Inhomogeneous Loop Quantum Cosmology with Matter

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Introduction

- Loop Quantum Cosmology (LQC)
  - Satisfactory quantization of several homogeneous cosmological models
  - New quantum phenomenology $\Rightarrow$ Resolution of initial singularity
- Hybrid quantization: Inhomogeneous models
  - Reduced model with only global constraints
  - It assumes that the most relevant effects of loop quantum geometry are in the homogeneous degrees of freedom
  - Combine LQC quantization for this homogeneous sector with a Fock quantization for inhomogeneities
- Linear polarized Gowdy $\mathbb{T}^3$ in vacuo
  - Homogeneous sector $\equiv$ Vacuum Bianchi I
  - Inhomogeneous sector $\equiv$ Linear polarized gravitational waves

Motivation

- Inclusion of a massless scalar field in the Gowdy $\mathbb{T}^3$ model
  - Minimally coupled
  - Same symmetries of the geometry
- Motivation
  - Inclusion of matter inhomogeneities in LQC.
  - Study of more realistic models, closer to the observed universe.
  - Scenario in which one can study some interesting features
    - Quantum effects of the inhomogeneities and the anisotropies on an FRW background.
    - Robustness of the Big Bounce scenario of LQC.
    - Changes in the evolution of the matter inhomogeneities due to quantum geometry effects.
    - *projection* to more symmetric quantum models.

Classical Settings

- Reduced phase space
  - Homogeneous sector: Bianchi I + homogeneous massless scalar field ϕ
  - Inhomogeneous sector: Matter inhomogeneities and gravitational waves (propagating in $\theta \in S^1$)

- Ashtekar-Barbero variables for Bianchi I
  - $su(2)$ connection: c^j; densitized triad: p_j $i \in \{\theta, \sigma, \delta\}$

- Satisfactory Fock quantization of the inhomogeneities:
  - Unitary dynamics + Vacuum invariant under S^1 translations.
  - Parametrization of the matter φ and gravitational ξ inhomogeneities
  - Creation-annihilation variables (free m. s. f.): $\left(a_m^{(\alpha)*}, a_m^{(\alpha)}\right)$, $\alpha = \xi, \varphi$

- Two global constraints remain:
  - Diffeomorphism constrain: $C_\theta = C_\theta^\xi + C_\theta^\varphi$
  - Densitized Hamiltonian constraint: $\mathcal{C} = \mathcal{C}_{\text{hom}} + \mathcal{C}_{\text{inh}}$

Hybrid Quantization: Kinematics

Kinematical Hilbert space

$$\mathcal{H}_{\text{kin}} = \mathcal{H}_{\text{kin}}^{\text{hom}} \otimes \mathcal{H}_{\text{kin}}^{\text{inh}} = \mathcal{H}_{\text{kin}}^{\text{BI}} \otimes L^2(\mathbb{R}, d\phi) \otimes \mathcal{F}^{\xi} \otimes \mathcal{F}^{\varphi}$$

- Fock Spaces $\mathcal{F}^{\alpha}$:
  - $a_m^{(\alpha)*}, a_m^{(\alpha)} \;\rightarrow\; \hat{a}_m^{(\alpha)\dagger}, \hat{a}_m^{(\alpha)}$: creation-annihilation operators.
  - n-particle states: $|\mathfrak{n}^{\alpha}\rangle = |\ldots, n_{-m}^{\alpha}, \ldots, n_m^{\alpha}, \ldots\rangle$, $n_m^{\alpha} \in \mathbb{N}, \sum_m n_m^{\alpha} < \infty$

- $\mathcal{H}_{\text{kin}}^{\text{hom-mat}} = L^2(\mathbb{R}, d\phi)$:
  - Standard Schrödinger quantization: $\hat{\phi},\; \hat{p}_{\phi} = -i\hbar\partial_{\phi}$

- Bianchi I kinematical Hilbert space:
  - Improved dynamics: minimum length, $\bar{\mu}_j$, in the holonomies.
  - $\mathcal{H}_{\text{kin}}^{\text{BI}} = \overline{\text{span}\{|\lambda_{\theta}, \lambda_{\sigma}, v\rangle : \lambda_{\theta}, \lambda_{\sigma}, v \in \mathbb{R}\rangle\}} \qquad v = 2\lambda_{\theta}\lambda_{\sigma}\lambda_{\delta}$
  - $\hat{\mathcal{N}}_{\pm\bar{\mu}_i}$: Scale λ_i such that shift v in ± 1; $\quad \hat{p}_j : \; p_j \propto \text{sgn}(\lambda_j)\lambda_j^2$.

Operators on the inhomogeneous Hilbert space

- Diffeomorphism constraint operator

$$\widehat{\mathcal{C}}_\theta = \sum_{m=1}^{\infty} m\left(\widehat{X}^\xi_m + \widehat{X}^\varphi_m\right), \qquad \widehat{X}^\alpha_m = \hat{a}^{(\alpha)\dagger}_m \hat{a}^{(\alpha)}_m - \hat{a}^{(\alpha)\dagger}_{-m} \hat{a}^{(\alpha)}_{-m}.$$

  - $\widehat{\mathcal{C}}_\theta |\mathfrak{n}^\xi\rangle \otimes |\mathfrak{n}^\varphi\rangle \quad \Rightarrow \quad \sum_{m=1}^{\infty} m(X^\xi_m + X^\varphi_m) = 0, \quad X^\alpha_m = n^\alpha_m - n^\alpha_{-m}.$
  - $\mathcal{F}_{\mathsf{p}} \equiv$ proper subspace of $\mathcal{F}_\xi \otimes \mathcal{F}_\varphi$.

