Remarks on Resonant Scalars in the AdS/CFT Correspondence *

M. Bañados[a], A. Schwimmer[b] and S. Theisen[c]

[a] *Departamento de Física, P. Universidad Católica de Chile, Casilla 306, Santiago 22, Chile*

[b] *Department of Physics of Complex Systems, Weizmann Institute, Rehovot 76100, Israel*

[c] *Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, 14476 Golm, Germany*

Abstract

The special properties of scalars having a mass such that the two possible dimensions of the dual scalar respect the unitarity and the Breitenlohner-Freedman bounds and their ratio is integral ("resonant scalars") are studied in the AdS/CFT correspondence. The role of logarithmic branches in the gravity theory is related to the existence of a trace anomaly and to a marginal deformation in the Conformal Field Theory. The existence of asymptotic charges for the conformal group in the gravity theory is interpreted in terms of the properties of the corresponding CFT.

April 2006

* Partially supported by GIF, the German-Israeli Foundation for Scientific Research, the Minerva Foundation, DIP, the German-Israeli Project Cooperation, by grants FONDECYT 1060648 and 7020832 (M.B.) and by the European Research and Training Networks 'Superstrings' (MRTN-CT-2004-512194) (A.S.) and 'Forces Universe' (MRTN-CT-2004-005104) (S.T.)

1. Introduction

The coupling of scalars to gravity plays an important role in the study of the AdS/CFT correspondence. The scalar fields in gravity are related to scalar operators in the CFT and therefore test its detailed structure. Consider the general Lagrangian which admits an AdS background solution coupling gravity to a scalar field Φ in D dimensions:

$$S = \int d^d x d\rho \sqrt{G} \left[R + \Lambda - G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - m^2 \Phi^2 - V(\Phi) \right] \tag{1.1}$$

where we separated the D coordinates into $d = D - 1$ "boundary" coordinates and the "radial" coordinate ρ. For simplicity we did not include higher order curvature terms in the gravity part. The potential of the scalar is separated into a "mass term" and an interaction term $V(\Phi)$ which will be specified later. Neglecting the scalar interaction term the possible dimensions of the dual operators in the CFT are are:

$$\Delta_\pm = \frac{d}{2} \pm \nu \tag{1.2}$$

where

$$\nu = \sqrt{\frac{d^2}{4} + m^2}\,.$$

We will be interested in this note in the situation when both Δ_+ and Δ_- represent normalizable modes in the gravity corresponding to operators which satisfy the unitarity bound in the CFT. The condition for this to happen is:

$$0 \leq \nu < 1 \tag{1.3}$$

where the lower limit is the "Breitenlohner-Freedman" [1] bound. As discussed in [2],[3],[4],[5],[6],[7],[8] this situation corresponds to two possible CFT's in which only one of the operators $\mathcal{O}_\pm$ with dimensions $\Delta_\pm$, respectively, exists. A flow produced by the relevant perturbation $f \int \mathcal{O}_-^2 d^d x$ takes the two theories into each other. Moreover, there is an interesting "duality" between the two theories, the generating functionals for correlators of the $\mathcal{O}_\pm$ operators in the two theories being Legendre transforms of each other.

Special, very interesting features appear when the scalars are "resonant" i.e. if

$$\frac{\Delta_+}{\Delta_-} = n \tag{1.4}$$

where n is an integer. In the gravity theory solutions with logarithmic dependence on the radial coordinate appear. Some implications for the AdS/CFT correspondence of the resonant scalars in the $n = 1$ case were discussed in [9],[2]. Recently an in depth study of the gravity theory in this situation was undertaken in [10],[11],[12]. In particular the existence of asymptotic charges of the conformal group was studied. In order that these charges would be well defined, certain "integrability conditions" relating the coefficients of the expansion near the boundary should be imposed. In the present note we study the general implications of "resonant scalars" for the AdS/CFT correspondence. When the resonance condition (1.4) is fulfilled, if the operator $\mathcal{O}_-$ exists in the theory, necessarily also the operator $\mathcal{O}_+$ is present in the same theory since it appears in the OPE of n $\mathcal{O}_-$ operators [1]. As a consequence of the presence of both operators, two features single out this type CFT:

a) there is a "type B" conformal anomaly involving one energy-momentum tensor and $n+1$ $\mathcal{O}_+$ operators;

b) there is a marginal deformation generated by the interaction [2] $f \int \mathcal{O}_-^{n+1} d^d x$.

