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QUANTUM GEOMETRY AND ITS IMPLICATIONS FOR BLACK HOLES*

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General relativity successfully describes space–times at scales that we can observe and probe today, but it cannot be complete as a consequence of singularity theorems. For a long time, there have been indications that quantum gravity will provide a more complete, non-singular extension which, however, was difficult to verify in the absence of a quantum theory of gravity. By now there are several candidates which show essential hints as to what a quantum theory of gravity may look like. In particular, loop quantum gravity is a non-perturbative formulation which is background independent, two properties which are essentially close to a classical singularity with strong fields and a degenerate metric. In cosmological and black hole settings, one can indeed see explicitly how classical singularities are removed by quantum geometry: there is a well-defined evolution all the way down to, and across, the smallest scales. As for black holes, their horizon dynamics can be studied showing characteristic modifications to the classical behavior. Conceptual and physical issues can also be addressed in this context, providing lessons for quantum gravity in general. Here, we conclude with some comments on the uniqueness issue often linked to quantum gravity in some form or another.

Keywords: Quantum gravity; black holes; singularities; horizons; uniqueness.

1. General Relativity

General relativity successfully describes the gravitational field, in terms of space–time geometry, on large scales. However, on small scales it is hardly being probed by direct observations. Nevertheless, there are indirect indications, and all of them point to its failure: singularity theorems imply not only infinite densities but even a complete breakdown of evolution after a finite amount of proper time for most realistic solutions.[1] The best known examples are cosmological situations and black holes.

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Traditionally, such a breakdown of evolution has been interpreted as corresponding to the beginning, or end, of the universe. In a situation like this, it is instructive to remember what is probably best expressed by the quote[2] "The limits of my language mean the limits of my world." Currently, our best language to speak about the universe is general relativity, but it clearly has its limitations. The description it provides of the world is incomplete when curvature becomes large, and such limitations should not be mistaken for the limits of the actual world it is supposed to describe.

Such strong curvature regimes are reached when relevant length scales become small such as in black holes or the early universe. One can understand the failure of general relativity as a consequence of extrapolating the well-known long-distance behavior of gravity to unprobed small scales. According to general relativity, the gravitational force is always attractive such that, at some point, there will be nothing to prevent the total collapse of a region in space–time to a black hole, or of the whole universe.

Moreover, this is a purely classical formulation, and further expected modifications arise on small scales when they approach the Planck length $\ell_{\mathrm{P}} = \sqrt{8\pi G\hbar} \approx 10^{-35}$ m. Possible spatial discreteness, also an expectation from well-known quantum properties, would imply a radically different underlying geometry. These lessons from quantum mechanics can be made more precise, although they remain at a heuristic level until they are confirmed by a theory of quantum gravity (see, e.g., Ref. 3).

First, the hydrogen atom is known to be unstable classically since the electron falls into the nucleus in a finite amount of time, a situation not unlike that of the collapse of a universe into a singularity. Upon quantization, however, the atom acquires a finite ground state energy and thus becomes stable. The value, $E_0 = -\frac{1}{2}me^4/\hbar^2 \stackrel{\hbar\to 0}{\longrightarrow} -\infty$, also shows the importance of quantization and the necessity of keeping the Planck constant non-zero. In quantum gravity, one can expect on similar dimensional grounds that densities are bounded by an inverse power of the Planck length such as $\ell_{\mathrm{P}}^{-3} \stackrel{\hbar\to 0}{\longrightarrow} \infty$ which again diverges in the classical limit but is finite in quantum gravity.

The second indication comes from black body radiation where the classical Rayleigh–Jeans law, which would imply diverging total energy, is modified by quantum effects on small (wavelength) scales. The resulting formula for its spectral energy density, due to Planck, gives a finite total energy.

This observation gives us an indication as to what could happen when one successfully combines general relativity with quantum theory. According to Einstein's equations, matter energy density back-reacts on geometry. If matter energy behaves differently on small scales due to quantum effects, the classical attractive nature of gravity may change to repulsion. In addition, also the non-linear gravitational interaction itself, without considering matter, can change.