- Operators in $\widehat{\mathcal{C}}_{\mathsf{inh}}$

  - $\widehat{H}_0 = \sum_{\alpha\in\{\xi,\varphi\}} \sum_{m=1}^{\infty} m\widehat{N}^\alpha_m, \qquad \widehat{N}^\alpha_m = \hat{a}^{(\alpha)\dagger}_m \hat{a}^{(\alpha)}_m + \hat{a}^{(\alpha)\dagger}_{-m} \hat{a}^{(\alpha)}_{-m}.$
  - $\widehat{H}_{\mathsf{int}} = \sum_{\alpha\in\{\xi,\varphi\}} \sum_{m=1}^{\infty} \frac{1}{m}\left(\widehat{N}^\alpha_m + \hat{a}^{(\alpha)\dagger}_m \hat{a}^{(\alpha)\dagger}_{-m} + \hat{a}^{(\alpha)}_m \hat{a}^{(\alpha)}_{-m}\right).$

Hamiltonian constraint operator $\widehat{\mathcal{C}} = \widehat{\mathcal{C}}_{\text{hom}} + \widehat{\mathcal{C}}_{\text{inh}}$

- $\widehat{\mathcal{C}}_{\text{hom}} = -\sum_{i\neq j}\sum_{j}\frac{\widehat{\Theta}_i\widehat{\Theta}_j}{16\pi G\gamma^2} - \frac{\hbar^2}{2}\left[\frac{\partial}{\partial\phi}\right]^2, \qquad i,j\in\{\theta,\delta,\sigma\}.$
- $\widehat{\mathcal{C}}_{\text{inh}} = 2\pi\hbar\widehat{|p_\theta|}\widehat{H}_0 + \hbar\left[\widehat{\frac{1}{|p_\theta|^{\frac{1}{4}}}}\right]^2\frac{\left(\widehat{\Theta}_\delta+\widehat{\Theta}_\sigma\right)^2}{16\pi\gamma^2}\left[\widehat{\frac{1}{|p_\theta|^{\frac{1}{4}}}}\right]^2\widehat{H}_{\text{int}}.$

- Symmetric factor ordering:
  - Triad operators: $v=0$ states decouple (kin. singularity resolution)
  - $\widehat{\Theta}_j$ operators do not mix states with different sign of $\lambda_\theta,\lambda_\sigma,v$.
- $\widetilde{\mathcal{H}}^{\text{BI}}_{\text{kin}}$: states such that $\lambda_\theta,\lambda_\sigma,v>0 \;\Rightarrow\; \Lambda_\theta=\log\lambda_\theta,\ \Lambda_\sigma=\log\lambda_\sigma.$

- Superselection sectors:
  - in v: $v\in\mathcal{L}_\epsilon=\{\epsilon+4k;\ k\in\mathbb{N}\}$
  - in Λ_a: Given an initial $\Lambda_a^\star \;\Rightarrow\; \Lambda_a=\Lambda_a^\star+z_\epsilon,\ z_\epsilon\in\mathcal{Z}_\epsilon$

Physical Hilbert space

- Action of the Hamiltonian constraint
  - The coefficients do not depend on Λ_σ
  - It is a difference equation in $v \Rightarrow$ evolution equation in v
  - The solutions can be determined by a set of initial data on the section of minimum homogeneous volume

- Physical Hilbert space $\mathcal{H}_{\text{phy}} \Leftrightarrow$ Hilbert space of initial data

$$\mathcal{H}_{\text{p}} = \mathcal{H}^{\text{BI}}_{\text{phys}} \otimes L^2(\mathbb{R}, d\phi) \otimes \mathcal{F}_{\text{p}}$$

  - $\mathcal{H}^{\text{BI}}_{\text{phys}} \equiv$ Physical Hilbert space of Bianchi I

Projection to LRS-Gowdy

- The model is symmetric under the interchange of σ and δ directions.
- Classical solutions with local rotational symmetry (LRS)
- General state: $|\Psi\rangle = \sum_{\Lambda_\theta,\Lambda_\sigma,v} |\Psi(\Lambda_\theta,\Lambda_\sigma,v)\rangle \otimes |\Lambda_\theta,\Lambda_\sigma,v\rangle$
- *Projection map:*

$$|\Psi(\Lambda_\theta,\Lambda_\sigma,v)\rangle \qquad \longrightarrow \qquad \sum_{\Lambda_\sigma} |\Psi(\Lambda_\theta,\Lambda_\sigma,v)\rangle \equiv |\psi(\Lambda_\theta,v)\rangle$$

Quantum Gowdy Model $\xrightarrow{\text{projection}}$ Quantum LRS-Gowdy Model

- *projection* over Λ_θ to get the isotropic Gowdy model fails
  - There is no classical inhomogeneous and isotropic solutions.

Conclusions

- Satisfactory quantization of the Gowdy T^3 model with linearly polarized gravitational waves and a massless scalar field.
- Hybrid quantization applied as in the vacuum model.
- Inclusion of the matter field:
  - Classical isotropic solutions of the homogeneous sector.
  - Two "copies" of inhomogeneities (mathematically speaking).
  - Matter inhomogeneities in LQC.
- Same results as in the vacuum model.
  - Standard Fock quantization of the inhomogeneities is recovered.
  - Classical singularity resolved at the kinematical level.
- Study of the *projection* to more symmetric systems.
- Possibility of analyzing the effect of the anisotropies and the inhomogeneities on a flat FRW model. (Work in progress)

Thanks for your attention!