PHYSICAL REVIEW D **84**, 101501(R) (2011)

Graviton mass bounds from space-based gravitational-wave observations of massive black hole populations

Emanuele Berti,[1,2,*] Jonathan Gair,[3,†] and Alberto Sesana[4,‡]

[1]Department of Physics and Astronomy, The University of Mississippi, University, Mississippi 38677, USA
[2]California Institute of Technology, Pasadena, California 91109, USA
[3]Institute of Astronomy, University of Cambridge, Cambridge, CB3 0HA, UK
[4]Max-Planck Institut für Gravitationasphysik (Albert-Einstein-Institut), Am Mühlenberg 1, D-14476, Potsdam, Germany

(Received 18 July 2011; published 4 November 2011)

We study the bounds that space-based gravitational-wave detectors could realistically place on the graviton Compton wavelength $\lambda_g = h/(m_g c)$ by observing multiple inspiralling black hole binaries. Observations of *individual* inspirals will yield mean bounds $\lambda_g \sim 3 \times 10^{15}$ km, but the *combined* bound from observing ~ 50 events in a two-year mission is about ten times better: $\lambda_g \simeq 3 \times 10^{16}$ km ($m_g \simeq 4 \times 10^{-26}$ eV). The bound improves faster than the square root of the number of observed events, because typically a few sources provide constraints as much as three times better than the mean. This result is only mildly dependent on details of black hole formation and detector characteristics. The bound achievable in practice should be an order of magnitude better than this figure, because our calculations ignore the merger/ringdown portion of the waveform.

DOI: 10.1103/PhysRevD.84.101501 PACS numbers: 04.30.Tv, 04.50.−h, 04.70.Bw, 04.80.Cc

The formulation of gravitational theories with nonzero mass for the graviton that are consistent with cosmological observations is an important open problem. Attempts to construct such theories led to well-known conceptual difficulties, such as the so-called van Dam-Veltman-Zakharov discontinuity [1–3], due to the fact that the helicity-0 component of the graviton does not decouple from matter when the putative mass of the graviton $m_g \to 0$. To circumvent pathologies related to the van Dam-Veltman-Zakharov discontinuity, various versions of Lorentz-violating massive graviton theories have been proposed in recent years [4]. Massive graviton signatures in the CMB and possible constraints on m_g from cosmological observations are an active area of research (see, e.g., [5]).

In this paper we are interested in hypothetical massive graviton theories as "straw men" for alternative theories of gravity in which the propagation speed of gravity differs from that of electromagnetic waves, leading to a modified dispersion relation. Therefore we will adopt a phenomenological point of view and ask the following question: *if* the graviton mass were nonzero, what upper bounds on m_g could we set by gravitational-wave (GW) observations of inspiralling compact binaries with future space-based detectors? Using $\lambda_g = h/(m_g c)$, upper limits on the graviton mass m_g (in eV) can be expressed as lower limits on its Compton wavelength λ_g (in km), since

$$\lambda_g[\text{km}] \times m_g[\text{eV}] = 1.24 \times 10^{-9}. \tag{1}$$

*berti@phy.olemiss.edu
†jgair@ast.cam.ca.uk
‡alberto@aei.mpg.de

Our analysis will show that, in hierarchical models of massive black hole (BH) formation [6,7], the bound on λ_g from GW observations of a population of merger events is about an order of magnitude better than the mean bound from GW observations of individual mergers.

To put our results in context, in Table I we present a *nonexhaustive* summary of current and proposed bounds on λ_g that do not rely solely on GW observations.

These bounds can be roughly divided into three classes.

(i) *Static bounds.* If the graviton has nonzero mass, it is reasonable to expect that the Newtonian gravitational potential will be modified to the Yukawa form in the nonradiative near zone of any body of mass M: $V(r) = (GM/r)\exp(-r/\lambda_g)$. Talmadge et. al. [9] investigated deviations from Kepler's third law for the inner planets of the Solar System. By translating the uncertainties in these

TABLE I. Graviton mass bounds. For proposed methods we quote the *best* achievable bounds. In the notation of the main text, a dagger (†) denotes static bounds; a number sign (#) dynamical bounds; an asterisk (*) bounds that could be achieved by comparing GW and electromagnetic observations.

