

# Asteroseismology of Neutron Stars and Black Holes

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**Abstract.** One of the goals of the large gravitational wave detectors is eventually to observe radiation from oscillations of neutron stars and black holes. These objects have characteristic frequencies of what are called “quasi-normal” mode oscillations, and these frequencies reveal important information about the source. The frequency spectrum of black holes is very different from that of any stars, so if one or more modes are observed then one can conclusively identify the source as a black hole. For neutron stars the spectrum is similar to that of main-sequence stars, but observing a single mode is enough to put strong constraints on the nuclear-matter equation of state, something which is still highly uncertain. Current detectors could make these observations only if the source were exceptionally close. But planned upgrades could make the first relativistic asteroseismological observations; in particular the GEO600 detector will be optimised for these observations by 2010.

## 1. Introduction

Helioseismology has revolutionised the study of the interior structure of the Sun by testing models against an enormous wealth of data on oscillation frequencies. Asteroseismology is on the threshold of making an equally important transformation of our understanding of main-sequence stars. Each of these fields has been able to advance through the development of new technologies, particularly in space-based observatories. In the next decade, a radically different kind of technology may begin to provide asteroseismological information: gravitational wave detection. The oscillations of compact relativistic objects – neutron stars and black holes – emit gravitational waves that carry the signature of their characteristic frequencies. Detecting these waves will for the first time allow us to positively identify a black hole and to see into the interior of a neutron star.

The first detections of these oscillations may provide only sparse information, perhaps just one oscillation mode per object. Yet even this would be enough to provide key information about these objects. Detecting the radiation from a vibrating black hole is the only direct observation we can make of a black hole; all other evidence we have for them is indirect and circumstantial, and always allows the (unlikely) possibility that some exotic form of matter is producing a strong central gravitational field. But, as for stars, the dynamical oscillations of a black hole are its unique signature. And, as we shall see below, the spectrum of oscillations of a black hole is bizarre and would not be mimicked by any material object, no matter how exotic its equation of state.

For neutron stars, observations of the characteristic oscillation frequencies is our best way of getting information about the interior, and especially the equation of state of neutron matter.

Neutron stars are probably the most complex and, to my mind, fascinating objects in nature. From the magnetosphere, created by magnetic fields between  $10^9$  and  $10^{15}$  G spinning at speeds up to  $0.1c$ , inward through the outer crust, made up of exotic neutron-rich nuclei found nowhere else in nature, to the dense interior, where at temperatures of millions of degrees the neutrons are superfluid and the small number of remaining protons are superconducting – a neutron star is an incomparable laboratory for exotic physics. Even the most basic property of neutron matter, its equation of state, remains a matter of considerable uncertainty. We shall see that the observation in gravitational waves of just one characteristic vibration frequency of a neutron star would put strong constraints on the equation of state that could eliminate 90% of existing candidates. More sensitive observations could reveal frequency sub-structure due to rotation, superfluidity, and magnetic fields.

In this paper I will review the physics of oscillations of these objects and current plans for the development of gravitational wave detectors sensitive enough to see these oscillations. For a more in-depth review see [1]. I begin by introducing the principal difference between oscillations of main-sequence stars and compact objects: the effect of gravitational wave damping.

## 2. Quasi-normal modes of neutron stars

The energy emitted in gravitational waves by the oscillations of neutron stars and black holes is so large that it is the main effect responsible for damping the vibration; the characteristic frequencies therefore have real and imaginary parts that can be calculated from models of the objects. To understand the spectrum one has to think about two coupled dynamical systems: the radiating object and the wave field.

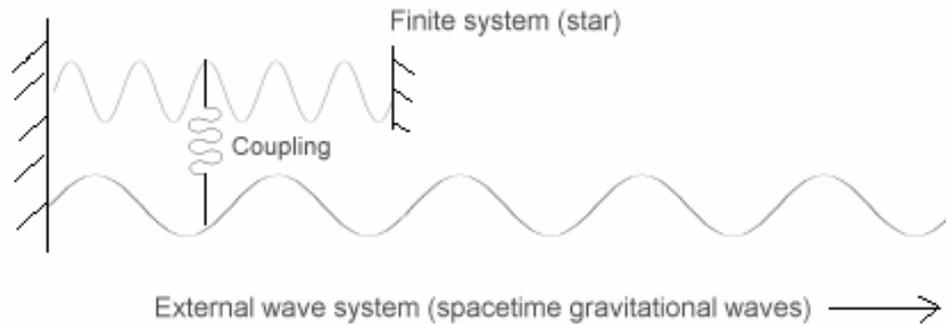
### 2.1. Understanding complex eigenfrequencies

