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Black holes with gravitational hair in higher dimensions

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Abstract

A new class of vacuum black holes for the most general gravity theory leading to second order field equations in the metric in even dimensions is presented. These space-times are locally AdS in the asymptotic region, and are characterized by a continuous parameter that does not enter in the conserve charges, nor it can be reabsorbed by a coordinate transformation: it is therefore a purely gravitational hair. The black holes are constructed as a warped product of a two-dimensional space-time, which resembles the $r-t$ plane of the BTZ black hole, times a warp factor multiplying the metric of a $D-2$-dimensional Euclidean base manifold, which is restricted by a scalar equation. It is shown that all the Noether charges vanish. Furthermore, this is consistent with the Euclidean action approach: even though the black hole has a finite temperature, both the entropy and the mass vanish. Interesting examples of base manifolds are given in eight dimensions which are products of Thurston geometries, giving then a nontrivial topology to the black hole horizon. The possibility of introducing a torsional hair for these solutions is also discussed.

1 Introduction

One of the most important class of theorems in General Relativity are the so-called *no-hair* theorems. The importance of this type of results comes from the fact that they provide one with a very nice and effective description of the asymptotic degrees of freedom of a black hole. Originally, the no hair theorems stated that a four dimensional black hole, which is asymptotically flat, regular outside and on the horizon should be completely determined by its mass and angular momentum (for references see the book by Heusler [1]). However, nowadays it is known that this theorem does not extend neither to asymptotically Anti de Sitter (AdS) space-times [2, 3, 4] nor to higher dimensions [5]. The non uniqueness in higher dimensions is actually expected to be vast. Indeed, in five dimensions an asymptotically flat black hole is not uniquely characterized, even when its angular momentum vanishes, by its conserved charges. As it is explicitly shown by the black Saturn [6]; the angular momentum of the inner black hole can exactly cancel the angular momentum of the black ring and therefore the only conserved charge at infinity is the ADM mass. Although it is very likely that there are black rings, black Saturns and a very large variety of black holes when the cosmological constant is included or when higher dimensions are considered no analytic construction of such black holes has been performed so far. It is therefore interesting to explore to what extent no-hair conjectures can be violated in higher dimensions.

The main objective of this paper is to construct a family of black holes which are not completely characterized by its mass and angular momentum: we will show that, within our family of black holes, both the mass and the angular momentum vanish. The $r-t$ part of the metric looks like a three dimensional Bañados-Teitelboim-Zanelli (BTZ) black hole [7]. This space-time has proved to be very important since despite its simplicity, it encodes all the basic features of a black hole providing one with a theoretical laboratory to try to answer many difficult questions in black holes physics. Due to the fact that the BTZ black hole is a quotient of the maximally symmetric anti-de Sitter (AdS) space-time [8], it differs from it only globally[1] and the local integrability properties of fields propagating on it remain after the identification. Still, the global differences between AdS and the BTZ space-times manifest themselves quite dramatically when one considers the semiclassical approach to the quantization of gravity [9] since the Faddeev-Popov determinant in the De Donder gauge vanishes identically on AdS while it does not on the BTZ space-time. In the context of AdS/CFT the BTZ metric was the first, non supersymmetric black hole whose entropy was correctly reproduced by the asymptotic counting of microstates of the CFT at the boundary through Cardy's formula [10]. Furthermore, quasi-normal modes of fields with different spins can be obtain exactly on this metric, and the quasi-normal frequencies are in exact agreement with the location of the poles of the retarded correlation function of the dual perturbations in the CFT at the boundary [11].

In any dimension higher than three, general relativity with a negative cosmological constant Λ, namely with field equations

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=0\ , \tag{1}$$

admits the following topological black hole solution, which resembles three-dimensional BTZ black hole although it is no longer of constant curvature

$$ds_D^2=-\left(\frac{\hat{r}^2}{l^2}-\mu\right)d\hat{t}^2+\frac{d\hat{r}^2}{\frac{\hat{r}^2}{l^2}-\mu}+\hat{r}^2d\hat{\Sigma}_{D-2}^2\ . \tag{2}$$

The squared AdS radius is defined by $l^2:=-\frac{(D-1)(D-2)}{2\Lambda}$ and $\hat{\Sigma}_{D-2}$ stands for the line element of a $D-2$-dimensional Euclidean base manifold, which, due to the Einstein equations, has to satisfy

$$\hat{R}^i_j=-\mu\left(D-3\right)\delta^i_j\ , \tag{3}$$

i.e. $\hat{\Sigma}_{D-2}$ is an (Euclidean) Einstein manifold. Here the $\{i,j\}$ indices are internal indices on $\hat{\Sigma}_{D-2}$, and $\hat{R}^i_j$ is the Ricci tensor of $\hat{\Sigma}_{D-2}$. It is clear that for positive values of μ, the metric (2) describes a black hole with event horizon located at $\hat{r}=\hat{r}_+$ with

$$\hat{r}_+:=l\sqrt{\mu}\ , \tag{4}$$

As it is well known, in order for such a black hole to have interesting thermodynamical properties, μ should be a non-trivial integration constant otherwise it would represent an isolated point in the solution space so that the thermodynamics would be trivial. However, in this case it is easy to see that *μ is not an integration constant* [12], [13] since one can perform the following change of coordinates

$$\hat{r}=\sqrt{\mu}r,\ \hat{t}=\frac{t}{\sqrt{\mu}}\ , \tag{5}$$

[1] Consequently, any gravity theory in three dimensions admitting an AdS vacuum, will also contain the BTZ black hole as a solution.

supplemented with the following rescaling on the base manifold

$$d\hat{\Sigma}_{D-2}^2 = \frac{1}{\mu} d\tilde{\Sigma}_{D-2}^2 \ , \tag{6}$$

and obtain the equivalent metric in which there is not anymore the parameter μ

$$ds_D^2 = -\left(\frac{r^2}{l^2} - 1\right) dt^2 + \frac{dr^2}{\frac{r^2}{l^2} - 1} + r^2 d\tilde{\Sigma}_{D-2}^2 \ , \tag{7}$$

where also the new base manifold $\tilde{\Sigma}_{D-2}$ is an Einstein manifold but with rescaled curvature

$$\tilde{R}_j^i = -\left(D-3\right)\delta_j^i \ . \tag{8}$$

The constant μ can be eliminated by a coordinate transformation plus a rescaling since it enters not only in the lapse function but also as the curvature scale of the base manifold in Eq. (3). The only exception is in three dimensions where the base manifold has no intrinsic curvature scale so that the previous argumentation does not apply and therefore μ becomes a genuine integration constant.

