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A scientific case study of an advanced LISA mission

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Abstract

A brief status report of an ongoing scientific case study of the Advanced Laser Interferometer Antenna (ALIA) mission is presented. Key technology requirements and primary science objectives of the mission are covered in the study. Possible descope options for the mission and the corresponding compromise in science are also considered and compared. Our preliminary study indicates that ALIA holds promise in mapping out the mass and spin distribution of intermediate mass black holes possibly present in dense star clusters at low redshift as well as in shedding important light on the structure formation in the early Universe.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The LISA mission has a goal of providing detailed gravitational wave observations over the frequency range from 3×10^{-5} to 1 Hz. However, a number of possible future missions have been proposed that would have considerably higher sensitivity, with emphasis on the frequency range from 0.1 to 10 Hz [1–8]. Among these, the ALIA mission proposed in [1, 2] is conceivably in many ways the simplest adaptation of the LISA mission to a frequency band one order of magnitude higher than that of LISA. Compared with other such proposals, ALIA seems to be more realistic from a technological perspective and offers hope to be realized in the next few decades. Though modest in terms of improvement of the strain sensitivity (see figure 1 below) in comparison with the proposed BBO and DECIGO missions, the ALIA mission promises to provide significant insights into the physics of intermediate mass black holes (IMBHs) possibly harbored in dense star clusters [9–13] or formed from the remnants of metal-free Pop III stars with masses as light as $10^2 M_\odot$, as predicted by certain hierarchical cosmological scenarios [14, 15].

A feasibility study has been ongoing for ALIA for the past two years by an informal gravitational wave study group in China, with technical assistance generously offered by the Albert Einstein Institute, Max Planck Society, Hannover. The objective of the study is to have a more in-depth understanding of the mission design, on both the technological as well as scientific fronts. The aim of this paper is to give mainly a report on the scientific case study of the proposed mission.

The outline of the paper may be described as follows. In section 2 the mission design including a few descope options will be briefly sketched. Section 3 is devoted to a certain preliminary understanding of the science ALIA is likely to offer. The paper is concluded by some brief remarks in section 4.

2. Mission design

Table 1 gives the baseline design parameters and the total noise budget of the ALIA mission as suggested by Bender and Begelman [2]. With the prescribed noise budget, a slight variation of the parameters may also be considered.

Though modest in terms of the improvement in strain sensitivity from that of LISA, many questions remain to be answered concerning the viability of various key technologies of the ALIA mission. One obvious major challenge is the sub-picometer precision in laser interferometry. As we shall see in section 3, even for a descoped mission to be discussed in a moment, sub-picometer precision in laser interferometry is forced on us if we want to probe intermediate mass binaries not easily accessible to LISA. The development of technologies beyond LISA is needed and we will venture into uncharted territory. Utmost care is needed

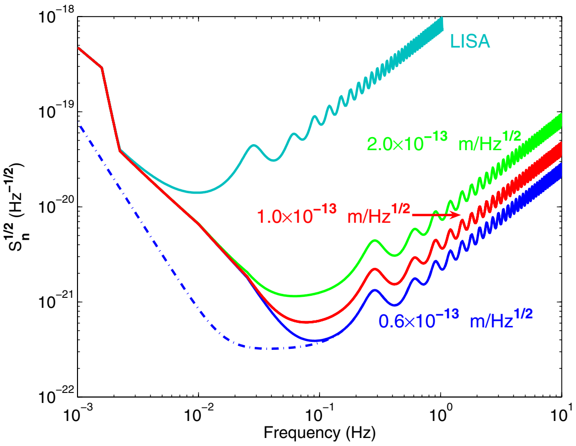


Figure 1. Sensitivity of a few representative mission options whose sensitivity curves are labeled by their corresponding total position noise budget levels. Also shown is the standard single Michelson sensitivity of LISA for comparison [20]. The optimistic confusion noise level generated by both galactic and extra-galactic cosmological compact binaries (mainly WD-WD) given in [17] is adopted in drawing the solid lines, and the dash-dotted curve stands for the pure instrumental noise for the most ambitious design.

Table 1. Baseline design parameters.