Current bounds	λ_g[km]	m_g[eV]	Reference
Binary pulsars#	1.6×10^{10}	7.6×10^{-20}	[8]
Solar system†	2.8×10^{12}	4.4×10^{-22}	[9,10]
Clusters†	$6.2 \times 10^{19} h_0$	$2.0 \times 10^{-29} h_0^{-1}$	[11]
Weak lensing†	1.8×10^{22}	6.9×10^{-32}	[12]
Proposed bounds	λ_g[km]	m_g[eV]	Reference
Pulsar timing#	4.1×10^{13}	3.0×10^{-23}	[13]
White dwarfs*	1.4×10^{14}	8.8×10^{-24}	[14]
EM counterparts*	10^{15}–10^{16}	10^{-24}–10^{-25}	[15]

measurements in terms of accelerations of test bodies resulting from a Yukawa potential, Will [10] found that the strongest bound on λ_g comes from the very nearly Keplerian orbit of Mars: $\lambda_g > 2.8 \times 10^{12}$ km.

Some bounds on m_g quoted by the Particle Data Group (PDG) [16] similarly assume Yukawa corrections to the Newtonian potential in the weak-field limit. The strongest bounds are naturally obtained from observations on the largest (cosmological) scales. As early as 1974, Goldhaber and Nieto assumed that the graviton mass would produce a Yukawa-type correction to the standard Newtonian potential and argued that the evidence for bound clusters and tidal interactions between galaxies should imply a range for gravity at least as large as a few Mpc, so that $\lambda_g > 6.2 \times 10^{19} h_0$ km [11]. However, the need to include dark matter to explain galactic rotation curves indicates that there are complications in the nature of gravity on those scales that are not necessarily well characterized by a Yukawa-type potential. An even stronger bound $\lambda_g > 1.8 \times 10^{22}$ km comes from weak gravitational lensing, because no distortions are observed in the measured values of the variance of the power spectrum [12]. Because of uncertainties in the amount and dynamics of dark matter in the Universe (and in the absence of a consistent massive graviton theory compatible with cosmology) these bounds should be regarded as model-dependent and viewed with some caution.

(ii) *Dynamical bounds.* All bounds listed in part i) are static, in the sense that they do not probe features related to the *propagation* of the gravitational interaction when $m_g \neq 0$. The best available dynamical bounds come from the (indirect) observations of GWs from binary pulsars, that are in excellent agreement with general relativity [17]. Finn and Sutton [8] observed that the consistency of the orbital decay rates of binary pulsars PSR B1534 + 12 and B1913 + 16 with general relativistic predictions yields a "dynamical" bound $\lambda_g > 1.6 \times 10^{10}$ km. A recent idea is to set bounds on m_g using pulsar timing arrays [13]. The best bound achievable in the near future would be $\lambda_g \simeq 4.1 \times 10^{13}$ km, but this figure could worsen by an order of magnitude depending on the number of pulsars used for the test, timing accuracy and observation time.

(iii) *Comparisons of gravitational and electromagnetic observations.* If $m_g \neq 0$, the modified dispersion relation for GWs would result in different arrival times of GWs and electromagnetic waves emitted by the same astrophysical source. Cutler et. al. [14] proposed to correlate electromagnetic observations and future space-based GW observations of white dwarf binaries. The best bound that could be obtained in this way is $\lambda_g \sim 1.4 \times 10^{14}$ km, but realistic bounds would probably be worse by about an order of magnitude.

Kocsis et. al. proposed to correlate LISA observations of GWs from massive BH binary mergers with their possible electromagnetic counterparts [15]. An intrinsic limitation of this method is related to timing uncertainties in the GW burst, which are comparable to the dynamical time scale for the coalescing binary during merger, and can be estimated as the inverse of the orbital frequency at the innermost stable circular orbit. For binaries in the mass range $M = 10^5$–10^7 at redshift $z = 1$, this uncertainty leads to a *best* bound $\lambda_g \sim 10^{15}$–10^{16} km, worse than bounds coming from GW observations *alone* (as we will see below). Systematic, model-dependent uncertainties in the electromagnetic counterpart will further weaken graviton mass bounds achievable in this way.