It is worth noting that in $D < 6$, this constraint on $\tilde{\Sigma}_{D-2}$ implies that the base manifold, being of dimension $D - 2 < 4$ must be of constant curvature, therefore homeomorphic to a quotient of the hyperbolic spaces H_2 for $D = 4$ and H_3 for $D = 5$. In dimensions greater the five, due to the "nontriviality" of the Weyl tensor of the base manifold, the Einstein restriction (8) does not fix the Riemann curvature locally (see references [14]).

In higher dimensions, the most natural generalization of the Einstein-Hilbert action corresponds to the Lovelock actions [15] which are constructed following the same principles of general covariance and of the requirement to have second order field equations for the metric. Thus, if the possibility to have more than four dimensions is considered, then it is important to analyze the black holes arising in Lovelock gravities.

We will prove in this work the existence of solutions of the form (2) in arbitrary Lovelock theories, such that the restrictions on the base manifold $\tilde{\Sigma}_{D-2}$, do not allow to rescale away the parameter μ which turns out to be a true integration constant which however does not appear in any of the conserved charges of the system and it is therefore a *pure gravitational hair*. It is worth emphasizing here that in the five dimensional Ricci flat black holes it is still possible to differentiate the black holes by the topology of the horizon: as it will be shown in the following sections this is no longer true in the cases studied here.

Some specific examples of this type have been found in [16]: in the black holes constructed in the quoted reference the constant μ cannot be rescaled away. The idea is that when the base manifold is the direct product of two constant curvature manifolds then two independent curvature scales may appear and one can get rid of only one of the two scales with a rescaling. As it will be shown below, in order to find the most general Lovelock action where the base manifold of the BTZ-like ansatz in Eq. (2) possesses more than one curvature scale, one needs to avoid tensor restrictions on $\tilde{\Sigma}_{D-2}$, allowing at most a reduced set of scalar constraints on it. It is clear that in order to be the direct product of two manifolds with independent curvature scales, the base manifold can not be Einstein. We will show that this analysis naturally singles out two possible Lovelock theories. In odd dimension the Lovelock theory reduces to the Chern-Simons (CS) case, in which all the vacua coincide. This theory can be written as a gauge theory for the AdS group $SO\left(D-1,2\right)$, and the base manifold is not restricted at all by the field equations. This implies that the constant μ can not be reabsorbed and is actually the mass parameter of the CS black hole. Since this case has been already extensively explored in the literature [17] [18], we will focus on even dimensional cases in which *it does not* occur that base manifold is not restricted by the field equations. In the latter situation, all the vacua of Lovelock theory have to coincide: the theory obtained is known as the Born-Infeld (BI) theory, since the Lagrangian can be written as a Pfaffian form [19], being then the square root of a determinant. We will show that the base manifold is then fixed by a scalar equation, which in $6N+2$ dimensions can have as solutions direct products of Thurston geometries, which determine the nontrivial topology of the horizon. Remarkably, by applying the formalism for constructing conserved charges given in [20], specially developed for asymptotically locally AdS black holes in even dimensional Lovelock theories, we will show that in the theories considered all the Noether charges vanish for any black hole of the form (2). Therefore the "would be mass parameter" can be interpreted as a purely gravitational hair, i.e. a parameter in a family of black hole geometries, that does not modify the asymptotic behavior, neither contribute to the conserved charges. These results are confirmed also by applying the Euclidean action approach.

This paper is organized as follows: In the next section, we will single out the BI theories from the Lovelock class, in order to avoid tensor restrictions on the horizon geometries of the black holes of the form (2) in even dimensions. Then in section III it will be shown that in these theories all the black holes of the form (2) have zero Noether charges, in particular the one associated to the Killing vector $\xi = \partial_t$, i.e. the mass. In section IV, we will concentrate on the eight-dimensional case, and we will show specific examples of six-dimensional base manifolds constructed as direct products of two Thurston geometries. In section V we consider the existence of torsional hair parameters, and finally in section VI we conclude and give further comments.

2 BTZ-like black holes in Lovelock gravities

The field equations for Lovelock theory in arbitrary dimensions can be conveniently written as

$$\varepsilon^{\alpha}{}_{\beta} = \delta^{\alpha\gamma_1...\gamma_{2k}}_{\beta\rho_1...\rho_{2k}} \underbrace{\left(R^{\rho_1\rho_2}{}_{\gamma_1\gamma_2} - \lambda_1 \delta^{\rho_1\rho_2}_{\gamma_1\gamma_2}\right) ... \left(R^{\rho_{2k-1}\rho_{2k}}{}_{\gamma_{2k-1}\gamma_{2k}} - \lambda_k \delta^{\rho_{2k-1}\rho_{2k}}_{\gamma_{2k-1}\gamma_{2k}}\right)}_{k-\text{factors}} = 0 \ , \tag{9}$$

where $R^{\alpha\beta}{}_{\gamma\delta}$ is the Riemann tensor, $\delta^{\gamma_1...\gamma_p}_{\rho_1...\rho_p}$ stands for the generalized Kronecker delta, and λ_i with $i = \{1, ..., k\}$ are the curvatures of the k different possible maximally symmetric solutions of the theory, with $0 \leq k \leq \left[\frac{D-1}{2}\right]$. The greek indices will split as $\{\alpha, \beta, \gamma\} = \{t, r, \Sigma\}$ where the indices corresponding to Σ will be denoted by the Latin symbols $\{i, j, k, l, m\}$. Note that the requirement for the field equations, and thus the action, to be real allows for pairs of then λ_i in principle to be complex conjugate numbers. We do not need to be worried about this at the moment.