Neutron stars have families of oscillation modes in general relativity that are similar to those of Newtonian gravity. The spherical and non-radial  $\ell = 1$  oscillations of nonrotating stars emit no gravitational waves, and so they are not of interest here. Generally the strongest emission comes from  $\ell = 2$ . For non-radial oscillations of spherical harmonic index  $\ell \geq 2$  there are f-, p-, and g-modes, with similar physical mechanisms as in Newtonian stars. In particular, the f-mode frequency still scales with  $\sqrt{\pi G \rho}$ . However, the eigenfrequencies are modified by the coupling to gravitational waves, and this coupling even creates new families of modes and new instabilities.

To understand the main effect of this coupling, it is helpful to consider a simple dynamical system that duplicates the important features of a neutron star emitting gravitational waves. This model system is illustrated in Figure 1[2]. It consists of two strings, one of finite length (the star) and the other of semi-infinite length (the wave field). These are coupled by a spring, whose stiffness represents how relativistic the system is: the stronger the spring, the more effectively a disturbance on the finite string (the star) is transmitted to the infinite string (the wave field).

To model outgoing radiation the semi-infinite string must have an outgoing-wave boundary condition. This is not a time-symmetric condition and therefore the resulting eigenvalues will not be real. In the absence of coupling, the finite string has the usual modes, analogues of the p-modes of a star. If the coupling is weak, these eigenfrequencies pick up a small imaginary part the leads to damping, as the wave energy runs out along the infinite string. At the same time, the system gains a completely new class of modes, called w-modes[2]. These have most of their amplitude on the infinite string and can be thought of as trapped resonances between the attachment point of the spring and the fixed finite end of the string. These resonances have a weak effect on the finite string and leak energy out along the infinite string, damping away quickly.

It is now known[3] that realistic relativistic stars exhibit not only damped p-modes but also damped w-modes as well. If they are not isentropic then they also have damped g-modes. One further difference to the Newtonian-gravity case is important. An outgoing wave on the



**Figure 1.** A model system that illustrates the coupling of a relativistic star to a gravitational wave field.

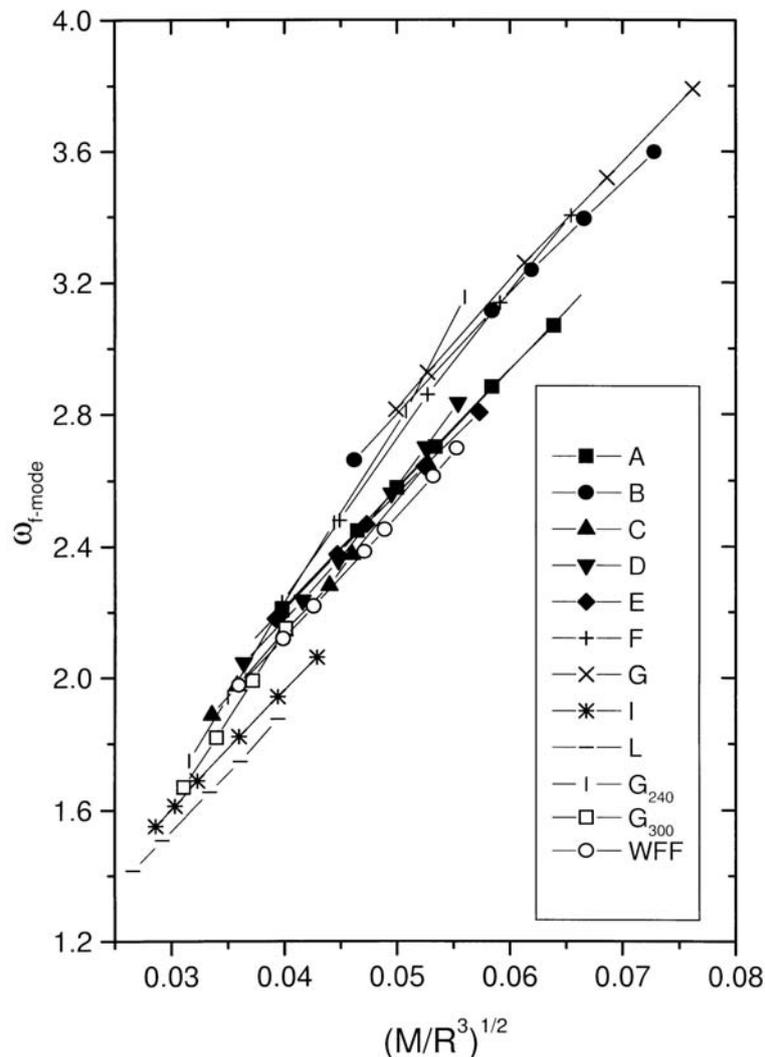
semi-infinite string depends on distance  $x$  and time  $t$  in the combination  $x - ct$ , where  $c$  is the wave speed. If it is damped in time, then for a fixed  $x$  the amplitude of the wave decreases exponentially as  $t$  increases. But because  $x$  and  $t$  enter with opposite sign, it follows that at any fixed time  $t$  the amplitude of the wave *grows* with increasing  $x$ , i.e. as one goes toward infinity. This is sensible: the waves further away represent the oscillations of the system at an earlier time, when the amplitude was larger.