Armlength (L)	Telescope diameter (D)	Laser power (P_t)	Shot noise ($h_{\rm shot}$)	1-way position noise ($h_{\rm posi}$)	Acceleration noise ($h_{\rm acc}$)
5×10^8 m	1.0 m	30 W	$3\times10^{-14}\,\frac{\rm m}{\sqrt{\rm Hz}}$	$6\times10^{-14}\,\frac{\rm m}{\sqrt{\rm Hz}}$	$3\times10^{-16}\,\frac{\rm m\,s^{-2}}{\sqrt{\rm Hz}}$

Table 2. Descoped position noise budgets.

$h_{\rm posi}$ (m/$\sqrt{\rm Hz}$)	0.6×10^{-13}	1.0×10^{-13}		2.0×10^{-13}		4.0×10^{-13}	1.0×10^{-12}	
D (m)	1.0	1.0	0.8	1.0	0.6	0.6	0.5	0.4
P_t (W)	30	10	25	3	20	5	2	5

when confronted by the uncertainties ahead. See also [16] for a further discussion on noise sources for ALIA.

In view of the potential technological challenges posed by ALIA, it is natural to ask whether it is feasible to descope the mission in order to minimize the possible risks involved in the mission, shorten the R&D period and yet at the same time make as little compromise as possible in the science goals of the mission. To this end, the following descope options displayed in table 2 have been considered.

For the acceleration noise, in the descoped missions we still adopt the same noise budget as that originally proposed for ALIA (see table 1). According to our present understanding of the confusion generated by cosmological compact binaries [17, 18], part of the improvement may be lost because of the noise generated by the unresolvable foreground of compact binaries (both galactic and extra-galactic, see figure 1). Thus, it might make little difference if we adopted a more conservative LISA level sensitivity of $3\times10^{-15}\,{\rm m\,s^{-2}}/\sqrt{\rm Hz}$ instead, with the sensitivity floor shifted to the 0.01 Hz range. However, in view of the highly uncertain estimate of the confusion level generated by the unresolvable foreground of compact binaries, the technological readiness of a more sensitive sensor might enable us to plan a mission with better science when we have a more thorough understanding of the confusion noise in the future [18]. Further, it is conceivable that further sensor development would be useful for future missions in gravitational wave astronomy.

Though an increase in frequency in the most sensitive detection window means that most acceleration noises known in LISA will be less severe (see however [19]), various aspects of the design of the gravitational reference sensor will have to be looked at and studied carefully. The outcome of the LISA Pathfinder will help immensely to clarify the tasks to be tackled.

3. IMBHs as the primary science driver

Compared with LISA, a distinctive feature of ALIA is its capability to detect 10–$10^2 M_\odot$ black holes coalescing with IMBHs, even at large redshifts. Such events possibly occur in low redshift dense star clusters and in the structure formation process in the early Universe. Given the mission design parameters delineated in the previous section and the sensitivity (see figure 1), we will try next to understand better the possible science ALIA might offer.

3.1. Science reach

To understand the designed sensitivity in relation to the target science, we calculate the redshift ALIA is capable of reaching for intermediate mass binary black holes with various mass ratios. In the calculations that follow, a one-way position noise level of $1.0 \times 10^{-13}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ will be used and the detector sensitivity will be cut off at $f_{\mathrm{low}} = 10^{-3}$ Hz. We assume a threshold SNR of 7 in the sense of single Michelson (which corresponds to a double-channel SNR of 10) and a 1 year observation before merger. Since for most of the sources we are considering the redshifts cannot be neglected, whenever we talk about 'a final-year observation' or 'a 1 year observation prior to merger' we are referring to the local time at the detector, which corresponds to a much shorter merger duration in the rest frame at the source. The confusion level in figure 1 will be adopted in all calculations that follow. Both the PN waveform and phenomenological hybrid waveform [21, 22] will be used, with the latter further incorporating the phases of merger and ringdown. As we shall see in a moment, by comparing the SNRs calculated using two different waveforms, we will be able to assess how the three stages of an intermediate mass binary coalescence contribute to the overall SNR.

For an intermediate mass ratio inspiral (IMRI) whose mass ratio is above a certain level, it is generally expected that the waveforms used in the calculations are not accurate enough. Still we hope that calculations based on our present state of the art in waveform construction would enable us to gain at least a qualitative understanding of IMRIs as possible gravitational wave sources. Further improvement will then be made when more accurate waveforms are available from numerical relativity.

The results of the calculations are shown in figure 2 below.