We want to have the following metric as a solution of the system (9)

$$ds^2 = -\left(-\lambda r^2 - \mu\right) dt^2 + \frac{dr^2}{-\lambda r^2 - \mu} + r^2 d\tilde{\Sigma}^2_{D-2} \ , \tag{10}$$

with $\tilde{\Sigma}_{D-2}$ a $D-2$-dimensional (arbitrary at the moment) Euclidean base manifold. Analyzing the Lovelock equations (9) along $\tilde{\Sigma}_{D-2}$, $\varepsilon^{i}{}_{j} = 0$, considering separations of variables, one obtains the following set of $k+1$ equations labelled by the index p which transform as a tensor under diffeomorphisms on $\tilde{\Sigma}$:

$$A_p \ \delta^{il_1...l_{2(k-p)}}_{jm_1...m_{2(k-p)}} \underbrace{\left(\tilde{R}^{m_1m_2}{}_{l_1l_2} + \mu\delta^{m_1m_2}_{l_1l_2}\right) ... \left(\tilde{R}^{m_{2(k-p)-1}m_{2(k-p)}}{}_{l_{2(k-p)-1}l_{2(k-p)}} + \mu\delta^{m_{2(k-p)-1}m_{2(k-p)}}_{l_{2(k-p)-1}l_{2(k-p)}}\right)}_{k-p-\text{factors}} r^{2p} = 0 \ , \ \forall p \in [0, k] \ . \tag{11}$$

where $\tilde{R}^{ij}{}_{kl}$ is the intrinsic Riemann tensor of $\tilde{\Sigma}_{D-2}$. Here the factor A_0 is given by

$$A_0 = (D - 2k - 1)(D - 2k - 2) \ , \tag{12}$$

while the remaining A_p with $0 < p \leq k$ are given by the sum of products

$$A_p = \sum_{i_1 \neq i_2 \neq \ldots \neq i_p} (\lambda - \lambda_{i_1})(\lambda - \lambda_{i_2}) \ldots \left(\lambda - \lambda_{i_p}\right) \ . \tag{13}$$

Therefore, as it is shown below, the only way to avoid tensor restriction on the base is to require the vanishing of all the factors A_p.

Indeed, if the base manifold would satisfy tensorial constraints, then the "would be mass parameter" μ could be rescaled away in very much the same way as in Eqs. (4), (5) and (6), the only difference being that the tensorial constraint Eq. (8) would be replaced by suitable Euclidean Lovelock equations for the base manifold itself.

To fix ideas, let us consider the cubic Lovelock theory in arbitrary dimensions, i.e. $k = 3$. The vanishing of $A_0 = 0$ implies that the dimension must be fixed to $D = 7$ or $D = 8$. The vanishing of A_1 reduces to

$$(\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) = 0 \ , \tag{14}$$

so the constant λ in the metric (10) must be equal to one of the λ_i's of the theory, let's say equal to $\lambda = \lambda_1$. The vanishing of A_2 reads

$$(\lambda - \lambda_2)(\lambda - \lambda_3) + (\lambda - \lambda_1)(\lambda - \lambda_2) + (\lambda - \lambda_1)(\lambda - \lambda_3) = 0 \ , \tag{15}$$

but since $\lambda = \lambda_1$, the last two terms at the left hand side vanish and we are left with $(\lambda - \lambda_2)(\lambda - \lambda_3) = 0$, which is solved, without loss of generality by $\lambda = \lambda_2 \, (= \lambda_1)$. Finally, the equation $A_3 = 0$ reduces to

$$(\lambda - \lambda_1) + (\lambda - \lambda_2) + (\lambda - \lambda_3) = 0 \ , \tag{16}$$

which by virtue to $A_2 = 0$ and $A_1 = 0$ implies that $\lambda = \lambda_3 = \lambda_1 = \lambda_2$.

In seven dimensions we are left with a special Lovelock theory which can be written as a CS theory. The field equations along the time ($\varepsilon^t{}_t = 0$) and radial ($\varepsilon^r{}_r = 0$) direction vanish identically, without imposing any restriction on the base manifold.

In eight dimensions the theory obtained is the BI theory. Here the base manifold turns out to be restricted by a single scalar equation, so the theory does not have the degeneracy of the CS case in the metric sector we are considering. The field equations $\varepsilon^t{}_t = 0$ and $\varepsilon^r{}_r = 0$ are compatible and they imply the scalar constraint (no free index on $\tilde{\Sigma}$)

$$\delta^{i_1 \ldots i_6}_{j_1 \ldots j_6} \left(\tilde{R}^{j_1 j_2}{}_{i_1 i_2} + \mu \delta^{j_1 j_2}_{i_1 i_2}\right) \left(\tilde{R}^{j_3 j_4}{}_{i_3 i_4} + \mu \delta^{j_3 j_4}_{i_3 i_4}\right) \left(\tilde{R}^{i_5 i_6}{}_{j_5 j_6} + \mu \delta^{i_5 i_6}_{j_5 j_6}\right) = 0 \ . \tag{17}$$

In section IV, we will find non-trivial explicit examples of base manifolds which fulfill this equation.