The "running" of the interaction b) is related to the anomaly a). The gravity theory faithfully reproduces these features. In particular the conditions for the existence of asymptotic conformal charges can be understood as a cancellation of the conformal anomaly a) by a non-polynomial "Wess-Zumino-Green-Schwarz" term or, equivalently, b), by the existence of a marginal perturbation which could be made "truly marginal" by the addition of a non-polynomial term.

The paper is organized as follows: In section 2 we discuss the structure of CFT having resonant scalar operators. We analyze the general structure of the type B trace anomaly and its implications. We study the marginal perturbation appearing, we calculate its lowest order running and we relate it to the trace anomaly. In section 3 we study the gravity theory in Fefferman-Graham gauge in dimensional regularization. We show the appearance of the trace anomaly and we discuss in detail the relation between the existence of asymptotic conformal charges and the trace anomaly and the running of multitrace deformations. In section 4 we summarize our results and we discuss possible future applications.

---

[1] We limit ourselves to the discussion of this setup, though in principle one could have a theory where starting with the $\mathcal{O}_+$ operator the $\mathcal{O}_-$ operator is not present.

[2] The role of marginal deformations in the AdS/CFT correspondence in d=4 and d=3 examples were studied by [2] and [13].

2. Trace anomalies and marginal perturbations in CFT with resonant scalars

Let's consider a CFT in which both operators $\mathcal{O}_+$ and $\mathcal{O}_-$ are present with dimension given by (1.2) with conditions (1.3) and (1.4) fulfilled. We start with presenting the mechanism for a trace anomaly [14] which is "type B" following the classification of [15]. Consider the correlator $G(x_1, ..., x_{n+1})$ of $n+1$ operators $\mathcal{O}_+$

$$G(x_1, \ldots, x_{n+1}) = \langle 0|T(\mathcal{O}_+(x_1) \ldots \mathcal{O}_+(x_{n+1}))|0\rangle \,. \tag{2.1}$$

Since $\mathcal{O}_+$ appears in the OPE of n operators $\mathcal{O}_-$ it can be written as:

$$\mathcal{O}_+(x) \;\equiv\; :\mathcal{O}_-^n(x): \tag{2.2}$$

where the normal ordering sign is used symbolically in order to indicate that inside (2.2) there are no short distance singularities. The correlator G can now be evaluated using the two point function of $\mathcal{O}_-$:

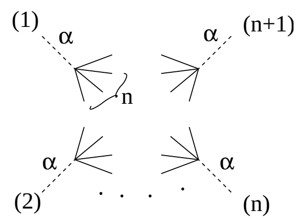
$$\langle \mathcal{O}_-(x)\mathcal{O}_-(y)\rangle = \frac{c}{|x-y|^{2\Delta_-}} \,. \tag{2.3}$$

The expression obtained does not have a well defined Fourier transform: indeed, after removing an overall δ-function for momentum conservation and taking into account that $(n+1)\Delta_- = d$, the dimension is 0 indicating a logarithmic ultraviolet divergence. A subtraction at a scale μ is needed and the correlator will contain a factor $\log(p^2/\mu^2)$ where p^2 is the overall scale of the external momenta. Clearly, the presence of a scale shows that dilation invariance is violated by the correlator (2.1) or that, equivalently, when an energy-momentum tensor is inserted, the Ward identity following from the tracelessness of the energy-momentum tensor is violated [14].

The same information can be summarized by looking at the generating functional when the energy-momentum tensor of the theory is coupled to an external gravitational field g_{ij} and the operator $\mathcal{O}_+$ to a source α of dimension Δ_-:

$$\exp[iW(g,\alpha)] = \int \mathcal{D}\varphi \, e^{iS_0(g,\varphi)+i\int d^dx\sqrt{g}\alpha\mathcal{O}_+} \tag{2.4}$$

where φ represent the fields of the CFT, S_0 is the action coupled to the external metric and W is the generating functional. Now the diagram



will contribute a term

$$\int d^dx\, \alpha^p \log(\Box_0/\mu^2)\alpha^{n+1-p} \tag{2.5}$$

to W before coupling to g. $\Box_0$ is the Laplacian in flat space. Since general coordinate invariance is preserved this term generates in the full W a term

$$\int d^dx\sqrt{g}\, \alpha^p \log(\Box/\mu^2)\alpha^{n+1-p} \tag{2.6}$$

where this time $\Box$ is the full, covariant Laplacian and additional, higher order terms. Performing a Weyl transformation on W:

$$\begin{aligned} g_{ij}(x) &\to g_{ij}(x)e^{2\sigma(x)} \\ \alpha(x) &\to \alpha(x)e^{-\Delta_-\sigma(x)} \end{aligned} \tag{2.7}$$

we obtain for the variation of W, $\delta_\sigma W$:

$$\delta_\sigma W = -2\int d^dx\sqrt{g}\,\sigma(x)\alpha^{n+1}(x) \tag{2.8}$$

due to the transformation of the covariant Laplacian.

