Double Neutron Star Binaries: A "Foreground" Source for the Gravitational-Wave Stochastic Background

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Cosmological sources of gravitational wave backgrounds can be hidden by "foreground" sources: signals produced recently by normal astrophysical sources. Here, we briefly examine one such foreground source: the signals produced by the inspiral of Double Neutron Star (DNS) binaries. We do simple estimates of the spectrum and statistical properties of the resulting foreground, and discuss when the signals might be removable and when they form a central-limit-theorem confusion noise.

§1. Introduction

Many years ago, I wrote a review article about the stochastic background of gravitational waves.[1)] The article concentrated on so-called "cosmological" sources: gravitational waves produced by processes taking place in the early Universe. Examples included the decay of a network of cosmic strings, or the non-adiabatic growth of perturbations from inflationary models. Such a cosmological background can carry information about processes that take place at very early times, e.g. (at 100 Hz) 10^{-22} s after the big bang.

The article also mentioned that, as with the cosmic microwave background radiation, this interesting cosmological background might be obscured by contaminating "foreground" sources. In this paper, I take a brief look at one such source: the background produced by inspiraling pairs of binary neutron stars.

Currently in the frequency band around 100 Hz, the strongest upper limits on a gravitational-wave stochastic background come from analysis of data from the LIGO gravitational wave detector.[2)] In recognition of this, and to make this work more accessible to experimental colleagues, I have kept the units in the equations, so G denotes Newton's gravitational constant, and c denotes the speed of light.

§2. Binary neutron stars

Neutron Stars (NSs) are formed as the end-point of the evolution of massive stars. If those massive stars have a mass exceeding a few solar masses, the core undergoes gravitational collapse and a subsequent supernova explosion. This leaves behind a NS: a giant atomic nucleus, with a mass in the range from 1.3 to 2 solar masses.

In binary star systems, containing two massive stars, it is possible to form pairs of NS. Such systems are typically referred to in the literature as DNS (Double Neutron Stars) or BNS (Binary Neutron Stars). About a half-dozen examples are known in our galaxy, where one or both of the NS are radio pulsars. These systems are very interesting; they provide high-precision tests of general relativity, and important information about stellar populations and evolution.

DNS typically have orbital periods of hours or tens of hours. The most famous example is the Hulse-Taylor binary pulsar, PSR B1913 + 16, which has an orbital period of 7.8 h and an eccentricity of 0.6. The system radiates gravitational waves; with each orbit some potential energy is converted into gravitational waves and the stars move about 3 mm closer. In about 300 My the two NSs will move so close together that they merge. In the last minute before this merger, they will orbit each other many times per second, emitting a "chirp" of gravitational radiation in the band of ground-based detectors like LIGO. In the last minute, this chirp signals increases in frequency from ≈ 10 Hz to some kHz, and also increases in amplitude.

This kind of DNS coalesence takes place about once every 100 s within our Hubble sphere, providing a "contemporary" foreground of gravitational radiation, which can serve to obscure the more fundamental underlying cosmological background. I call this background contemporary because this inspiral process only takes place in the relatively recent history of the Universe, meaning for redshifts $z \lesssim 5$.

In this short paper, I estimate the properties of this foreground and show how it can obscure the cosmological background below certain frequencies.

§3. Estimated foreground from DNS

Here we estimate two quantities. First, the characteristic frequency below which the signals from different DNS binaries merge together to form (from the central limit theorem) an unresolvable confusion-noise foreground. Second, the approximate energy-density in such a foreground. The masses of the stars in the binary pair are denoted by M_1 and M_2.

3.1. *Number of DNS sources per frequency-bin*

The signals from DNS can be characterized as follows. Let f denote gravitational-wave frequency, and consider a system of two NS in a circular obit. In the simplest post-Newtonian approximations, the time it takes for the system to inspiral, starting from frequency f, is

$$t = kc^5 f^{-8/3} \left(GM_{\text{chirp}}\right)^{-5/3} = k\tau_\odot (\tau_\odot f)^{-8/3} \left(\frac{M_\odot}{M_{\text{chirp}}}\right)^{5/3}, \tag{3·1}$$

where $k = \frac{5}{256}\pi^{-8/3} \approx 0.0009$ is a numerical constant, $M_\odot$ is the solar mass, $\tau_\odot = \frac{GM_\odot}{c^3} = 5\,\mu$s is the mass of the sun, expressed in units of time, and $M_{\text{chirp}} = (M_1 M_2)^{3/5}(M_1 + M_2)^{-1/5}$ is the chirp mass. Taking the derivative of this expression, one can see that the system remains in a frequency band of width df for a time

$$dt = \frac{8}{3} k\tau_\odot^2 (\tau_\odot f)^{-11/3} \left(\frac{M_\odot}{M_{\text{chirp}}}\right)^{5/3} df. \tag{3·2}$$

If this frequency band is the smallest one that can be resolved by an experiment, and a large number of sources are simultaneously within this band, then according to the central limit theorem, they form an unresolvable confusion-noise background. (Some caveats are given in the Conclusion!)