For arbitrary dimensional Lovelock theories, a similar analysis shows that the vanishing of the factors A_p imply that

$$\lambda = \lambda_1 = \ldots = \lambda_k \ , \tag{18}$$

and also that the dimensions has to be fixed by $D = 2k + 1$ or $D = 2k + 2$, where the Lovelock theory reduces to the CS or BI theories respectively. In the former (CS) case, the rest of the field equations leave the base manifold undetermined. Let us focus in the latter (BI) case. The equations along the radial direction and time, are compatible and imply that the base manifold must fulfill

$$\delta^{i_1...i_{2k}}_{j_1...j_{2k}} \underbrace{\left(\tilde{R}^{j_1 j_2}{}_{i_1 i_2} + \mu\delta^{j_1 j_2}_{i_1 i_2}\right) ... \left(\tilde{R}^{i_{2k-1} i_{2k}}{}_{j_{2k-1} j_{2k}} + \mu\delta^{i_{2k-1} i_{2k}}_{j_{2k-1} j_{2k}}\right)}_{k=\frac{D-2}{2}-\text{factors}} = 0 \ . \tag{19}$$

This is a scalar equation on $\tilde{\Sigma}_{D-2}$.

In summary, the metric (10) with negative λ describes a BTZ-like black hole which solves the Lovelock field equations (9) in the BI case defined by (18) in $D = 2k + 2$, provided the base manifold $\tilde{\Sigma}_{D-2}$ fulfills the scalar equation (19).

This solution has μ as a genuine integration constant and also all the other possible integrations constants associated to the base manifold which fulfills (19). In the next section, we will show that all the Noether charges $Q(\xi)$, associated to diffeomorphisms generated by a Killing field ξ, vanish identically, in particular the mass $Q(\xi = \partial_t)$.

3 Conserved charges for BTZ-like metrics in Lovelock theories

In order to simplify the computations, here after we will use the first order formalism.

For the metric (10) we can choose the vielbein e^A as

$$e^0 = f(r)dt \quad ; \quad e^1 = 1/f(r)dr \quad ; \quad e^{a_i} = r\tilde{e}^{a_i} \ , \tag{20}$$

where the Lorentz index A have been split as $A = \{0, 1, a_i\}$, $\tilde{e}^{a_i}$ are the intrinsic vielbeins of the base manifold $\tilde{\Sigma}_{D-2}$, and provided we define $l^2 := -1/\lambda$

$$f^2(r) := \frac{r^2}{l^2} - \mu \ . \tag{21}$$

If R^{AB} is the curvature two-form, then we define the concircular curvature by

$$\bar{R}^{AB} := R^{AB} + \frac{1}{l^2}e^A e^B \ . \tag{22}$$

Note that this curvature vanishes on locally AdS space-times of curvature radius l, since they satisfy $R^{AB} = -\frac{1}{l^2}e^A e^B$. For the metric (10), the only non-vanishing components of $\bar{R}^{AB}$, are the one with Lorentz indices along $\tilde{\Sigma}_{D-2}$, and read

$$\bar{R}^{ab} = \tilde{R}^{ab} + \mu\tilde{e}^a\tilde{e}^b \ , \tag{23}$$

which in terms of the Riemann tensor in the coordinate basis read

$$\bar{R}^{ij}_{kl} = \frac{\tilde{R}^{ij}_{kl} + \mu\delta^{ij}_{kl}}{r^2} \ . \tag{24}$$

Note that all the components of $\bar{R}^{AB}$ vanish in the asymptotic regions ($r \to +\infty$), therefore the black hole metric is asymptotically locally AdS. The scalar equation on $\tilde{\Sigma}_{D-2}$ (19), in terms of differential forms reads

$$\epsilon_{a_1...a_{2k}} \underbrace{\left(\tilde{R}^{a_1 a_2} + \mu\tilde{e}^{a_1}\tilde{e}^{a_2}\right) ... \left(\tilde{R}^{a_{2k-1} a_{2k}} + \mu\tilde{e}^{a_{2k-1}}\tilde{e}^{a_{2k}}\right)}_{k\text{-factors}} = 0 \ . \tag{25}$$

The action of the Lovelock theory in the BI case in $D = 2k + 2$ dimensions can be conveniently written as

$$I_{BI} = \kappa \int_{M_{2k+2}} \epsilon_{A_1 \ldots A_D} \underbrace{\bar{R}^{A_1 A_2} \ldots \bar{R}^{A_{D-1} A_D}}_{k+1\text{-terms}} , \tag{26}$$

where $\epsilon_{A_1 \ldots A_D}$ is the completely antisymmetric Levi-Civitta Lorentz tensor and $\bar{R}^{AB}$ is defined in equation (22). In the $2k+2$-dimensional BI action in Eq. (26) there is a total derivative term (the generalized Euler density) of order $k+1$ in the curvature which, obviously, does not contribute to the field equations. For example, in four dimensions, $k = 1$ and the action corresponds to a combination of the Einstein-Hilbert action with a cosmological term, supplemented with the Gauss-Bonnet density (which is a topological density in $D = 4$ cuadratic in the curvature). In the same manner, the theory in six dimensions reduces to the Einstein-Gauss-Bonnet theory with a cosmological term, supplemented by the cubic Lovelock term (topological in $D = 6$). In any even dimension $D = 2k + 2$, the BI Lagrangian has only one independent coupling constant since the couplings are fixed in such a way that there is a single maximally symmetric solution and the action can be written as a combination of $\bar{R}^{AB}$ of order $k + 1$.

It was shown in [20] that in even dimensions if one adds to the Lagrangian the generalized Euler density (even though it does not contribute to the field equations) then one gets a well defined action principle for asymptotically locally AdS spaces. In this way the variation of the action vanishes identically, the action has a finite value, and by applying directly Noether's theorem, one can construct finite conserved charges associated to a diffeomorphism generated by the Killing field ξ and by local Lorentz transformation. These charges vanish on space-times which are locally AdS. The expression for these charges is given by [20]

$$Q(\xi) = \int I_\xi \omega^{AB} T_{AB} , \tag{27}$$

where

$$T_{AB} := \epsilon_{ABC_1 \ldots C_{2k}} \bar{R}^{C_1 C_2} \ldots \bar{R}^{C_{2k-1} C_{2k}} . \tag{28}$$

Here $I_\xi \omega^{AB}$ denotes the inner product of the diffeomorphism generator ξ and the spin connection ω^{AB}.

