out in the previous sections that under these circumstances societal attempts to reduce climate change by imposing levies on emissions lead to reduced societal welfare.

Although these results conform with those of standard neoclassical cost-benefit analysis [IPCC 2001c], they may have been derived from a counterfactual assumption: what, if the uniform discounting of future climate damages would not comply with the perceptions of current societies? What, if the governments that have until recently signed the Kyoto protocol act on behalf of societies that give future climate damages greater value than future consumption?

To address this issue, the definition (3.34) offers the possibility to express an increase in the relative valuation of  $d_i$  by setting  $\lambda_{d,d} > 0$ , thereby creating an exponentially growing share of intangible damages in the societal welfare function. The growth rate  $\lambda_{d,d}$  should not be too large and eventually drop to zero to avoid mathematical problems (see the case ' $\lambda_{d,d} = 0.015$ ' below), but this mechanism in equation (3.34) is a useful first guess. There have been other propositions for the exact formulation of this differential discounting approach (see e.g. Hasselmann [1999]), which, however, follow the same principle.

In the following, the effect of several positive  $\lambda_{d,d}$  values (0.005, 0.01, 0.015) on  $u_s$  and  $U_s$  is examined. Note that  $\lambda_{d,d}$  must not be confused with the discount rate  $\lambda_d$  that describes the devaluation of utility, while  $\lambda_{d,d}$  denotes the increase of utility that is assigned to a given



**Figure 5.10:** Discounted, logarithmized societal welfare  $u_s(t)$  where the relative value of intangible damages increases at the rate  $\lambda_{d,d} = 0.005$ . Results are shown for the Constrained World Baseline without levies (solid line) and with 10 \$/tC levy rate used either for consumption ( $\sigma_s = 0$ , dotted line), or for investments in non-fossil energy capital ( $\sigma_s = 1$ , dashed line). The lower panel is a magnification of the time interval between model year 85 (where levies are imposed) and model year 385. Due to the normalization of  $b_s$  in the definition of  $u_s$ , ordinate units are arbitrary.

intangible damage, or, rather, its monetary value. The valuation of damages thus increases relative to that of general consumption.

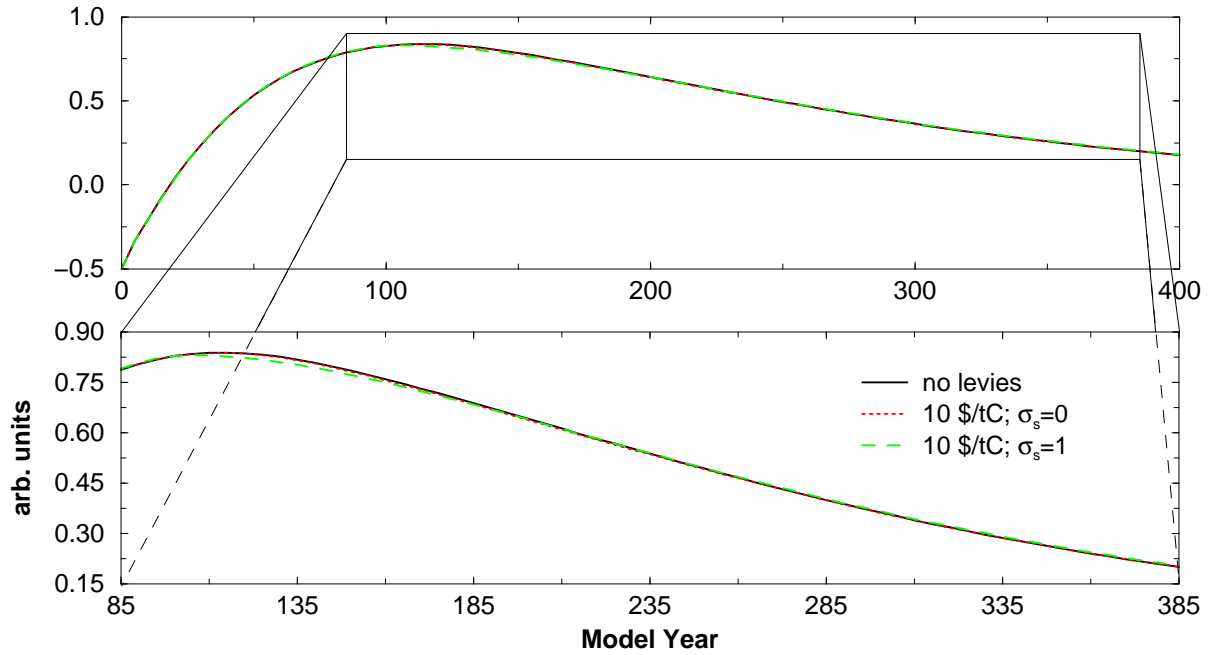
### 1. $\lambda_{d,d} = 0.005$

The effect of  $\lambda_{d,d} = 0.005$  on the CWB and the CWB with 10 \$/tC levy and alternating re-use of levy revenues is shown in fig. 5.10. Note that, as  $b_s$  and therefore also  $u_s$  and  $U_s$  are purely diagnostic variables with respect to the business optimization, the economic scenarios from figs. 4.6, 5.2, 5.6 are unchanged when  $\lambda_{d,d}$  varies. Comparing fig. 5.10 to the case where  $\lambda_{d,d} = 0$  in fig 5.8 does not yield large differences, although the increase in damage values slightly lowers the level of  $u_s$  particularly after the maximum. This is also reflected in the integral values<sup>7</sup>

$$\begin{aligned} U_s^{0,0,0.005} &= 214.0, \\ U_s^{10,0,0.005} &= 213.6, \\ U_s^{10,1,0.005} &= 213.0, \end{aligned} \tag{5.4}$$

which are reduced compared to the respective results derived from  $\lambda_{d,d} = 0$  in section 5.2.2. Note, however, that the smallest reduction is achieved for  $U_s^{10,1,0.005}$ , i.e. in the

<sup>7</sup>The indexes read as  $U_s^{y_e, \sigma_s, \lambda_{d,d}}$ , i.e. the CWB scenario from section 4.3.2 can be written as  $U_s^{CWB} = U_s^{0,0,0}$ .



**Figure 5.11:** Discounted, logarithmized societal welfare  $u_s(t)$  where the relative value of intangible damages increases at the rate  $\lambda_{d,d} = 0.01$ . See fig. 5.10 for details on the plot organization.

scenario where levy revenues are re-invested to start the transformation of the energy system. This caused the smallest climate change of all hitherto calculated scenarios. Consequently, an increase of damage valuation has the least negative effect on welfare, because damages are smallest.