All those expectations, some of them long-standing, have to be verified in explicit calculations which requires a *non-perturbative* (due to strong fields) and *background* *independent* (due to degenerate geometry) framework of quantum gravity. Nonperturbativity can be implemented by adopting a canonical quantization, which when done in Ashtekar variables[4,5] also allows one to deal with background independence: there are natural smearings of the basic fields as holonomies and fluxes which do not require the introduction of a background metric and nonetheless result in a well-defined kinematical algebra.[6,7]

2. Loop Quantum Gravity

Instead of using the spatial metric and extrinsic curvature for the canonical formulation following Arnowitt *et al.*,[8] the Ashtekar formulation[4,5] expresses general relativity as a constrained gauge theory with connection $A_a^i = \Gamma_a^i + \gamma K_a^i$ and its momenta given by a densitized triad E_i^a expressing spatial geometry. In the connection, Γ_a^i is the spin connection compatible with the densitized triad and thus a function of E_a^i, while K_a^i is extrinsic curvature. The Barbero–Immirzi parameter[5,9] $\gamma > 0$ is free to choose and does not have classical implications.

2.1. *Representation*

These basic fields (A_a^i, E_j^b) must now be smeared in order to obtain a well-defined Poisson algebra (without delta functions) suitable for quantization. However, the common three-dimensional smearing of all basic fields is not possible because the spatial metric is now dynamical and there is no other background metric. Fortunately, Ashtekar variables allow a natural smearing without background geometry by using

$$\begin{aligned} &\text{holonomies } h_e(A) = P\exp\int_e \tau_i A_a^i \dot{e}^a dt \quad \text{for edges } e,\\ &\text{and fluxes } F_S(E) = \int_S \tau^i E_i^a n_a d^2y \quad \text{for surfaces } S, \end{aligned} \tag{1}$$

as smeared objects. Indeed, their Poisson algebra closes with well-defined structure constants. Edges e and surfaces S arising in this definition play the role of labels of the basic objects and chosen freely in a given 3-manifold Σ. Alternatively, edges and surfaces can be introduced as abstract sets with a relation showing their intersection behavior, which determines the structure constants of the algebra.

This algebra can now be represented on a Hilbert space to define a quantum theory. Requiring diffeomorphism covariance of the representation, i.e. the existence of a unitary action of the diffeomorphism group, even fixes essentially the available representation to that used in loop quantum gravity.[10,11] It is most easily constructed in terms of states being functionals of the connection such that holonomies become basic multiplication operators. Starting with the simplest possible state which does not depend on the connection at all, holonomies "create" spin network states[12] upon action. Most generally, states are then of the form $T_{g,j,C}(A) = \prod_{v\in g} C_v \cdot \prod_{e\in g} \rho_{j_e}(h_e(A))$ where g is an oriented graph formed by the edges e used in multiplication operators. Each edge carries a label j corresponding to an irreducible SU(2) representation and arising from the fact that the same edge can be used several times in holonomies. Finally, C_v are contraction matrices to multiply the matrix elements of $\rho_{j_e}(h_e(A))$ for edges containing the vertex v to a complex number. These contraction matrices can be chosen such that the state becomes invariant under local SU(2) gauge transformations.

2.2. *Discrete geometry*

Flux operators can be derived using the fact that they are conjugate to holonomies, and thus become derivative operators on states. Replacing triad components by functional derivatives and using the chain rule, we obtain

$$\begin{aligned}\hat{F}_S f_g &= -i\gamma\ell_{\mathrm{P}}^2 \int_S d^2y \tau^i n_a \frac{\delta}{\delta A_a^i(y)} f_g(h(A)) \\ &= -i\gamma\ell_{\mathrm{P}}^2 \sum_{e\in g} \int_S d^2y \tau^i n_a \frac{\delta h_e}{\delta A_a^i(y)} \frac{df_g(h)}{dh_e}, \end{aligned} \tag{2}$$

with non-zero contributions only if S intersects the edges of g. Moreover, each such contribution is given by the action of an SU(2) derivative operator with discrete spectrum. The whole spectrum of flux operators is then discrete implying, since fluxes encode spatial geometry, discrete spatial quantum geometry. This translates to discrete spectra also of more familiar spatial geometric expressions such as area or volume.[13–15]

2.3. *Dynamics*

It does, however, not directly imply discrete space–time geometry since this requires dynamical information encoded, in a canonical formulation, in the Hamiltonian constraint. There are classes of well-defined operators for this constraint,[16,17] which usually change the graph of the state they act on. Their action is therefore quite complicated in full generality, not unexpectedly so for an object encoding the quantized dynamical behavior of general relativity.