In the present case, due to the fact that for the metric in Eq. (10) the only nonvanishing components of $\bar{R}^{AB}$ have indices along the base manifold $\tilde{\Sigma}_{D-2}$ we have that

$$T_{0a} = T_{1a} = T_{ab} = 0 , \tag{29}$$

the only nonvanishing component of T_{AB} being

$$T_{01} = \epsilon_{01 a_1 \ldots a_{2k}} \bar{R}^{a_1 a_2} \ldots \bar{R}^{a_{2k-1} a_{2k}} . \tag{30}$$

Then, by virtue of Eq. (23), one can see that the expression in Eq. (30) reduces to the scalar restriction on the base manifold in Eq. (25). Thus we conclude that all the conserved charges evaluated according to Eq. (27) corresponding to the solutions in Eq. (10) of BI theories, vanish identically. Therefore the integration constant μ can be interpreted as a purely gravitational hair parameter.

Since in four dimensions the BI theory reduces to Einstein theory, as mentioned in the introduction, the parameter μ can be reabsorbed and the metric is locally AdS, so it differs from it only at a topological level, and all the Noether charges vanish as well.

3.1 Black hole entropy as a Noether charge and Euclidean action

The approach developed by Wald in [21], [22] and further studied for Lovelock gravities in [23], provides one with a definition of black hole entropy as a Noether charge. In the present case, the black hole entropy is given by

$$S = -2\pi \int_{\Sigma_h} \frac{\delta L}{\delta R_{\alpha\beta\gamma\delta}} \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \bar{\epsilon} \ , \tag{31}$$

where L is the Lagrangian, $R_{\alpha\beta\gamma\delta}$ the Riemann tensor, and $\bar{\epsilon}$ is the volume form of the bifurcation surface Σ_h, with binormal $\epsilon_{\alpha\beta}$ normalized by $\epsilon_{\alpha\beta}\epsilon^{\alpha\beta} = -2$.

As in the case of the conserved charges evaluated in the previous subsection, one can see that the entropy of the black holes in Eq. (10) in the Born-Infeld theory, is proportional to the scalar constraint on $\tilde{\Sigma}_{D-2}$ given in Eq. (25), and consequently the entropy vanishes identically.

Since the black holes considered here have finite temperature given by

$$T = \frac{r_+}{2\pi l^2} = \frac{\sqrt{\mu}}{2\pi l} \ , \tag{32}$$

the first law of black hole thermodynamics is trivially satisfied, since $dS = 0$ and $dM = 0$, therefore

$$dM = TdS \ . \tag{33}$$

It is worth to point out that in order to get zero entropy it is necessary to include also the topological term to the action in the form (26). The reason is that the entropy would be non-vanishing [16] unless one includes this term (which, as far as the equations of motion are concerned, is irrelevant) in the Wald formula. Therefore the present results together with [16] disclose the topological origin of the hairy parameter μ.

This is also confirmed if one takes into account that the Euclidean extension of the finite BI action (26) I^E_{BI}, vanishes identically on the family of black hole solutions considered here, i.e. $I^E_{BI} = 0$. Then both the mass and the entropy computed in the canonical ensemble from the free energy corresponding to the Euclidean action vanish as well

$$M = -\frac{\partial I^E_{BI}}{\partial \beta} = 0 \ , \tag{34}$$

$$S = I^E_{BI} - \beta \frac{\partial I^E_{BI}}{\partial \beta} = 0 \ . \tag{35}$$

As it has been already mentioned, in four dimensional BI theory, i.e. Einstein gravity, the solution (10) turns out to be locally equivalent to AdS, and the constant μ can always be rescaled to 1. Consequently the mass and entropy vanish and the metric is know as the massless topological black hole. For BI theory in dimensions higher than four, the black hole metric (10) is not of constant curvature in general (since $\tilde{\Sigma}_{D-2}$ is restricted by a scalar equation), still the mass and the entropy also vanish identically. We will further comment on this in the conclusions.

In the next section, we will focus on the case of eight-dimensional BI theory. We will show that if the behavior of the metric at infinity is fixed, then the parameter μ cannot be generically absorbed as it is the case in general relativity.

4 BI theory in eight dimensions and examples of base manifolds

In $D = 8$, we will focus then on black hole metrics of the form

$$ds_8^2 = -\left(\frac{r^2}{l^2} - \mu\right) dt^2 + \frac{dr^2}{\frac{r^2}{l^2} - \mu} + r^2 d\tilde{\Sigma}_6^2 \ , \tag{36}$$

where the base manifold $\tilde{\Sigma}_6$ solves the scalar restriction in Eq. (17), which in first order formalism, reads

$$\epsilon_{a_1 \dots a_6} \left(\tilde{R}^{a_1 a_2} + \mu \tilde{e}^{a_1} \tilde{e}^{a_2}\right) \left(\tilde{R}^{a_3 a_4} + \mu \tilde{e}^{a_3} \tilde{e}^{a_4}\right) \left(\tilde{R}^{a_5 a_6} + \mu \tilde{e}^{a_5} \tilde{e}^{a_6}\right) = 0 \ . \tag{37}$$