I. BOUNDS ON λ_g FROM INDIVIDUAL GRAVITATIONAL-WAVE OBSERVATIONS

Tight dynamical bounds on the graviton mass can be obtained using GW observations in space. This is due to two reasons: (1) the larger mass of observable BH binaries, as compared to ground-based GW observations; (2) the statistical increase in the bound that would result from observing *populations* of individually resolved binaries. To illustrate point (1), in this section we review existing work on massive graviton bounds from GW observations of individual binaries with LISA. In Sec. II we will present the first attempt to quantify the statistical improvement of bounds on λ_g that could be achievable in reality by observing several events in the context of hierarchical models of massive BH formation.

Will [10] first pointed out that interesting bounds on λ_g could come from a careful monitoring of the phase of GWs emitted by binaries of compact objects, such as BHs and/or neutron stars. In hypothetical massive graviton theories, the GW damping formulas and dispersion relation would be modified. As a consequence, the GW phasing $\Psi_{\rm GR}(f)$ would acquire an additional term

$$\Psi_{\rm MG}(f) = \Psi_{\rm GR}(f) - \beta_g (\pi M f)^{-1}, \tag{2}$$

where $\beta_g \equiv \pi^2 D M / [(1+z)\lambda_g^2]$ and D is a distance parameter, similar to (but not quite the same as) the luminosity distance D_L (here and below we assume a ΛCDM model with $H_0 = 70\,{\rm km}\cdot{\rm s}^{-1}\,{\rm Mpc}^{-1}$, $\Omega_M = 0.3$, $\Omega_M = 0.7$).

Using the restricted post-Newtonian approximation and neglecting binary spins, Will estimated that stellar-mass binary inspirals to be observed with LIGO would yield a bound $\lambda_g \simeq 5 \times 10^{12}$ km, only slightly better than Solar System bounds. For an "optimal" LISA system, i.e., an equal-mass BH binary of total mass $M = 2 \times 10^6 M_\odot$ at $D_L = 3$ Gpc ($z \simeq 0.5$), the bound would be 4 orders of magnitude better: $\lambda_g \simeq 5 \times 10^{16}$ km $\simeq 1.6$ kpc.

These initial estimates were refined in various papers. Will and Yunes [18] showed that bounds from binary BH inspirals at $D_L = 3$ Gpc would range between 10^{15} km and 5×10^{16} km for M in the range 10^4–$10^7 M_\odot$. They found that the bound on λ_g is proportional to $\sqrt{L}$ (where L is the LISA armlength) and to

(LISA Aaccelerationnoise)$^{-1/2}$, and that it scales in the following way:

$$\lambda_g \propto \left(\frac{D}{(1+z)D_L}\right)^{1/2} \frac{\mathcal{M}^{11/12}}{S_0^{1/4} f_0^{1/3}}, \tag{3}$$

where $\mathcal{M} = (m_1 m_2)^{3/5} M^{-1/5}$ is the "chirp mass", S_0 (in Hz^{-1}) sets the scale of the noise power spectral density (PSD) of the detector, and f_0 is a characteristic "knee" frequency where the PSD has a minimum. Distance dependence is weak because the effect of the massive graviton and measurement errors both grow with distance.