The expression (2.8) represents the type B trace anomaly in this case. Though the effective action contains a scale μ, indicating an explicit breaking of scale invariance, the variation (2.8) itself is scale free and has the usual properties of an anomaly, i.e. it is a local variation fulfilling the Wess-Zumino condition of a nonlocal effective action. The Wess-Zumino condition is fulfilled in a way characteristic to all type B anomalies, i.e. the variation $\delta_\sigma W$ itself is Weyl invariant. We stress that the presence of this anomaly does not spoil the conformal invariance of the theory: the energy-momentum tensor is traceless as a consequence of the Heisenberg equations of motion and the Ward identity following from the tracelessness of the energy-momentum tensor is obeyed in all correlators except one class of anomalous ones whose coefficients are, however, related by the Wess-Zumino condition. A special important feature of the type B anomaly is that due to the presence of a scale in the effective action not only the Ward identity involving the dilation current is violated but also the conservation of the dilation charge (unlike for type A trace anomalies, or axial anomalies, for that matter).

The anomaly (2.8) is the analogue of the type B trace anomaly involving only energy-momentum tensors. An important difference is that (2.8) exists in any integer dimension while the one involving only energy-momentum tensors exists only in even dimensions.

In dimensional regularization type B anomalies are signaled by the presence in the effective action of terms having a factor $\frac{1}{(d-p)}$ with nonzero residue where p is an integer.

Though we will not use them in this note, we add for completeness a short discussion of the generalizations of (2.8). As mentioned above the anomaly satisfies the Wess-Zumino condition by being Weyl invariant. Therefore a complete list of trace anomalies can be obtained by construction of Weyl invariant, local expressions involving a scalar source γ with dimension Γ [16]. If $\frac{d}{\Gamma} = n+1$ where d is an integer dimension and n is an integer the anomaly is given by (2.8) with α replaced by γ. Let's consider the situation $\frac{(d-2)}{\Gamma} = p+1$ with d and p integers, which would illustrate the general procedure. Consider a "composite" metric $\bar{g}_{ij}$ defined by

$$\bar{g}_{ij} \equiv g_{ij}\gamma^{\frac{2(p+1)}{(d-2)}} . \tag{2.9}$$

By construction $\bar{g}_{ij}$ is invariant under a Weyl transformation and therefore any expression constructed from it will be also Weyl invariant. Consider therefore as a candidate for the anomaly

$$\int d^d x\, \sigma(x)\sqrt{\bar{g}}\bar{R} \tag{2.10}$$

where $\bar{R}$ is the curvature scalar constructed from $\bar{g}_{ij}$. Expressing $\bar{R}$ as a function of R and g_{ij} we obtain finally for the anomaly:

$$\int d^d x \sigma(x)\sqrt{g}\left\{\gamma^{p+1}R + \frac{2(p+1)(d-1)}{d-2}\gamma^p\Box\gamma + \frac{(p^2-1)(d-1)}{d-2}\gamma^{p-1}\partial_j\gamma\partial^j\gamma\right\} . \tag{2.11}$$

This particular anomaly is generated by the non-leading logarithmic divergence in the quadratically divergent correlator of $p+1$ operators whose source is γ. Similarly, by considering quadratic expressions in the curvatures we can construct the anomalies when the dimensions of the source fulfill $\Gamma = \frac{d-4}{p+1}$ etc.

We discuss now the other characteristic feature of CFT having resonant scalars. For such a theory

$$S_{\rm int} \equiv \frac{f}{(n+1)!}\int d^d x \; :\mathcal{O}_-^{n+1}: \tag{2.12}$$

represents a marginal perturbation. Again the definition of the operator appearing is given by the OPE of $n+1$ $\mathcal{O}_-$ operators. Due to the associativity of the OPE the operator can be given alternatively as

$$:\mathcal{O}_-\mathcal{O}_+: \; . \tag{2.13}$$

Generically the coupling constant will be renormalized. If $n+1$ is a prime number the lowest contribution is again given by Fig. 1 with $\mathcal{O}_-$ replacing the source α. The calculation is identical with the one giving the trace anomaly and the renormalized coupling f_R is