This scalar restriction is trivially solved when $\tilde{\Sigma}_6$ is of constant curvature $\tilde{R}^{ab} = -\mu \tilde{e}^a \tilde{e}^b$, and in this case the constant μ can be absorbed as it occurs in Einstein gravity. In order to depart from maximally symmetric base manifolds, we will focus in the case in which $\tilde{\Sigma}_6$ can be written as the direct product of two three-dimensional Euclidean manifolds, i.e.,

$$\tilde{\Sigma}_6 = M_1 \times M_2 \ . \tag{38}$$

where each of the M_1 and M_2 is of three-dimensional. Euclidean three-dimensional closed orientable geometries, can be canonically decomposed according to the eight Thurston geometries: the Euclidean three-dimensional space E^3, the three-sphere S^3, the three dimensional hyperbolic space H_3, $S^1 \times H_2$ and $S^1 \times S^2$ with the trivial metrics on them are the simplest ones. In addition one has the following three representative metrics (for a review see [24])

$$Sol : e^{2z} dx^2 + e^{-2z} dy^2 + dz^2 \ , \tag{39}$$

$$Nil : dx^2 + dy^2 + (dz - x dy)^2 \ , \tag{40}$$

$$SL2R : \frac{1}{x^2}\left(dx^2 + dy^2\right) + \left(dz + \frac{1}{x} dy\right)^2 \ . \tag{41}$$

These last three geometries are non-trivial in the sense that they are homogeneous but neither constant curvature nor a product of constant curvature manifolds.

These metrics are constructed as invariant metrics on group manifolds, through a symmetric quadratic combination of the corresponding Maurer-Cartan forms. Due to the homogeneity property of these three-dimensional geometries, the intrinsic curvature two-form $\tilde{R}^{ab}$ will have constant components when expressed as a exterior product of two dreibein, and so, from the scalar constraint in Eq. (37), we will obtain a single algebraic equation, involving μ and the radii of the base manifold factors. If we scale the last three Thurston metrics by a constant square radius R_0^2, the natural dreibein and the components of the curvature two-form in each case are given by:

- Sol geometry:

$$\tilde{e}^1 = R_0 \exp(z)\, dx,\ \tilde{e}^2 = R_0 \exp(-z) dy,\ \tilde{e}^3 = R_0 dz \ ,$$
$$\tilde{R}^{12} = \frac{1}{R_0^2} \tilde{e}^1 \tilde{e}^2,\ \tilde{R}^{13} = -\frac{1}{R_0^2} \tilde{e}^1 \tilde{e}^3,\ \tilde{R}^{23} = -\frac{1}{R_0^2} \tilde{e}^2 \tilde{e}^3,$$

- Nil geometry

$$\tilde{e}^1 = R_0 dx,\ \tilde{e}^2 = R_0 dy,\ \tilde{e}^3 = R_0 (dz - x dy) \ ,$$
$$\tilde{R}^{12} = -\frac{3}{4R_0^2} \tilde{e}^1 \tilde{e}^2,\ \tilde{R}^{13} = \frac{1}{4R_0^2} \tilde{e}^1 \tilde{e}^3,\ \tilde{R}^{23} = \frac{1}{4R_0^2} \tilde{e}^2 \tilde{e}^3,$$

- SL2R geometry

$$\tilde{e}^1 = \frac{R_0}{x}dx,\ \tilde{e}^2 = \frac{R_0}{x}dy,\ \tilde{e}^3 = R_0\left(dz + \frac{1}{x}dy\right)\ ,$$
$$\tilde{R}^{12} = -\frac{7}{4R_0^2}\tilde{e}^1\tilde{e}^2,\ \tilde{R}^{13} = \frac{1}{4R_0^2}\tilde{e}^1\tilde{e}^3,\ \tilde{R}^{23} = \frac{1}{4R_0^2}\tilde{e}^2\tilde{e}^3,$$

Let us then consider the following metric on the six-dimensional base manifold

$$d\tilde{\Sigma}_6^2 = R_1^2 dM_1^2 + R_2^2 dM_2^2\ , \tag{42}$$

where R_1 and R_2 are the radii of the three dimensional space-times with line elements dM_1 and dM_2. Each of the two factors dM_1 and dM_2 will be chosen as one of the non-trivial Thurston geometries Nil, Sol and $SL2R$. Then, the field equations through the scalar constraint in Eq. (37) determine a scalar relation between μ, R_1^2 and R_2^2. In the following table there is the complete list of possible pairs of non-trivial Thurston geometries together with the corresponding scalar relation:

$$R_1^2 Nil \times R_2^2 Nil \Rightarrow R_1^2 = \frac{-1+12\mu R_2^2}{12\mu(20\mu R_2^2 - 1)} \tag{43}$$
$$R_1^2 Sol \times R_2^2 Sol \Rightarrow R_1^2 = \frac{-1+3\mu R_2^2}{3\mu(-1+5\mu R_2^2)} \tag{44}$$
$$R_1^2 SL2R \times R_2^2 SL2R \Rightarrow R_1^2 = \frac{-5+12\mu R_2^2}{12\mu(-1+4\mu R_2^2)} \tag{45}$$
$$R_1^2 Nil \times R_2^2 SL2R \Rightarrow R_1^2 = \frac{-5+12\mu R_2^2}{60\mu(-1+4\mu R_2^2)} \tag{46}$$
$$R_1^2 Nil \times R_2^2 Sol \Rightarrow R_1^2 = \frac{-1+3\mu R_2^2}{12\mu(-1+5\mu R_2^2)} \tag{47}$$
$$R_1^2 Sol \times R_2^2 SL2R \Rightarrow R_1^2 = \frac{-5+12\mu R_2^2}{15\mu(-1+4\mu R_2^2)} \tag{48}$$

It is worth emphasizing that when $\tilde{\Sigma}_6$ is constructed out of two factors, one can fix the value of the curvature of only one of the two factors, let's say R_1. Of course, this fixes part of the freedom of the geometry at infinity but then one cannot rescale μ anymore, which is therefore a non-trivial integration constant.

This construction of a base manifold product of Thurston geometries can be extended naturally to dimension $6N+2$ where the base manifold is the product of of $2N$ Thurston geometries.

