

Leptogenesis in models with keV sterile neutrino dark matter

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We analyze leptogenesis in gauge extensions of the Standard Model with keV sterile neutrino dark matter. We find that both the observed dark matter abundance and the correct baryon asymmetry of the Universe can simultaneously emerge in these models. Both the dark matter abundance and the leptogenesis are controlled by the out of equilibrium decays of the same heavy right handed neutrino.

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I. INTRODUCTION

The modern cosmology provides at least two experimental evidences for the existence of new physics beyond the Standard Model (SM). One of them is the baryon asymmetry of the Universe (BAU). The other one is the Dark Matter (DM) which constitutes most of the gravitating mass in the Universe. To be consistent with the observations, the DM candidate should be a very weakly interacting particle. For a sterile neutrino DM-candidate there are two ways to achieve the observed DM abundance. The first possibility is that the sterile neutrino interacts so weakly, that it never enters thermal equilibrium after the reheating (ν MSM situation [1–6] or models with extra-dimensions [7], see also section 1 D of [8] for a more comprehensive review). Here we are going to pursue another interesting possibility, described in [9]. We assume in this paper that the SM gauge group is embedded in some larger group (for concreteness we consider a left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)$) and the right handed neutrinos are charged under this group:

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \sum_a (W_L^\mu \bar{l}_{aL} \gamma_\mu \nu_{aL} + W_R^\mu \bar{l}_{aR} \gamma_\mu N_{aR}) + \text{h.c.}$$

This additional gauge interaction brings all the right handed neutrinos into the thermal equilibrium at some high temperature. In such a scenario the right-handed neutrinos are expected to be heavy and the active-sterile mixings, which are important for our scenario, are generally expected to be tiny. If one of the sterile neutrinos (called N_1) is light (keV scale), it can be stable enough to

be a DM candidate by a reasonable tuning of the coupling constants. However, being a light thermal relic, one may expect that it is significantly overproduced. This problem is, however, not an issue if there is a later dilution of the overproduced amount of the DM neutrino by a subsequent out-of-equilibrium decay of another massive particle. A good and most natural candidate for such a particle is a heavier right handed neutrino (N_2). If it also has a sufficiently small mixing angle with left handed neutrinos then it freezes out at high temperatures together with N_1 (due to identical gauge interactions). Later it decays to the SM particles while being significantly out of equilibrium, thus diluting the abundance of the DM sterile neutrino N_1 . Models of this type are able to reproduce the experimentally observed DM abundance provided that certain constraints on the model parameters are imposed [9]. In short, there are very strict bounds on the Yukawa couplings of the DM sterile neutrino which are required to suppress its radiative decay constrained by the X-ray observations (see [3] for a review and [10–16] for details) and bounds on the Yukawa couplings of the other sterile neutrino appearing from the requirement of the entropy generation (which controls the DM density). In addition there are bounds on the masses of the sterile neutrinos and additional gauge bosons, appearing from the fact that the entropy generation should happen before the Big Bang Nucleosynthesis (see section 2.7 of [9]).

In addition to providing a DM candidate, any viable extension of the SM should also be able to explain the observed BAU. In models with right-handed sterile neutrinos it is rather natural to expect additional CP -violation in the neutrino sector leading to the generation of a lepton asymmetry (leptogenesis), which later on is transferred to the baryon asymmetry by the sphaleron processes. The simplest possibility here is the usual thermal leptogenesis at high temperatures, associated with CP -asymmetric decays of the heavy sterile neutrinos. The main goal of this paper is to demonstrate that the constraints imposed by the requirement of successful

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dark matter production are compatible with leptogenesis. Thus both the dark matter of the Universe and the BAU can be explained together in a rather minimalistic scenario where heavy sterile neutrinos decay to one light sterile neutrino. We also find additional constraints on the neutrino masses, appearing from the requirement of successful leptogenesis and analyze possible patterns of masses and decay widths of the sterile neutrinos in the model.