$$f_R = f + c f^{n+1}\log(\Lambda^2/\mu^2) \tag{2.14}$$

where f is the bare coupling, producing a β function:

$$\beta(f_R) = c f_R^{n+1}\,. \tag{2.15}$$

This relation between the trace anomaly and the renormalization of the coupling is a manifestation of the well known Ward identity [17]

$$T_j{}^j = \beta(f_R)\mathcal{O}_-^{n+1}\,. \tag{2.16}$$

A convenient way to define the renormalized coupling $\bar{f}$ is through the effective potential of $\mathcal{O}_-$ [18], $V_{\rm eff}(O_-)$:

$$\bar{f} \equiv \mu^{-d} V_{\rm eff}(\mathcal{O}_-)\Big|_{\mathcal{O}_-=\mu^{\Delta_-}}\,. \tag{2.17}$$

Then following Coleman-Weinberg [18] the Callan-Symanzik equation for the effective potential can be integrated. We use the β function given by (2.15) and we don't include a possible anomalous dimension for $\mathcal{O}_-$ which would enter at higher order. The solution of the Callan-Symanzik equation gives the form of the effective potential to the order we are working:

$$V_{\rm eff}(\mathcal{O}_-) = \bar{f}\mathcal{O}_-^{n+1} + \frac{c\bar{f}^{n+1}}{\Delta_-}\mathcal{O}_-^{n+1}\log\left(\frac{\mathcal{O}_-}{\mu^{\Delta_-}}\right) \tag{2.18}$$

which we will use in the comparison with the gravity treatment.

3. Resonant scalars in the gravity theory

The results of the analysis described in this section agree with the ones described in [12].

We consider a gravity theory which admits an AdS solution coupled to a scalar Φ with the action (1.1). We will analyze the solutions of (1.1) in the Fefferman-Graham gauge where the metric has the form:

$$ds^2 = \frac{(d\rho)^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j \, . \tag{3.1}$$

This gauge is suited for the studies of anomalies signaled in dimensional regularization by the appearance of poles in $d-N$ where N is an integer, in the solutions for various fields and the action evaluated on solutions [19],[20],[21]. Since the integration measure contains $\rho^{-1-d/2}$, the integration can produce poles if the power $\rho^{N/2}$ appears in the expansion of the action. This condition will be guiding us in setting up the regularization. By requiring that we are in the resonant scalar situation (i.e. equations (1.2),(1.3) and (1.4) are fulfilled) the expansion of the fields will have the general form:

$$\begin{aligned} g_{ij}(x,\rho) &= \overset{(0)}{g}_{ij}(x) + \ldots + \overset{(N/2)}{g}_{ij}(x)\rho^{N/2} + \ldots \\ \Phi(x,\rho) &= \alpha(x)\rho^{\Delta} + \ldots + \beta(x)\rho^{n\Delta} + \ldots \end{aligned} \tag{3.2}$$

where the dots include additional terms which should be present due to the coupled equations of motion (like $\rho^{2\Delta}$, possibly half-integer powers, etc.) which do not play, however, a role in the anomaly structure we are studying. In (3.2) and from now on we denote:

$$\Delta \equiv \frac{\Delta_-}{2} \, . \tag{3.3}$$

In order that the critical ρ dependence appears we require:

$$(n+1)\Delta = \frac{N}{2} \, . \tag{3.4}$$

The regularization choice (3.4) violates (1.4) by a term of order $d-N$ and therefore it is recovered only in the limit.

The linear part in the Φ equation of motion fixes then m through eq.(1.2) to be:

$$m^2 = \frac{N^2}{(n+1)^2}\left[1 - \frac{(n+1)d}{N}\right] \, . \tag{3.5}$$

Finally we choose $V(\Phi)$ in eq.(1.1) such that the second term in the expansion of Φ in (3.2) appears in leading order in the nonlinearity:

$$V(\Phi) = \frac{h}{n+1}\Phi^{n+1} \,. \tag{3.6}$$

Generically the potential may have additional terms which do not change the anomaly structure.

The procedure is now straightforward: We study the coupled equations of motion following from the action (1.1) with the choice (3.5) for the expansions (3.2); once the solutions are known we use them in the action isolating the terms which have poles in $d-N$. We defer a more general discussion for arbitrary actions involving g_{ij} and Φ based on diffeomorphisms in $d+1$ dimensions to another publication.

The case $n=1$ when $\mathcal{O}_+$ and $\mathcal{O}_-$ coincide requires a special treatment which will be given after the general discussion.

