New Relations for Three-Dimensional Supersymmetric Scattering Amplitudes

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We provide evidence for a duality between color and kinematics in three-dimensional supersymmetric Chern-Simons matter theories. We show that the six-point amplitude in the maximally supersymmetric $\mathcal{N} = 8$ theory can be arranged so that the kinematic factors satisfy the fundamental identity of three-algebras. We further show that the four- and six-point $\mathcal{N} = 8$ amplitudes can be squared into the amplitudes of $\mathcal{N} = 16$ three-dimensional supergravity, thus providing evidence for a hidden three-algebra structure in the dynamics of the supergravity.

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Introduction.—Scattering amplitudes have provided a rich vein of insight into the hidden structures underlying theories of gauge and gravitational interactions. One particularly suggestive result is the color-kinematics duality discovered by Bern, Carrasco, and Johansson (BCJ) [1]. At tree-level, color-dressed scattering amplitudes in Yang-Mills (YM) theories can, quite generally, be written as a sum over cubic graphs

$$\mathcal{A}_n = g^{n-2} \sum_{i \in \text{graphs}} \frac{n_i c_i}{\prod_{\ell_i} p_{\ell_i}^2}, \tag{1}$$

where the c_i's are color structures made from the usual Lie algebra structure constants, and the n_i's are kinematic factors from which we have removed products of inverse propagators $p_{\ell_i}^2$ associated to internal lines of the respective cubic diagram. BCJ proposed that there exists a representation of the amplitude such that for any set of color structures related by a Jacobi identity, there is a corresponding relation between their numerator factors:

$$c_s + c_t + c_u = 0 \Rightarrow n_s + n_t + n_u = 0. \tag{2}$$

This duality implies nontrivial relations between different tree-level color-ordered subamplitudes, the so-called BCJ relations [1].

Moreover, such Yang-Mills amplitudes can be used to express tree-level scattering in related gravity theories via the well-known Kawai-Lewellen-Tye (KLT) relations [2,3]. BCJ [1,4] proposed that it is possible to express gravity amplitudes in terms of the gauge theory data by simply replacing the color factors by another copy of the kinematic numerators and summing over the same cubic diagrams.

Scattering amplitudes have also been recently studied in the context of supersymmetric Chern-Simons matter theories. These theories are of great interest as they describe the low-energy dynamics of multiple membranes and have played an important role in recent studies of the AdS/CFT correspondence including applications as toy models for condensed matter physics. In this Letter, we propose, and provide evidence for, a nontrivial analog of the color-kinematics duality in the maximally supersymmetric case and for a corresponding "double-copy" construction leading to $E_{8(8)}$ symmetric, three-dimensional $\mathcal{N} = 16$ supergravity. This three-dimensional gravity is of particular interest as it is related to the dimensional reduction of $\mathcal{N} = 8$ four-dimensional gravity, and plays a key role, after further reduction to two dimensions, in understanding the classical "hidden symmetries," e.g., [5,6], of this theory, which in turn will likely be important in understanding the quantum theory.

$\mathcal{N} = 8$ *supersymmetric Chern-Simons scattering amplitudes.*—The maximally supersymmetric Chern-Simons theory, Bagger-Lambert-Gustavsson (BLG) theory, constructed in [7–9], is the unique three-dimensional gauge theory with OSp(8|4) superconformal symmetry. The on-shell physical states comprise eight scalars X^I in the $\mathbf{8_v}$ and eight fermions $\Psi^{\dot{I}}$ in the $\mathbf{8_c}$ of SO(8). An important feature of the original construction was the appearance of three-algebras. Briefly, a three-algebra is a vector space T^a, $a = 1, \ldots, N$, with a trilinear product,

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d,$$

where the structure constants $f^{abc}{}_d$ satisfy the fundamental three-algebra identity,

$$f^{efg}{}_d f^{abc}{}_g = f^{efa}{}_g f^{bcg}{}_d + f^{efb}{}_g f^{cag}{}_d + f^{efc}{}_g f^{abg}{}_d. \tag{3}$$