The two curves in the left panel of figure 2 describe the redshifts reached by ALIA when a roughly $10M_\odot$ stellar mass black hole spirals into an IMBH using both PN and hybrid waveforms. As we may see from the figure, they become distinguishable only in the high-mass end, when the contributions in SNR by merger and ringdown begin to be non-negligible. Merger and ringdown hardly contribute to the SNR for intermediate mass binary black hole coalescence so long as the redshifted total mass is not too large. This is a distinct feature of ALIA compared with DECIGO and ET in the detection of GWs in the intermediate mass sources regime and would hopefully result in less loss in SNR due to inaccurate waveforms

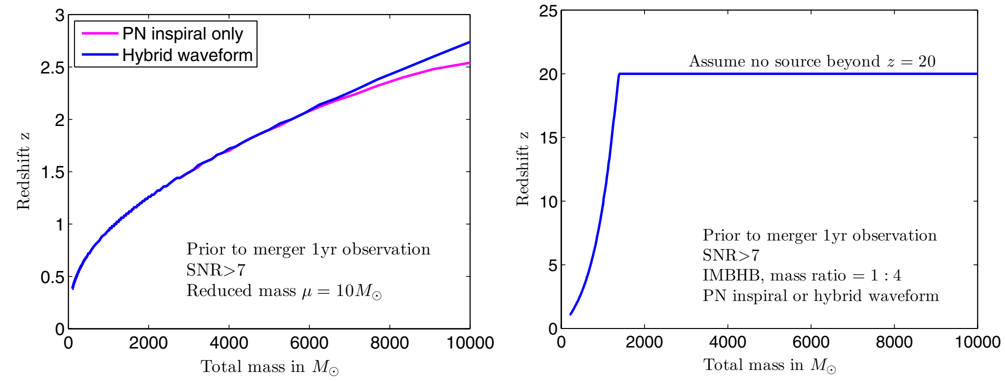


Figure 2. All-angle averaged detection range under a single Michelson threshold SNR of 7, with a prior to merger 1 year observation. Left: stellar mass black hole spiralling into IMBH with a reduced mass of $10M_\odot$. Right: 1:4 mass ratio IMBH–IMBH binary.

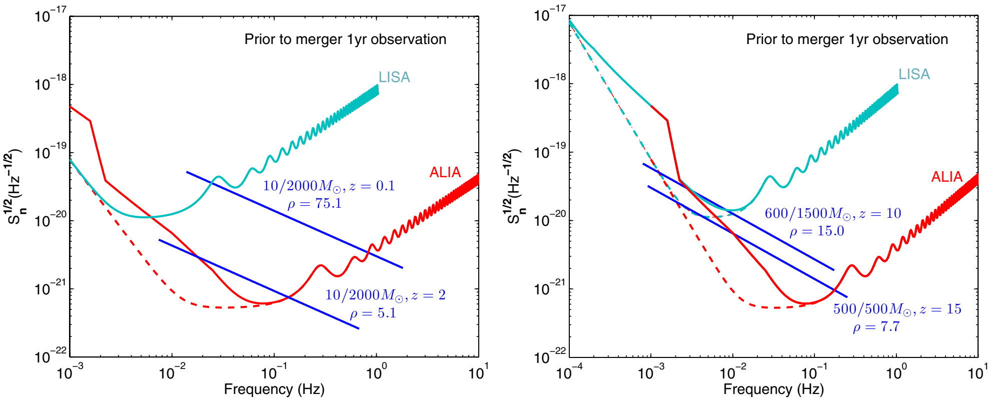


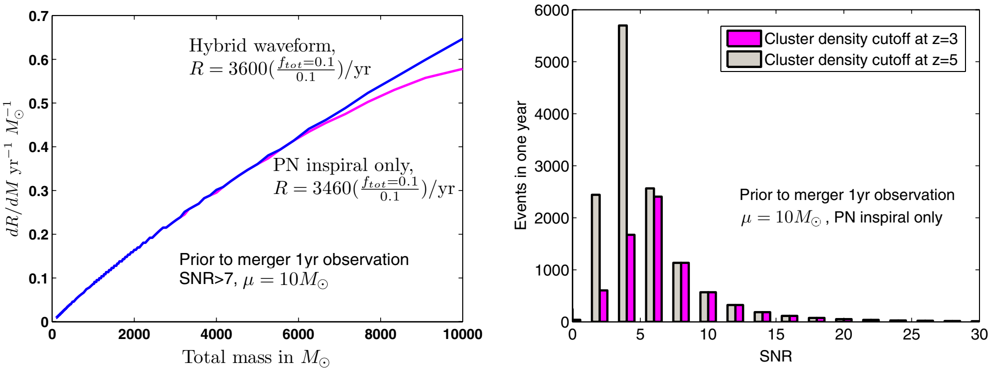
Figure 3. Typical intermediate-mass binary coalescences in a 1 year observation right before merger. Left: IMRIs in low redshift. Right: almost equal mass IMBHBs at high redshift. SNRs given are calculated using hybrid waveforms based on the $1.0 \times 10^{-13}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ position noise budget, single Michelson. The virtual duration of a signal in the source's rest frame depends on its redshift.

