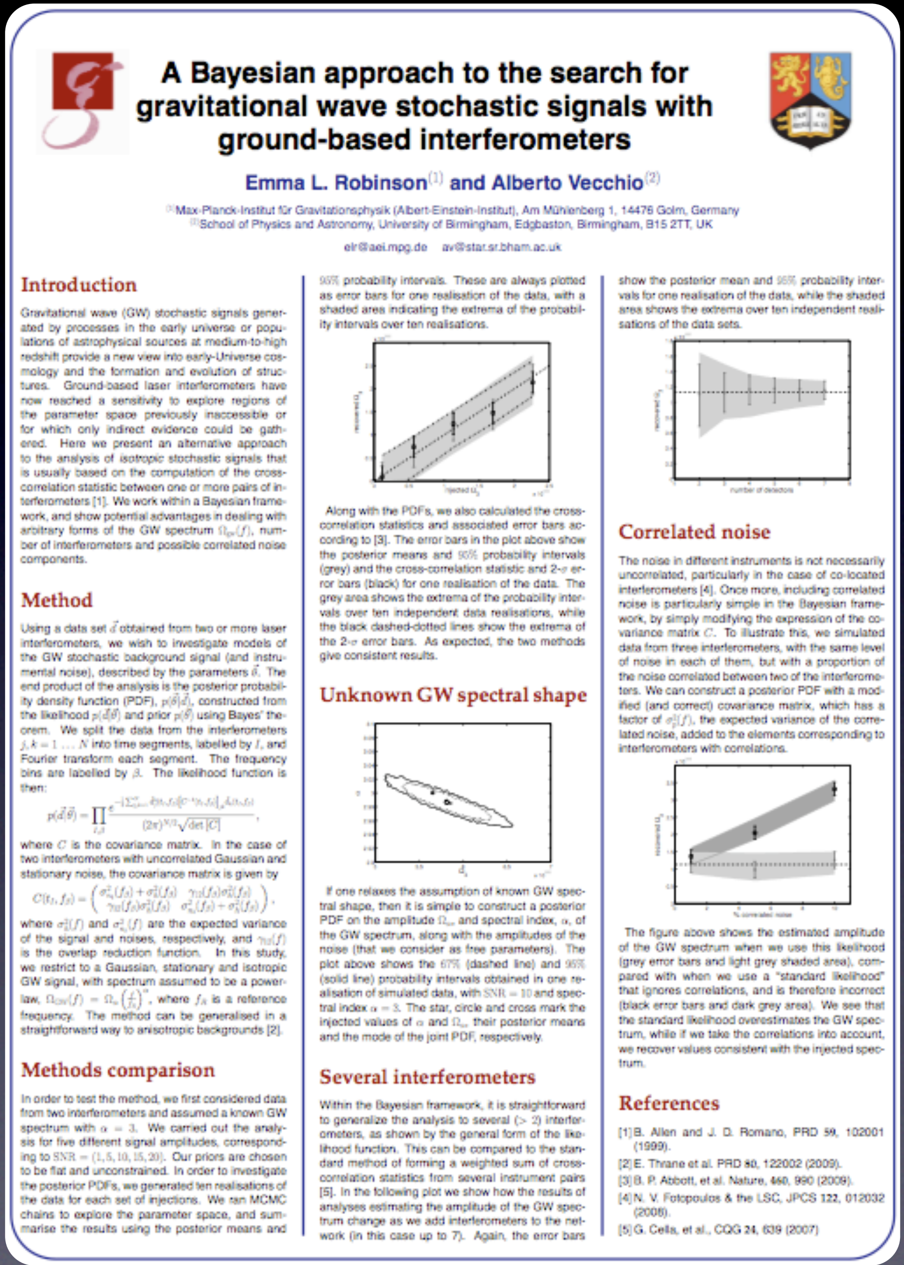

A Bayesian approach to the search for gravitational wave stochastic signals with ground-based interferometers

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Introduction

Gravitational wave (GW) stochastic signals generated by processes in the early universe or populations of astrophysical sources at medium-to-high redshift provide a new view into early-Universe cosmology and the formation and evolution of structures. Ground-based laser interferometers have now reached a sensitivity to explore regions of the parameter space previously inaccessible or for which only indirect evidence could be gathered. Here we present an alternative approach to the analysis of isotropic stochastic signals that is usually based on the computation of the cross-correlation statistic between one or more pairs of interferometers [1]. We work within a Bayesian framework, and show potential advantages in dealing with arbitrary forms of the GW spectrum $\Omega_{gw}(f)$, number of interferometers and possible correlated noise components.