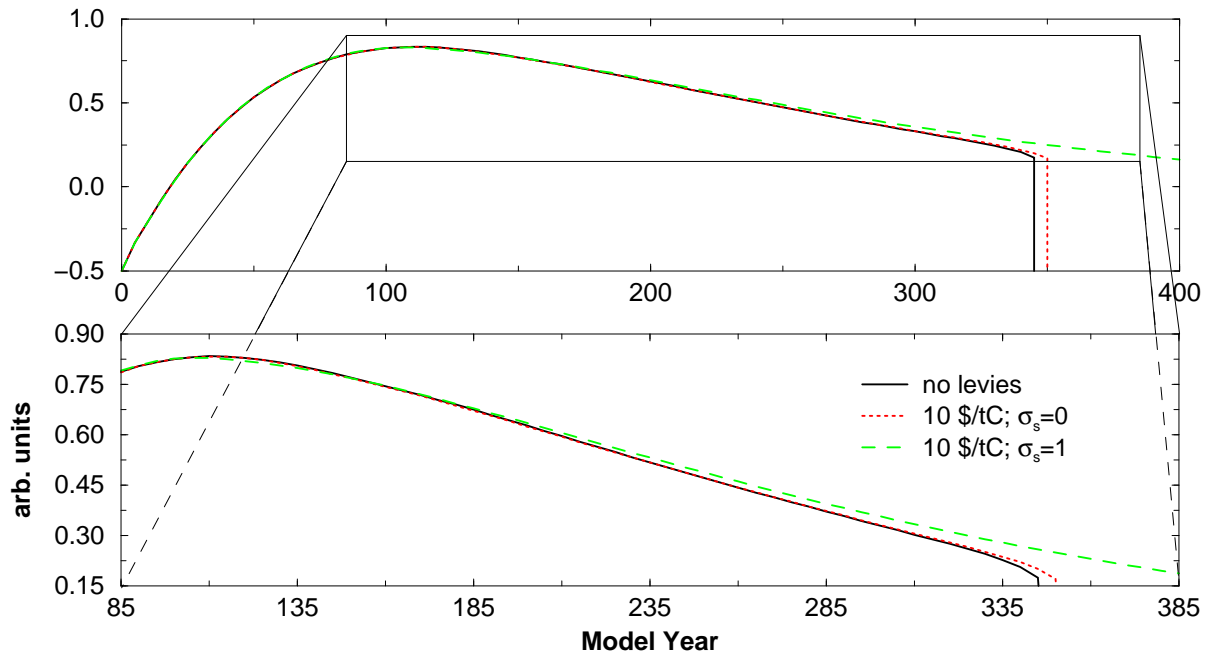
## 2. $\lambda_{d,d} = 0.01$

When  $\lambda_{d,d}$  is increased to 0.01 the changes in societal welfare become slightly more obvious (see fig. 5.11). Apparently, the difference between the  $\sigma_s = 1$  scenario (dashed) and the others becomes less pronounced after the  $u_s$  maximum. More notably, the ordering of the three scenarios reverts over time, i.e. at the end of the time horizon the  $\sigma_s = 1$  scenario is greater than the others. The crossing occurs around model year 235.

This behaviour is simply an amplification of what has already been discussed for  $\lambda_{d,d} = 0.005$ . The different damage amplitudes in the three plotted scenarios now grow at the exponential rate  $\lambda_{d,d} = 0.01$ , while consumption levels are unchanged, so that the welfare reduction is greatest where damages are greatest, i.e. in the scenarios with  $\sigma_s = 0$  (solid and dotted lines). The integrated welfare values amount to

$$\begin{aligned} U_s^{0,0,0.01} &= 211.6, \\ U_s^{10,0,0.01} &= 211.3, \\ U_s^{10,1,0.01} &= 212.2. \end{aligned} \tag{5.5}$$

Note that the integrated welfare in the  $\sigma_s = 1$  case is already larger than for the  $\sigma_s = 0$  scenarios for the reason of the greater  $u_s$  after model year 235 (recall from section 5.1 that the integration is carried out over the full 600 year time horizon).



**Figure 5.12:** Discounted, logarithmized societal welfare  $u_s(t)$  where the relative value of intangible damages increases at the rate  $\lambda_{d,d} = 0.015$ . See fig. 5.10 for details on the plot organization.

### 3. $\lambda_{d,d} = 0.015$

Finally, consider the case where  $\lambda_{d,d} = 0.015$  in fig. 5.12. Again, the result for the  $\sigma_s = 1$  case (dashed line) with its small damage levels is hardly affected by the even faster growth of damage value relative to consumption value. For the scenarios with  $\sigma_s = 0$  (solid and dotted curves), the greater damage levels cause greater reductions in  $u_s$  and lead to an earlier depression below the  $\sigma_s = 1$  scenario at around model year 160.

Figure 5.12 reveals one technical shortcoming of the approach of exponentially increasing the relative value of damages. When  $\lambda_{d,d} = 0$ , both consumption and damages grow at virtually the same rate as production. When  $\lambda_{d,d} > 0$ , then damages grow at a faster rate than consumption and it is only a matter of time when the initial difference in amplitude is over-compensated by the exponential growth. At that time, damages become greater than consumption and the argument of the logarithm in equation (5.1) becomes negative, so that  $u_s(t \geq t') \rightarrow -\infty$ , where  $t'$  denotes the time where  $d_i(t')e^{\lambda_{d,d}t'} > c(t')$ . In the two previous examples, the additional growth rate  $\lambda_{d,d}$  was too small to have  $t'$  within the time horizon of the numerical model ( $t_{max} = 600$ ). Now, with  $\lambda_{d,d} = 0.015$ , it turns out that  $t' \approx 345$  for the  $\sigma_s = 0$  cases (solid and dotted line), while for the  $\sigma_s = 1$  case (dashed line)  $t' = 430$ , which is outside the displayed time interval in fig. 5.12.

The integral  $U_s$  cannot meaningfully be compared for the different scenarios when  $u_s \rightarrow -\infty$  within the integration time horizon<sup>8</sup>. This problem can be circumvented in the current context by integrating over a shorter time interval where  $u_s > -\infty$  and examining the solutions, although these can only qualitatively be compared with the solutions (5.3)-

<sup>8</sup>When an optimization is performed, the optimizing agent would choose the control variables in order to push  $t'$  further into the future, preferably beyond the time horizon.



(5.5). Integrating over the interval  $0 \leq t \leq 345$  for all three scenarios yields

$$\begin{aligned} \int_0^{345} u_s^{0,0,0.015} dt &= 176.2, \\ \int_0^{345} u_s^{10,0,0.015} dt &= 176.4, \\ \int_0^{345} u_s^{10,1,0.015} dt &= 179.8. \end{aligned} \tag{5.6}$$

Note that the integrated societal welfare no longer decreases when levies are applied, instead, welfare increases when climate is better preserved, even though this preservation is the result of emission levies. This supports the findings from the previous calculations with  $\lambda_{d,d} > 0$  and confirms that levies only have adverse effects on societal welfare when the value of climate damages remains small compared to consumption.

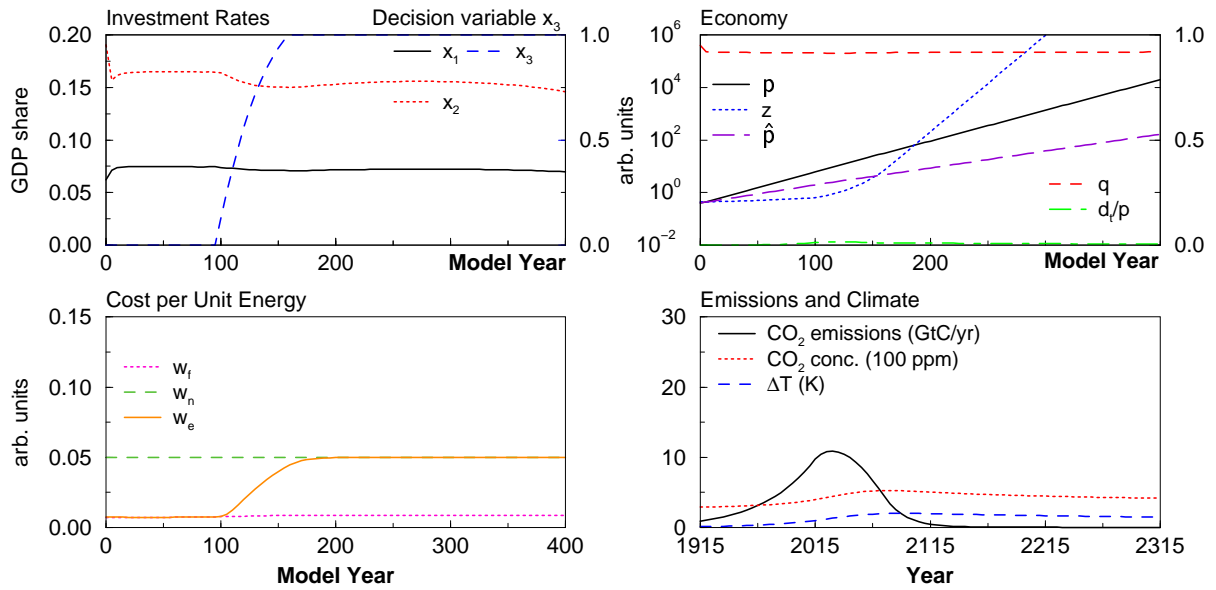
The above calculations indicate that emission levies have a positive effect on societal welfare when societal valuation of future damages increases relative to future consumption, and if this increase in relative damage value is not too slow. The resulting relevance of differential discounting in valuing climate change complies with the findings of Hasselmann *et al.* [1997] and Hasselmann [1999]. Put differently, from the viewpoint of a welfare-optimizing society it makes sense to impose levies when the preferences of society exhibit a sufficiently fast increasing valuation of intangible damages. If this were not the case, any levy would only unnecessarily reduce the welfare below the level determined by the pure business optimization in the CWB and would therefore be avoided.