Berti et. al. [19] performed a more detailed Monte Carlo calculation of LISA's parameter estimation capabilities. They still considered restricted post-Newtonian inspiral waveforms, but they used Cutler's model [20] to take into account the motion of the detector (all previous analyses assumed a *static* LISA constellation). In this paper we will do the same. In Cutler's model, LISA can be seen as one (two) independent "LIGO-like" Michelson interferometers depending on whether 4 (5/6, respectively) laser links are available among the three satellites forming the constellation. For the "optimal binary" (a nonspinning, equal-mass binary with $M = 2 \times 10^6 M_\odot$ at $D_L = 3$ Gpc), Monte Carlo averages over the binary position and orientation yield a bound $\lambda_g = 5.0 \times 10^{16} (3.7 \times 10^{16})$ km when using two (one) Michelsons, in excellent agreement with Will's original results. For masses in the range $M = 2 \times 10^4$–$2 \times 10^7 M_\odot$, the mean bound using two Michelsons at $D_L = 3$ Gpc is in the range $\sim 5 \times 10^{15}$–7×10^{16} km.

Later work used mostly Cutler's model and investigated the effect of more complex inspiral waveforms. Arun and Will [21] studied the effect of higher harmonics and PN amplitude corrections. They found that bounds on λ_g improve by a factor of a few for systems of total mass $\gtrsim 10^6 M_\odot$, and that this improvement is more significant for binaries with large mass ratios. If one ignores spin precession, adding spins to the waveforms generally introduces degeneracies between the binary parameters, degrading parameter estimation accuracy [19]. Stavridis and Will [22] showed that modulations induced by spin precession can break these degeneracies. For the "optimal binary," for example, the average bound including spin precession is $\lambda_g \sim 5 \times 10^{16}$ km, basically the same as in the case of nonspinning binaries. Yagi and Tanaka [23] included *both* spin precession and eccentricity. Performing simulations for a $(10^7 + 10^6) M_\odot$ binary at 3 Gpc, they found an average bound $\lambda_g = 3.1 \times 10^{16}$ km. This is again consistent with [19] to within a factor of 2.

More recently, Keppel and Ajith [24] revisited this problem using phenomenological waveforms for nonspinning BH binaries that include the merger/ringdown phase. Their analysis of LISA bounds is similar to the original work by Will [10] in that it ignores the motion of the detector, using an effective non-sky-averaged noise PSD. This simplification should not affect their main conclusions: (i) for the "optimal binary," the bound on λ_g *improves by about an order of magnitude*, up to $\sim 4 \times 10^{17}$ km; (ii) a comparable bound is obtained also for binaries of larger mass, up to $M \sim 10^8 M_\odot$. At $D_L = 3$ Gpc, the *best* bound using the full merger is $\lambda_g = 5.9 \times 10^{17}$ km for $M = 4.8 \times 10^7 M_\odot$, to be compared with a best bound $\lambda_g = 6.3 \times 10^{16}$ km for $M = 1.9 \times 10^6 M_\odot$ if we consider only inspiral waves. A similar order-of-magnitude improvement is expected for Earth-based detectors.

Del Pozzo et. al. [25] recently revisited graviton mass bounds using Bayesian inference (see also [26–29] for similar work including other possible alternatives to general relativity). They focused on ground-based observations of sources within 150 Mpc and found results consistent with Will's original analysis [10]: for example, their Fig. 6 shows that binary observations with Advanced LIGO would yield typical bounds $\sim$few $\times 10^{12}$ km.

In this study we consider quasicircular, nonspinning, restricted post-Newtonian inspiral waveforms. We consider an observation time of two years. We take into account weak lensing errors on the redshift following [30], and we compute the individual graviton mass bounds by generalizing the Fisher matrix formalism described in [31], which takes into account correlations with the other waveform parameters. The results of Refs. [22,24] suggest that our bounds will be (i) very close to the bounds we would obtain for spinning, precessing inspirals, and (ii) about an order of magnitude worse than the bounds achievable if we used "full" merger waveforms.