But what this means is that the eigenfunctions do not belong to a simple Hilbert space: they are not square-integrable, and cannot be normalized in amplitude. They are therefore usually called *quasi-normal* modes[4]. Because the eigenfunctions do not belong to a Hilbert space, it has not proved possible yet to demonstrate useful completeness relations: it is not known, for example, if all initial perturbations of a neutron star can be written as superpositions of the quasi-normal modes. In fact, for the wave field this is not very likely. This means that studies of the stability of objects are often difficult: knowing that all the modes are damped, for example, does not by itself establish stability.

## 2.2. Neutron-star asteroseismology

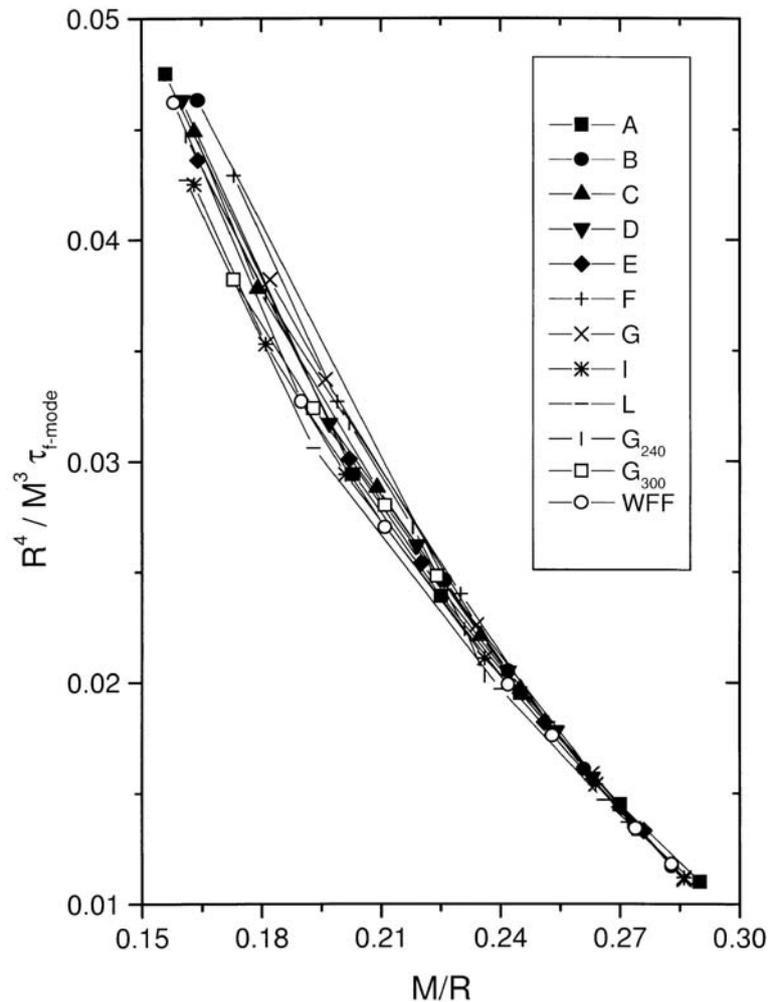
For the purpose of neutron-star asteroseismology, it is important to understand that the real and imaginary parts of the f- and p-mode frequencies exhibit systematic scaling with the mass and radius of the neutron star, which will allow us to make important inferences when these modes are observed. In a remarkable paper, Andersson and Kokkotas[5] showed that, for all proposed equations of state, the frequencies could be scaled in such a way to fall in a very narrow band, illustrated in Figure 2 and Figure 3. If we can observe the frequency and damping time of the f-mode of just one neutron star, then the tight fit in both figures would allow us to infer both the mass  $M$  and radius  $R$  of the star to accuracies of order 10% or better. The importance of this can be seen in Figure 4, which shows the mass-radius curves for the various equations of state used in the previous figures. In this graph, the curves are well separated, so that knowing the mass and radius (from the measurement of just one frequency) is enough to pin down the equation of state with striking accuracy. Since the main uncertainty in the equation of state is the many-body nuclear interaction at high density, this measurement has the potential to provide important clues to this physics.