II. CONSTRAINTS FROM LEPTOGENESIS

The lepton asymmetry generated in the decay of the heavy neutrino [17, 18] is converted into the BAU by sphaleron processes [19]. This implies that the generation of the lepton asymmetry should happen before sphaleron freezeout ($T \simeq 100$ GeV), and that the sterile neutrinos should be out of thermal equilibrium at higher temperatures. These two requirements are similar to the BBN constraints found in [9], where it was required that the heavy sterile neutrinos decay before the BBN. In complete analogy with [9] we require that sterile neutrino N_2 decays into the SM particles before sphaleron freezeout and generates sufficient entropy to dilute the DM sterile neutrino (N_1) abundance. Replacing the BBN temperature by the sphaleron freezeout temperature in Eq. (20–22) of [9] we find:

$$M_2 > 200 \left(\frac{M_1}{1 \text{ keV}} \right) T_r \sim 2 \times 10^4 \text{ GeV}. \quad (1)$$

In addition, the requirement of efficient entropy production in the decays of N_2 implies that it should freeze out while being still relativistic, $T_f > M_2$. This constraints the strength of the gauge interaction of the right handed neutrinos or, equivalently, the mass M_R of the additional gauge bosons. Although this requirement is similar to those in [9], the lower bound on the neutrino mass M_2 is now larger and the lowest possible value for M_R is higher

$$\frac{M_R}{g_R} > \frac{1}{g_{*f}^{1/8}} \left(\frac{M_2}{2 \times 10^4 \text{ GeV}} \right)^{3/4} 10^4 \text{ TeV}, \quad (2)$$

where g_R is the gauge coupling constant for the additional gauge group. All other constraints formulated in [9] for successful DM generation are left intact.

To achieve proper entropy dilution, the entropy generating neutrino N_2 should decouple while relativistic and has decay width

$$\Gamma_2 \simeq 0.50 \times 10^{-6} \frac{g_{*f}^2}{g_*^2} g_*^{-1/2} \frac{M_2^2}{M_{\text{Pl}}} \left(\frac{1 \text{ keV}}{M_1} \right)^2, \quad (3)$$

which can be used to calculate the Yukawa couplings of N_2 .

III. LEPTON ASYMMETRY

In Friedman-Robertson-Walker Universe the evolution equation for the particle number density in the comoving volume, $Y \equiv \sqrt{-g_3} n$, has the form [20]

$$\frac{dY}{dz} = \frac{\sqrt{-g_3}}{\dot{z}} \int \frac{\hat{C}[f]}{E} d\Omega_p,$$

where $\sqrt{-g_3} = a^3 \propto s$ is the determinant of the spatial part of the metric, s is the entropy density, $z = M/T$ is the dimensionless inverse temperature, \dot{z} is its derivative with respect to the proper time τ and finally \hat{C} the collision term. Under the usual assumption that the distribution function of the Majorana neutrino is proportional to the equilibrium one, the contribution of the decay and inverse decay processes to the lepton asymmetry reads [20]:

$$\int \frac{\hat{C}[f_L]}{E} d\Omega_p = \sum_i \langle \Gamma_{N_i} \rangle [\epsilon_i (n_{N_i} - n_{N_i}^{eq}) - n_L n_{N_i}^{eq} / n_L^{eq}], \quad (4)$$

where ϵ is the CP -violating parameter and $\langle \Gamma_{N_i} \rangle$ is the thermally averaged decay width of the heavy neutrino. The first term in the square brackets describes the generation of the lepton asymmetry and is proportional to the deviation of the Majorana neutrinos from thermal equilibrium. The second term describes washout effects due to the inverse decays of the heavy neutrinos and is proportional to the generated lepton asymmetry n_L . The contribution of the same processes to the right-hand side of the rate equations for the Majorana number density is given by [20]:

$$\int \frac{\hat{C}[f_{N_i}]}{E} d\Omega_p = -\langle \Gamma_{N_i} \rangle (n_{N_i} - n_{N_i}^{eq}),$$

In the model under consideration the right-handed neutrinos are efficiently equilibrated by the $SU(2)_R$ gauge bosons. Before the freezeout of the gauge interactions the right-handed neutrinos are in thermal equilibrium and no asymmetry is generated. After the freezeout the requirement that one of the Majorana neutrinos is dark matter together with the universality of the $SU(2)_R$ gauge interactions implies that all three heavy species are completely out of equilibrium¹ [9]. In this case the washout processes play no role and the second term on

¹ Note that strictly speaking the strong Yukawa interactions could alter the freezeout temperature of the sterile neutrinos. For the dark matter and the entropy generating sterile neutrinos the Yukawas are constrained by the upper bound on their lifetimes. The Yukawa couplings of the third sterile neutrino can in principle be large, which would lead to a decrease of the generated asymmetry.