4.1 Torsional hairs

An interesting feature of Lovelock gravity is that in the first order formalism the equations of motion do not imply the vanishing of torsion in vacuum as in General Relativity. This means that torsion may also have propagating degrees of freedom. However the consistency between the equations of motion coming from variations with respect to the vielbein e^A and the spin connection ω^{AB}, imposes very strong constraints on the torsion, so that in most cases one obtains an over-constrained system of equations. It was proved in [19] that in even dimensions the BI case is the one in which torsion is less restricted, while in odd dimensions this case corresponds to the CS Lagrangian. For other values of the coupling constants, using the ansatz for the torsion introduced in [25] (see Eq. (49) below), it has been possible for the first time to construct solutions with non-vanishing torsion [26],[27]. This ansatz works very naturally on a three-dimensional constant curvature sub-manifold in which case the torsion two-form is proportional to the corresponding three-dimensional Levi-Civita tensor contracted with two dreibein. It has been also used to include a non-vanishing torsion on eight dimensional BTZ-like black holes when the base manifold was the product of two constant curvature three-dimensional manifolds [16].

Interestingly enough, the constant curvature condition is not necessary in order for the ansatz introduced in [25] to work, since it is enough to have a three-dimensional sub-manifold to have a properly defined fully antisymmetric torsion. Therefore, also when the base manifold is the product of two Thurston geometries it is possible to consider on one of the two factors the following ansatz for the torsion:

$$T^a = \frac{\delta}{r}\epsilon^{abc}e_b e_c \ , \quad K^{ab} = -\frac{\delta}{r}\epsilon^{abc}e_c \tag{49}$$

where δ is an integration constant constant and K^{ab} is the contorsion. Due to the fact that the torsion in Eq. (49) is fully antisymmetric [25] the field equations can be satisfied even when the torsion is non-vanishing. Indeed, the Riemann tensor modified by the presence of torsion reads

$$R^{01} = \hat{R}^{01} \quad ; R^{1i} = \hat{R}^{1i} \quad ; \quad R^{ij} = \hat{R}^{ij} \tag{50}$$

$$R^{1a} = \hat{R}^{1a} - \frac{f}{r}T^a \quad ; \quad R^{ab} = \hat{R}^{ab} - \left(\frac{\delta_2}{r}\right)^2 e^a e^b \quad . \tag{51}$$

where f is the lapse function, the indices i, j, k,... correspond to the Thurston factor without torsion while the indices a, b, c,... correspond to the Riemann curvature of the Thurston factor supporting the fully antisymmetric torsion in Eq. (49). The notation introduced in Eqs. (50) and (51) has the following meaning: $\hat{R}^{AB}$ represents the Riemannian curvature without the inclusion of the contributions coming from the contorsion (computed in the previous sections) while R^{AB} represents the total curvature including the contorsion contributions. Only the components with at least one index of type a are modified. It is also easy to see that the modification of the R^{1a} components drops out from the field equations (see the discussion in [25]). The modifications of the R^{ab} components are easy to describe in terms of the following replacements in the components of the Riemann curvature of the Thurston geometries

- Sol geometry:

$$\tilde{R}^{12} = \frac{1}{R_0^2}\tilde{e}^1\tilde{e}^2 \rightarrow \quad R^{12} = \left(\frac{1}{R_0^2} - \delta^2\right)\tilde{e}^1\tilde{e}^2$$
$$\tilde{R}^{13} = -\frac{1}{R_0^2}\tilde{e}^1\tilde{e}^3 \rightarrow \quad R^{13} = \left(-\frac{1}{R_0^2} - \delta^2\right)\tilde{e}^1\tilde{e}^3$$
$$\tilde{R}^{23} = -\frac{1}{R_0^2}\tilde{e}^2\tilde{e}^3 \rightarrow \quad R^{23} = \left(-\frac{1}{R_0^2} - \delta^2\right)\tilde{e}^2\tilde{e}^3$$

- Nil geometry

$$\tilde{R}^{12} = -\frac{3}{4R_0^2}\tilde{e}^1\tilde{e}^2 \rightarrow \quad R^{12} = \left(-\frac{3}{4R_0^2} - \delta^2\right)\tilde{e}^1\tilde{e}^2$$
$$\tilde{R}^{13} = \frac{1}{4R_0^2}\tilde{e}^1\tilde{e}^3 \rightarrow \quad R^{13} = \left(\frac{1}{4R_0^2} - \delta^2\right)\tilde{e}^1\tilde{e}^3$$
$$\tilde{R}^{23} = \frac{1}{4R_0^2}\tilde{e}^2\tilde{e}^3 \rightarrow \quad R^{23} = \left(\frac{1}{4R_0^2} - \delta^2\right)\tilde{e}^2\tilde{e}^3$$

- SL2R geometry

$$\tilde{R}^{12} = -\frac{7}{4R_0^2}\tilde{e}^1\tilde{e}^2 \rightarrow \quad R^{12} = \left(-\frac{7}{4R_0^2} - \delta^2\right)\tilde{e}^1\tilde{e}^2$$
$$\tilde{R}^{13} = \frac{1}{4R_0^2}\tilde{e}^1\tilde{e}^3 \rightarrow \quad R^{13} = \left(\frac{1}{4R_0^2} - \delta^2\right)\tilde{e}^1\tilde{e}^3$$
$$\tilde{R}^{23} = \frac{1}{4R_0^2}\tilde{e}^2\tilde{e}^3 \rightarrow \quad R^{23} = \left(\frac{1}{4R_0^2} - \delta^2\right)\tilde{e}^2\tilde{e}^3$$

Obviously, in this case, Eq. (37) (which, of course, has to be written in terms of the total curvatures including the torsional contributions) is still a single scalar equation with the important difference of the presence of a further integration constant (namely δ) related to the torsion. Therefore, one can still write down R_1^2 in terms of R_2^2 as in the previous section (the modified expression are not particularly illuminating) but now one may wonder whether the new integration constant contribute to the charges. When torsion is included the Noether charges can be calculated in a Lorentz invariant manner following the construction in [28]. One can prove that a torsion of the form (49) does not affect the value of the charges since, once again, they are proportional to the equations of motion, so they vanish identically. This means that the integration constant δ could be interpreted as a torsional hair.