II. BOUNDS ON λ_g FROM GRAVITATIONAL-WAVE OBSERVATIONS OF MASSIVE BLACK HOLE POPULATIONS

All studies of massive graviton bounds so far analyzed isolated systems, such as Will's "optimal binary" of mass $M = 2 \times 10^6 M_\odot$ at $z \sim 0.5$. Unfortunately, hierarchical models of massive BH formation and evolution predict that *typical* systems observable by space-based interferometers would have masses smaller than this, and be located at redshift $z \sim 4$ or higher (see, e.g., [6,7] and Fig. 2 of [31]). The main uncertainties in these models concern the seeding mechanism and the role of accretion in BH growth [31,32]. In this work we ignore spins in the gravitational waveforms. Accretion mostly influences the spin magnitude [33], so we will focus on the role of seeding. Following work by the LISA Parameter Estimation Task Force [34], we will consider two "extreme" scenarios: small seeds, efficient accretion (SE) and large seeds, efficient accretion (LE). These models being extreme, we expect that our results should bracket the constraints that would be obtained using other population models.

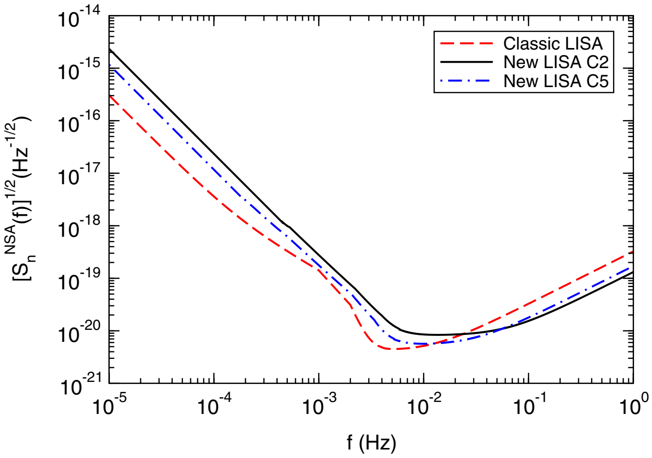

FIG. 1 (color online). Non-sky-averaged noise power spectral density $S^{NSA}(f)$ for New LISA C2 (black, solid line), New LISA C5 (blue, dash-dotted line) and Classic LISA (red, dashed line).

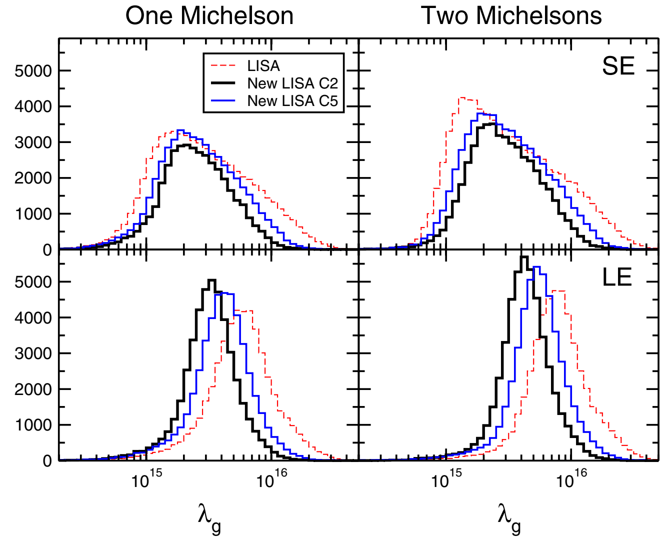

FIG. 2 (color online). Distribution of bounds on the graviton Compton wavelength for individual observations. We consider 1000 realizations of the massive BH population and three different detector designs: New LISA C2 (black, thick line), New LISA C5 (blue, medium line) and Classic LISA (red, dashed line).

Here we consider the "Classic LISA" design along with two different designs for the proposed ESA-led space-based detector, that we will call by the working name of "New LISA." "Classic LISA" consists of three spacecraft forming an equilateral triangle with laser power $P = 2$ W, telescope diameter $d = 0.4$ m and armlength $L = 5 \times 10^9$ m, trailing 20° behind the Earth at an inclination of 60° with respect to the ecliptic. The authors are members of a Science Performance Task Force that is considering several different LISA-like configurations with different characteristics and sensitivities. The configurations that we call New LISA C2 (C5, respectively) consist of three spacecraft forming an equilateral triangle with armlength $L = 10^9$ m ($L = 2 \times 10^9$ m), laser power $P = 2$ W and telescope diameter $d = 0.4$ m ($d = 0.28$ m). "New LISA" should be deployed 10° behind the Earth, gradually drifting to ~25° behind the Earth in five years.