Following these findings, the behaviour of the signatory states of the Kyoto protocol makes perfect sense: by imposing levies they increase their future societal welfare because it discounts climate damages differently from consumption. It should, however, be noted, that the actual decision of these governments could also be inspired by the normative decision to avoid or reduce climate change instead of performing the cost-benefit approach taken in SDIAM. In that case,  $\lambda_{d,d} > 0$  can be regarded as a mechanism to translate such normative decisions into a cost-benefit framework like SDIAM, which are then simply the two sides of the same coin. In fact, the origin of the actual setting of  $\lambda_{d,d}$  cannot be resolved within the model; it is thus taken as an external parameter.

## 5.4 Reduced Non-fossil Energy Costs

The previous sections focused on the effect of levies that are recycled either in a lump-sum fashion as general consumption or as direct investments in non-fossil energy systems to speed up the energy system transformation. The model currently is not designed to examine other concepts to prevent climate change. Namely learning curves are not implemented yet, so that there is no way to decrease the unit cost price, e.g. by levy revenues that are specifically invested in non-fossil technology. This section addresses the question whether this concept provides a suitable way to reduce climate change and/or fosters societal welfare.

The concept of 'learning curves' describes the common observation that investment costs for a given amount of desired output are initially high but decrease with the amount that has already



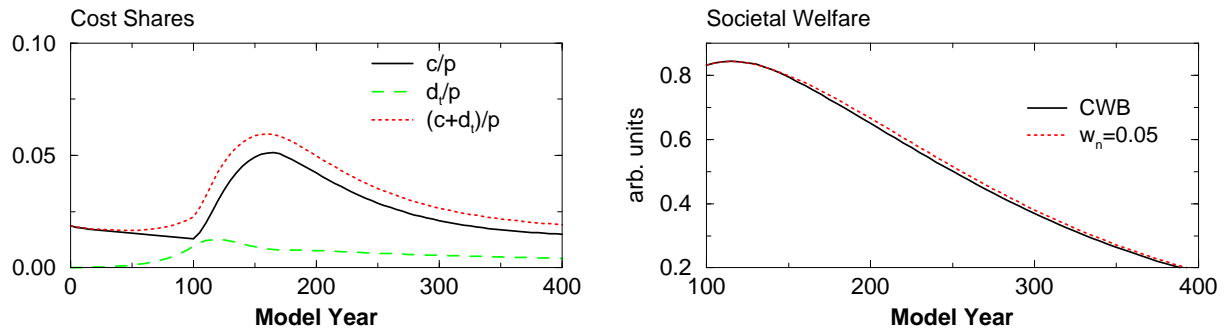
**Figure 5.13:** Business optimization results for the Constrained World Baseline with reduced non-fossil energy unit costs ( $w_n = 0.05$ ) and no levies applied.

been invested in the respective technology. This reduction is mostly an effect of learning-by-doing, which requires that the technology is installed and worked with. Since learning curves are generally determined empirically and differ for each technology, it is difficult to make predictions on their evolution.

As mentioned throughout chapters 4 and 5, the difference in fossil and non-fossil energy unit costs is the major determinant both for the start of the transformation of the energy system and the costs associated with it. While fossil technologies are mature and exhibit virtually constant investment costs, non-fossil technologies have more or less dynamic learning curves. It can therefore be expected that the non-fossil price declines in the future. This decline could be accelerated when levy revenues are spent specifically in non-fossil technology. Although SDIAM is not yet capable of representing the dynamic cost reduction, the effect of a decreased unit cost difference can be examined.

Figure 5.13 shows the result of a business optimization where the only difference to the CWB from chapter 4.3.2 is that non-fossil unit costs are set to  $w_n = 0.05$  corresponding to 70 \$/bboe. The unit cost difference is thus reduced to only seven times the fossil unit costs, while in the CWB this difference was a factor 14 (see chapter 3.3). Note that this result has been generated with no levies applied and with uniform discount rates.

Obviously, the reduced unit cost difference leads to a much earlier start of the transition, as the associated energy costs are significantly reduced even without the effect of improved energy efficiency  $f_p$  (see left panel in fig. 5.14). Since the earlier transition also leads to a significant reduction of the  $\text{CO}_2$  emissions, damage costs are also reduced compared to the standard CWB. Both effects lead to less disturbances in the investment rates and thus to faster economic growth. This in turn increases consumption so that societal welfare also benefits from reduced non-fossil energy costs. This can be seen in the right panel of fig. 5.14, where before  $t = 100$  the curves are identical and have been omitted to allow for a magnification of the remaining time interval.



**Figure 5.14:** Further variables for the scenario with reduced  $w_n = 0.05$ . Left: energy (solid) and damage (dashed) costs and their sum (dotted). Right: societal welfare in the present scenario (dotted) and in the CWB from chapter 4.3.2. For magnification only the interval from model year 100 . . . 400 is displayed.

The integral  $U_s^{w_n=0.05} = 218.0$  over the full 600 year time horizon is significantly greater than the well-known  $U_s^{CWB} = 214.6$ .

Apparently, a reduction of  $w_n$  would be a beneficial option, both in terms of climate change and of societal welfare. Recall, however, that the shown results are obtained assuming a constantly reduced non-fossil unit cost. When a learning curve is assumed,  $w_n$  would initially be as high as in the standard CWB and would only decline after considerable investments in energy transformation. Therefore, the optimal transition would occur somewhere between the timing in the standard CWB and the timing obtained here. Since the start of the transition causes the greatest relative cost increase of the whole transformation process, the result would presumably lie closer to the standard CWB result, both in timing and magnitude. However, as the learning curve leads to reduced costs once the transition has started, the cost peak would be more narrow.

These reflections imply that the incentive for society to use levy revenues as investments to bring the non-fossil unit costs down will not be overly great, at least when uniform discounting is applied. These investments would again lead to a premature increase of energy costs and thereby to cuts in investment rates, wages, and consumption, very much like in the case without learning curve. Additionally, the levy payments would also have to be compensated, which has not been considered in the reasoning above. However, these cuts would not last as long when a learning curve is applied because energy costs decline earlier, so that there might well be a slightly positive net benefit for societal welfare. The actual effect will depend on the particular parameterization of the learning curve and can hardly be assessed without explicit modelling.



# Chapter 6

## Conclusions and Outlook

The main focus of this study was the relation of societal measures to prevent anthropogenic climate change and their effect on the societal welfare. To that end, the economic system and its evolution as well as its interaction with society and the climate system have been represented by newly developed structural dynamic models. These models aim at a dynamic description of socio-economic systems by focusing on the dynamic interaction of the main actors in the respective systems. Utilizing these models, the integrated assessment (IA) of climate change mitigation measures was performed within the basic framework of a cost-benefit analysis. The major new concept in the IA model was the introduction of a societal welfare function which is treated and can be optimized independently from the economic welfare function. Below, these new concepts as well as the main findings are summarized and discussed.

### 6.1 Approach

#### 6.1.1 Structural Dynamic Models

The Structural Dynamic Economic Model (SDEM) is the basic structural dynamic model in this study. It features endogenous economic growth as the result of profit-driven investments in capital and productivity by the entrepreneurs. The dynamic component is the striving of workers for higher wages, modelled as an explicit adaptation of the wage rate, or factor costs, respectively. Wages follow any singular increase in labour productivity with a time lag and thereby erode the profits over time. A continuous productivity increase, however, leads to persistently positive profits per worker, but reduces the number of employed workers through rationalization. The absolute amount of profit grows when at the same time the capital stock is increased, since this provides additional workplaces. Due to the assumed fixed proportions production function, this basic model is very sensitive to changes in the dynamic parameters. This sensitivity is substantially reduced by introducing an additional feedback of the employment rate on the wage rate, which simulates the competition between unemployed and employed for workplaces.