Let's denote the expected rate of binary inspiral events in our Hubble sphere by $R = 300\,000/\mathrm{yr}$ (corresponding to one inspiral every 100 seconds[3]) and suppose that the experimental observation lasts a time $T = 1$ yr. Then the frequency resolution of the experiment is $df = 1/T$ and the number of sources within this narrow frequency band is

$$N = R\,dt = \frac{8kR\tau_\odot^2}{3T}(\tau_\odot f)^{-11/3}(M_\odot/M_{\mathrm{chirp}})^{5/3}. \tag{3·3}$$

This is shown in Fig. 1 for equal mass stars $M_1 = M_2 = 1.4M_\odot$.

One can see that at low frequencies, the number of "foreground sources" per resolvable frequency bin becomes much larger than one. These then form a confusion-noise stochastic signal. Thus, at frequencies below a fraction of a Hz, this "astrophysical" or "contemporary" foreground has the nature of stochastic confusion noise, which would be indistinguishable from a cosmological or fundamental background. If the energy-density in such a foreground were larger than that of a cosmological background, it would mask it in a way that could not be removed.

3.2. *Approximate amplitude of DNS background*

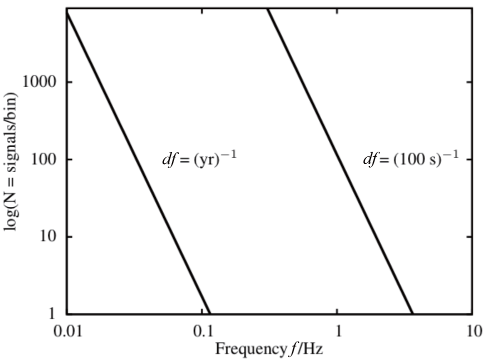

Fig. 1. The number of DNS signals per frequency bin df is shown for two different widths of frequency bin. The larger bin width, on the right, is appropriate for data analysis schemes that search for stochastic background using 100 s data segments. The smaller bin width, shown on the left, is when the maximum possible frequency resolution is used, corresponding to a (presumed) year-long experiment.

We now estimate the DNS contribution to the energy-density of the gravitational-wave foreground. This is characterized, as in 1), by the dimensionless spectral function

$$\Omega_{\mathrm{gw}}(f) = \frac{f}{\rho_{\mathrm{c}}}\frac{d\rho_{\mathrm{gw}}}{df}, \tag{3·4}$$

where $\rho_{\mathrm{c}} = \frac{3c^2H_0^2}{8\pi G}$ is the critical energy-density required to close the Universe, $H_0 = 2.4 \times 10^{-18}\,\mathrm{s}^{-1}$ is the present-day Hubble expansion rate, and $d\rho_{\mathrm{gw}}$ is the energy-density in gravitational waves in the frequency range $[f, f + df]$.

Using the simplest post-Newtonian approximations, a single DNS deposits a total energy

$$dE = \frac{1}{3}(\pi^2G^2M_{\mathrm{chirp}}^5)^{1/3}f^{-1/3}df \tag{3·5}$$

into a frequency-band of width df.

The contribution made to the total gravitational-wave energy in this frequency band, in the entire Universe, is larger by a factor RT_0. This factor is the total number of such systems created since the Universe was formed, where $T_0 = \frac{2}{3}H_0^{-1}$ is the age of the Universe. Assuming a spatially-flat Universe, we can divide this quantity by the volume of the Universe (the volume inside a sphere with the Hubble radius)

$V = \frac{4}{3}\pi(cT_0)^3$ to obtain the *average* energy-density contributed by DNS systems to Eq. (3·4):

$$\begin{aligned}\Omega_{\mathrm{DNS}}(f) &= \frac{RT_0}{V}\frac{f}{\rho_{\mathrm{c}}}\frac{dE}{df} \\ &= \frac{3}{2}\pi^{2/3}R\left(\frac{GM_{\mathrm{chirp}}}{c^3}\right)^{5/3} f^{2/3} \\ &= \frac{3}{2}\pi^{2/3}R\tau_\odot\left(\frac{M_{\mathrm{chirp}}}{M_\odot}\right)^{5/3}(f\tau_\odot)^{2/3}.\end{aligned}$$

This DNS contribution to the stochastic background is shown in Fig. 2. We assume that the spectrum cuts off sharply at a frequency of a few kHz, corresponding to the time at which the two stars merge together and stop emitting gravitational waves.

One can see that, particularly at low frequencies, it is a significant foreground source that might obscure interesting background sources such as a gravitational-wave stochastic background from inflation.

§4. Conclusion

This short paper presents crude estimates and ignores many important effects that may change the results somewhat (although not by orders-of-magnitude). Here we mention some of these.

The expansion of the Universe: DNS inspiral signals coming from redshifts of order 1 or greater will be reduced in frequency and in amplitude. This will change both the shape and the amplitude of the spectrum.

Mass distribution of binary systems: These estimates assume that all DNS binaries have stars with exactly $1.4\,M_\odot$. The distribution of these masses will affect both the amplitude and spectrum of the resulting background.