when IMRI is concerned. The right panel of the figure illustrates the large redshift ALIA is capable of reaching for signals of coalescing IMBH binaries (IMBHBs), which goes up rapidly with the total mass. For a total mass in excess of 1000 $M_\odot$, redshift beyond $z = 20$ is also within the detection range for ALIA.

Figure 3 illustrates the results of the SNR calculations for signals from typical intermediate mass binaries in a final-year observation, low redshift IMRIs in the left panel and high redshift IMBHBs in the right. As may be seen from the plot here, the most sensitive detection band of ALIA matches the frequency of GWs generated by the final inspirals of intermediate mass binaries.

3.2. IMBHs harbored in dense star clusters

Though not yet conclusive, mounting evidence from x-ray observations and numerical simulations suggest the existence of IMBHs in globular clusters or dense young clusters,



(*a*) (*b*)

Figure 4. Detection rates and SNR distributions of cluster-harbored IMRIs. (a) Event rates of IMRIs in clusters assuming a reduced mass of 10 $M_\odot$, prior to merger 1 year observation using PN and hybrid waveforms. (b) SNR distributions of events assuming redshift cutoffs for n_c, two cases $z_c = 3$, $z_c = 5$ are considered.

and in some cases, even multi-IMBHs might be expected to form in a single dense cluster [9–13, 23]. While tracking these sources proves difficult by electromagnetic means, IMRIs are expected to form in such a crowded stellar environment and so do IMBHBs when they form in a single cluster. It seems that observations through gravitational waves provides a promising avenue to understand the astrophysics of the IMBH and the underlying evolution and growth dynamics of the dense star clusters.

With a given luminosity distance $D_L(M, \mu)$ accessible by a gravitational wave detector (see figure 2), the event rate may be calculated using the following formula [11, 24]:

$$R = \frac{4\pi}{3}\int_{M_{\min}}^{M_{\max}} [D_L(M, \mu)]^3\ \nu(M, \mu, z)\, n_c\, f(M)\, \mathrm{d}M\,.$$

As far as the IMRI event rate is concerned, $\nu(M, \mu, z)$ is the capture rate of stellar mass black holes by the IMBH with mass M converted to the detector time, n_c is the number density of clusters in the universe and $f(M)$ describes the mass distribution of IMBHs. We take the values of these parameters from the dynamical formation scenario suggested in [11]. Assume IMBHs form in $f_{\mathrm{tot}} = \int_{M_{\min}}^{M_{\max}} f(M)\,\mathrm{d}M = 10\%$ of globular clusters and a $f(M) \propto M^{-1}$ distribution in the IMBH mass spectrum. The rest frame capture rate of stellar mass black holes by an IMBH is taken to be proportional to the mass of the central IMBH (in the mass range $M_{\min} = 10^2\,M_\odot$ to $M_{\max} = 10^4\,M_\odot$) and inversely to the reduced mass. As ALIA is able to reach for IMRIs at moderate redshift, we further adopt a constant number density in comoving volume for n_c. Calculations using the PN and hybrid waveforms both suggest that an integrated total event rate of $\sim 3000\,(f_{\mathrm{tot}}/0.1)$ per year is expected in the dynamical formation scenario considered, the results are schematically presented in (a) of figure 4. The SNR distribution of IMRI sources is shown using the histogram in (b) of figure 4. With a view to calculate the total contribution of IMRI events to the possible foreground for the detection of IMBHB at high redshift in what follows, we have also included events below the detection threshold here. Two simple cases assuming a cutoff of n_c at redshift $z = 3$ and $z = 5$ are considered.