The Structural Dynamic Integrated Assessment Model (SDIAM) is also a structural dynamic model, as it employs SDEM as its dynamic economic submodule. Dynamic extensions include

the dynamic growth of fossil energy costs due to the depletion of fossil fuel resources, while non-fossil energy unit costs are currently exogenous. Entrepreneurs decide which fraction of capital investments shall be used to build a non-fossil energy system. SDIAM is aggregated as it derives climate change from the nonlinear impulse-response climate model NICCS [Hooss *et al.*] and calculates the resulting climate impacts using a simple aggregated damage function. Furthermore, the model consists of only one region and one sector.

The main advantage of dynamic models is that there is an explicit formulation of every process considered. This offers, on the one hand, the possibility to keep the model as simple as possible by considering only the most relevant and avoiding unnecessary processes. On the other hand, it enables the examination of the dynamic behaviour at any time and in any state of the system, without the restriction to a particular state like the general equilibrium in usual economic models. In SDEM, one important consequence of this enhanced freedom is the possibility to model endogenous economic growth as the result of an explicit, profit-driven 'disequilibrium' effect: investments in productivity and capital drive the system away from its stationary state with zero growth and zero profits. When economic growth can be understood in that sense as an inherently non-equilibrium process, it is no surprise that neoclassical equilibrium economics has had great difficulties in understanding it.

Another advantage is that well-known disequilibrium effects like unemployment follow easily from the dynamic model. They do not require exceptional assumptions such as 'market failures' in equilibrium economics and therefore do not spoil the completeness of the underlying theory. The same holds for any other effect which shall be considered: once the dynamic of the process is known, it can easily be added to the basic model.

The dynamic approach also has disadvantages: sufficient agreement with empirical data can only be obtained when the most important mechanisms have been included with sufficient precision. In SDEM, the model results did not agree well with empirical data until a feedback of the employment rate on the wage rate was included. This points to two principal problems. Firstly, it is never clear, whether *all* important mechanisms have been considered. Therefore, a result derived by a dynamic model can be based on an incomplete set of assumptions and therefore be misleading. Secondly, in social systems the involved mechanisms generally lack a mathematical description, which is required by a numerical model. The modeller therefore has to rely on (sometimes far-reaching) assumptions to get a tractable formulation of the considered mechanism. In the case of the unemployment feedback, these assumptions led to the effect that the employment rate in the extended SDEM appears overly rigid. Additional feedbacks would be required to refine this model behaviour, making the model again more and more complex.

### 6.1.2 Separate Welfare Functions

Besides the dynamic approach, SDIAM features two distinct welfare functions for the sub-modules economy and society. In the economic submodule, investment decisions are made by the entrepreneurs in order to maximize utility from profits. In contrast, the model society in SDIAM defines its welfare as a function of material consumption less the value of intangible climate damages. A levy on CO<sub>2</sub> emissions can be specified by society to force the entrepreneurs to reduce emissions. In response, entrepreneurs adapt their investments in order to maximize profits under the new conditions. This adaptation reduces emissions and consequently climate

damages, but it turned out that consumption is generally negatively affected as well. The effect of CO<sub>2</sub> levies on societal welfare thus depends on society's valuation of climate damages relative to consumption.

The advantages of this approach are as follows. Firstly, the cost structure becomes clearer. Tangible damages appear in the profit function and are treated like normal economic costs in economic decision making. Intangible damages appear in the societal welfare function where they can be valued independent from economic efficiency considerations or be subject to non-standard inter-temporal accounting. The problems of standard economics with the accounting and valuation of intangible damages are thereby greatly reduced.

Secondly, also the decision structure gets clarified. The accounting problems of intangible damages have not vanished - they still exist in the societal welfare function. However, they no longer affect the economic investment decisions taken by the entrepreneurs. The societal valuation of intangible damages is fed back to the entrepreneurs by means of levies, i.e. as simple economic costs, which can easily be taken into account for the entrepreneurial cost-benefit analysis that maximizes profits.

Thirdly, the patterns of action and response in the IA model become more transparent. A levy imposed by society may increase societal welfare, but reduce economic welfare, and vice versa. The corresponding mechanisms can be traced and addressed independently when welfare functions are not mingled together.

## 6.2 Main Findings

The limited fossil fuel resources in SDIAM cause production growth to either cease or decline within the next century and eventually production will have to stop. The way out is to switch to non-fossil energy sources, which have greater costs per unit energy than fossil fuels.

A Constrained World Baseline (CWB) scenario is defined by calibrating SDEM to empirical growth patterns to derive the economic parameters and using average literature values for the additional SDIAM parameters. The CWB scenario contains only an economic optimization without society interference. The transformation of the energy system starts in the year 2035. This leads to total cumulated emissions of 1670 GtC, a maximum climate change of 3 °C in 2120 and a maximum damages of 2.7% of GDP around 2050. The timing of the transformation is based on the properties of the energy cost dynamics (available resources, productivity growth, unit cost difference) and climate damages. The assumptions employed in the CWB include perfectly foresighted entrepreneurs and that non-fossil fuels are unlimited. The utilized fossil fuel estimates are based on recent assessments and are likely to change in the future, however, this will not affect the general dynamics of SDIAM.

Society uses a levy on CO<sub>2</sub> emissions after the year 2000 to influence entrepreneurial decisions in order to reduce these emissions and maximize societal welfare, which is defined as consumption less the value of intangible damages. In this thesis, only studies on the local behaviour of the societal welfare function by imposing levies on the CWB have been performed. The implementation of a full society optimization had to be omitted due to time constraints. Levy rates

are chosen comparable to those currently discussed for international emissions trading schemes, which is the appropriate reference mechanism for a globally averaged model like SDIAM. The application of levies has been examined for different relative discounting schemes for intangible damages and consumption to account for changes in the relative valuation of these quantities over time.

In the case of uniform discount rates for damages and consumption, two ways to recycle the levy revenues have been examined. The first is lump-sum recycling to increases total consumption. The resulting slightly positive effects on climate change are over-compensated by the strong reductions in investments and consumptions and lead to a negative net effect on societal welfare. Greater levy rates cause a stronger reduction of societal welfare. The second way of levy recycling is to invest levy revenues in the build-up of non-fossil energy technology. This leads to significant reductions of CO<sub>2</sub> emissions, but the economic distortions are also much greater than in the lump-sum approach, because energy costs increase inefficiently due to the premature start of the energy system transformation. The adverse effect on societal welfare is much stronger than in the lump-sum approach. Under the assumption of a uniform discounting scheme, levies in any form have negative effects on societal welfare.

These adverse effect of levies on societal welfare are, however, reversed when damages and consumption are discounted differently. When the relative value of intangible damages increases fast enough over time, the reduced climate damages that result from CO<sub>2</sub> levies over-compensate the consumption losses and lead to increased societal welfare. Under a differential discounting scheme, societal measures to reduce climate change have beneficial effects. This finding supports the approach of Hasselmann *et al.* [1997] and Hasselmann [1999]. It also explains the behaviour of signatory states to the Kyoto protocol, which could not be understood from the point of view of neoclassical cost-benefit analysis using uniform discounting.

### 6.3 General Remarks

Several general shortcomings remain. The question what the term “society” really stands for, is not tackled here. The model assumes that societal decisions and values are based on consensus. However, in actual policy making, this is not the case. Even in democracies, non-consensual decisions are not uncommon, and even democratically elected governments often follow particular interests instead of representing the average society. This leads to the question of power structures in general, which are greatly simplified in SDIAM. In particular, the assumption that the economic system accepts every levy and does not struggle for a levy setting that least affects its profits is only justified as a first approach. However, the structural dynamic approach is open to model extensions that aim at a better representation of such effects. Furthermore, the high aggregation of SDIAM may be criticized, which veils many effects that happen on the intersectoral and international level and are important for applied policy making. However, this is unavoidable within the conceptual character of the study.

Finally, as with every model study, in particular at this aggregation level, it should again be highlighted that the results derived are based on many, and many crucial, assumptions. In many cases, these assumptions are simply due to lack of knowledge of the functioning of social systems, or the lack of a formalized description. For the latter, joint efforts of social and natural



scientists could bring integrated modelling forward. One should, however, always be aware of the general limitation that deterministic models of optimal control are based on 'Laplacian demons' and the fiction that the state of the world can be known. Therefore, the model results and figures show only how the model behaves, which is an approximation of the real-world system dynamics, and must not be confused with a prediction of future evolution.