As usually, symmetries can be used to obtain simpler expressions while still allowing access to the most interesting phenomena in gravity such as cosmology or black holes. In loop quantum gravity with its well-developed mathematical techniques,[7] moreover, such symmetries can be imposed *at the quantum level*[18] by inducing a reduced quantum representation from the unique one in the full theory. This is particularly useful because in symmetric situations usually no uniqueness theorems hold, for instance in homogeneous models where the widely used Wheeler–DeWitt representation is inequivalent to the representation arising from loop quantum gravity. The loop representation in such models is then distinguished by its relation to the unique representation of the full theory. On the loop representation one can then construct more complicated operators such as the Hamiltonian constraint in analogy to the full construction. Often, the operators simplify considerably and can sometimes be used in explicit calculations.

2.4. *Loop quantum cosmology*

Following this procedure in cosmological situations of homogeneous spatial slices, the usual Wheeler–DeWitt equation[19] is replaced by a *non-singular* difference equation.[20–23] For this equation, the wave function of a universe model is uniquely defined once initial conditions are imposed at large volume. In particular, the difference equation continues to determine the wave function even at and beyond places where classical singularities would occur and also the Wheeler–DeWitt equation would stop. For a mathematical discussion of properties of the resulting difference equations, see. Ref. 24.

As in the full setting, there are currently different versions (such as symmetric and non-symmetric orderings, or other ambiguity parameters) resulting in non-singular behavior. Some versions, however, are singular which means that ambiguities are already restricted by ruling them out. Nevertheless, some aspects can change also between different non-singular versions, such as the issue of *dynamical initial conditions*[25,26] which are consistency conditions for wave functions provided by the dynamical law rather than being imposed by hand. They arise with stronger restrictions on wave functions in a non-symmetric ordering compared with a symmetric one. Such ambiguities in simple models have to be constrained by studying more complicated situations (see, e.g., Refs. 27–29). Through this procedure, the theory becomes testable because it is highly non-trivial that one and the same mechanism (including, e.g., the same ordering choice) applies to all situations.

Intuitively, the behavior can be interpreted as giving a well-defined evolution to a new branch *preceding the big bang*; see Ref. 3 for a general discussion. This new branch is provided by a new, binary degree of freedom given by the orientation of the triad. We are naturally led to this freedom since triads occur in the background independent formulation. It is precisely the orientation change in triad components which presents us with two sides to classical singularities. Unlike the Wheeler–DeWitt equation which abruptly cuts off the wave function at vanishing metric components, loop quantum cosmology has an equation connecting the wave function on both sides of a classical singularity. This is a very general statement and applies to *all* possible initial conditions at large volume, compatible with potential dynamical initial conditions. It is thus independent of complicated issues such as which of the solutions to the difference equation are normalizable in a physical inner product (see Ref. 30 for more details).

For more precise information on the structure of classical singularities in quantum gravity, however, one needs additional constructions. For instance, the evolution of semiclassical states can be studied in detail in some special models such as a flat isotropic model coupled to a free, massless scalar.[31–33] Here, the classical singularity turns out to be replaced by a bounce at small scales, connecting two semiclassical phases at larger volume. It remains open, however, how general this scenario is when potentials or anisotropies are included.

When a physical inner product or precise semiclassical states are unavailable, one can make use of effective equations of the classical type which are ordinary differential equations in coordinate time but incorporate some prominent quantum effects. They have been introduced in Ref. 34 and applied in the context of bounces in Refs. 35–37. Recently, it has been shown, using a geometrical formulation of quantum mechanics,[38,39] that this is part of a general scheme which agrees with effective action techniques common from quantum field theory where both approaches can be applied.[40,41] Thus, these equations allow an effective analysis of quantum theories in the usual sense. Also those equations show bounces, sometimes even in semiclassical regimes, but not generically. Effects can depend on quantization choices as well as on which quantum corrections are included. There are different types of corrections which in general are all mixed without any one being clearly dominant. A full study, including all possible quantum correction terms, is complicated and still to be done.[41]

All these bounce scenarios can be seen intuitively as confirmation of our expectation that quantum gravity should contribute a repulsive component to the gravitational force on small scales. Such repulsion can stop the collapse of a universe and turn it into a bounce, after which the weakening repulsion will contribute to accelerated expansion in an inflationary scenario.[34]