Now, the Φ equation of motion is:

$$\Box\Phi = m^2\Phi + \frac{h}{2}\Phi^n \,. \tag{3.7}$$

The linear term determines the relation between m and Δ which, due to our condition (3.4), gives (3.5). From the leading nonlinear matching we obtain:

$$\beta(x) = \frac{h(n+1)}{2N(n-1)}\frac{\alpha(x)^n}{N-d} + \beta_0(x) \,. \tag{3.8}$$

The appearance of the pole in $d-N$ signals the existence of the logarithmic branch. The function β_0 is undetermined by the expansion and it is the analogue of the so called "Fefferman-Graham ambiguity" in this context.

The gravitational equation of motion is:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R - \frac{\Lambda}{2}G_{\mu\nu} - \partial_\mu\Phi\partial_\nu\Phi + \frac{1}{2}G_{\mu\nu}\left(\partial_\alpha\Phi\partial^\alpha\Phi + m^2\Phi^2 + \frac{h}{n+1}\Phi^{n+1}\right) = 0 \,. \tag{3.9}$$

Instead of using it directly we will concentrate on the terms of the action which get contributions from the scalar field. First, using the trace of eq. (3.9) we can express R through the other terms and the action becomes:

$$S = \int d^dx d\rho\, \rho^{-1-d/2}\sqrt{g}\left[d + \frac{1}{d-1}\left(m^2\Phi^2 + \frac{h}{n+1}\Phi^{n+1}\right)\right] \tag{3.10}$$

where we used the relation between Λ and the AdS radius in the parametrization (3.1):

$$\Lambda = -d(d-1)\,. \tag{3.11}$$

Now, in (3.10) there are two contributions depending on the scalar field: the one appearing explicitly and the other in back-reaction to $\sqrt{g}$. The interesting term in the back-reaction after expanding $\sqrt{g}$ around $g^{(0)}$ is contributed by $\frac{1}{2}\overset{(0)}{g}_{ij}\overset{(N/2)}{g}_{ij}$ and can be calculated through its relation to R where δR is the part of the coefficient of $\rho^{N/2}$ in the expansion of R depending on Φ :

$$\frac{1}{2}\mathrm{tr}\big(\overset{(N/2)}{g}\big) = \frac{\delta R}{2N(N-d-1)}\,. \tag{3.12}$$

Putting the two contributions together we have to isolate the coefficient of $\rho^{N/2}$ in the expression

$$\frac{d}{2N(N-d-1)}\partial_\alpha\Phi\partial^\alpha\Phi + \frac{1}{d-1}\left[1+\frac{d(d+1)}{2N(N-d-1)}\right]\left(m^2\Phi^2+\frac{h}{n+1}\Phi^{n+1}\right)\,. \tag{3.13}$$

The result is:

$$\begin{aligned}&\frac{(N-d)}{(N-d-1)(d-1)(n+1)^2}\Big[d^2(n+1)+2N(N-1)-d(n+1)(2N-1)\Big]\alpha\beta\\&\qquad+\frac{d(d+1-2N)+2N(N-1)}{2N(N-d-1)(d-1)}\frac{h}{n+1}\alpha^{n+1}\,.\end{aligned} \tag{3.14}$$

We remark the $(d-N)$ factor in the $\alpha\beta$ term which shows that even though there is a pole term in the expression (3.8) of β in terms of α the integrand of S finally does not have any explicit pole terms. The pole term is the result of the integration over ρ. Keeping the pole and finite term the structure of the result is:

$$S[\overset{(0)}{g}_{ij},\alpha,\beta_0] = \int\sqrt{g^{(0)}}d^dx\left[\frac{c_1}{d-N}h\alpha^{n+1}+c_2\alpha\beta_0\right] \tag{3.15}$$

where $c_{1,2}$ are numerical coefficients.

We discuss now briefly the exceptional case $n=1$ when $\Delta = N/4$. In this case the expansion (3.2) is replaced by

$$\Phi(x,\rho) = \left[\frac{\alpha(x)}{d-N}+\beta_0(x)\right]\rho^{N/4} \tag{3.16}$$

and it is easy to verify, following the steps outlined above, that again (3.15) is obtained with $n=1$ [9].

