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Reply To: MRI Resolution Enhancement: How Useful are Shifted Images Obtained by Changing the Demodulation Frequency?

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In a recent article, Tieng et al. studied a super-resolution MRI technique, which relies on the combination of several low-quality images (same field-of-view and resolution) that are shifted against each other by a fraction of a pixel (1). Because additional (new) information is required in these images to improve the in-plane resolution by super-resolution principles, the authors investigated whether small changes in the demodulation frequency of the MRI acquisition may lead to images with different information content. As already pointed out by Scheffler (2), the fundamentals of MRI state that—in the above situation and neglecting noise and other errors—the information of all shifted images is identical because all sampled positions in k-space are the same, while the respective signals only differ by known phase factors. Nevertheless, Tieng et al. tried to demonstrate that shifted low-resolution images can be composed into a more densely sampled image that exhibits higher quality than achievable by conventional interpolation methods. They conclude that this improvement indicates the existence of new information due to the applied change in demodulation frequency. In our opinion, this conclusion is supported neither by the theoretical arguments nor by the experimental data presented.

As correctly observed in Ref. 1, the visual appearance of images that are affected by truncation artifacts (i.e., blurring and ringing) can be improved, if the discrete image intensities are displayed on a finer grid. This even applies if the point-spread function as determined by the *k*-space sampling pattern is not changed. The situation is illustrated for an ideal point source in Fig. 1 and experimentally confirmed in Fig. 2a,b. The authors implicitly assume that the additional grid points carry new information, which may be obtained from measurements with different demodulation frequencies. However, this assumption is not correct because all values at arbitrary points in image space can be computed by evaluating the Fourier transform of the k-space data of a single measurement. A well-known and widely used procedure to resolve the image on a finer grid is to expand the *k*-space with zeros before applying a discrete Fourier transform. Neglecting noise, the calculated grid points in image space are identical to those obtained by combining multiple images with a shifted field-of-view. Again, this is demonstrated in Fig. 1 for an ideal point source and experimentally confirmed in Fig. 2b,c. While the procedure is mathematically equivalent to a trigonometric (Dirichlet, i.e., periodic sinc) interpolation, the comparative sinc interpolation performed in Ref. 1 does not seem to correctly restore the low-resolution signal between grid points. Unfortunately, the exact method has not been described.

In the presence of noise, a zero-filled image and a composite image of combined shifted images may indeed differ. The zero-filled image contains the same low-spatial frequencies as the original measurement without adding high-frequency components. In contrast, the composite image presents with an equal noise distribution over all frequencies and a coherent sum of low spatial frequencies, i.e., the image content. Despite the same overall signal-to-noise ratio of 19.3 and 19.4, respectively, this difference is demonstrated in Fig. 3a,b with the composite image having more noise in the high-spatial frequencies and higher signal in the low-spatial frequencies than the zero-filled image. Because the high-

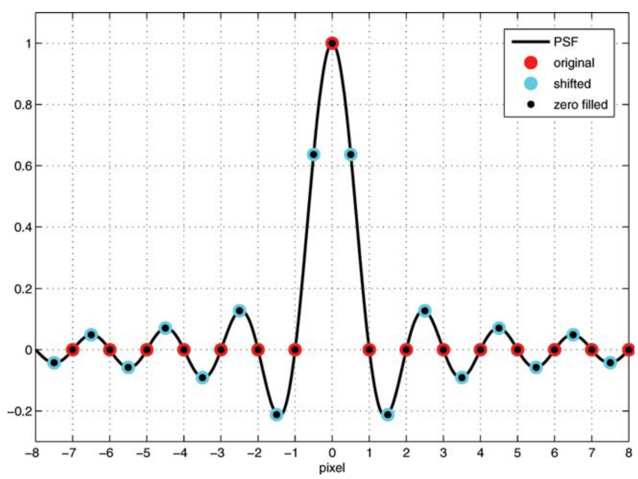

FIG. 1. Point-spread function (Dirichlet kernel) for a measurement with 256 *k*-space samples (continuous black line, only the central part of the field-of-view is shown). For an ideal point source at the origin, the values at integer positions (red points) were obtained by a direct application of a discrete Fourier transformation (DFT), while half-integral positions were obtained for a point source shifted by one half unit and have been shifted back after DFT (cyan points). A second set of values at all positions was obtained using 2-fold zero filling of the original *k*-space and DFT (small black points).