Other types of stars: A large fraction of the stars in the Universe are members of binary systems and will also contribute to the background. For example at frequencies below 1 mHz it is believed that white dwarf binary star systems will dominate the foreground. At substantially lower frequencies, binaries formed from super-massive black holes will also contribute to the foreground.

High-frequency cutoffs: The spectra that we estimate here assume that the signal spectra of the DNS systems is well-described by the simplest post-Newtonian approximation, right up to the merger. A more detailed investigation would show that the shape of the spectra changes and that the shape is affected at frequencies above about 1 kHz.

Low-frequency cutoffs: The spectra that we estimate here assume that the orbital energy loss of DNS systems are dominated by gravitational radiation, and that the systems form bound orbits with lifetimes of millions of years. In fact other effects such as tidal friction with nearby stars and similar effects will cut off the gravitational-wave background at low frequencies.

Effects of Signal-to-Noise Ratio (SNR): These estimates are done by taking an average over the entire Universe. In fact the real DNS sources will be distributed at different distances, and thus have different amplitudes at the detectors. The

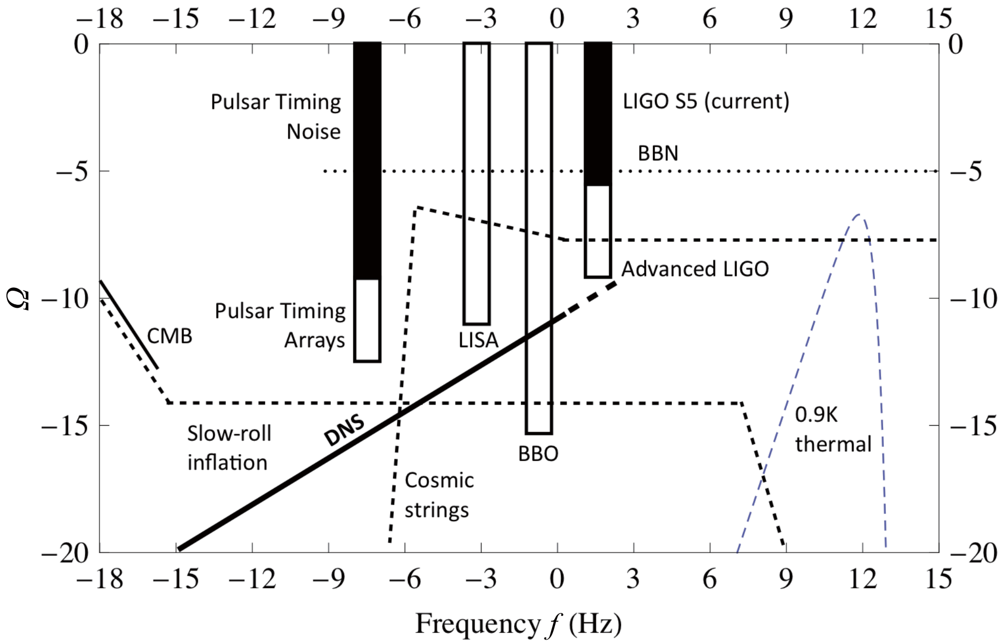

Fig. 2. A landscape plot showing the gravitational-wave stochastic background. The horizontal and vertical axes are $\log_{10}(f/\mathrm{Hz})$ and $\log_{10} \Omega_{gw}(f)$. The solid line labeled DNS is the estimated contribution of Double Neutron Star binaries in the "confusion noise" region. Depending upon the frequency resolution of the instrument and data analysis method, the the dotted part of this curve is not confusion noise. The solid (open) boxes show the current (expected future) sensitivity of pulsar-timing arrays, a space-based LISA gravitational-wave detector, a proposed Big Bang Observatory (BBO) and the ground-based LIGO detectors. The CMB line is an upper limit from observations of the Cosmic Microwave Background; the dashed lines give typical spectra from slow-roll inflation and cosmic-string models. The 0.9K curve is the expected background if the gravitational perturbations had been in equilibrium with an early hot Universe. The dotted line labeled BBN is the upper limit from big-bang nucleosynthesis *if* the stochastic background were contained in a single exponential octave of width $\ln(df/f) = 1$.

strongest of those signals will be "loud" enough to be clearly distinguished in the detectors and subtracted away. This might in fact permit individual signals to be resolved well enough to remove the "confusion noise" entirely. A nice paper by Cutler and Harms[4)] presents a method to use with the proposed Big Bang Observatory mission to do exactly this.

These issues and effects are also discussed in the literature, for example in work by Farmer and Phinney[5)] and Regimbau and collaborators.[6),7),8),9)]

A more complete study of these different effects is being carried out by my student at AEI, Pablo A. Rosado Gonzalez.[10)]

Acknowledgements

The author gratefully acknowledges the support of the Yukawa Institute and the organizers of the YKIS2010 Workshop "Cosmology — the next generation"and the support of National Science Foundation grants Nos. 0701817 and 0970074 and the Max Planck Society.

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