3. Black Holes

Unlike cosmological models, black holes require inhomogeneous situations. There are currently several techniques to get hints for the resulting behavior and for typical quantum effects in the physics of black holes.[42]

3.1. *The Kantowski–Sachs model*

Inside the horizon, the Schwarzschild solution is homogeneous because the Killing vector field which is timelike in the static outside region turns spacelike. With the three rotational Killing vectors this combines to a four-dimensional symmetry group corresponding to Kantowski–Sachs models. Densitized triads with this symmetry can be written as (see Refs. 43 and 44 for details on this part)

$$E = p_c\tau_3 \sin\vartheta \frac{\partial}{\partial x} + p_b\tau_2 \sin\vartheta \frac{\partial}{\partial\vartheta} - p_b\tau_1 \frac{\partial}{\partial\varphi} \tag{3}$$

such that $\det E = p_c p_b^2$ and orientation, which is important for the singularity structure, is given by $\operatorname{sgn} p_c$. The sign of p_b is not relevant as there is a residual gauge transformation $p_b \mapsto -p_b$. From such a densitized triad, the spatial metric

$$ds^2 = \frac{p_b^2}{|p_c|} dx^2 + |p_c| d\Omega^2 \tag{4}$$

results. Comparison with the interior Schwarzschild metric suggests the following identification between space–time and minisuperspace locations: the *Schwarzschild singularity* at $p_c = 0$ and the *horizon* at $p_b = 0$.

When quantized, the densitized triad components become operators

$$\hat{p}_b|\mu,\nu\rangle = \frac{1}{2}\gamma\ell_{\mathrm{P}}^2\mu|\mu,\nu\rangle, \quad \hat{p}_c|\mu,\nu\rangle = \gamma\ell_{\mathrm{P}}^2\nu|\mu,\nu\rangle, \tag{5}$$

acting on orthonormal states $|\mu,\nu\rangle$ with $\mu,\nu \in \mathbb{R}$, $\mu \geq 0$. This Hilbert space is the analog of the spin network representation in the full theory, although most labels disappeared, thanks to the high symmetry. Also analogously to the full theory one can construct the Hamiltonian constraint operator which gives rise to the dynamical law

$$\begin{aligned} &2(\sqrt{|\nu+2|}+\sqrt{|\nu|})\,(\psi_{\mu+2,\nu+2}-\psi_{\mu-2,\nu+2})+(\sqrt{|\nu+1|}-\sqrt{|\nu-1|})\\ &\quad\times\left((\mu+2)\psi_{\mu+4,\nu}-(1+2\gamma^2)\mu\psi_{\mu,\nu}+(\mu-2)\psi_{\mu-4,\nu}\right)\\ &\quad+2(\sqrt{|\nu-2|}+\sqrt{|\nu|})\,(\psi_{\mu-2,\nu-2}-\psi_{\mu+2,\nu-2})=0 \end{aligned} \tag{6}$$

as a difference equation for the wave function depending on the triad components. This is singularity free as in the isotropic case, extending the wave function beyond the classical singularity. The situation is more complicated, however, because the classical minisuperspace now has two boundaries, one corresponding to the horizon at $p_b = 0$ and one at the singularity corresponding to $p_c = 0$. But only one direction can be extended given only one sign factor from orientation. Thus, the system provides an interesting consistency check of the general scheme by determining which boundary, the singularity or the horizon, is removed upon quantization. The horizon boundary should not be removed because at this place our minisuperspace approximation breaks down. Indeed it is just the classically singular boundary which is removed by including the sign of p_c in the analysis, providing a non-trivial test of the singularity removal mechanism of loop quantum cosmology. This rests crucially on the use of densitized triad variables which we are led to naturally in a full background independent formulation. While models also allow quantizations in terms of other variables, e.g. using co-triads or metrics,[45] classical singularities appear at different places of minisuperspace and general schemes of singularity removal do then not exist.

3.2. *Evaporation*

By extrapolating the extension of the interior Schwarzschild geometry through the classical singularity to dynamical situations in the presence of matter one can arrive at a new paradigm for *black hole evaporation*[46] founded on loop quantum gravity.