## 6.4 Outlook

The models and approaches in this thesis have been developed almost 'from scratch', therefore only some basic concepts could be elaborated. A stochastic version to address uncertainty is under way [Weber 2004], but much remains to be done for future research. This includes an extension to more than one region and one sector to investigate trade effects, the inclusion of learning curves in the determination of the non-fossil unit cost determination, and a better formulation of societal welfare to include e.g. equity aspects or unemployment. The latter would require a more realistic formulation of the unemployment mechanism.

What is also missing here due to time constraints is to complete the societal cost-benefit framework by providing the optimal levy setting that maximizes societal welfare. This could be implemented by the following algorithm which follows the approach already taken in this study:

1. Define an initial levy setting.
2. Run a business optimization, using these levies as side conditions.
3. At the optimal business solution, calculate societal welfare.
4. Test whether maximum societal welfare is reached.
5. If not, modify the levy settings and proceed with the second step. If yes, exit.

This 'nested' optimization leads to a combined optimal state: When maximum societal welfare is reached, the economic subsystem has also reached the optimal state that can be reached within that given framework. This is not necessarily the global entrepreneurial optimum, i.e. greater entrepreneurial welfare might be obtained at different levy settings.

Besides the straightforward task of determining the optimal levy rate, such a modelling framework could also be used to determine time-dependent levy schemes. By further extending the structural dynamic approach it becomes even possible to address power structures, e.g. by introducing feedbacks from entrepreneurial welfare to the societal determination of levy rates.



# Appendix A

## Optimizing SDEM for Constant Investment Rates

Although the model is primarily intended to be solved numerically, we first examine the model analytically in order to gain some insight on the model dynamics. We maximize  $U$  subject to equations (2.5)-(2.8), where initial values for  $k(0)$ ,  $\hat{w}(0)$ , and  $l(0)$  are exogenously given as well as the parameters  $\mu, \nu$ , the various time constants ( $\lambda_k, \lambda_w, \lambda_d$ ) and the population,  $l_{max}(t)$ . The initial value of productivity is determined by  $k(0)$  and  $\nu$ .

We simplify our analytic analysis by splitting up the capital investments in the capital replacement part, which we assume to be always undertaken, and the part, that increases the capital stock. Therefore, entrepreneurs can only decide how fast the capital stock grows, but they cannot decide to let it shrink by investing less than needed to replace depreciated capital. The advantage for our analysis is that we may consider the replacement investments as simple costs that accrue but which are no longer under the decision of the entrepreneurs. We may thus write

$$i_k = i'_k + \lambda_k k, \quad (\text{A.1})$$

We do not attempt to provide a complete analytical solution of the whole model here, which appears to be quite complicated. In the following we restrict ourselves to the solution of the model with linear and logarithmic utility function under the assumption that capital and productivity grow exponentially at constant rates. This is equivalent to the assumption that the investments  $i'_k, i_{\hat{p}}$  are defined as constant fractions  $x_1, x_2$  of annual production. We set

$$i'_k = x_1 p \quad (\text{A.2})$$

$$i_{\hat{p}} = x_2 p. \quad (\text{A.3})$$

This assumption is supported by the results of the numerical model provided in section 2.4, which exhibit almost constant investment rates when spin-up effects are excluded.

With (2.5), (2.10), (2.8) the profit function (2.12), (2.16) simplifies to

$$\begin{aligned} b &= p - \lambda_k - w - i'_k - i_{\hat{p}} \\ &= p \left( \pi - \frac{\hat{w}}{\hat{p}} - x_1 - x_2 \right), \end{aligned} \quad (\text{A.4})$$

$$\text{with } \pi = 1 - \frac{\lambda_k k}{p} = 1 - \frac{\lambda_k}{\nu} = \text{const.} \quad (\text{A.5})$$

Note that although  $\pi$  equals  $\alpha_w$  from (2.11), these two parameters have different origins:  $\pi$  is the fraction of production left after capital replacement and interest payment, while  $\alpha_w$  describes the maximum wage rate that is compatible with a stationary economy. While  $\pi$  is a datum when the depreciation rate is given,  $\alpha_w$  may be subject to political decisions and might then deviate from  $\pi$ . In such a case, the profits that arise when no investments above capital replacement are made would be different from zero. For the purposes of this work we stick to the assumption of zero profits without investments above replacement, and therefore have  $\alpha_w = \pi$ .

Integrating the equations of motion (2.1), (2.2) and (2.3) with constant  $x_1, x_2$  we get

$$k = k^* \exp(\lambda_a t), \quad \text{where } \lambda_a = x_1 \nu \quad (\text{A.6})$$

$$\hat{p} = \hat{p}^* \exp(\lambda_b t), \quad \text{where } \lambda_b = x_2 \mu \quad (\text{A.7})$$

$$\hat{w} = \hat{w}^* \exp(\lambda_b t) + c_1^* \exp(-\lambda_w t) \quad (\text{A.8})$$

where variables with an asterisk (\*) denote constant amplitudes and  $\lambda_a, \lambda_b$  are the growth rates of capital and productivity, respectively. The amplitudes  $k^*, \hat{p}^*$  can be arbitrarily chosen and are set to the initial values  $k(0), \hat{p}(0)$ . The prognostic equations imply that the remaining two amplitudes are determined through the relations

$$\hat{w}^* = \frac{\lambda_w \alpha_w}{\lambda_w + \lambda_b} \hat{p}^*. \quad (\text{A.9})$$

$$c_1^* = \hat{w}(0) - \hat{w}^*. \quad (\text{A.10})$$

The amplitude  $c_1^*$  covers the initial mismatch between the amplitude of the wage rate in the long run,  $\hat{w}^*$ , and the initial wage rate  $\hat{w}(0)$ . The latter is set exogenously, while the former is governed by  $\lambda_b$ , which is a result of the optimization and thus not known *a priori*. According to (A.8), this initial mismatch decays exponentially at the rate  $\lambda_w$ . Since we are free to choose the initial value  $\hat{w}(0)$  we will choose it always such that  $c_1 = 0$  in order to simplify our analytic analysis.

Finally, it follows from (2.6) that employed labor evolves according to

$$l = l^* \exp[(\lambda_a - \lambda_b)t], \quad \text{where } l^* = \nu \frac{k^*}{\hat{p}^*}. \quad (\text{A.11})$$

Employed labor either grows or declines exponentially, or is constant. This depends on the growth rates of capital and productivity, which are determined by the respective investment rates  $x_1, x_2$ . In the following we will examine how the maximization of  $U$  determines the choice of  $x_1, x_2$  and the behaviour of employment in our model economy.

## A.1 Linear Utility Function

We first consider the linear utility function

$$u = \tilde{b} = \frac{b}{p(t_0)}. \quad (\text{A.12})$$

Using this utility function and the results for constant investment rates (A.6)-(A.11) to integrate (2.13) we obtain

$$U = \frac{k^*}{p(t_0)} \frac{\nu}{\lambda_d - \lambda_a} \left( \pi - \frac{\hat{w}^*}{\hat{p}^*} - x_1 - x_2 \right) \quad (\text{A.13})$$

where  $\lambda_a < \lambda_d$  must hold to ascertain convergence of the welfare integral. Substituting for  $\lambda_a, \lambda_b$  and  $\hat{w}^*$  yields

$$U = \frac{\mathcal{A}}{\mathcal{B} - x_1} \left( \pi - \frac{\mathcal{C}}{\mathcal{D} + x_2} - x_1 - x_2 \right), \quad (\text{A.14})$$

where

$$\mathcal{A} = \frac{k^*}{p(t_0)} \quad (\text{A.15})$$

$$\mathcal{B} = \frac{\lambda_d}{\nu} \quad (\text{A.16})$$

$$\mathcal{C} = \frac{\lambda_w}{\mu} \alpha_w \quad (\text{A.17})$$

$$\mathcal{D} = \frac{\lambda_w}{\mu} \quad (\text{A.18})$$

$$\mathcal{E} = \pi - x_2 - \frac{\mathcal{C}}{\mathcal{D} + x_2} = \mathcal{E}(x_2). \quad (\text{A.19})$$

The last expression allows us to write (A.14) in the form

$$U = \mathcal{A} \frac{\mathcal{E}(x_2) - x_1}{\mathcal{B} - x_1} \quad (\text{A.20})$$

that summarizes the two ways to increase  $U$ : either by maximizing the profit function at a given production level without considering future growth of  $p$  (this is represented by the numerator  $\mathcal{E} - x_1$ ), or by maximizing the growth of production (or capital) and thereby extending the effective time horizon of the  $U$ -integral (this is represented by the denominator  $\mathcal{B} - x_1$ ). Equation (A.20) also shows that the problem of maximizing  $U$  can be solved by treating the optimization separately for  $x_1$  and  $x_2$ . For any  $x_1$ , maximizing  $\mathcal{E}(x_2)$  simultaneously maximizes  $U$  and thus determines the optimal choice for  $x_2$ . When the maximum  $\mathcal{E}$  is found, we maximize  $U$  by variation of  $x_1$ , taking  $\mathcal{E}_{max}$  and  $\mathcal{B}$  as given.