the right-hand side of Eq.(4) can be neglected. Therefore, the rate equation for the lepton asymmetry takes a simple form:

$$\frac{dY_L}{dz} = - \sum_i \epsilon_i \frac{dY_{N_i}}{dz}.$$

Since the initial lepton asymmetry and the final number densities of $N_{2,3}$ are zero, its solution reads:

$$Y_L^{fin} = \sum_i \epsilon_i Y_{N_i}^{in}. \quad (5)$$

To convert the particle yield (5) into the leptonic asymmetries we should take into account that after the freeze-out at T_f additional entropy was generated by the decays of the heavy neutrinos. Using (5) we get for the lepton number density to the entropy ratio after the decay of $N_{2,3}$

$$\Delta_L \equiv \frac{n_L}{s} = \frac{\epsilon_i Y_{N_i}^{in} s_f a_f^3}{s_f a_f^3 s_e a_e^3} = \frac{\epsilon_i n_{N_i,f}}{S s_f},$$

where n_L and n_N are lepton and sterile neutrino number densities, index f corresponds to the freezeout and index e to some moment after the heavy neutrino has decayed. Assuming that the sterile neutrino decouples while still being relativistic its initial abundance is

$$\frac{n_{N_i,f}}{s_f} = \frac{1}{g_*} \frac{135\zeta(3)}{4\pi^4}.$$

The entropy generation factor S is due to the out of equilibrium decay of heavy sterile neutrino N_i [21]

$$S \simeq 0.76 \frac{g_*^{-1/4} M_i}{g_* \sqrt{\Gamma_i M_{\text{Pl}}}}. \quad (6)$$

Here we would like to stress two points. First, the formula (6) is derived for $S \gg 1$, which is motivated by the fact that $S \sim 10 - 100$ is required to obtain a proper DM density [9]

$$S \simeq 100 \left(\frac{10.75}{g_{*f}} \right) \left(\frac{M_1}{1 \text{ keV}} \right).$$

Second, we have in principle two entropy generating neutrinos. It is easy to check that if the decay rates Γ_2 and Γ_3 are significantly different (while the masses are similar), then the resulting entropy generation is dominated by the neutrino with the smallest decay width, see (6). If both decay widths or entropy generation factors S_1 and S_2 are of the same order the result can be obtained by numerical solution of the differential equations from [21], generalized to the multi-species case.

The generated lepton asymmetry is transferred to the baryon asymmetry by sphalerons with the coefficient $\Delta B = -0.54 \Delta L$ [19, 22–25]. Taking $S \approx \mathcal{O}(10)$ we then find

$$\Delta B \approx -1.5 \times 10^{-4} \epsilon,$$

which is to be compared with the experimentally measured value $\Delta B \approx 0.86 \times 10^{-10}$. Since the washout processes are strongly suppressed in the considered model, successful leptogenesis is possible if $\epsilon \approx -6 \times 10^{-7}$.

The CP -violating parameter in the decay of the i 'th neutrino receives two contributions. The self-energy contribution is given by [26, 27]:

$$\epsilon_i^S = -\eta_{ij} \frac{R_{ij}}{R_{ij}^2 + A_{ij}^2}, \quad R_{ij} \equiv \frac{\Delta M_{ji}^2}{M_i \Gamma_j}, \quad (7)$$

where η_{ij} is defined by $\text{Im}(h^\dagger h)_{ij}^2 \equiv \eta_{ij} (h^\dagger h)_{ii} (h^\dagger h)_{jj}$. For the on-shell quasiparticle approximation that we use here the ‘regulator’ A is given by $A_{ij} = M_i/M_j$, see [27, 28] for more details. The vertex contribution to the CP -violating parameter reads [18, 29]:

$$\epsilon_i^V = \eta_{ij} \frac{\Gamma_j}{M_j} f\left(\frac{M_j^2}{M_i^2}\right),$$

where

$$f(x) = \sqrt{x} \left[1 - (1+x) \ln\left(\frac{1+x}{x}\right) \right] \xrightarrow{x \gg 1} \frac{-1}{2\sqrt{x}}.$$

Since the DM candidate has a mass of order of a few keV, the corresponding CP -violating parameter is strongly suppressed by the mass ratio and its contribution to the lepton asymmetry can be neglected. Furthermore, the effective in-medium masses of the Higgs and leptons are of order of one tenth of the temperature, so that the decays of the lightest Majorana neutrino are kinematically forbidden. Therefore only the two heavier neutrinos $N_{2,3}$ are relevant for the asymmetry generation.