Moreover, there is a trace form, $h^{ab} = \mathrm{Tr}(T^a T^b)$, which can be used to raise and lower indices. The structure constants with all indices raised are completely antisymmetric, $f^{abcd} = f^{[abcd]}$. All on-shell fields are three-algebra-valued fundamental fields, e.g., $X^I = \sum_{a=1}^N (X^I)^a T^a$. The only known finite-dimensional example is where the three-algebra is four dimensional, while the structure constants are proportional to the invariant four-index tensor $f^{a_1 a_2 a_3 a_4} \propto \epsilon^{a_1 a_2 a_3 a_4}$.

As we are interested in scattering amplitudes, it is convenient to make use of the spinor-helicity formalism, whereby three-momenta are expressed as the product of two-component real spinors [10]: $p^{\alpha\beta} = \lambda^\alpha \lambda^\beta$, where $\alpha, \beta = 1, 2$. The on-shell fields can be grouped into a single superfield [11], by introducing four Graßmann parameters γ^i, $i = 1, \ldots, 4$. This construction breaks manifest SO(8) R symmetry by rewriting the $\mathbf{8}_v$ scalars as $X^I = \{\bar{X}, X^{[ij]}, X\}$ and similarly for the fermions, $\Psi^{\dot{A}} = \{\psi_i, \bar{\psi}^i\}$, so that the on-shell superfield is

$$\Phi_{\rm BLG} = \bar{X} + \gamma^i \psi_i + \frac{1}{2}\epsilon_{ijkl}\gamma^i\gamma^j X^{[kl]} + \frac{1}{3!}\epsilon_{ijkl}\gamma^i\gamma^j\gamma^k\bar{\psi}^l + \frac{1}{4!}\epsilon_{ijkl}\gamma^i\gamma^j\gamma^k\gamma^l X.$$

An OSp(8|4) invariant four-point scattering amplitude,

$$\mathcal{A}_4 = \frac{4\pi i}{k}\frac{\delta^{(3)}(P)\delta^{(8)}(Q)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} f^{a_1a_2a_3a_4}, \tag{4}$$

has previously been constructed [13]. In this formula the delta functions impose conservation of momenta, $P^{\alpha\beta} = \sum_{j=1}^4 p_j^{\alpha\beta}$, and supermomenta, $Q^{\alpha i} = \sum_{j=1}^4 \lambda_j^\alpha \gamma_j^i$, while the kinematic invariants are defined as $\langle jk\rangle = \epsilon_{\alpha\beta}\lambda_j^\alpha\lambda_k^\beta$. The overall form of the amplitude is fixed by the superconformal symmetries, while the normalization, dependence on the Chern-Simons coupling k, and color structure are fixed by the explicit Feynman diagram calculation of any component amplitude.

Quite generally, we can write an n-point amplitude in the BLG theory in the form (1), but with the c_i corresponding to three-algebra color structures [14]. The sum is now over diagrams with quartic vertices, and the color structures are found by associating to each vertex a factor f^{abcd}, and to each internal line a metric h_{ab}. For example, Fig. 1 corresponds to $c_{(123)(456)} := f^{a_1a_2a_3b}h_{bc}f^{ca_4a_5a_6}$.

A key feature is that due to the fundamental identities (3) not all of the color structures are independent. Namely, given $c_s = \ldots f^{efg}{}_d f^{abc}{}_g \ldots$, $c_t = \ldots f^{efa}{}_g f^{bcg}{}_d \ldots$, $c_u = \ldots f^{efb}{}_g f^{cag}{}_d \ldots$, and $c_v = \ldots f^{efc}{}_g f^{abg}{}_d \ldots$, where the "..." denote factors common to all diagrams, it follows that $c_s = c_t + c_u + c_v$. Our first proposal is that corresponding numerators n_s, n_t, n_u, and n_v can always be found such that (see Fig. 2)

$$c_s = c_t + c_u + c_v \Rightarrow n_s = n_t + n_u + n_v. \tag{5}$$

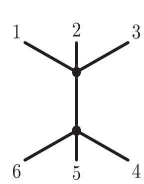

FIG. 1. Six-point quartic diagram

We do not have a general proof for these relations; instead, we will provide evidence for their existence by considering the first nontrivial case, i.e., six points.