The maximization of  $\mathcal{E}$  with respect to  $x_2$  is straightforward and yields

$$\mathcal{E}_{max} = \pi - 2\sqrt{\mathcal{C}} + \mathcal{D} \quad (\text{A.21})$$

$$\text{at } x_2 = \sqrt{\mathcal{C}} - \mathcal{D}. \quad (\text{A.22})$$

When the model parameters are such that  $\mathcal{D} > \sqrt{\mathcal{C}}$  this would result in  $x_2 = 0$ , because investment rates cannot be negative. In an economy described by these parameters the reduction of the wage rate caused by an investment in productivity would be smaller than that investment itself. In such an economy it would never be possible to lower the wage rate below the asymptotic value  $\hat{w} = \hat{w}^0$ . This, in turn, also prevents investments in capital, and therefore the economy would remain static, neither growing nor shrinking, because depreciation is balanced at  $\hat{w} = \hat{w}^0$ .

When the model parameters, however, allow for positive  $x_2$ , we obtain one distinct  $\mathcal{E}$ -maximum as defined by (A.21), (A.22) that allows for positive  $x_1$  and/or positive profit  $b$ . We will proceed on the assumption that the model parameters allow for a solution with positive  $x_2$ .

Using the result (A.21) for  $\mathcal{E}(x_2)$  we find that the derivative of  $U$  with respect to  $x_1$  does not vanish. The maximum value for  $U$  will thus be found on the boundary of the permissible  $x_1$  interval and will depend on the fraction  $\mathcal{E}/\mathcal{B}$ . Note that (A.20) has a pole at  $x_1 = \mathcal{B}$  corresponding to  $\lambda_a = \lambda_d$ . However, since we assumed that  $\lambda_a < \lambda_d$ , this pole is excluded from the allowed range of  $x_1$  by setting  $\lambda_d$  appropriately.

To understand the behaviour of  $x_1$ , we distinguish the following three cases:

**Case 1:**  $\mathcal{E}_{max} < \mathcal{B}$

$U$  is maximized when  $x_1$  lies on its lower boundary,  $x_1 = 0$ , and has the value  $U = \mathcal{A}\mathcal{E}/\mathcal{B}$ . Investing in capital does not pay, since the gain of a longer effective time horizon from faster  $k$  growth is smaller than the profit loss caused by that investment. Since  $k$  does not grow in this case, but  $\hat{p}$  does, it follows that in the course of time employment decreases, production remains constant and profits increase. This case simulates a pure strategy of rationalization.

**Case 2:**  $\mathcal{E}_{max} = \mathcal{B}$

In this case  $U$  is independent of  $x_1$  and has the constant value  $U = \mathcal{A}$ . Here, the profit reduction through investments is exactly balanced by the gain from the extended time horizon, therefore the level of  $x_1$  does not matter to the entrepreneur's integrated profit. Thus, the capital growth rate may take any value in the interval  $0 \leq \lambda_a < \lambda_d$ , and unemployment is unpredictable for the same reason.

**Case 3:**  $\mathcal{E}_{max} > \mathcal{B}$

This is the opposite of Case 1 and  $U$  is maximized by choosing the highest possible value of  $x_1$ . As long as the employment rate  $q = l/l_{max} < 1$ , the only applicable restriction to capital growth is set by the discount rate that requires  $x_1 < \mathcal{B}$ . It then depends on the value of  $x_2$  how the employment rate develops. If capital grows faster than productivity,  $\lambda_a > \lambda_b$ , absolute employment  $l$  increases, and if  $l$  grows also faster than  $l_{max}$ , eventually full employment is reached. At this point, a second restriction applies to  $x_1$ , since  $\nu k > \hat{p}l_{max}$  would result in idle, unproductive capital. Therefore, the capital growth rate  $\lambda_a$  adapts such that  $\lambda_a - \lambda_b$  equals the growth rate of  $l_{max}$  (cf. (2.6), (A.11)). Since  $x_2$  and thus  $\lambda_b$  are defined by the  $\mathcal{E}$  maximum,  $x_2$  is not altered.

The linear utility model thus exhibits a bistable solution regarding capital investments, which are either zero or at the highest possible value. In contrast to that, the productivity investment rate is clearly defined, unless the model parameters prevent the productivity-wage rate-mechanism to work properly.

This results in a capital growth rate almost equal to the discount rate (below full employment), or in coupled growth of capital and productivity (at full employment). This means that the major determinant for the economic growth rate in the linear utility model is the discount rate.

## A.2 Logarithmic Utility Function

For comparison we now examine the logarithmic utility function

$$u = \ln \left( \frac{b}{p(0)} \right), \quad (\text{A.23})$$

where the constant denominator  $p(0) = \nu k(0)$  is only used to make the argument of the logarithm dimensionless. This utility function describes the case when entrepreneurs do have a certain preference about when their profits occur. We proceed similar to section A.1, utilize constant investment rates that allow us to rewrite the profit function (2.16) as

$$b = \Omega \exp(\lambda_a t), \quad \text{where} \quad \Omega = k^* \nu (\mathcal{E} - x_1) = \text{const.} \quad (\text{A.24})$$

and integrate (2.13) using (A.24) to get

$$U = \frac{1}{\lambda_d} \left( \ln \frac{\Omega}{p(0)} + \frac{x_1}{\mathcal{B}} \right). \quad (\text{A.25})$$

Note that to derive (A.25) only  $\lambda_d > 0$  is required to guarantee convergence of the welfare integral, thus the capital growth rate  $\lambda_a$  is no longer restricted by the discount rate. Remember, however, that the interpretation of  $\lambda_d$  is different for the logarithmic utility model and the 'linear' model, as in the 'logarithmic' model part of the discounting is hidden in the nonlinear utility function and  $\lambda_d$  in this section only describes the 'pure rate of time preference'.

Again, the optimization of  $U$  can be carried out separately for  $x_1$  and  $x_2$ . The latter turns out to have the same result as for the linear welfare function, namely (A.21), (A.22). For the former optimization, differentiating  $U$  partially with respect to  $x_1$ , keeping in mind that  $k^* = k(0)$ , and equating to zero yields

$$x_1 = \mathcal{E}_{max} - \mathcal{B} \quad (\text{A.26})$$

$$\text{and } U = \frac{1}{\lambda_d} \left( \ln \mathcal{B} + \frac{\mathcal{E}_{max}}{\mathcal{B}} - 1 \right). \quad (\text{A.27})$$

As  $x_1$  cannot become negative it is set to zero when  $\mathcal{E}_{max} < \mathcal{B}$ . Note that the sign of  $U$  is of no importance here, since it is only determined by the choice of the normalization constant in (A.23). In the current context only the existence if a defined maximum is important.