Now the meaning of (3.15) is obvious [19],[20]: the presence of the pole requires a subtraction before the limit $d \to N$ is taken and as a consequence the subtracted action is nonlocal and it is no longer Weyl invariant. The Weyl variation can be calculated before taking the limit and it is finite and nonvanishing:

$$\delta_\sigma S[\overset{(0)}{g}, \alpha, \beta_0] = -c_1 h \int d^d x \sqrt{g^{(0)}} \sigma \alpha^{n+1} = \delta_\sigma \left\{ \frac{c_1}{2} h \int d^d x \sqrt{g^{(0)}} \alpha^{n+1-p} \log(\Box/\mu^2) \alpha^p \right\} . \tag{3.17}$$

The structure of the basic result (3.15) agrees completely with the structure of the CFT discussed in section 2 where α in the gravity theory should be identified with the source of the $\mathcal{O}_+$ operator in the CFT. In particular the trace anomaly (2.8) is reproduced by (3.17). Therefore we conclude that also in the case of resonant scalars there is a relation between two theories: a CFT with operators $\mathcal{O}_-$ and $\mathcal{O}_+$ and a gravity theory with action (1.1) and mass parameter such that the conditions (1.2),(1.3) and (1.4) are satisfied. The exact duality between the two theories requires a matching of the numerical coefficients in the above formulae which would depend on the exact formulation of the CFT and more specification of the gravity background including the additional compactified dimensions, etc. We stress that both theories are well defined and conformal, the logarithmic branches signaling, as we discussed in detail above, the matched conformal anomalies.

We discuss now the interesting observation made in [10],[11],[12] that in the gravity theory the asymptotic conformal charges are well defined only if a certain integrability condition is obeyed expressing the function β_0 appearing in (3.15) in terms of α. The integrability condition in our notation is the condition that the total Weyl variation of (3.15) vanishes:

$$\delta_\sigma S = \int d^d x \sqrt{g^{(0)}} \Big\{ \sigma(x) c_1 h \alpha^{n+1} + \alpha c_2 \delta_\sigma \beta_0 + \alpha n \Delta_- c_2 \sigma(x) \beta_0 \Big\} = 0 \tag{3.18}$$

or, more explicitly:

$$c_2 \delta_\sigma \beta_0 = -c_1 \sigma(x) h \alpha^n(x) - n \Delta_- c_2 \sigma(x) \beta_0 \,. \tag{3.19}$$

A particular solution of (3.19) proposed in [10],[11],[12] is

$$\beta_0 = \frac{c_1}{c_2 \Delta_-} h \alpha^n(x) \log\left(\frac{\alpha(x)}{\mu^{\Delta_-}} \right) \tag{3.20}$$

where the choice of the scale μ includes the ambiguity of adding to (3.20) a term α^n with an arbitrary coefficient. From the point of view of the gravity this condition is quite natural:

the existence of well defined asymptotic charges implies that under the solutions which satisfy the requirement the action is invariant on the full conformal group. Of course, once (3.19) is satisfied the two terms in (3.15), after the subtraction, combine, the scale μ disappears from the expression and S becomes:

$$S = \int d^d x \sqrt{g^{(0)}} \left\{ c_1 h \alpha^{n+1-p} \log \left(\frac{\Box}{\alpha^{2/\Delta_-}} \right) \alpha^p + \bar{c}_1 \alpha^{n+1} \right\} \tag{3.21}$$

which is Weyl invariant. In (3.21) we made explicit the freedom of adding a conformally invariant term as a consequence of the ambiguity in (3.20).

From the point of view of the CFT dual to the gravity theory the interpretation is obvious: if the generating functional W is identified with S as given by (3.21) a term was added to the anomalous, one loop term (2.5). This term which originates in the $\alpha\beta_0$ expression with the choice (3.20) for β_0 is classical and it is tuned such as to cancel the anomaly. Therefore it is the conformal version of a Green-Schwarz-Wess-Zumino term. Of course the term added, though local, is non-polynomial so in a usual QFT situation we would not consider it legal.

An alternative, closely related interpretation of the integrability condition is offered by looking at the solution (3.2),(3.8),(3.20) as representing a perturbation of the original CFT by a "multiple trace deformation" [2],[3],[4],[22],[23],[24],[25],[26],[27],[28],[29]. The deformation is given by the integral of the defining relation of β_0 :

$$S_{\rm pert} = \int d^d x \left\{ \frac{h c_1}{c_2 \Delta_-} \left[\frac{\mathcal{O}_-^{n+1}}{n+1} \log \frac{\mathcal{O}_-}{\mu^{\Delta_-}} - \frac{\mathcal{O}_-^{n+1}}{(n+1)^2} \right] + \frac{f}{(n+1)!} \mathcal{O}_-^{n+1} \right\} \tag{3.22}$$