The *non-sky-averaged* noise power spectral densities for all three configurations are shown in Fig. 1; they are related to the sky-averaged power spectral density by $S^{NSA}(f) = \frac{3}{20} S^{SA}(f)$ (see [19] for a discussion of sky averaging). These curves include galactic confusion noise, estimated using methods similar to [34] (which in turn was based on [35]). In our study we consider the noise power spectral density to be infinite below a cutoff frequency $f_{cutoff} = 10^{-5}$ Hz. As shown in [19], bounds on λ_g drop significantly at masses $\gtrsim 2 \times 10^6 M_\odot$ if the noise cannot be trusted below 10^{-4} Hz. This assumption has a mild effect on our results, because most binaries in our models have mass lower than this.

TABLE II. Top: mean (in parentheses: median) bound on λ_g for different BH formation models, using one or two detectors, in units of 10^{15} km. Bottom: mean (in parentheses: median) of the *combined* bound on λ_g over 1000 realizations of the massive BH population, in units of 10^{16} km.

Mean (median) of individual events (10^{15} km)				
Detector	SE, 1 Mich.	LE, 1 Mich.	SE, 2 Mich.	LE, 2 Mich.
Classic LISA	4.26(2.60)	6.83(5.77)	4.87(2.72)	9.13(7.72)
New LISA C2	3.03(2.44)	3.62(3.27)	3.60(2.80)	4.76(4.29)
New LISA C5	3.41(2.53)	4.63(4.13)	4.02(2.84)	6.15(5.48)
Mean (median) of combined bound (10^{16} km)				
Detector	SE, 1 Mich.	LE, 1 Mich.	SE, 2 Mich.	LE, 2 Mich.
Classic LISA	4.93(4.87)	5.67(5.59)	6.51(6.45)	7.50(7.37)
New LISA C2	2.29(2.25)	2.73(2.71)	3.09(3.04)	3.66(3.64)
New LISA C5	3.10(3.07)	3.64(3.62)	4.16(4.12)	4.85(4.82)

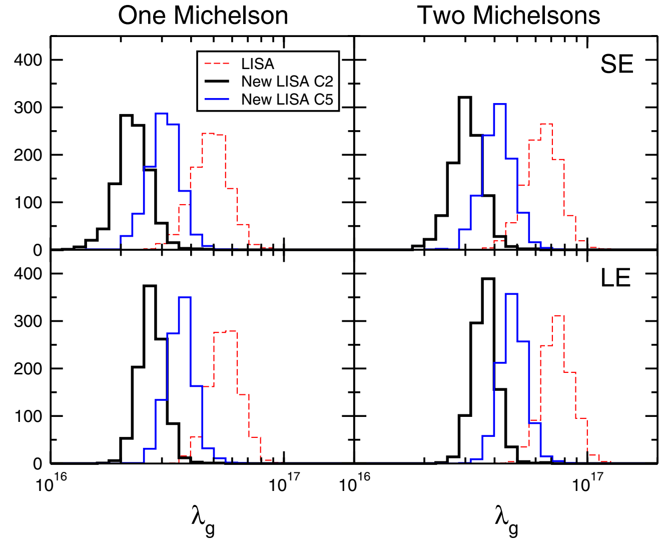

FIG. 3 (color online). Distribution of *combined* bounds over 1000 realizations of the MBH population. Line styles are as in Fig. 2.

For each model we consider 1000 realizations of the Universe. Each of these realizations typically produces ~30–50 events observable with signal-to-noise ratio larger than 8. The distribution of bounds on λ_g resulting from individual observations is shown in Fig. 2.

The top half of Table II shows that the mean bound over individual observations is $\sim 3 \times 10^{15}$ km for New LISA C2, and only slightly better for the other designs. This conclusion is quite robust, in the sense that numbers vary only mildly for different seeding mechanisms and different detector characteristics.