Using  $\mathcal{E}_{max}$  from (A.21) and the explicit assumption that the maximum wage rate only allows for capital replacement,  $\pi = \alpha_w$ , we may also write this result completely in terms of exogenous parameters:

$$x_1 = \frac{1}{\alpha_w} (\alpha_w - \sqrt{\mathcal{C}})^2 - \mathcal{B} \quad (\text{A.28})$$

$$\text{and } U = \frac{1}{\lambda_d} \left( \ln \mathcal{B} + \frac{(\alpha_w - \sqrt{\mathcal{C}})^2}{\alpha_w \mathcal{B}} - 1 \right) \quad (\text{A.29})$$

As long as the economy is below full employment the growth rate of employed labor at the optimal solution is then given as

$$\frac{\dot{l}}{l} = \lambda_a - \lambda_b = \nu(\mathcal{E} - \mathcal{B}) - \mu(\sqrt{\mathcal{C}} - \mathcal{D}) \quad (\text{A.30})$$

$$= \alpha_w \nu - \sqrt{\mathcal{C}}(2\nu + \mu) + \mathcal{D}(\nu + \mu) - \nu \mathcal{B} \quad (\text{A.31})$$

where the last equality holds only in the case  $\pi = \alpha_w$ . When full employment is reached, the same restrictions for  $x_1$  apply as described in Case 3 in section A.1. Note that the growth rate of  $l$  is a unique function of exogenous parameters and is constant over time. In particular it does not change its value unless the parameters are changed. The growth rate of the employment rate is again simply the difference of the growth rates of  $l$  and  $l_{max}$ . It follows that a constant employment rate  $q$  requires

$$\frac{\dot{l}}{l} = \lambda_a - \lambda_b = \nu x_1 - \mu x_2 \stackrel{!}{=} \frac{\frac{d}{dt}(l_{max})}{l_{max}}. \quad (\text{A.32})$$



# Appendix B

## Model Equations

### B.1 SDEM Equations

The basic SDEM from chapter 2.2 is formed by the following equations and defines an optimization problem:

Objective:

$$(2.13) \quad \text{Max.} \quad U = \int_0^{\infty} u(b) e^{-\lambda_d t} dt$$

$$\text{where} \quad u(b) = \begin{cases} \ln \left[ \frac{b}{p(t_0)} \right], & \text{logarithmic utility} \\ \frac{b}{p(t_0)}, & \text{linear utility} \end{cases}$$

$$(2.12) \quad \text{and} \quad b = p - w - i_k - i_{\hat{p}}$$

subject to the following constraints:

Time Evolution:

$$(2.1) \quad \dot{k} = i_k - \lambda_k k$$

$$(2.2) \quad \dot{\hat{p}} = \mu \frac{i_{\hat{p}}}{\hat{p}}$$

$$(2.3) \quad \dot{\hat{w}} = \lambda_w (\hat{w}^0(\hat{p}) - \hat{w})$$

Production and Labour:

$$(2.4) \quad \hat{p} = \nu \frac{k}{l}$$

$$(2.6) \quad l = \nu \frac{k}{\hat{p}} \leq l_{max}$$

$$(2.5) \quad p = \hat{p} \cdot l$$

$$(2.7) \quad q = \frac{l}{l_{max}}$$

Wages:

$$(2.11) \quad \alpha_w = 1 - \frac{\lambda_k}{\nu}$$

$$(2.10) \quad \hat{w}^0(\hat{p}) = \hat{p} \alpha_w$$

Investments:

$$(A.2) \quad i'_k = x_1 p$$

$$(A.1) \quad i_k = i'_k + \lambda_k k$$

$$(A.3) \quad i'_p = x_2 p$$

Unemployment Feedback (Extended SDEM):

$$(2.26) \quad F = F_{min} + (1 - F_{min}) q^{\alpha_q}$$

$$(2.28) \quad \hat{w}^\dagger = \hat{w} F$$

$$(2.27) \quad \dot{\hat{w}}^\dagger = \lambda_w (\hat{w}^0 F - \hat{w}^\dagger) \quad \text{replaces (2.3)}$$

## B.2 SDIAM Equations

SDIAM consists of SDEM embedded in the integrated assessment framework formed by the following equations. The climate module NICCS, which provides  $\Delta T(e)$  is not listed here. For details on the equations in NICCS see Hooss *et al.* [2001].

Carbon Efficiency:

$$(3.5) \quad e = \frac{p}{z}$$

$$(3.6) \quad z = f_e f_p = z(t_0) \frac{f_e f_p}{f_e(t_0) f_p(t_0)}$$

Energy-Carbon Ratio:

$$(3.7) \quad k_e = \kappa E := E(t_0) \frac{p/p(t_0)}{f_p/f_p(t_0)}$$

$$(3.8) \quad k_{e,n} = \kappa E_n$$

$$(3.9) \quad E_n(t_0) = \epsilon E(t_0)$$

$$(3.11) \quad i_e = \dot{k}_e + \lambda_k k_e \left[ -\sigma_s \frac{s_e}{\beta} \right]$$

$$(3.12) \quad \dot{k}_{e,n} = x_3 i_e - \lambda_k k_{e,n} \left[ +\sigma_s \frac{s_e}{\beta} \right]$$

$$(3.13) \quad f_e = \frac{E}{e} = 1 + \frac{k_{e,n}}{k_e - k_{e,n}}$$

$$(3.14) \quad \beta = \frac{k_e(t_0)}{k(t_0)}$$

Production-Energy Efficiency:

$$(3.16) \quad f_p = f_{p,max} \left( 1 - \frac{\phi_1}{\phi_2 + \hat{p}} \right)$$

Energy Costs:

$$(3.20) \quad w_f(t_0) \ll w_n = \text{const.}$$

$$(3.21) \quad w_f = w_f(t_0) \left( \frac{c_{res}(t_0)}{c_{res}} \right)^{\alpha_e}$$

$$(3.24) \quad \dot{c}_{res} = -e$$

$$(3.25) \quad w_e = w_f \frac{k_e - k_{e,n}}{k_e} + w_n \frac{k_{e,n}}{k_e}$$

$$(3.26) \quad c = w_e E$$

Climate Damage Costs:

$$(3.29) \quad d = \delta_b p \cdot \frac{1}{2} \left( \left( \frac{\Delta T}{\tau_b} \right)^2 + \left( \frac{d}{dt} \frac{\Delta T}{\tau_b} \right)^2 \right)$$

$$(3.33) \quad d_t = 0.8 d$$

$$(3.33) \quad d_i = 0.2 d$$

Emission Levies and the Role of Society:

$$(3.34) \quad b_s = w + b + (1 - \sigma_s) s_e - d_i e^{\lambda_a, at}$$

$$(3.35) \quad s_e = (y_e p) e$$

$$(3.36) \quad y_e = 0 \quad , \text{ for } t < t_1$$

$$(3.37) \quad y_e = \frac{y_{e,2}}{t_2 - t_1} t \quad , \text{ for } t_1 \leq t \leq t_2$$

$$(3.38) \quad y_e = y_{e,2} \quad , \text{ for } t > t_2$$

$$(3.39) \quad U_s = \int_0^\infty \ln(\tilde{b}_s) e^{-\lambda_a t} dt$$

Adaptations to SDEM Equations:

$$(3.40) \quad b = p - w - c - d_t - i_k - i_{\hat{p}} - s_e \quad \text{replaces (2.12)}$$

$$(3.41) \quad \hat{w}^0 = \hat{p} a_w \quad \text{replaces (2.10)}$$

$$(3.42) \quad a_w = \alpha_w - \frac{c}{p} - \frac{d_t}{p} - \frac{s_e}{p} = 1 - \frac{\lambda_k}{\nu} - \frac{c}{p} - \frac{d_t}{p} - \frac{s_e}{p}$$



# Appendix C

## Variable Index

The variables and parameters used in SDEM and SDIAM are listed below. In order to keep the model general, variables in SDEM/SDIAM are normalized to three basic units:

1. Monetary units ('dollars') are expressed in quantities of the initial capital stock,  $k(t_0)$ .
2. Workforce units ('number of workers') are expressed in units of initial labour force,  $l(t_0)$ .
3. Energy units ('Gigajoule') are expressed in units of initial energy use,  $E(t_0)$ .

The remaining units are left unaltered:

1. Time  $t$  is expressed in years ( $yr$ ).
2. Carbon units are expressed in Gigatonnes Carbon ( $GtC$ ).
3. Global temperature change is measured in degrees centigrade ( $^{\circ}C$ ).

In the following tables all parameters and variables are given in their 'natural' as well as in their normalized units as used in the model. For parameters the default value or Constrained World Baseline (CWB) value is also given. A hat ( $\hat{\phantom{x}}$ ) indicates that the corresponding variable is divided by the number of employed workers.