This deformation has two pieces: a marginal perturbation with arbitrary strength which has a running as we discussed in section 2 and the term containing the logarithm of $\mathcal{O}_-$ which violates conformal invariance. The first term will produce an effective potential at lowest order of the form (2.18). Treating the second term of (3.22) as classical and choosing appropriately the bare coupling and the scale the two logarithmic terms could cancel exactly making the perturbation truly marginal at lowest order. Therefore, at least at lowest order, we defined a new CFT which is a deformation of the original one. To this new CFT corresponds the same conformally invariant action (3.21) possibly with a new interpretation. Indeed, we expect that due to the perturbation the dimensions of the operators $\mathcal{O}_+$ and $\mathcal{O}_-$ will change such that they will not be anymore resonant and as a consequence the trace anomaly will no longer be present. Correspondingly the interpretation of (3.21) will change which now represents the full generating functional of the theory. The exact correspondence between the operators in the perturbed CFT and α in (3.21) is an unsolved problem.

Summarizing, we offer two possible holographic interpretations of the requirement of well defined conformal charges in the gravity theory: a) the original CFT with a Green-Schwarz-Wess-Zumino term added to the partition function in order to cancel the trace anomaly and b) a perturbation of the original CFT by a marginal operator made truly marginal by canceling the one loop running of the perturbation by a fine tuned term treated classically. Given the connection between the trace anomaly and the running of the marginal perturbation discussed in section 2 the two interpretation are closely related.

4. Discussion

The presence of resonant scalars shows new features of the AdS/CFT correspondence. The two possible CFT's corresponding to the same gravity theory with different boundary conditions are replaced by a single CFT where both scalar operators are present. As a consequence the CFT has a type B trace anomaly and related to it a marginal perturbation. The gravity theory accommodates naturally this structure: the trace anomaly is reproduced and the boundary conditions allow the presence of the marginal perturbation through the "multiple trace" mechanism. The action of the gravity theory corresponding to the situation when the asymptotic conformal charges are well defined and conserved corresponds to two possible CFT: one with the addition of a classical term tuned to compensate the trace anomaly, the other a perturbation of the first by a marginal operator made truly marginal by the addition of another term treated classically.

The purpose of this note was to discuss the general structure of the AdS/CFT correspondence in the above set up. We intend to study further this set up in concrete realizations. At the level of the general structure several problems remain open which require further study:

a) the calculation of correlators for both operators on the gravity side and their consistency;

b) the validity of building truly marginal perturbations of CFT through cancellation with terms treated classically beyond one loop and the general consistency of such an approach;

c) the detailed relation between the observables of a CFT built along the lines mentioned in b) and the gravity action;

d) the general structure of theories with scalars and higher order curvature terms following form their D dimensional diffeomorphism invariance;
e) the appearance of scalar operators which obey the Breitenlohner-Freedman bound and produce higher trace anomalies of the type discussed in section 2.

Acknowledgments: Very useful discussions with O. Aharony, M. Berkooz and M. Henneaux are gratefully acknowledged. AS thanks the Humboldt Foundation for its support and the Albert Einstein Institute for its hospitality. ST thanks the Einstein Center and the Weizmann Institute for their hospitality and support;

References

[1] P. Breitenlohner and D. Z. Freedman, "Positive Energy In Anti-De Sitter Backgrounds And Gauged Extended Supergravity," Phys. Lett. B **115**, 197 (1982); "Stability In Gauged Extended Supergravity," Annals Phys. **144**, 249 (1982).

[2] E. Witten, "Multi-trace operators, boundary conditions, and AdS/CFT correspondence," arXiv:hep-th/0112258.

[3] I. R. Klebanov and E. Witten, "AdS/CFT correspondence and symmetry breaking," Nucl. Phys. B **556**, 89 (1999) hep-th/9905104].

[4] T. Hartman and L. Rastelli, "Double-trace deformations, mixed boundary conditions and functional determinants in AdS/CFT," hep-th/0602106.

[5] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. 2 (1998) 253 [hep-th/9802150].

[6] D. Z. Freedman, S. D. Mathur, A. Matusis and L. Rastelli, "Correlation functions in the CFT(d)/AdS($d+1$) correspondence," Nucl. Phys. B **546**, 96 (1999) [hep-th/9804058].

[7] S. S. Gubser and I. R. Klebanov, "A universal result on central charges in the presence of double-trace deformations," Nucl. Phys. B **656**, 23 (2003) [hep-th/0212138].