Method

Using a data set $\vec{d}$ obtained from two or more laser interferometers, we wish to investigate models of the GW stochastic background signal (and instrumental noise), described by the parameters $\vec{\theta}$. The end product of the analysis is the posterior probability density function (PDF), $p(\vec{\theta}|\vec{d})$, constructed from the likelihood $p(\vec{d}|\vec{\theta})$ and prior $p(\vec{\theta})$ using Bayes' theorem. We split the data from the interferometers $j, k = 1 \dots N$ into time segments, labelled by I, and Fourier transform each segment. The frequency bins are labelled by β. The likelihood function is then:

$$p(\vec{d}|\vec{\theta}) = \prod_{I,\beta} \frac{e^{-\frac{1}{2}\sum_{j,k} \tilde{d}_{jI}(f_\beta)\,[C^{-1}(f_\beta)]_{jk}\,\tilde{d}_{kI}(f_\beta)}}{(2\pi)^{N/2}\sqrt{\det[C]}},$$

where C is the covariance matrix. In the case of two interferometers with uncorrelated Gaussian and stationary noise, the covariance matrix is given by

$$C(f_1, f_2) = \begin{pmatrix} \sigma_{n_1}^2(f_\beta) + \sigma_h^2(f_\beta) & \gamma_{12}(f_\beta)\sigma_h^2(f_\beta) \\ \gamma_{12}(f_\beta)\sigma_h^2(f_\beta) & \sigma_{n_2}^2(f_\beta) + \sigma_h^2(f_\beta) \end{pmatrix},$$

where $\sigma_h^2(f)$ and $\sigma_n^2(f)$ are the expected variance of the signal and noises, respectively, and $\gamma_{12}(f)$ is the overlap reduction function. In this study, we restrict to a Gaussian, stationary and isotropic GW signal, with spectrum assumed to be a power-law, $\Omega_{GW}(f) = \Omega_\alpha \left(\frac{f}{f_R}\right)^\alpha$, where f_R is a reference frequency. The method can be generalised in a straightforward way to anisotropic backgrounds [2].

Methods comparison

In order to test the method, we first considered data from two interferometers and assumed a known GW spectrum with $\alpha = 3$. We carried out the analysis for five different signal amplitudes, corresponding to $\mathrm{SNR} = (1, 5, 10, 15, 20)$. Our priors are chosen to be flat and unconstrained. In order to investigate the posterior PDFs, we generated ten realisations of the data for each set of injections. We ran MCMC chains to explore the parameter space, and summarise the results using the posterior means and 95% probability intervals. These are always plotted as error bars for one realisation of the data, with a shaded area indicating the extrema of the probability intervals over ten realisations.

Along with the PDFs, we also calculated the cross-correlation statistics and associated error bars according to [3]. The error bars in the plot above show the posterior means and 95% probability intervals (grey) and the cross-correlation statistic and 2-σ error bars (black) for one realisation of the data. The grey area shows the extrema of the probability intervals over ten independent data realisations, while the black dashed-dotted lines show the extrema of the 2-σ error bars. As expected, the two methods give consistent results.

Unknown GW spectral shape

If one relaxes the assumption of known GW spectral shape, then it is simple to construct a posterior PDF on the amplitude Ω_α and spectral index, α, of the GW spectrum, along with the amplitudes of the noise (that we consider as free parameters). The plot above shows the 67% (dashed line) and 95% (solid line) probability intervals obtained in one realisation of simulated data, with $\mathrm{SNR} = 10$ and spectral index $\alpha = 3$. The star, circle and cross mark the injected values of α and Ω_α, their posterior means and the mode of the joint PDF, respectively.

Several interferometers

Within the Bayesian framework, it is straightforward to generalize the analysis to several (> 2) interferometers, as shown by the general form of the likelihood function. This can be compared to the standard method of forming a weighted sum of cross-correlation statistics from several instrument pairs [5]. In the following plot we show how the results of analyses estimating the amplitude of the GW spectrum change as we add interferometers to the network (in this case up to 7). Again, the error bars show the posterior mean and 95% probability intervals for one realisation of the data, while the shaded area shows the extrema over ten independent realisations of the data sets.

Correlated noise

The noise in different instruments is not necessarily uncorrelated, particularly in the case of co-located interferometers [4]. Once more, including correlated noise is particularly simple in the Bayesian framework, by simply modifying the expression of the covariance matrix C. To illustrate this, we simulated data from three interferometers, with the same level of noise in each of them, but with a proportion of the noise correlated between two of the interferometers. We can construct a posterior PDF with a modified (and correct) covariance matrix, which has a factor of $\sigma_n^2(f)$, the expected variance of the correlated noise, added to the elements corresponding to interferometers with correlations.

The figure above shows the estimated amplitude of the GW spectrum when we use this likelihood (grey error bars and light grey shaded area), compared with when we use a "standard likelihood" that ignores correlations, and is therefore incorrect (black error bars and dark grey area). We see that the standard likelihood overestimates the GW spectrum, while if we take the correlations into account, we recover values consistent with the injected spectrum.

References

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