As illustrated in Fig. 1, the quantum region around the classical black hole singularity is not a future boundary of space–time. Correlations between infalling matter components are then not destroyed during evaporation. Instead, matter will be able to leave the black hole region, defined by the presence of trapped surfaces

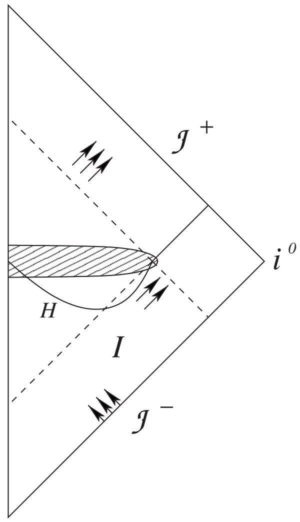

Fig. 1. Evaporating non-singular black hole. The hatched quantum region replaces the classical singularity such that infalling matter can propagate through it and re-emerge later. The horizon H evaporates due to Hawking radiation.

and a horizon H,[47–50] after the evaporation process restoring all correlations. There are thus trapped surfaces which as in the usual singularity theorems of general relativity implies geodesic incompleteness. But here, this does not lead to a singularity; rather the space–time continuum is replaced by a discrete, quantum geometrical structure. Only classical concepts break down but not the quantum gravitational description. There is then also no need to continue the horizon H beyond the point where it meets the strong quantum region, although there can well be past trapped surfaces in the future of the quantum regime. In this sense, the picture is similar to closed horizons enclosing a bounded space–time region which have been suggested earlier.[51–53]

There are several underlying assumptions for this scenario which are now being tested. For instance, for this picture space–time has to become semiclassical again after evolving through the quantum regime where discrete geometry is essential. Otherwise, there will be no asymptotically flat future space–time to detect remaining correlations. In the Schwarzschild interior discussed above, one can verify that solutions to the difference equation have to be symmetric under $\mu \mapsto -\mu$ as a consequence of consistency conditions in the recurrence scheme.[54] Note that this is not a condition imposed on solutions but follows from the dynamical law, similarly to dynamical initial conditions for cosmology.[25,26] In the non-interacting, empty Schwarzschild solution the future is thus the time reverse of the past and in particular, becomes semiclassical also to the future. Whether semiclassical behavior in the future is also achieved in more complicated collapse scenarios is not known so far.

3.3. *Outside the horizon*

The behavior is more complicated if matter is present or when inhomogeneities are considered. Then, back-reaction of Hawking radiation[55] on geometry and scattering of matter leads to a future behavior different from the past of the quantum region. For such cases, it has not been shown that space–time becomes semiclassical after all fields are settled down.

To complete the picture, access to the outside of the horizon is needed as well as a handle on field degrees of freedom of matter or gravity itself. Moreover, the horizon dynamics must be understood taking into account quantum effects. There are two main ways to approach this complex issue:

- *Effective equations*,[34,56,39,40] which have been successful in understanding qualitative aspects of homogeneous models (e.g., Refs. 34, 35, 57–63), are not yet available for inhomogeneous situations, but the homogeneous forms can be exploited in matchings.
- *Midisuperspace models* and their quantum dynamics close to classical singularities or at horizons, related to properties of difference equations, are being developed and have already given initial promising insights.

3.3.1. *Matching*

Gravitational collapse is often modeled by matching a homogeneous matter distribution (such as a star) to an inhomogeneous exterior geometry, following the work of Oppenheimer and Snyder.[64] Modeling the collapsing body by an isotropic interior solution

$$ds^2 = -dt^2 + \frac{a(t)^2}{(1+\frac{1}{4}r^2)^2}\left(dr^2 + r^2 d\Omega^2\right) \tag{7}$$

with the scale of the body determined by $a(t)$, and matching it to a generalized Vaidya spherically symmetric outside region

$$ds^2 = -\left(1 - \frac{2M(v,\chi)}{\chi}\right) dv^2 + 2dv d\chi + \chi^2 d\Omega^2 \tag{8}$$

with a function $M(v,\chi)$ determining the outside matter flux, leads to the conditions

$$\begin{aligned} \chi(v) &= Ra(t)/(1+R^2/4), \\ 2M &= aR^3(\dot{a}^2+1)/(1+R^2/4)^3, \end{aligned} \tag{9}$$

at the matching surface. Here, R is the coordinate value for r where the interior solution is cut off.