So far we have considered only IMRIs harbored in dense clusters, numerical simulations also suggest the possibility of formation of IMBHBs in dense star clusters. Based on

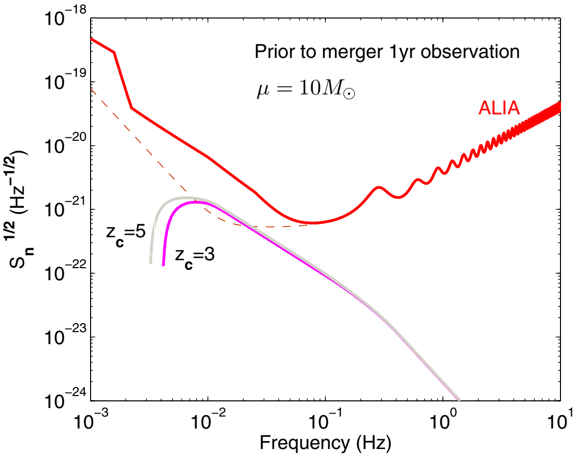


Figure 5. Total amplitude spectrum of IMRIs in clusters. Two simple cluster distribution scenarios are considered, prior to merger 1 year observation.

calculations and assumptions similar to those in [25, 26] and our chosen sensitivity, an event rate of approximately 1000 $(g_{\rm tot}/0.1)$ $(g_{\rm cl}/0.1)$ per year would be expected, where $g_{\rm tot}$ is the fraction of dense clusters that form binary IMBHs, and $g_{\rm cl}(z)$ is the fraction of star forming mass that eventually goes into star clusters within the mass range of 10^5–10^7 $M_\odot$. Three-star formation rate (SFR) models are considered as in [25], and it turns out the result is not sensitive to the chosen SFR.

Though the high event rate estimate is subject to many uncertainties common in astrophysics, like for instance the precise value of $f_{\rm tot}$ and the validity of the dynamical formation scenario among many, we are inclined to be cautiously optimistic about the prospect of ALIA as a probe of the IMRI sources at low to moderate redshift.

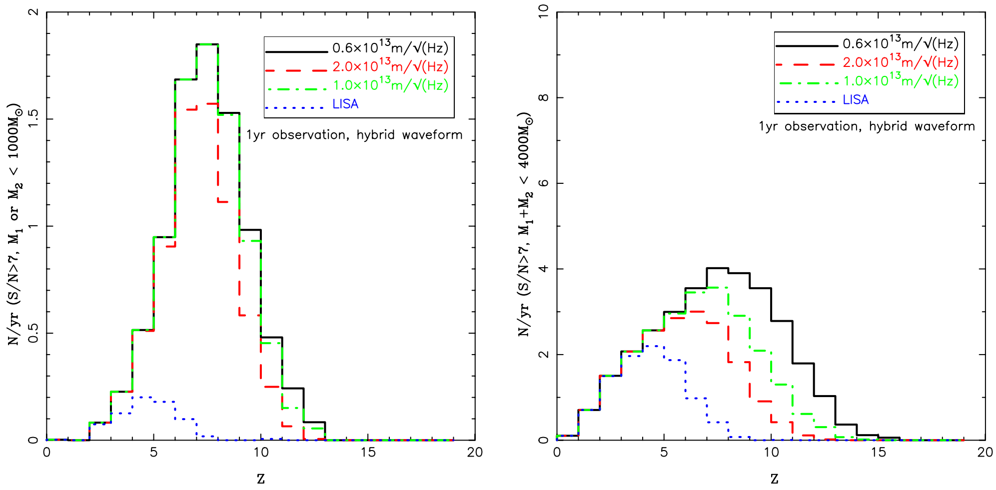
Given the high event rate, one natural question to ask is whether the foreground generated by the low redshift IMRIs will generate a source of confusion in the detection of their high redshift counterparts. To address this question, we calculate the total power spectrum of the cluster-harbored IMRI events with the cluster distribution assumed to be cut off at a certain redshift $z_{\rm c}$. The results are shown in figure 5.

Though the foreground level is dependent on $z_{\rm c}$, the SNRs of high redshift events are fairly low. An increase in $z_{\rm c}$ will not increase the foreground level significantly in the frequency window of interest. As may be seen from figure 5, the confusion generated by compact binaries still dominates and the presence of an IMRI foreground will have little impact on the SNR of high redshift IMBHB events.