Although the payoff for observing the characteristic frequencies of neutron stars will be large, it is not clear how long we will have to wait for the first observation. The main uncertainty is how often neutron star modes may be excited into emitting strong radiation. The prime candidates



**Figure 2.** The f-mode frequencies of stars constructed from various equations of state (symbols), plotted as a function of the mean stellar density ( $M$  and  $R$  are in km and  $\omega_{f\text{-mode}}$  in kHz). (Reproduced from Andersson and Kokkotas[5].)

are glitches in young neutron stars, which are commonly seen in pulsars like the Crab and Vela, and X-ray outbursts from soft gamma-ray repeaters, which are accreting neutron stars in our Galaxy. Glitches are sudden changes in the rotation rate of pulsars, which presumably come about because of a rearrangement of the angular momentum within the star, likely between the crust and the core. If this happens on a neutron-star dynamical timescale (milliseconds) then it is very likely that the quadrupolar f-mode will be excited strongly, and the energy available in the glitch would make detection by next-generation instruments possible (see Section 4 below). But if the glitch occurs more adiabatically over several rotation periods, then the f-mode probably remains small and the radiation will be undetectable. Even more energetic are outbursts such as that which occurred in SGR 1900+14 in August 1998, which dramatically increased the ionization level in the Earth's ionosphere. These events are thought to originate in thermonuclear explosions that rapidly spread through accreted hydrogen on the surface of the star. If this starts with a detonation in one place, say under the accretion column, then this could also transfer



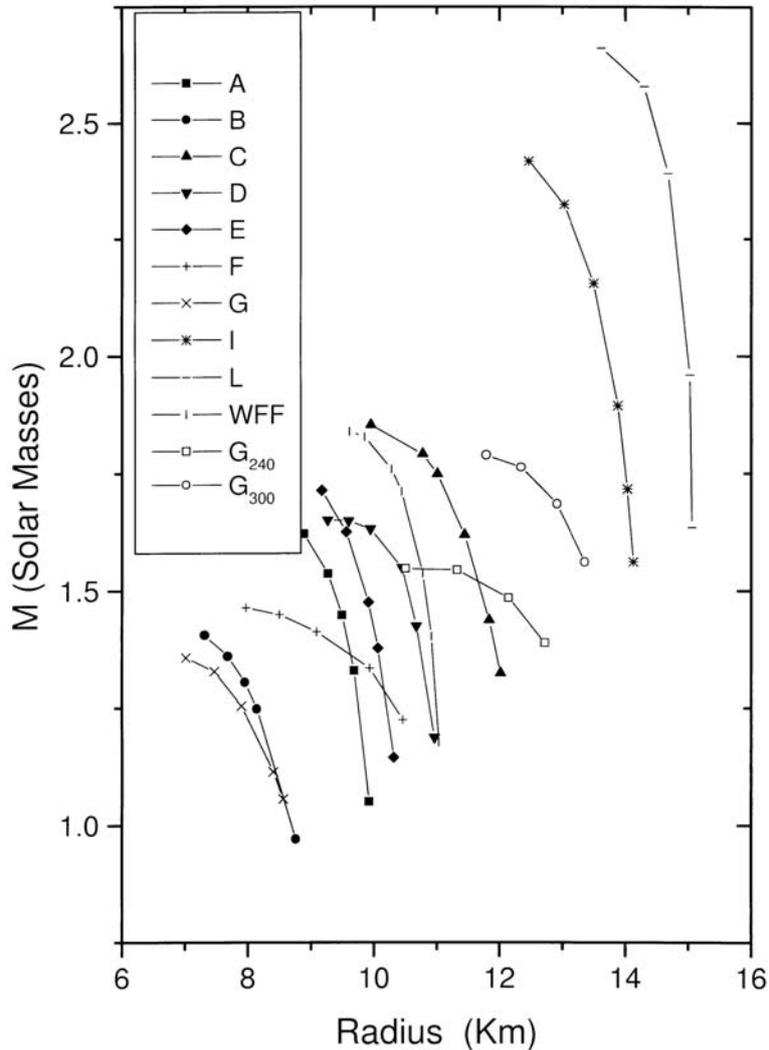
**Figure 3.** The normalized damping time of the f-modes of the previous figure as functions of the stellar compactness ( $M$  and  $R$  are in km and  $\tau_{f\text{-mode}}$  in s). (Reproduced from Andersson and Kokkotas[5].)