Table C.1: Parameters used in SDIAM

Parameter	Description	'Natural' Dimension	Normalized Dimension	Default/CWB value
$\alpha_w$	maximum wage- $\hat{p}$ ratio	1	1	0.875
$\alpha_q$	unempl. feedback exponent	1	1	60
$\alpha_e$	resource cost exponent	1	1	2
$\beta$	energy capital share	1	1	0.1
$c_{res}(t_0)$	initial carbon resources	$GtC$	$GtC$	$10^4 GtC$
$\delta_b$	benchmark damage per GDP	1	1	0.02
$\epsilon$	$E_n(t_0)/E(t_0)$	1	1	0.01
$F_{min}$	min. unempl. feedback	1	1	0.66
$\phi_1$	$f_p$ function parameter	$\frac{\$}{worker \cdot yr}$	$\frac{1}{yr}$	$25 \frac{1}{yr}$
$\phi_2$	$f_p$ function parameter	$\frac{\$}{worker \cdot yr}$	$\frac{1}{yr}$	$27.5 \frac{1}{yr}$
$f_{p,max}$	max. energy efficiency	1	1	0.7
$\kappa$	energy-energy capital ratio	$\frac{GJ}{\$}$	1	1
$\lambda_d$	economic discount rate	$\frac{1}{t}$	$\frac{1}{t}$	$0.01 \frac{1}{yr}$
$\lambda_{d,d}$	$d_i$ value growth rate	$\frac{1}{t}$	$\frac{1}{t}$	0
$\lambda_k$	depreciation rate	$\frac{1}{t}$	$\frac{1}{t}$	$0.05 \frac{1}{yr}$
$\lambda_w$	rate of wage adaptation	$\frac{1}{t}$	$\frac{1}{t}$	$0.15 \frac{1}{yr}$
$\mu$	$i_{\hat{p}}$ invest. efficiency	1	1	0.1
$\nu$	GDP-capital ratio	$\frac{1}{t}$	$\frac{1}{t}$	$0.4 \frac{1}{yr}$
$\sigma_s$	levy redistribution	1	1	0
$t_0$	model initialization year	1	1	1915 (model year 0)
$t_1$	start year for levies	1	1	2000 (model year 85)
$t_2$	end of levy increase	1	1	2020 (model year 105)
$\tau_b$	benchmark temperature change	$^{\circ}C$	$^{\circ}C$	$2.5^{\circ}C$
$\frac{d}{dt}\tau_b$	benchmark temp. change rate	$\frac{^{\circ}C}{yr}$	$\frac{^{\circ}C}{yr}$	$0.02 \frac{^{\circ}C}{yr}$
$w_f(t_0)$	initial foss. energy unit cost	$\frac{\$}{GJ}$	1	0.007
$w_n(t_0)$	initial non-foss. energy unit cost	$\frac{\$}{GJ}$	1	0.1
$y_{e,2}$	final emission levy rate	$\frac{1}{GtC}$	$\frac{1}{GtC}$	$3.3 \cdot 10^{-4} \frac{1}{GtC}$

**Table C.2:** Variables used in SDIAM

Variable	Description	'Natural' Dimension	Normalized Dimension	Remark/ Initial value
$a_w$	$\hat{w}^0$ as fraction of $\hat{p}$	1	1	
$b$	annual profit	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$b_s$	societal welfare	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$c$	annual energy cost	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$c_{res}$	available carbon resources	$GtC$	$GtC$	$c_{res}(t_0) = 10^4 GtC$
$d$	annual climate damages	$\frac{\$}{yr}$	$\frac{1}{yr}$	$d(t_0) = 0$
$d_t$	tangible damages	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$d_i$	intangible damages	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$e$	annual emissions	$\frac{GtC}{yr}$	$\frac{GtC}{yr}$	$e(t_0) = 0.9 \frac{GtC}{yr}$
$E$	annual energy use	$\frac{GJ}{yr}$	$\frac{1}{yr}$	
$E_n$	non-fossil ann. energy use	$\frac{GJ}{yr}$	$\frac{1}{yr}$	
$f_e$	energy per carbon unit	$\frac{GJ}{GtC}$	1	normalized to $\frac{E-E_n}{e}$
$f_p$	energy efficiency	1	1	
$F$	wage reduction factor	1	1	
$i_e$	ann. investments in $k_e$	$\frac{\$}{yr}$	$\frac{1}{yr}$	diagnostic
$i_k$	ann. capital investment	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$i_{\hat{p}}$	ann. productivity inv.	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$k$	capital	$\$$	1	$k(t_0) = 1$
$k_e$	capital for energy supply	$\$$	1	
$k_{e,n}$	capital for non-fossil en. supp.	$\$$	1	
$l$	number of workers	<i>worker</i>	1	
$l_{max}$	max. number of workers	<i>worker</i>	1	exogenous
$p$	annual GDP	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$\hat{p}$	productivity	$\frac{\$}{worker \cdot yr}$	$\frac{1}{yr}$	$\hat{p}(t_0) = \nu$
$q$	employment rate	1	1	$q(t_0) = 0.95$
$s_e$	ann. emission levy revenues	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$t$	time	<i>yr</i>	<i>yr</i>	
$\Delta T$	change of global temperature	$^{\circ}C$	$^{\circ}C$	

**Table C.2** (continued): Variables used in SDIAM

Variable	Description	'Natural' Dimension	Normalized Dimension	Remark/ Initial value
$u$	annual business utility	$\frac{\$}{yr}$	1	normalized to $p(t_0)$
$u_s$	utility of $b_s$	$\frac{\$}{yr}$	1	normalized to $p(t_0)$
$U$	net present value of $u$	$\$$	$yr$	integrand $u$ normalized
$U_s$	net present value of $b_s$	$\$$	$yr$	integrand $u_s$ normalized
$w$	annual wage sum	$\frac{\$}{yr}$	$\frac{1}{yr}$	
$\hat{w}$	wage rate	$\frac{\$}{worker \cdot yr}$	$\frac{1}{yr}$	
$\hat{w}^0$	max. wage rate	$\frac{\$}{worker \cdot yr}$	$\frac{1}{yr}$	
$\hat{w}^\dagger$	wage rate reduced by $F$	$\frac{\$}{worker \cdot yr}$	$\frac{1}{yr}$	
$w_f$	init. foss. en. unit cost	$\frac{\$}{GJ}$	1	$w_f(t_0) = 0.007$
$w_n$	init. non-foss. en. unit cost	$\frac{\$}{GJ}$	1	$w_n(t_0) = 0.1$
$w_e$	average energy unit cost	$\frac{\$}{GJ}$	1	
$x_1$	investment rate in $k$	1	1	control variable
$x_2$	investment rate in $\hat{p}$	1	1	control variable
$x_3$	share of $i_e$ used for $k_{e,n}$	1	1	control variable
$y_e$	levy per $GtC$ as GDP share	$\frac{1}{GtC}$	$\frac{1}{GtC}$	
$z$	carbon efficiency	$\frac{\$}{GtC}$	$\frac{1}{GtC}$	



# Appendix D

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# Acknowledgements

I am grateful to all people who helped in many respects during the preparation of this study. In particular, I would like to thank

- Prof. Dr. Klaus Hasselmann for giving me the opportunity and the necessary freedom to enter into the fascinating field of integrated assessment and to conduct this stimulating and mind-broadening study. I learned a lot during the past years.
- Prof. Dr. Carlo Jaeger for his immediate interest in my work and his kind support during the genesis and the review phase of this thesis.
- the GELENA team in Oldenburg and Berlin for patience, generosity and support during the final phase of this thesis.
- the participants of the Oberseminar “Environmental Change and Management” at the University of Hamburg for helpful comments that improved the argumentation.
- the many colleagues I met in Hamburg and elsewhere, who made life and work more agreeable and helped in creating new ideas.
- the flexible, busy, friendly and competent support crews at the MPI: administration and CIS. Only someone who knows life without can assess the importance of their presence.
- my parents for their tacit but always present and well-minded support. Without them I would have never come here.
- Ines and Erik: for your love, trust, motivation, support, and shared experiences – and for pointing me to the important things in life, again and again.



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ISSN 0938 - 5177