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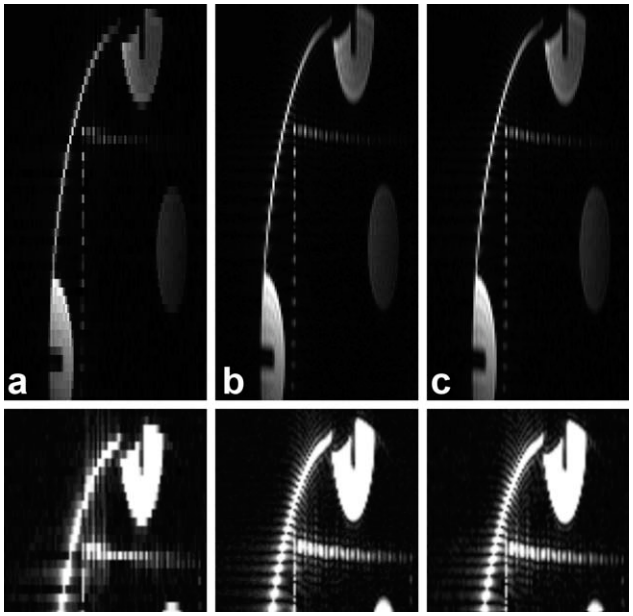

FIG. 2. MRI of a phantom at high signal-to-noise ratio using data from a single channel of an acquisition with a 32-element head coil at 3 T. **a**: Original image with pixels stretched by a factor of 4 in the (vertical) frequency-encoding direction; **b**: Composite image from the combination of four images with subpixel shifts; **c**: Original image with 4-fold zero filling. The bottom row presents views of a region of interest with altered windowing to emphasize blurring and ringing effects.

frequency noise does not interfere with the actual image content (which in this case resides in the low frequencies only) one may even get the illusion of a higher resolution of the composite image. The higher signal-to-noise ratio for the low-spatial frequencies becomes further evident when removing this high-frequency noise as shown in Fig. 3c, yielding an image that is almost identical to a 4-fold average of the zero-filled image in Fig. 3d.

In conclusion, the improvements in image quality demonstrated in Ref. 1 and reproduced here are not caused by additional information obtained from changes in the demodulation frequency but are due to (i) a better visualization of exactly the same information on a finer image grid and (ii) a higher signal-to-noise ratio for the low-spatial frequencies of a composite image obtained from multiple measurements.

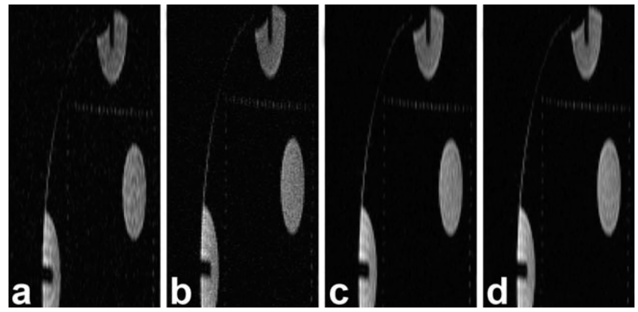

FIG. 3. MRI of a phantom at low signal-to-noise ratio using a different channel of the same acquisition as shown in Fig. 2. **a**: Original image with 4-fold zero filling (overall signal-to-noise ratio = 19.3); **b**: composite image from the combination of four images with subpixel shifts (overall signal-to-noise ratio = 19.4); **c**: composite image as in (b) but with high-frequency noise removed with a rectangular low-pass filter yielding signal-to-noise ratio = 38.0; **d**: original image with 4-fold zero filling as in (a) but with four averages yielding signal-to-noise ratio = 38.6.

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