At six points, all color structures consist of the contractions of two tensors as in Fig. 1. Accounting for the antisymmetry of f^{abcd}, there are ten distinct color structures c_i, labeled by partitions of the six color labels into groups of three, e.g., $c_1 = c_{(123)(456)}$ [15]. There are five independent three-algebra relations between the ten different color structures. Our claim is that there is a choice of numerators such that they satisfy the same three-algebra fundamental identities. However, the numerators are not uniquely defined and finding explicit forms is not straightforward. Instead, we will show the existence of such numerators, and give a recipe for calculating them, by considering the color-ordered subamplitudes of $\mathcal{N} = 6$ Aharony-Bergman-Jafferis-Maldacena (ABJM) theory [16].

New relations for color-ordered subamplitudes.—As is well known, the BLG theory can be rewritten [17] as a special case ($N = 2$) of the $SU(N) \times SU(N)$ $\mathcal{N} = 6$ Chern-Simons theories with bi-fundamental matter, which are ABJM-theories. The ABJM on-shell fields can be grouped into two superfields: $\hat{\Phi}^A_{\bar{A}}$, transforming as $(\mathbf{N}, \bar{\mathbf{N}})$, and $\hat{\bar{\Phi}}^{\bar{B}}_B$, transforming as $(\bar{\mathbf{N}}, \mathbf{N})$ [18]. This formalism is manifestly U(3) symmetric, making use of three Graßmann parameters $\gamma^{\hat{\imath}}$, $\hat{\imath} = 1, 2, 3$. For $N = 2$ the conjugate representations are equivalent and the two superfields can be combined: $\Phi_{\rm BLG} = \hat{\Phi} + \gamma^4\hat{\bar{\Phi}}$.

Scattering amplitudes in BLG theory can be found from those of ABJM by identifying the appropriate fields and color structures. ABJM scattering amplitudes can, however, be decomposed into color-ordered subamplitudes. Each color-ordered subamplitude will contribute to several kinematical coefficients of the BLG color structures c_i. We claim that every color-ordered ABJM subamplitude can be written as a certain combination of numerators n_i with propagators, in such a way that the corresponding BLG amplitudes take the form (1), with the numerators satisfying the three-algebra identities (5). This implies nontrivial relations among the color-ordered ABJM subamplitudes for any N, and thus is a slightly stronger claim than the existence of three-algebra-satisfying numerators for BLG amplitudes. In the following, we provide evidence for our claim by examining the six-point amplitudes.

Four-point amplitudes in ABJM [19] were considered in [20]; the six-point color-ordered subamplitudes for ABJM were first calculated in [18] (see also [21]). As a representative component amplitude, we consider the six-point

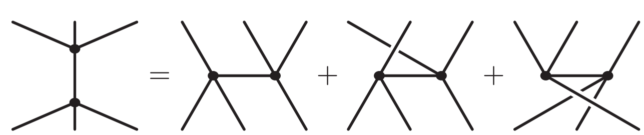

FIG. 2. Graphical expression of the fundamental identity

amplitude involving a single flavor of a complex scalar $\phi(p)^A_{\bar{A}}$ and its conjugate $\bar{\phi}(p)^{\bar{B}}_B$,

$$\hat{A}_{6\phi} = A(1,2,3,4,5,6)\delta^{\bar{B}_2}_{\bar{A}_1}\delta^{A_3}_{B_2}\delta^{\bar{B}_4}_{\bar{A}_3}\delta^{A_5}_{B_4}\delta^{\bar{B}_6}_{\bar{A}_5}\delta^{A_1}_{B_6} + \ldots, \quad (6)$$

with the ellipses denoting other color orderings.