The abundance of events raises the prospect that ALIA will help to map out the mass function and spin distributions of IMBHs in dense star clusters. This will significantly enhance our knowledge of the mass and number distributions of core collapsed dense clusters up to moderate redshift together with the underlying dynamics. These problems will be further looked at in our future mission study.

3.3. In search of light seed black holes at high redshift

In the standard bottom up scenario in cosmology, it is suggested that the first generation of classical black holes, formed either as remnants of zero metallicity Pop-III stars, typically of



(a) M_1 or $M_2 < 1000M_\odot$ (b) $M_1 + M_2 < 4000M_\odot$

Figure 6. Event rates for light seed black holes in the MC simulations for representative mission options.

$\sim 10^2\,M_\odot$ [14, 15], or direct gas collapse with much bigger masses [27, 28], serve as seeds for the subsequent massive black hole growth through mergers and accretions. Due to a lack of observational constraints, different models, many with big discrepancies among them in many respects, may be tuned to fit the observations on galaxies and quasars.

Through direct detection LISA will be able to distinguish the growth models for supermassive black holes provided the seed black holes are heavy [29–31]. With the designed sensitivity of ALIA and its descoped versions worked out in the previous section, it is worth understanding the potential capability of ALIA to detect light seed black holes beyond those of LISA and the corresponding event rates. Even the detection of a single event of a light seed black hole will be of significant astrophysical importance for the understanding of the hierarchical assembling process, the lifecycle and destination of first stars and gas dynamics in the earliest galaxies.

As a preliminary step, we carry out a simple Monte Carlo simulation of black hole merger histories based on the realizations of EPS formalism and semi-analytical dynamics. We place equal mass $150\,M_\odot$ black holes at $z = 20$ in highly biased 3.5σ halos, and by prescribing VHM-type dynamics [32–34], we trace downwards the black hole merging history. In major mergers, $M-\sigma$ relation calibrated accretion at Eddington rate is adopted, and we assume efficient gaseous alignment of the black holes and as a result only moderate gravitational radiation recoils [35]. The low density core is assumed to continue growing in successive mergers and a mass-to-energy conversion efficiency of $\epsilon = 0.1$ is used. The SNR calculation is based on a prior to merger 1 year observation and the threshold SNR being set to 7.

The results are schematically given in figure 6. For certain representative descope options, a plot of the event rate against the redshift distributions of the IMBHBs is given. Only light IMBHBs with either one black hole in the binary having a mass $< 1000\,M_\odot$ or the total binary mass being under $4000\,M_\odot$ are included in the plots. The black solid curve refers to the corresponding event rate for LISA level of sensitivity.

We have considered only very simple merger history of supermassive black holes. The investigation of more sophisticated scenarios such as for instance more realistic seed black

Table 3. Prior to merger 1 year observation SNR for typical high-redshift sources. Numbers outside (inside) the parenthesis are calculated using phenomenological hybrid (PN) waveform.

h_{posi} (m/$\sqrt{\mathrm{Hz}}$)	$(350, 350)M_\odot$ @$z = 18$	$(500, 500)M_\odot$ @$z = 15$	$(800, 500)M_\odot$ @$z = 12$	$(1800, 600)M_\odot$ @$z = 12$	$(3200, 800)M_\odot$ @$z = 10$
0.6×10^{-13}	**7.04** (7.01)				
1.0×10^{-13}	5.4	**7.70** (7.67)			
2.0×10^{-13}	3.7	5.4	**7.01** (7.00)		
4.0×10^{-13}	2.6	3.8	5.0	**7.35** (7.29)	
1.0×10^{-12}	1.7	2.4	3.2	4.7	**7.10** (7.04)
LISA					4.3

hole mass and redshift distributions, black hole spin evolutions and recoils under different accretion models will be presented elsewhere. Still the simple simulation suggests that ALIA and to a certain extent its descoped versions are very likely to be good probes of the growth mechanisms of a supermassive black hole.

In table 3 we also present the SNRs of IMBHB mergers with typical masses and mass ratios at different redshifts. From the table, we may see that sub-picometer interferometry is indeed needed to probe high redshift intermediate mass binaries.

4. Concluding remarks

We have briefly described a very preliminary mission concept study of ALIA. Primitive as it may seem, it does suggest that the mission concept is worth studying in greater depth. Needless to say there are countless tasks ahead on many fronts if we want to deepen our understanding on various aspects of the mission and aspire to realize the mission eventually. We hope to report upon our progress again in the not too distant future.