considerable energy into the f-mode and make it detectable.

Interestingly, neutron-star modes may already have been observed in X-rays. Analysis of the X-ray emission from the giant outburst of SGR 1900+14 has revealed lower-frequency structure (around 100 Hz) that might be explained by torsional oscillations in the crust of the accreting neutron star, which could shake the magnetic field and produce a visible effect on the burning gas[6]. While these modes do not penetrate the interior of the star and therefore contain little information about neutron-matter nuclear physics, they could reveal much about the neutron-rich nuclei of the crust, and it is possible that their excitation mechanism could, if understood, have something to say about the excitation of the interior modes during this event. In any case, the age of neutron-star asteroseismology has already dawned!

### 2.3. Instabilities driven by gravitational wave emission

Rotating neutron stars have a remarkable property when coupled to gravitational radiation: they are all *unstable*[7, 8], at least within the perfect-fluid approximation. Although the emission of gravitational radiation removes energy from the star, it also removes angular momentum, and



**Figure 4.** The mass-radius relation for the twelve EOS in the previous two figures. If one knows the mass and radius of a star to better than 10% then one can discriminate strongly among these sequences and infer what nuclear physics plays a key role in determining conditions in the interior of neutron stars. (Reproduced from Andersson and Kokkotas[5].)

this allows the perturbation to tap the reservoir of energy contained in the rotation, which is not available within Newtonian gravity because of the conservation of angular momentum. It turns out always to be possible to find a perturbation that gains more energy from rotation than it radiates in gravitational waves, and which therefore grows exponentially. The time-scale for this is normally very long, and realistic levels of viscosity also can damp it out, so the instability exists only in very relativistic situations. One of these is the r-mode instability[9], where Rossby wave modes on a neutron star radiate gravitational waves through gravitomagnetic radiation (not the standard quadrupole radiation) and can grow on very short timescales. This might be responsible for the apparent difficulty of spinning up accreting neutron stars beyond about 600 Hz in low-mass X-ray binary systems[10], the birthplace of the millisecond pulsars. As originally suggested by Wagoner[11], gravitational wave emission could remove enough angular momentum to balance the accreted angular momentum, leading to a steady rotation rate. While there are

several possible ways this might happen in an accreting neutron star, the r-mode instability is a prime candidate.

### 3. Quasi-normal modes of black holes

Black holes do not form gradually: they result either from a turbulent gravitational collapse or from the collision and merger of two other compact objects, either neutron stars or black holes. These events are probably accompanied by the emission of strong gravitational radiation, whose waveform will reflect the dynamics of the formation event. But numerical simulations show that the radiation quickly becomes dominated by quasi-normal oscillations of the final black hole, which damp away after a few cycles. By observing these few cycles, we can do asteroseismology on the black hole, which is sometimes called *bothroseismology*.

