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Black holes surrounded by uniformly rotating rings

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This paper provides a brief summary of a talk on rings surrounding Black Holes that was given at the spring meeting 2005 of the German physical society (DPG). A detailed discussion of the topics covered in the talk can be found in [1].

Motivation

Black Holes have grown to become a well accepted member of the canon of astrophysical objects. Nonetheless, there is still very little known about spacetimes containing both matter and Black Holes. But for a few exceptions (e.g. the Oppenheimer-Snyder collapse), such situations have been considered using approximation methods, perturbation techniques or pioneering, but still somewhat inaccurate numerical codes [2–11]. Here a numerical method is introduced that can handle the rather simple model of a uniformly rotating fluid ring around a Black Hole in axially symmetric, stationary spacetimes with extremely high accuracy. Such configurations are often proposed as probable sources of gamma ray bursts, have been seen as an intermediate stage in the collapse of stars and can be used to model the central regions in galaxies [12–14]. Together with the opportunity to study the effect of matter on the properties of Black Holes, the calculation of these configurations to extremely high accuracy will allow one to address many interesting questions and provide initial data for time evolution codes.

Numerical methods

The coordinates in which we choose to formulate the problem of the ring surrounding a Black Hole, the boundary conditions describing the Black Hole as well as the Einstein equations themselves are all discussed in detail in [15]. The coordinates are tailored to the symmetries, thus resulting in a two-dimensional problem, and are chosen such that the event horizon is a coordinate sphere. What results is an elliptic free-boundary problem (the shape of the ring is unknown), which is solved by using a multi-domain quasi spectral method in which the infinite two-dimensional space of interest is compactified onto a small number of squares. Two of the domain boundaries coincide with those of the ring and the Black Hole and the method is shown to converge to machine accuracy for typical configurations. The details of the methods used as well as a discussion of various issues specific to the given situation can be found in [1].

Results

Before turning our attention to Black Holes surrounded by rings, we briefly discuss (self-gravitating) rings revolving around a point mass in Newtonian theory. Not only is this limiting case important in its own right,

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it also turned out to be a crucial step in constructing the necessary initial guess for “igniting” the relativistic code.

After choosing an equation of state to describe the matter in the fluid, a ring surrounding a point mass in Newtonian theory is characterized by three parameters. In this paper, we concentrate on a particularly simple equation of state and discuss homogeneous rings. If the total mass of the system and the ratio of the inner to the outer radius of the ring are held constant, then a sequence of configurations results, which can be parameterized by the ratio of the mass of the central object to that of the ring. When this ratio tends to zero, we recover the Dyson rings discussed in [16–22]. As the mass ratio increases, the inner edge of the ring becomes increasingly sharp, until an inner mass-shed is reached. If the inner mass-shed sequence is again parameterized by mass ratio, then one finds for a vanishing point mass configuration ‘(H)’ of Fig. 6 in [22], which marks the transition from toroidal to spheroidal topologies. As the mass ratio tends to infinity, the ratio of inner to outer radius of the ring is forced to tend to one (i.e. the ring becomes infinitely thin).

An inner mass-shed turns out to be a typical feature in the relativistic situation as well. This marks a bound from above for the ratio of the mass of the Black Hole to that of the ring. The lower limit for vanishing Black Holes is much more interesting than for Newtonian point masses however, since the Black Hole has structure whereas a point mass does not. To be more specific, properties of the Black Hole can be defined via the behaviour of the metric functions and their derivatives on the horizon and a comparison with the Kerr metric enables one to talk about changes brought about by the influence of matter. It turns out that the ratio of the proper polar to equatorial circumference for the horizon of a Black Hole with zero angular momentum surrounded by a ring of finite extent tends to one in this limit. An analogous statement cannot be made for every limit of vanishing central mass, however.

As a proof of principle, a configuration was calculated for which the ratio of the Black Hole’s angular momentum to the square of its mass exceeded one. A value of 20/19 was chosen for this example and machine accuracy was reached.

References

[1] M. Ansorg and D. Petroff, Phys. Rev. D **72**, 024019 (2005).
[2] C. M. Will, Astrophys. J. **191**, 521 (1974).
[3] C. M. Will, Astrophys. J.**196**, 41 (1975).
[4] M. A. Abramowicz, M. Calvani, and L. Nobili, Nature **302**, 597 (1983).
[5] M. A. Abramowicz, A. Curir, A. Schwarzenberg-Czerny, et al., Mon. Not. R. Astron. Soc. **208**, 279 (1984).
[6] G. Bodo and A. Curir, Astron. Astrophys. **253**, 318 (1992).
[7] L. Rezzolla, S. Yoshida, and O. Zanotti, Mon. Not. R. Astron. Soc. **344**, 978 (2003).
[8] P. J. Montero, L. Rezzolla, and S. Yoshida, Mon. Not. R. Astron. Soc. **354**, 1040 (2004).
[9] O. Zanotti, J. A. Font, L. Rezzolla, et al., Mon. Not. R. Astron. Soc. **356**, 1371 (2005).
[10] A. Lanza, Astrophys. J. **389**, 141 (1992).
[11] S. Nishida and Y. Eriguchi, Astrophys. J. **427**, 429 (1994).
[12] N. L. Shapiro, Astrophys. J. **444**, 306 (1995).
[13] M. Shibata, K. Taniguchi and K. Uryu, Phys. Rev. D **68**, 084020 (2003).
[14] M. Ruffert and H.-T. Janka, Astron. Astrophys. **344**, 573 (1999).
[15] J. M. Bardeen, in: Black Holes, Les astres occlus, pp. 241–289, edited by C. DeWitt and B. DeWitt (Gordon and Breach Science Publishers, New York, 1973).
[16] H. Poincaré, Acta mathematica **7**, 259 (1885).
[17] S. Kowalewsky, Astronomische Nachrichten **111**, 37 (1885).
[18] F. W. Dyson, Philos. Trans. R. Soc. London, Ser. A **184**, 43 (1892).
[19] F. W. Dyson, Philos. Trans. R. Soc. London, Ser. A **184**, 1041 (1893).
[20] C. Y. Wong, Astrophys. J. **190**, 675 (1974).
[21] Y. Eriguchi and D. Sugimoto, Prog. Theor. Phys. **65**, 1870 (1981).
[22] M. Ansorg, A. Kleinwächter, and R. Meinel, Mon. Not. R. Astron. Soc. **339**, 515 (2003).