A. Hierarchical mass spectrum

In the considered model the washout processes are strongly suppressed. Therefore the usual argument that the asymmetry generated by the heavier Majorana is washed out by the inverse decay and scattering processes mediated by the lighter one is not applicable. This means that even for a hierarchical mass spectrum we have to consider contributions of both Majorana neutrinos. For a hierarchical mass spectrum the sum of the self-energy and vertex contributions can be approximated by:

$$\begin{aligned} \epsilon_i &\approx -\frac{3}{2} \eta_{ij} \frac{M_i \Gamma_j}{M_j M_j} \quad \text{if } M_i \ll M_j, \\ \epsilon_i &\approx \eta_{ij} \frac{M_j \Gamma_j}{M_i M_j} [2 + \ln M_j^2/M_i^2] \quad \text{if } M_i \gg M_j. \end{aligned} \quad (8)$$

For $M_i \gg M_j$ the $\ln M_j^2/M_i^2$ term is a large negative number and therefore both expressions have the same overall sign. In the following we enumerate the heavy neutrinos such, that the leading contribution to the entropy generation is due to N_2 , i.e. $M_2/\sqrt{\Gamma_2} > M_3/\sqrt{\Gamma_3}$.

Then the requirement of sufficient entropy dilution, see Eq. (3), implies that

$$\frac{\Gamma_2}{M_2} \sim 10^{-5} \frac{M_2}{M_{Pl}}.$$

On the other hand the ratio

$$\frac{\Gamma_3}{M_3} \equiv \frac{\tilde{m}_{33} M_3}{8\pi v^2},$$

where $\tilde{m}_{33} = (hh)_{33} v^2/M_3$ is the see-saw contribution to the effective mass of the active neutrino, is essentially unconstrained. Combined with (8) this implies that for reasonable masses of the right-handed neutrinos ϵ_3 is always strongly suppressed as compared to ϵ_2 . In other words, most of the lepton asymmetry is generated by N_2 . Depending on the choice of the model parameters there are three distinct situations:

- a) If $\Gamma_2 \gg \Gamma_3$ then $M_2 \gg M_3$. Assuming maximal CP -violating phase, i.e. assuming that $\eta_{ij} \sim 1$, we find for the CP -violating parameter:

$$\begin{aligned} \epsilon_2 &\sim \frac{\Gamma_3}{M_2} [2 + \ln M_3^2/M_2^2] \\ &\ll \frac{\Gamma_2}{M_2} [2 + \ln M_3^2/M_2^2] \sim 10^{-5} \frac{M_2}{M_{Pl}}. \end{aligned}$$

Therefore successful leptogenesis is only possible if $M_2 \sim M_{Pl}$ and this case is excluded.

- b) If $\Gamma_3 \sim \Gamma_2$ then $M_2 > M_3$. The resulting expression for the CP -violating parameter is the same as in the previous case and successful leptogenesis is again possible only if $M_2 \sim M_{Pl}$.
- c) If $\Gamma_3 \gg \Gamma_2$ then both $M_2 > M_3$ and $M_2 < M_3$ are possible. In the former case the CP -violating parameter is given by:

$$\begin{aligned} \epsilon_2 &\sim \frac{\Gamma_3}{M_2} [2 + \ln M_3^2/M_2^2] \\ &= \frac{\tilde{m}_{33} M_3}{8\pi v^2} \frac{M_3}{M_2} [2 + \ln M_3^2/M_2^2], \end{aligned}$$

whereas in the latter case we obtain

$$\epsilon_2 \sim \frac{M_2}{M_3} \frac{\Gamma_3}{M_3} \equiv \frac{\tilde{m}_{33} M_3}{8\pi v^2} \frac{M_2}{M_3}.$$

Since the see-saw contributions of the first and second heavy neutrinos should be small, see [9], \tilde{m}_{33} should not contribute significantly to the neutrino masses, i.e. should not exceed the atmospheric mass difference². We can see that if the hierarchy between M_2 and M_3 is mild, the required asymmetry

is produced for $M_{2,3} \sim 10^9$ GeV, which is well below the GUT scale. That is, in this case the CP -violating parameter can be large enough to generate the required amount of the baryon asymmetry. Note also that the gauge interaction scale bound (2), corresponding to this value, is also below the GUT scale, $M_R \gtrsim 10^{10}$ GeV.