#### 3.1. Spectrum of black hole oscillations

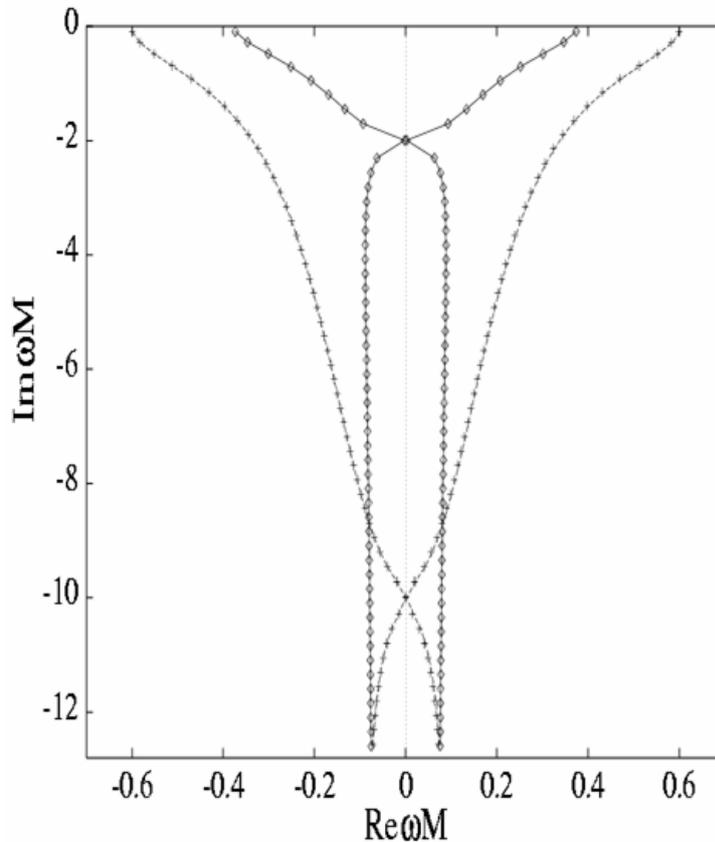
Black holes are not like stars: they do not have an interior through which oscillations move and form resonances. Any disturbance that moves toward the inside of a black hole just keeps going and will never be heard from again. The oscillations of a black hole, therefore, are a property of the geometry surrounding it. Spacetime in general relativity is a dynamical entity, so when the spacetime containing a black hole is disturbed, it vibrates. Some of these vibrations are trapped for a while in the deep gravitational well surrounding the hole, and resonate much like the w-modes of a neutron star. But unlike for a neutron star, the resonance is not set up by waves crossing the object; instead, the waves seem to orbit the horizon for a short time while leaking out.

The result is a bizarre spectrum, shown in Figreffig:bhspec for a Schwarzschild black hole[12]. Along the sequence it is the *imaginary* part of the frequency that keeps increasing, while the real part asymptotes to the constant value  $M\omega = 0.437$  for any value of  $\ell$ [13]. There are even modes that have zero real part, so that they damp away without oscillating at all.

There has been less work on the spectrum of modes of a Kerr black hole, because the equations are more complicated to handle. But one important result is known: the modes are all damped. This is not obvious: one might expect that the rotational energy of the Kerr metric could be tapped in the same way as that of a rotating star can be, to supply more energy to the mode than it radiates to infinity. But apparently this does not happen: the horizon absorbs enough of this energy to ensure that no mode of the Kerr metric grows exponentially[14]. But it is still an open question whether some modes might grow linearly with time, and it is also not known whether the Kerr metric is stable against all perturbations, because there is (as remarked in Section 2 above) no completeness proof for quasi-normal modes. The absence of exponentially growing modes does, however, give confidence that Kerr is in fact stable, even if it has not been possible to prove it yet. There is a full proof of stability for Schwarzschild[15].

#### 3.2. Asteroseismology of black holes

It is clear that if gravitational waves are detected from a disturbed or recently formed black hole, and if there is enough sensitivity to detect a few of the quasi-normal modes in the decaying tail of the radiation, then the frequencies will not be explainable in terms of any material object: even an exotic equation of state will lead to eigenfrequencies that arise from waves travelling across the object, and which therefore will form a spectrum in which the real part of the frequency steadily increases. In contrast, the higher modes of the black hole have progressively smaller real parts. The observation of these low-order modes is likely to be the first way in which we can demonstrate beyond doubt that black holes really exist. In addition, the numerical values of the frequencies will allow us to measure the mass and angular momentum of the emitting hole.