The homogeneous interior can then be described by effective equations including repulsive quantum correction terms as discussed in Sec. 2.4.[65] With such a term, the interior bounces which, through the matching, also influences the exterior geometry and its horizons. In the absence of effective equations for inhomogeneous situations, the full outside behavior cannot be determined. But at least in the neighborhood

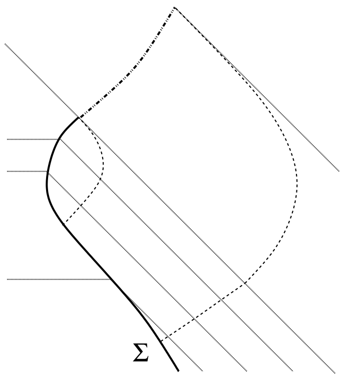

Fig. 2. Matching surface Σ of a bouncing interior (left) matched to an inhomogeneous outside region with evaporating horizons.

of the matching surface one can study the formation and possible disappearance of horizons. A marginally trapped spherical surface forms when $2M(v,\chi) = \chi$ is satisfied which, with the matching conditions, implies $|\dot{a}| = (1 - R^2/4)/R$ for the interior.

Classically, $\dot{a}$ is unbounded and monotonic such that there is always exactly one solution to the condition of a marginally trapped surface at the matching surface. There is thus only a single horizon covering the classical singularity.

With repulsive quantum effects, the situation changes: First, $\dot{a}$ starts to decrease before the bounce in the effective dynamics, implying the existence of a second solution corresponding to an inner horizon as illustrated in Fig. 2. Secondly, $\dot{a}$ is bounded and there are cases, depending on parameters and initial conditions, without any solution for $2M(v,\chi) = \chi$. The classical singularity is then replaced by a bounce which is covered by horizons only for larger mass. This scenario thus indicates the presence of a threshold for black hole formation.

3.3.2. *Midisuperspaces*

The quantum behavior across horizons can only be seen in inhomogeneous models. For spherically symmetric ones, the loop representation leads to states of the form[66]

$$|\psi\rangle = \sum_{\mathbf{k},\boldsymbol{\mu}} \psi(\mathbf{k},\boldsymbol{\mu}) \underset{\mu_-}{\overset{\cdots\quad k_-}{\bullet}} \underset{\mu}{\overset{k_+}{\bullet}} \underset{\mu_+}{\overset{\cdots}{\bullet}} \quad \text{with } k_e \in \mathbb{Z},\ 0 \le \mu_v \in \mathbb{R}, \tag{10}$$

subject to coupled difference equations (one for each edge)

$$\begin{aligned} &\hat{C}_0(\mathbf{k})\psi(\ldots,k_-,k_+,\ldots) + \hat{C}_{R+}(\mathbf{k})\psi(\ldots,k_-,k_+-2,\ldots) \\ &\quad + \hat{C}_{R-}(\mathbf{k})\psi(\ldots,k_-,k_++2,\ldots) + \hat{C}_{L+}(\mathbf{k})\psi(\ldots,k_--2,k_+,\ldots) \\ &\quad + \hat{C}_{L-}(\mathbf{k})\psi(\ldots,k_-+2,k_+,\ldots) + \cdots = 0 \end{aligned} \tag{11}$$

with coefficients which have been computed explicitly.[67] Also here, superspace is extended by the freedom of (local) orientation: sgn det E is determined by sgn k_e.

Again, the quantum equations are *non-singular*,[68] however much more crucially depending on the form (in particular on possible zeros) of the coefficients $\hat{C}_{R\pm}(\mathbf{k})$. Unlike in homogeneous models, a *symmetric ordering* is required to extend solutions. In isotropic models this removes any possible dynamical initial conditions, but in less symmetric models the solution space is still restricted. How strong the restrictions will be has to be seen from a more detailed analysis of the resulting initial/boundary value problem for difference equations.

Also unlike homogeneous models, the anomaly issue plays a bigger role: coupled difference equations must be consistent with each other for a well-defined initial/boundary value problem. While the anomaly issue in this model is open as of now, the existence and uniqueness of solutions in terms of suitable initial and boundary values has been shown. This has to be revisited, however, for the issue of semiclassical properties. In this model, there are further qualitatively different possibilities for the constraint operator. While the above discussion was based on a fixed number of labels (an operator not creating new spin network vertices), also other variants exist. For such a choice, the number of degrees of freedom would not be preserved when acting with the constraint, and a different type of recurrence problem arises. This freedom is analogous to quantization choices in the full theory which makes it possible to compare ambiguities and restrict choices by tight mathematical consistency conditions and the physical viability of scenarios. So far, the detailed quantization is far from unique (except for kinematical aspects), but there are characteristic and robust generic effects.