[8] S. S. Gubser and I. Mitra, "Double-trace operators and one-loop vacuum energy in AdS/CFT," Phys. Rev. D **67**, 064018 (2003) [hep-th/0210093].

[9] M. Bianchi, D. Z. Freedman and K. Skenderis, "How to go with an RG flow," JHEP **0108**, 041 (2001) [hep-th/0105276]; "Holographic renormalization," Nucl. Phys. B **631**, 159 (2002) [hep-th/0112119].

[10] T. Hertog and K. Maeda, "Black holes with scalar hair and asymptotics in N = 8 supergravity," JHEP **0407**, 051 (2004) [hep-th/0404261].

[11] M. Henneaux, C. Martinez, R. Troncoso and J. Zanelli, "Asymptotically anti-de Sitter spacetimes and scalar fields with a logarithmic branch," Phys. Rev. D **70**, 044034 (2004) [hep-th/0404236].

[12] M. Henneaux, C. Martinez, R. Troncoso and J. Zanelli, "Asymptotic behavior and Hamiltonian analysis of anti-de Sitter gravity coupled to scalar fields," arXiv:hep-th/0603185.

[13] S. Elitzur, A. Giveon, M. Porrati and E. Rabinovici, "Multitrace deformations of vector and adjoint theories and their holographic duals," JHEP **0602**, 006 (2006) [hep-th/0511061].

[14] S. Coleman and R. Jackiw , "Why Dilatation Generators Do Not Generate Dilatations," Ann. Phys. (N.Y.) **67**, (1971) 552.

[15] S. Deser and A. Schwimmer, "Geometric classification of conformal anomalies in arbitrary dimensions," Phys. Lett. B **309**, 279 (1993) [hep-th/9302047].

[16] B. Zumino, “Effective Lagrangians And Broken Symmetries,” in ‘Lectures on Elementary Particles and Quantum Field Theory’, S. Deser, M. Grisaru, H. Pendleton (eds.), M.I.T. Press 1970.

[17] J. C. Collins , “Renormalization of the energy-momentum tensor in Φ^4 theory,” Phys. Rev. D **14**, (1976) 1965.

[18] S. R. Coleman and E. Weinberg, “Radiative Corrections As The Origin Of Spontaneous Symmetry Breaking,” Phys. Rev. D **7**, 1888 (1973).

[19] C. Imbimbo, A. Schwimmer, S. Theisen and S. Yankielowicz, “Diffeomorphisms and holographic anomalies,” Class. Quant. Grav. **17**, 1129 (2000) [hep-th/9910267].

[20] A. Schwimmer and S. Theisen, “Diffeomorphisms, anomalies and the Fefferman-Graham ambiguity,” JHEP **0008**, 032 (2000) [hep-th/0008082].

[21] M. Henningson and K. Skenderis, “The holographic Weyl anomaly,” JHEP **9807**, 023 (1998) [hep-th/9806087].

[22] O. Aharony, M. Berkooz and E. Silverstein, “Multiple-trace operators and non-local string theories,” JHEP **0108**, 006 (2001) [hep-th/0105309]; “Non-local string theories on AdS(3) x S**3 and stable non-supersymmetric backgrounds,” Phys. Rev. D **65**, 106007 (2002) [hep-th/0112178].

[23] M. Berkooz, A. Sever and A. Shomer, “Double-trace deformations, boundary conditions and spacetime singularities,” JHEP **0205**, 034 (2002) [hep-th/0112264].

[24] W. Muck, “An improved correspondence formula for AdS/CFT with multi-trace operators,” Phys. Lett. B **531**, 301 (2002) [hep-th/0201100].

[25] P. Minces, “Multi-trace operators and the generalized AdS/CFT prescription,” Phys. Rev. D **68**, 024027 (2003) [hep-th/0201172].

[26] A. C. Petkou, “Boundary multi-trace deformations and OPEs in AdS/CFT correspondence,” JHEP **0206**, 009 (2002) [hep-th/0201258].

[27] A. Sever and A. Shomer, “A note on multi-trace deformations and AdS/CFT,” JHEP **0207**, 027 (2002) [hep-th/0203168].

[28] J. L. F. Barbon, “Multitrace AdS/CFT and master field dynamics,” Phys. Lett. B **543**, 283 (2002) [hep-th/0206207].

[29] I. R. Klebanov and A. M. Polyakov, “AdS dual of the critical O(N) vector model,” Phys. Lett. B **550**, 213 (2002) [hep-th/0210114].