**Figure 5.** The spectrum of quasi-normal modes for a Schwarzschild black-hole, for  $\ell = 2$  (diamonds) and  $\ell = 3$  (crosses). The 9<sup>th</sup> mode for  $\ell = 2$  and the 41<sup>st</sup> for  $\ell = 3$  are exceptional because the real part of the frequency is exactly zero. (Reproduced from Andersson and Linnaeus[12].)

#### 4. Gravitational wave detectors

Gravitational wave detectors have been under development for many decades, but have only recently reached a sensitivity where it would be possible, even though still unlikely, to see gravitational wave bursts from coalescing neutron stars or nearby supernova explosions. But upgrades to existing detectors that are already planned and funded will change that, and within less than a decade we will have instruments capable of doing asteroseismology on neutron stars and black holes.

The best current detectors are laser interferometers of the LIGO[16], VIRGO[17], and GEO[18] projects. These operate together, pooling data and doing data analysis jointly. Before mid-May 2007 only LIGO and GEO were operating in science mode, and as the LIGO Scientific Collaboration this group has published a number of upper limits and technical papers. Data from the most recent science run, which lasted almost 2 years, is still be analysed but the first papers are expected to be put on preprint servers before the end of 2007. This run is notable because the LIGO detectors have reached their first-level design sensitivity. This has demonstrated their ability to master the technology and has released funding for a series of upgrades in sensitivity. At its present sensitivity LIGO can reach out approximately to the Virgo Cluster for coalescences of binary neutron stars. While this volume of space contains thousands of galaxies, there is still

much less than 1% probability per year that such an event will occur in this volume of space. Similarly low event rates are expected for detectable black-hole coalescences and gravitational collapse.

Regarding asteroseismology, the best sensitivity of the LIGO and VIRGO detectors is around 100 Hz. This is far below the 2 kHz typical of the f-mode of a neutron star, and is near the fundamental black hole oscillation frequency only for masses around  $800M_{\odot}$ . While these detectors can detect radiation up to and above 1 kHz, their sensitivity gets progressively poorer as the frequency increases.

Right now (end of 2007) the LIGO and VIRGO detectors are performing the first of two planned upgrades. This first step, to so-called enhanced detectors, will increase their sensitivity (and range) by a factor of two. This may well give them enough sensitivity to see the coalescence of a pair of  $20M_{\odot}$  black holes at a very large distance, and this in turn could lead to the first bothroseismology observations. But it is by no means sure that this factor of two will be enough to produce a detection within the planned two-year observing run, let alone begin to do gravitational wave astronomy. Therefore a further upgrade is planned, with a bigger jump in sensitivity.

While LIGO and VIRGO are doing the enhancement upgrade, GEO and one of the LIGO detectors, joined occasionally by VIRGO, will observe in “astrowatch” mode, which means making sure that at least one detector is observing at any time, just in case a nearby supernova explodes. But after LIGO and VIRGO begin observing in enhanced mode, GEO will begin its own upgrade, to GEO-HF, a detector configuration aimed specifically at the higher frequencies at which the characteristic frequencies of compact objects in the Galaxy may be observed[19]. Not only will it have better broad-band sensitivity than the present GEO600 detector, but it will be configurable, so that it can operate with greatly increased sensitivity in a narrower band, say around 1.5 kHz. With a bit of luck, GEO-HF will open the field of neutron-star asteroseismology after it begins operating around 2010.

After that, around 2013, the final upgrades of LIGO and VIRGO will begin observing, called the advanced detectors. These will have as good sensitivity as GEO-HF in the high-frequency regime, and be configurable as well. An additional detector in Japan, with excellent low-frequency sensitivity, may join them[20]. They are sure to observe neutron-star coalescences, perhaps in coincidence with gamma-ray bursts, and they are likely to see many black-hole coalescences and their associated ringdown radiation. The neutron-star mergers will themselves form black holes, providing an additional opportunity to do bothroseismology. And as they accumulate observing time over several years, there is an excellent chance that a neutron-star glitch or an SGR will provide that unique insight into the interior of neutron stars: the frequency of the quadrupole f-mode.

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