3.3.3. *Quantum horizons*

In midisuperspace models, it is also easier than in a full setting to impose horizon conditions at the quantum level and study quantum horizon dynamics. Isolated horizons[50] are particularly useful because in spherical symmetry they simplify considerably in Ashtekar variables.[69] This is useful for loop quantum gravity where Ashtekar variables are basic, and here even the quantum dynamics simplifies in the neighborhood of horizons. Moreover, the simplifications occur approximately also for slowly evolving horizons[70] such that even dynamical situations are accessible. Alternatively, direct quantizations of classical expansion parameters have been formulated in Refs. 71–73 for fully dynamical situations.

Conclusions drawn so far confirm quantum fluctuations of horizons[69] as suggested often before by heuristic arguments. One can also easily count the degrees of freedom of exactly spherically symmetric horizons, but the symmetry reduction removes far too many of them for a faithful counting of black hole entropy. This is the one issue where symmetric models are clearly not reliable and one has to use the full theory. Fortunately, even the full dynamics simplifies if an isolated horizon is introduced as a boundary allowing the correct counting of entropy for all astrophysically relevant black holes.[74–76]

4. Non-Uniqueness

The framework of loop quantum gravity provides promising indications, but so far cannot be seen as a complete theory due to a large amount of ambiguities as they often occur in non-linear quantum theories (discussed, e.g., in Refs. 77–81). This fact belongs to a much broader issue about uniqueness in quantum gravity where often two very different kinds are envisaged: a unique theory versus a unique solution (see also Ref. 82). These are indeed very different concepts as the uniqueness of a theory as such is not testable even in principle and thus of metaphysical quality. When a theory has a unique solution, however, its properties can be compared with observations at least in principle.

In fact, both concepts may be contradictory for all we know so far: there are ambiguities in loop quantum gravity and thus no unique theory (although solutions may be restricted by dynamical initial conditions giving some degree of uniqueness at the level of solutions), while the supposedly unique string theory has a whole landscape of potentially admissible solutions.[83,84] From a philosophical point of view, this situation has a precedent which led to the following statement:[85]

> *. . . a vast new panorama opens up for him, a possibility makes him giddy, mistrust, suspicion and fear of every kind spring up, belief in m[· · ·], every kind of m[· · ·], wavers, — finally, a new demand becomes articulate. So let us give voice to this* new demand: *we need a* critique *of m[· · ·] values*, the value of these values should itself, for once, be examined —

Today, one may be tempted to complete the m-word by "M-theory," but in those days it was actually "morality." Historically, philosophers attempted to construct something which in the current terminology could be called "Grand Unified Morality" or GUM, most widely known in the form of Kant's categorical imperative. This was meant as a unique theory, but did have too many solutions. Nietzsche's lesson was that the overly idealistic approach had to be replaced by a rather phenomenological one, where he studied the behavior (i.e. the phenomenology of morality) in different cultures. Similarly, quantum gravity may currently be at such a crossroads where idealism has to be replaced by phenomenology.

5. Conclusions

Quantum effects are significant at small scales and lead to qualitatively new behavior. Intuitively, this can be interpreted as repulsive contributions to the gravitational force for which there are several examples in the framework of loop quantum gravity.[82] When such effects are derived rather than being chosen with phenomenological applications in mind, it is by no means guaranteed that their modifications take effect in the correct regimes. This gives rise to many consistency checks such as those discussed here in anisotropic and inhomogeneous models in the context of black holes. These effects, while not fixed in detail, are robust and rather direct consequences of the loop representation, with non-perturbativity and background independence being essential.

With these effects, the theory can resolve a variety of conceptual and technical problems from basic effects, without the need to introduce new ingredients. At the current stage we have a consistent picture of the universe, including the classically puzzling situations of the big bang and black holes, which is well-defined everywhere. From here, one can use internal consistency and potential contact with observations to constrain the remaining freedom and test the whole framework.

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