At six points, we propose that the color-ordered ABJM subamplitudes take the form

$$A(i,j,k,p,q,r) = \frac{n_{(ijk)(pqr)}}{p^2_{ijk}} + \frac{n_{(qri)(jkp)}}{p^2_{qri}} + \frac{n_{(rij)(kpq)}}{p^2_{kpq}}. \quad (7)$$

There are six independent subamplitudes, all others are related to those by cyclic double-shifts and by inversions, i.e., $A(k,p,q,r,i,j) = A(i,j,k,p,q,r)$, $A(i,r,q,p,k,j) = A(i,j,k,p,q,r)$. We will now give a recipe for constructing numerators n_i for which both (7) and the three-algebra relations (5) hold. The latter can be satisfied by setting $n_2 = n_1 + n_3 - n_4$, $n_8 = -n_3 + n_6 - n_{10}$, $n_5 = n_3 - n_4 + n_{10}$, $n_9 = -n_1 - n_3 + n_6$, $n_7 = -n_1 - n_3 + n_4 + n_6 - n_{10}$. We determine four further numerators n_3, n_4, n_6, and n_{10} in terms of known amplitude expressions by solving (7) for $A(1,2,3,4,5,6)$, $A(1,2,3,6,5,4)$, $A(1,2,5,4,3,6)$, and $A(1,4,3,6,5,2)$. Plugging these numerators into the remaining two relations (7), we obtain nontrivial identities among the six subamplitudes and the undetermined numerator n_1. We express the two relations as a six-term $U(1)$-decoupling relation and a four-term BCJ-type relation,

$$0 = \sum_{\sigma,\sigma'} A[1,\sigma(2),\sigma'(3),\sigma(4),\sigma'(5),\sigma(6)],$$
$$0 = p^2_{123}A(1,2,3,4,5,6) + s_1 A(1,2,5,4,3,6) + s_2 A(1,6,3,2,5,4) + s_3 A(1,6,5,2,3,4),$$

where $\sigma \in Z_3(2,4,6)$, $\sigma' \in Z_2(3,5)$, and the s_i's are some complicated kinematic factors. It is straightforward to confirm these relations by choosing explicit numerical values for the external momenta. This proves that the six-point BLG amplitude can indeed be expressed in terms of numerators satisfying the three-algebra relations. Importantly, the undetermined kinematical factor n_1 drops out of the relations and thus corresponds to a generalized gauge freedom analogous to that found in the YM case [1].

$E_{8(8)}$ supergravity theory.—The three-dimensional $\mathcal{N} = 16$ supergravity with $E_{8(8)}$ symmetry (E_8 theory), originally constructed by Marcus and Schwarz [22], consists of 128 scalar bosons and 128 fermions which are inequivalent real spinor representations of SO(16), the maximal compact subgroup of $E_{8(8)}$. As is well known, e.g., [23], this theory is related, on-shell, to the dimensional reduction of four-dimensional $\mathcal{N} = 8$ supergravity with $E_{7(7)}$ symmetry (E_7 theory) by performing a duality transformation of all the vector fields into scalars, which then combine with the scalars from dimensional reduction, including those originally in the $E_{7(7)}/SU(8)$ coset of the $\mathcal{N} = 8$ supergravity, to become those of the $E_{8(8)}/SO(16)$ coset.

Thus, for fields which are unchanged by the duality transformation, in particular the scalars originating in the $E_{7(7)}/SU(8)$ coset, the three-dimensional scattering amplitudes are just those of the four-dimensional theory evaluated on three-dimensional kinematics. However, due to the duality transformation this is not the case for all amplitudes; as a simple example, even for complex momenta, there is no E_8-theory three-point amplitude. Indeed, as explained in [22], all nontrivial scattering amplitudes must have an even number of external particles, as products of odd numbers of spinors cannot form a singlet.