It is interesting to note that in the examples considered above, torsion can be switched on in both three-dimensional submanifolds using ansatz similar to Eq. (49) along both Thurston factors:

$$T^a = \frac{\delta_1}{r}\epsilon^{abc}e_b e_c \ , \quad K^{ab} = -\frac{\delta_1}{r}\epsilon^{abc}e_c$$

$$T^i = \frac{\delta_2}{r}\epsilon^{ijk}e_j e_k \ , \quad K^{ij} = -\frac{\delta_2}{r}\epsilon^{ijk}e_k \ .$$

Also in this case the Noether charges vanish identically because their expressions are proportional to the equations of motion. However, in such a case, it is possible to construct the following six-form with support on $\tilde{\Sigma}_6$

$$\Omega = \Omega_1 \wedge \Omega_2 \ , \tag{52}$$

where

$$\Omega_1 = \frac{1}{r^2}\, T^a e_a\big|_{M_1} \ \text{ and } \Omega_2 = \frac{1}{r^2}\, T^j e_j\big|_{M_2} \ . \tag{53}$$

Using only the Bianchi identities, it can be shown that off-shell

$$D\Omega = 0 \tag{54}$$

where D is the exterior Lorentz covariant derivative. This equation implies that it is possible to associate a sort of "topological charge" to Ω. Then, when the torsion has support along both three dimensional factors of the base manifold, the constants δ_1 and δ_2 contribute to the charge constructed out from (54). In any case, the value of the charge is given by the product of these two integration constants, so that different values of the two integration constants may give rise to the same topological charge, therefore one of the two δ_i $(i = 1, 2)$ can still be interpreted as a topological hair.

5 Discussion

The main goal of this paper has been to construct a family of vacuum black holes in Lovelock gravity in even dimensions in which there is at least one non-trivial integration constant that can be interpreted as a purely gravitational hair. In the $t-r$ plane, these black holes defined in Eq. (10) look like BTZ black holes. It has been shown that it is necessary to select Lovelock theories where the tensor constraints on the base manifold reduce to a single scalar equation: this requirement singles out Born-Infeld gravity. The odd dimensional case actually leaves the base manifold completely undetermined.

Explicit solutions whose base manifolds are the direct product of two Thurston geometries have been constructed in eight dimensions (the extension to $6N+2$ dimensions being straightforward). It is possible to construct closed, smooth quotients of these metrics (see e.g. [39]), in order to obtain compact horizons with non-trivial topologies[2]. In reference [39] the authors constructed black hole solutions for Einstein gravity plus a negative cosmological constant in five dimensions in which all but one of the Thurston geometries appear (the Thurston geometry left outside being the SL2R model geometry). In the present framework, all the Thurston geometries can appear as factors of the base manifold. The black hole geometries considered here allow to find exactly the quasi-normal frequencies of fields with diverse spins (for $D \geq 4$ see e.g. [29]).

Remarkably, all the Noether charges corresponding to these solutions vanish identically. This implies that the integration constant in the lapse function which, due to the similarity with the BTZ black hole, could naively look like a mass parameter, is actually a purely gravitational hair, since it has no charge associated and moreover does not affect any other charge. The existence of purely gravitational hairs is of interest as these situations may represent the strongest possible counterexample to the no hair conjectures, in the sense that hairs arise already at purely gravitational level without the need of any matter field [3]. The charges are computed following the prescription of reference [20], in which a finite action principle that attains an extrema on asymptotically locally AdS space-times was constructed. It would be also interesting to obtain these charges in an independent manner, by extending to BI theories the results coming from Hamiltonian perturbation theory recently obtained in Lovelock gravities [34].

In the Lovelock theories considered here, the torsion does not vanish identically, and we have also explored some solutions with non-zero torsion. When the base manifold is the product of two three-dimensional geometries, we used the torsion ansatz in Eq. (49). It turns out to be that torsion has no effect on the value of the charges for the class of black holes considered, and therefore one may interpret the corresponding integration constant as a "torsional hair". On the other hand, when the torsion in nonvanishing on both three-dimensional factors of the base manifold, it is possible to construct a "topological charge" out of the torsion, the charge being proportional to the product of the two integration constants appearing in the torsion components.

To compare our findings with the usual situation in Einstein gravity can be illuminating. In Einstein gravity, the field equations force the base manifold to be an Einstein manifold. This condition implies that the base manifold is of constant curvature in $D = 4$ and 5, while in higher dimensions, due to the non-triviality of the Weyl tensor, does not fix the Riemann tensor. Thus, without loss of generality, one can normalize the constant μ in equation (10) to 1. Since we were looking for scenarios in which μ represents a nontrivial integration constant, the choice of the Lovelock family was the most natural option. In the cases in which the base manifold is an Einstein manifold, it was proven

[2]For an uncomplete list of references in which Thurston geometries have been used in gravitational theories see [40]-[46].

[3]For other example of gravitational hair in the three-dimensional BHT new massive gravity [30], see [31]. Also if not stated in the original papers other solutions that can be seen as gravitational hairs exist in the context of compactified Lovelock gravity [32] and in a two dimensional gravity models [33]

in [35] that Einstein-Gauss-Bonnet theory further imposes a quadratic restriction on the Weyl tensor of $\tilde{\Sigma}_{D-2}$. This analysis was extended in [36] were it was shown that in five dimensions for the EGB theory, one can actually get rid of the Einstein restriction in the case when there is a unique maximally symmetric solution, and the base manifold acquires more freedom. This analysis was extended to six and higher dimensions in [37]-[38] with similar results. For the cubic Lovelock theory, some specific black hole solutions with non constant curvature base manifolds were found in [16]. As it has been shown in the present paper, it is precisely this freedom on the base manifold, which allows to interpret μ as a true integration constant and, eventually, as a purely gravitational hair.

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