In most alternative theories, deviations from general relativity can be parametrized by one or more global parameters (such as λ_g) which are the same for every system. It is natural to expect that one can obtain better constraints on these parameters, as well as other universal constants, by combining multiple observations (see, e.g., [25,28,29,36–39]). Assuming that estimates for individual sources are independent and Gaussian posteriors for each source, consistent with the Fisher matrix approximation, the width σ^2 of the combined posterior on $1/\lambda_g$ is given by $\sigma^{-2} = \sum_i \sigma_i^{-2}$, where σ_i^2 is the width of the posterior for the ith source. The bound on λ_g can thus be obtained by adding the individual bounds in quadrature. The results are shown in Fig. 3 and in the bottom half of Table II. The combined bound obtained from the whole BH population is about an order of magnitude better than the average bound obtained from typical observations. A rough estimate would suggest that N identical sources should provide a bound $\sim\sqrt{N}$ times better than the bound from a single source. Our combined bound is typically about 3 times better than the bound from the best event, but the median bound is typically an order of magnitude worse than the best, and hence ~30 times worse than the combined bound. A typical realization has ~50 events, so our analysis shows that we can beat the $\sqrt{N}$ extrapolation from the median bound by a considerable margin. If 5/6 links (two Michelsons) are available instead of 4 links (one Michelson), the bound typically improves by a factor $\sim\sqrt{2}$.

III. CONCLUSIONS AND OUTLOOK

We assessed the capability of future space-based interferometers, such as "Classic LISA" and the proposed ESA-led "New LISA," to constrain the mass of the graviton by combining observations of a *population* of massive BH binaries. We found that: (1) by using a population of merging BH binaries we can obtain a bound on λ_g that is ~10 times better than the mean bound on individual observations; (2) quite independently of the detector's design and of details of the massive BH formation models, the combined bound from inspiral observations will be $\lambda_g \simeq 3 \times 10^{16}$ km. This figure is likely to underestimate the bound achievable in practice by about an order of magnitude, as we have ignored the merger and ringdown portion of the waveform [24], but further work is required to confirm this expectation.

In conclusion, space-based observations of a population of merging BHs should set bounds in the range $\lambda_g \in [2 \times 10^{16}, 10^{18}$ km$]$ on the graviton Compton wavelength, depending on details of the detector and on the specific waveform model used to set the bounds. This is comparable to the (static and model-dependent) bounds from cosmological-scale observations quoted in Table I but it is very different in nature, because gravitational radiation tests the dynamical regime of Einstein's general relativity.

ACKNOWLEDGMENTS

E. B. is supported by NSF Grant No. PHY-0900735 and by NSF CAREER Grant No. PHY-1055103. J. G.'s work is supported by the Royal Society. The authors wish to thank Martin Elvis and the "New LISA" Science Performance Task Force for discussions, Marta Volonteri for sharing her BH formation models, and the Aspen Centre for Physics (where this work was started) for providing a very stimulating environment.