The $E_{8(8)}$ algebra comprises 120 compact SO(16) generators X^{IJ}, $I, J = 1, \ldots, 16$, and 128 noncompact generators Y^A, $A = 1, \ldots, 128$. It is convenient to fix the unitary gauge, whereby a generic group element is written as $g = e^{\varphi^A Y^A}$ with φ^A being the physical scalars. The $E_{8(8)}/SO(16)$-coset action is constructed from the algebra-valued current $P_\mu = \frac{1}{2}(e^{-\varphi}\partial e^{\varphi} - e^{\varphi}\partial e^{-\varphi})$. The bosonic action is [22]

$$\mathcal{L}_{\rm bos} = \frac{1}{4\kappa^2}\sqrt{-g}R - \frac{1}{4\kappa^2}\sqrt{-g}g^{\mu\nu}P^A_\mu P^A_\nu, \quad (8)$$

where the first term is the usual gravity action. Using this action (the fermionic terms are also known) with appropriate gauge fixing, one can straightforwardly calculate scattering amplitudes using Feynman diagrams. At four points such amplitudes for four scalars receive contributions from graviton exchange and from contact interactions that arise upon expanding the coset term to quartic order in the fields, $\mathcal{L}_{\varphi^4} \sim (\varphi\Gamma^{IJ}\partial_\mu\varphi)(\varphi\Gamma^{IJ}\partial_\mu\varphi)$. In the simplest case we can consider the scattering of four scalars all carrying the same coset index, e.g., all fields being φ^1, in which case there is no contribution from contact terms. Combining all graviton exchange diagrams we find

$$M_4 = \frac{i\kappa^2}{4}\left(\frac{s^2+u^2}{t} + \frac{t^2+u^2}{s} + \frac{s^2+t^2}{u}\right). \quad (9)$$

It is not difficult to calculate other component amplitudes; however, we can make use of the supersymmetry to determine the full four-point superamplitude.

For the E_8 theory we can define an on-shell superfield by using eight Graßmann parameters η^I, $I = 1, \ldots, 8$, which breaks the SO(16) R symmetry to U(8). Splitting the 128 scalars φ^A into the fields $\{\xi, \bar{\xi}, \xi_{IJ}, \bar{\xi}^{IJ}, \xi_{IJKL}\}$ with, for example $\xi = \frac{1}{2}(\varphi^1 + i\varphi^2)$, and similarly for the fermionic fields, we can write the superfield [24]

$$\Xi = \xi + \eta^I\psi_I + \frac{1}{2}\eta^I\eta^J\xi_{IJ} + \ldots + \frac{1}{8!}\eta^8\bar{\xi}.$$

By using super-Poincaré symmetry and matching to the component amplitude, the four-point superamplitude is

$$\mathcal{M}_4 = \frac{i\kappa^2}{4} \frac{\delta^{(16)}(Q)\delta^{(3)}(P)}{(\langle 12\rangle\langle 23\rangle\langle 31\rangle)^2}. \quad (10)$$

Here, the 16-dimensional fermionic delta function is given by the product of two eight-dimensional fermionic delta functions, $\delta^{(16)}(Q) \sim \delta^{(8)}(Q^1)\delta^{(8)}(Q^2)$, such as appeared in (4). Stripping off the overall normalization and momentum delta function we see that this is the "square" of (4). This then suggests a relation between $\mathcal{N} = 8$ BLG and the E_8 theory analogous to that, due to KLT [2,3], between $\mathcal{N} = 4$ supersymmetric Yang-Mills and the E_7 supergravity theory. As zeroth-order checks, we note that the spectra of the E_8 theory and that of BLG theory squared match; furthermore, in both cases all nontrivial amplitudes have an even numbers of legs. As $\mathcal{N} = 8$ BLG theory can be found from supersymmetric three-dimensional Yang-Mills theory [25] via a Higgs mechanism reminiscent of the duality transformation, it is perhaps not surprising that it should be thus related to the E_8 supergravity theory.