It follows from this analysis that for a hierarchical mass spectrum the lepton asymmetry is generated by the same heavy neutrino which is responsible for the entropy production.

B. Quasidegenerate mass spectrum

For a quasidegenerate mass spectrum of the heavy neutrinos the CP -violating parameter is resonantly enhanced. In this regime the vertex contribution is much smaller than the self-energy one and can be neglected. The size of the CP -violating parameter is controlled by the difference of the right-handed neutrino masses, see Eq. (7). If the mass difference is not too small, $R \gtrsim 10^3$, then one can neglect medium corrections to the masses [27]. In this regime the A_{ij} term in (7) can still be neglected and we obtain

$$\epsilon_i^S \approx -\frac{\eta_{ij}}{R_{ij}} \approx -\eta_{ij} \frac{M_i \Gamma_j}{\Delta M_{ji}^2}. \quad (9)$$

Since we enumerate the right-handed Majorana neutrinos such that $M_2/\sqrt{\Gamma_2} > M_3/\sqrt{\Gamma_3}$ and $M_2 \sim M_3$ in the case under consideration then $\Gamma_3 > \Gamma_2$. Combined with (9) this implies that $\epsilon_2 > \epsilon_3$. In other words, just like for a strongly hierarchical mass spectrum, most of the lepton asymmetry is generated by the neutrino responsible for the entropy production. The required magnitude of the CP -violating parameter, $\epsilon \sim 10^{-7}$, can now be achieved even if η_{ij} is as small as $\sim 10^{-4}$. The condition (1) ensures that N_2 decays before the sphaleron freezeout so that the generated lepton asymmetry is converted into the baryon asymmetry. Therefore, successful leptogenesis is possible even if M_2 is as light as $\sim 10^4$ GeV.

For smaller values of R the enhancement of the CP -violating parameter is even stronger. However, one should keep in mind that for very small mass differences the usual quasiparticle approximation used in the present analysis breaks down. On one hand the effect of level crossing makes the sign of the CP -violating parameter temperature-dependent [27]. This leads to a partial cancellation of the asymmetry generated at the earlier and later stages of the Universe's evolution. On the other hand the oscillating behavior of the heavy neutrino propagators leads to a suppression of the final asymmetry [28].

IV. CONCLUSIONS

We have analyzed the generation of the baryon asymmetry of the Universe in scenarios with an extended

² Let us remind the reader that in this model the observed oscillation pattern can not be achieved with the type I see-saw and should follow from some other mechanism, eg. a type II see-saw as anyway expected in left-right symmetric models, see [9]. Then, if \tilde{m}_{33} exceeds the atmospheric mass difference, nontrivial cancellations between the type I and type II contributions are required.

gauge sector where a keV scale sterile neutrino is a DM particle. It was shown in [9] that the DM abundance can be controlled by the entropy produced in the out of equilibrium decay of one of the other two heavier neutrinos. We have found that the decays of the same neutrino can also lead to the generation of a significant lepton asymmetry, leading to the observed baryon asymmetry of the Universe, but at a price of a stronger bound on the extra gauge interaction scale and the heavy sterile neutrino masses. For reasonable values of the model parameters the third heavy neutrino does not contribute significantly to the asymmetry generation. For a hierarchical mass spectrum of the heavy neutrinos the resulting constraints on the masses of the model are stronger than those in [9], pushing the masses of the heavier sterile neutrinos and right handed gauge bosons to the 10^{10} GeV scale. For a quasidegenerate mass spectrum the CP -violating parameter is resonantly enhanced and the required amount

of the asymmetry can be produced even if the Majorana neutrinos are as light as $\sim 10^4$ GeV.

Note that our scenario is incomplete in the sense that some mechanism must explain the lightness of N_1 and the very tiny active-sterile mixings. Examples where these properties are explained by flavour symmetries [30, 31] show that there might be a very interesting connection between dark matter, the baryon asymmetry of the Universe and neutrino flavour properties.

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