[1] H. van Dam and M. J. G. Veltman, Nucl. Phys. **B22**, 397 (1970).
[2] V. I. Zakharov, JETP Lett. **12**, 312 (1970).
[3] T. Damour, I. I. Kogan, and A. Papazoglou, Phys. Rev. D **67**, 064009 (2003).
[4] A. S. Goldhaber and M. M. Nieto, Rev. Mod. Phys. **82**, 939 (2010).
[5] M. V. Bebronne and P. G. Tinyakov, Phys. Rev. D **76**, 084011 (2007); D. Bessada and O. D. Miranda, Classical Quantum Gravity **26**, 045005 (2009); S. Dubovsky, R. Flauger, A. Starobinsky, and I. Tkachev, Phys. Rev. D **81**, 023523 (2010); S. Basilakos, M. Plionis, M. E. S. Alves, and J. A. S. Lima, Phys. Rev. D **83**, 103506 (2011).
[6] M. Volonteri, F. Haardt, and P. Madau, Astrophys. J. **582**, 559 (2003).
[7] M. C. Begelman, M. Volonteri, and M. J. Rees, Mon. Not. R. Astron. Soc. **370**, 289 (2006).
[8] L. S. Finn and P. J. Sutton, Phys. Rev. D **65**, 044022 (2002).
[9] C. Talmadge, J. P. Berthias, R. W. Hellings, and E. M. Standish, Phys. Rev. Lett. **61**, 1159 (1988).
[10] C. M. Will, Phys. Rev. D **57**, 2061 (1998).
[11] A. S. Goldhaber and M. M. Nieto, Phys. Rev. D **9**, 1119 (1974).
[12] S. R. Choudhury, G. C. Joshi, S. Mahajan, and B. H. J. McKellar, Astropart. Phys. **21**, 559 (2004).
[13] K. Lee, F. A. Jenet, R. H. Price, N. Wex, and M. Kramer, Astrophys. J. **722**, 1589 (2010).
[14] C. Cutler, W. A. Hiscock, and S. L. Larson, Phys. Rev. D **67**, 024015 (2003).
[15] B. Kocsis, Z. Haiman, and K. Menou, Astrophys. J. **684**, 870 (2008).
[16] K. Nakamura *et al.* (Particle Data Group Collaboration), J. Phys. G **37**, 075021 (2010).
[17] T. Damour and J. H. Taylor, Astrophys. J. **366**, 501 (1991).
[18] C. M. Will and N. Yunes, Classical Quantum Gravity **21**, 4367 (2004).
[19] E. Berti, A. Buonanno, and C. M. Will, Phys. Rev. D **71**, 084025 (2005); Classical Quantum Gravity **22**, S943 (2005).
[20] C. Cutler, Phys. Rev. D **57**, 7089 (1998).
[21] K. G. Arun and C. M. Will, Classical Quantum Gravity **26**, 155002 (2009).
[22] A. Stavridis and C. M. Will, Phys. Rev. D **80**, 044002 (2009).
[23] K. Yagi and T. Tanaka, Phys. Rev. D **81**, 064008 (2010).
[24] D. Keppel and P. Ajith, Phys. Rev. D **82**, 122001 (2010).
[25] W. Del Pozzo, J. Veitch, and A. Vecchio, Phys. Rev. D **83**, 082002 (2011).
[26] N. Yunes and F. Pretorius, Phys. Rev. D **80**, 122003 (2009).
[27] N. Cornish, L. Sampson, N. Yunes, and F. Pretorius, Phys. Rev. D **84**, 062003 (2011).
[28] K. Yagi and T. Tanaka, Prog. Theor. Phys. **123**, 1069 (2010).
[29] K. Yagi, N. Tanahashi, and T. Tanaka, Phys. Rev. D **83**, 084036 (2011).
[30] Y. Wang, D. E. Holz, and D. Munshi, Astrophys. J. **572**, L15 (2002).
[31] A. Sesana, J. Gair, E. Berti, and M. Volonteri, Phys. Rev. D **83**, 044036 (2011).
[32] J. R. Gair, A. Sesana, E. Berti, and M. Volonteri, Classical Quantum Gravity **28**, 094018 (2011).
[33] E. Berti and M. Volonteri, Astrophys. J. **684**, 822 (2008).
[34] K. G. Arun, S. Babak, E. Berti, N. Cornish, C. Cutler, J. Gair, S. A. Hughes, B. R. Iyer *et al.*, Classical Quantum Gravity **26**, 094027 (2009).
[35] S. E. Timpano, L. J. Rubbo, and N. J. Cornish, Phys. Rev. D **73**, 122001 (2006).
[36] B. F. Schutz, Nature (London) **323**, 310 (1986).
[37] I. Mandel, Phys. Rev. D **81**, 084029 (2010).
[38] C. L. MacLeod and C. J. Hogan, Phys. Rev. D **77**, 043512 (2008).
[39] W. Zhao, C. Van Den Broeck, D. Baskaran, and T. G. F. Li, Phys. Rev. D **83**, 023005 (2011).