Three-dimensional gravity as the square of Chern-Simons.—Given the suggestion that the BLG amplitudes can be written in terms of numerators satisfying the three-algebra color-kinematics duality, it is natural to ask if the gravity theory amplitudes can be written as a "double-copy" as in [1],

$$M_n = i\left(\frac{\kappa}{2}\right)^{n-2} \sum_i \frac{n_i n_i}{\prod_{\ell_i} p_{\ell_i}^2}, \quad (11)$$

where the n_i's are the numerators appearing in the BLG amplitude (1) and the sum is over the same n-point quartic diagrams. This relation obviously holds at four points for the superamplitudes. At six points, we perform an explicit check by making use of the numerators calculated from the six-point color-ordered ABJM subamplitudes for specific components: We use the pure scalar ABJM amplitude $\hat{A}_{6\phi}$ (6) to calculate the numerators for $A_6(X_1\bar{X}_2X_3\bar{X}_4X_5\bar{X}_6)$ in the BLG theory, and find that it indeed squares into the $M_6(\xi_1\bar{\xi}_2\xi_3\bar{\xi}_4\xi_5\bar{\xi}_6)$ gravity amplitude. The latter could, in principle, be found by a direct Feynman diagram calculation. Instead, we take the complex scalar ξ to have originated in the $E_{7(7)}/SU(8)$ coset, so that $M_6(\xi_1\bar{\xi}_2\xi_3\bar{\xi}_4\xi_5\bar{\xi}_6)$ can be obtained by dimensional reduction of the six-scalar E_7 supergravity amplitude. The latter can be found by using the KLT relations [2,3] for a scalar component of the known $\mathcal{N} = 4$ supersymmetric Yang-Mills next-to-maximally helicity-violating amplitude [26]. It is then straightforward to check, again by choosing a range of numerical values for external momenta, that the resulting pure scalar amplitude in fact agrees with the squared BLG amplitude (11).

For higher-point amplitudes, it would be possible to prove, along the lines of [27], that (11) holds if there were Britto-Cachazo-Feng-Witten (BCFW) recursion relations [28,29] for the E_8 theory. Recursion relations for ABJM theories, and thus BLG theory, have been proven in [21]. The key step is proving that the superamplitude falls off sufficiently fast for large deformations of the momenta under a complex nonlinear shift: $\hat{\mathcal{A}}(\{\lambda_1(z), \lambda_l(z)\}) \sim \mathcal{O}(1/z)$ as $z \to \infty$ with $\lambda_1(z) = \frac{z+z^{-1}}{2}\lambda_1 - \frac{z-z^{-1}}{2i}\lambda_l$, $\lambda_l(z) = \frac{z-z^{-1}}{2i}\lambda_1 + \frac{z+z^{-1}}{2}\lambda_l$, and similar shifts for the Graßmann parameters. The proof of a sufficient falloff for E_8 superamplitudes does not currently exist. However, it is possible to naively apply the method of [21] and use the four-point amplitude (10) to construct a candidate six-point superamplitude in E_8 supergravity. We find that the relevant scalar component, $M_6(\xi_1\bar{\xi}_2\xi_3\bar{\xi}_4\xi_5\bar{\xi}_6)$, of this superamplitude agrees with the amplitude calculated by squaring the numerators (11). This shows that at least to six points, the BCFW recursion relations of [21] hold for the E_8 theory.

Outlook.—In order to confirm the proposed "double-copy" relations for the E_8 theory, it would be very useful to prove the BCFW relations for the three-dimensional supergravity. Relatedly, numerator identities for YM and squaring relations for gravity have been conjectured to extend to all-loop diagrams [4], and it would be interesting to check whether similar relations hold for the three-dimensional Chern-Simons and gravity theories. If this was the case, these relations would provide a useful practical tool for calculating amplitudes in three-dimensional supergravity at loop level. They would furthermore demonstrate the existence of a hidden three-algebra structure in three-dimensional gravity. This is interesting both as a nontrivial model for similar structures in four-dimensional gravity and as an important intermediary step to understanding the infinite "hidden symmetries" at the quantum level.

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