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References

[1] Bender P L 2004 *Class. Quantum Grav.* **21** S1203–8
[2] Bender P L and Begelman M C 2005 *Trends in Space Science and Cosmic Vision 2020* ESA SP-588 (Noordwijk: ESA Publications Division) pp 33–8
[3] Phinney S *et al* 2004 The big bang observer: direct detection of gravitational waves from the birth of the Universe to the present *NASA Mission Concept Study*
[4] Crowder J and Cornish N 2005 *Phys. Rev.* D **72** 083005
[5] Seto N, Kawamura S and Nakamura T 2001 *Phys. Rev. Lett.* **87** 221103
[6] Asto S *et al* 2008 *J. Phys.: Conf. Ser.* **120** 032004

[7] Freise A, Chelkowski S, Hild S, Del Pozzo W, Perreca A and Vecchio A 2009 *Class. Quantum Grav.* **26** 085012

[8] Hild S, Chelkowski S, Freise A, Franc J, Morgado N, Flaminio R and DeSalvo R 2010 *Class. Quantum Grav.* **27** 015003

[9] Portegies Zwart S F and McMillan S L W 2002 *Astrophys. J.* **576** 899–907

[10] Miller M C and Hamilton D P 2002 *Mon. Not. R. Astron. Soc.* **330** 232–40

[11] Miller M C 2002 *Astrophys. J.* **581** 438–50

[12] Miller M C and Colbert E J M 2004 *Int. J. Mod. Phys.* D **13** 1

[13] Gürkan M A, Fregeau J M and Rasio F A 2006 *Astrophys. J.* **640** L39–42

[14] Heger A and Woosley S E 2002 *Astrophys. J.* **567** 532–43

[15] Madau P and Rees M J 2001 *Astrophys. J.* **551** L27–30

[16] Bender P L, Begelman M C and Gair J R 2010 Talk presented at the *8th Int. LISA Symp. (LISA8), (Stanford University, California, USA, 28 June–2 July 2010)*

[17] Farmer A J and Phinney E S 2003 *Mon. Not. R. Astron. Soc.* **346** 1197–213

[18] Bender P L *email correspondences*

[19] Cavalleri *et al* 2009 *Phys. Rev. Lett.* **103** 140601

[20] Larson S L and Hiscock W A 2000 *Phys. Rev.* D **62** 062001
see also http://www.srl.caltech.edu/shane/sensitivity/index.html

[21] Ajith P *et al* 2008 *Phys. Rev.* D **77** 104017

[22] Ajith P *et al* 2009 Complete gravitational waveforms for black-hole binaries with non-precessing spins arXiv:0909.2867

[23] Colbert E J M and Miller M C 2004 Observational evidence for intermediate-mass black holes in ultra-luminous x-ray sources arXiv:astro-ph/0402677

[24] Will C M 2004 *Astrophys. J.* **611** 1080–3

[25] Fregeau J M, Larson S L, Miller M C, O'Shaughnessy R and Rasio F A 2006 *Astrophys. J.* **646** L135–8

[26] Gair J R, Mandel I, Miller M C and Volonteri M 2009 Exploring intermediate and massive black-hole binaries with the Einstein Telescope arXiv:0907.5450

[27] Koushiappas S M, Bullock J S and Dekel A 2004 *Mon. Not. R. Astron. Soc.* **354** 292–304

[28] Begelman M C, Volonteri M and Rees M J 2006 *Mon. Not. R. Astron. Soc.* **370** 289–98

[29] Sesana A, Volonteri M and Haardt F 2007 *Mon. Not. R. Astron. Soc.* **377** 1711–6

[30] Sesana A 2007 Extreme recoils: impact on the detection of gravitational waves from massive black hole binaries arXiv:0707.4677

[31] Koushiappas S M and Zentner A R 2006 *Astrophys. J.* **639** 7–22

[32] Volonteri M, Haardt F and Madau P 2003 *Astrophys. J.* **582** 559–73

[33] Volonteri M, Haardt F and Madau P 2003 *Astrophys. J.* **593** 661–6

[34] Volonteri M, Madau P, Quataert E and Rees M J 2005 *Astrophys. J.* **620** 69–77

[35] Bogdanović T, Reynolds C S and Miller M C 2007 *Astrophys. J.* **661** L147–50