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Black hole evaporation: A paradigm

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Abstract

A paradigm describing black hole evaporation in non-perturbative quantum gravity is developed by combining two sets of detailed results: i) resolution of the Schwarzschild singularity in loop quantum gravity [1]; and ii) time-evolution of black holes in the dynamical horizon framework [2, 3, 4]. Quantum geometry effects introduce a major modification in the traditional space-time diagram of black hole evaporation, providing a possible mechanism for recovery of information that is classically lost in the process of black hole formation. The paradigm is developed directly in the Lorentzian regime and necessary conditions for its viability are discussed. If these conditions are met, much of the tension between expectations based on space-time geometry and structure of quantum theory would be resolved.

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I. INTRODUCTION

In classical general relativity, a rich variety of initial data on past null infinity, $\mathscr{I}^-$, can lead to the formation of a black hole.[1] Once it is formed, space-time develops a new, future boundary at the singularity, whence one can not reconstruct the geometry and matter fields by evolving the data *backward* from future null infinity, $\mathscr{I}^+$. Thus, whereas an appropriately chosen family of observers near $\mathscr{I}^-$ has full information needed to construct the entire space-time, no family of observers near $\mathscr{I}^+$ has such complete information. In this sense, the classical theory of black hole formation leads to information loss. Note that, contrary to the heuristics often invoked (see, e.g. [5]), this phenomenon is not directly related to black hole uniqueness results: it occurs even when uniqueness theorems fail, as with 'hairy' black holes [6] or in presence of matter rings non-trivially distorting the horizon [7]. The essential ingredient is the future singularity, hidden from $\mathscr{I}^+$, which can act as the sink of information (see, in particular, Penrose's remarks in [8].)

A natural question then is: what happens in quantum gravity. Is there again a similar information loss? Hawking's celebrated work of 1974 [9] analyzed this issue in the framework of quantum field theory in curved space-times. In this approximation, three main assumptions are made: i) the gravitational field can be treated classically; ii) one can neglect the back-reaction of the spontaneously created matter on the space-time geometry; and iii) the matter quantum field under investigation is distinct from the collapsing matter, so one can focus just on spontaneous emission.[2] Under these assumptions, Hawking found that there is a steady emission of particles to $\mathscr{I}^+$ and the spectrum is thermal at a temperature dictated by the surface gravity of the final black hole. In particular, pure states on $\mathscr{I}^-$ evolve to mixed states on $\mathscr{I}^+$. In a next step, one can include back-reaction. To our knowledge, a detailed, systematic calculation is still not available. In essence one argues that, as long as the black hole is large compared to the Planck scale, the quasi-stationary approximation should be valid. Then, by appealing to energy conservation and the known relation between the mass and the horizon area of *stationary* black holes, one concludes that the area of the

[1] For simplicity of discussion, in this article we will consider only zero rest mass matter fields and assume that past null infinity is a good initial value surface. To include massive fields, one can suitably modify our discussion by adjoining past (future) time-like infinity to past (future) null infinity.

[2] Generally, only the first two assumptions are emphasized. However, we will see that the third also has a bearing on the validity of semi-classical considerations.

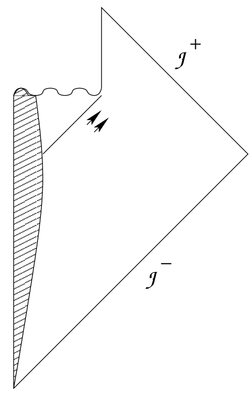

FIG. 1: The standard space-time diagram depicting black hole evaporation

event horizon should steadily decrease. This then leads to black hole evaporation depicted in figure 1 [9].

If one does not examine space-time geometry but uses instead intuition derived from Minkowskian physics, one may be surprised that although there is no black hole at the end, the initial pure state has evolved in to a mixed state. Note however that while space-time is now dynamical even after the collapse, *there is still a final singularity, i.e., a final boundary in addition to* $\mathcal{I}^+$. Therefore, it is not at all surprising that, in this approximation, information is lost —it is still swallowed by the final singularity [8]. Thus, provided figure 1 is a reasonable approximation of black hole evaporation and one does not add new input 'by hand', then pure states must evolve in to mixed states.

The question then is to what extent this diagram is a good representation of the physical situation. The general argument in the relativity community has been the following (see e.g. [10]). Figure 1 should be an excellent representation of the actual physical situation as long as the black hole is much larger than the Planck scale. Therefore, problems, if any, are associated *only* with the end point of the evaporation process. It is only here that the semi-classical approximation fails and one needs full quantum gravity. Whatever these 'end effects' are, they deal only with the Planck scale objects and would be too small to recover the correlations that have been steadily lost as the large black hole evaporated down to the Planck scale. Hence pure states must evolve to mixed states and information is lost.

Tight as this argument seems, it overlooks two important considerations. First, one would hope that quantum theory is free of infinities whence figure 1 can not be a good depiction of physics near the *entire singularity* —not just near the end point of the evaporation process. Second, the event horizon is a highly global and teleological construct. (For a recent discussion of limitations of this notion, see [11]). Since the structure of the *quantum* space-time could be very different from that of figure 1 near (and 'beyond') the singularity, the causal relations implied by the presence of the event horizon of figure 1 is likely to be quite misleading. Indeed, Hajicek [12] has provided explicit examples to demonstrate that the Vaidya solutions which are often used to model the evaporating black hole of figure 1 can be altered just in a Planck scale neighborhood of the singularity to change the structure of the event horizon dramatically and even make it disappear.

The purpose of this article is to point out that these considerations are important and conclusions drawn from figure 1 are therefore incomplete. More precisely, we will argue that *the loss of information is not inevitable even in space-time descriptions favored by relativists.* As in other discussions of the black hole evaporation process, we will not be able to present rigorous derivations. Rather, we will present a paradigm[3] by drawing on two frameworks where detailed and systematic calculations have been performed: i) analysis of the fate of the Schwarzschild singularity in loop quantum gravity; and ii) the dynamical horizon formalism which describes evolving black holes in classical general relativity. The result is a space-time description of black hole evaporation in the physical, Lorentzian setting in which one allows for a *quantum* extension of the space-time geometry *beyond* singularity. Since the space-time no longer has a future boundary at the singularity, pure quantum states on $\mathcal{I}^-$ can evolve to pure quantum states on $\mathcal{I}^+$.

The plausibility of this scenario is supported by the fact that its 2-dimensional version is realized [13] in the CGHS black hole [14]. There, it is possible to isolate the true degree of freedom and carry out an exact quantization using, e.g., Hamiltonian methods. On the resulting Hilbert space, one can in particular define the quantum (inverse) metric operator. The classical black hole metric arises as the expectation value in a suitable quantum state, i.e., in the *mean field approximation.* Hawking effect emerges through the study of small fluctuations on this mean field. One can explicitly check that this mean field approximation is good in a significant portion of the quantum space-time. However, the quantum fluctuations are very large near the entire singularity, whence the approximation fails there. The quantum (inverse) metric operator itself is well-defined everywhere; only its expectation value vanishes

[3] This paradigm was briefly sketched in section 8 of [11].

at the classical singularity. Thus, quantum geometry is defined on a manifold which is *larger* than the black hole space-time of the mean field approximation. The mean field metric is well-defined again in the asymptotic region 'beyond' the singularity.[4] Thus, there is a single asymptotic region in the distant past *and* distant future and pure states on $\mathscr{I}^-$ evolve to pure states on $\mathscr{I}^+$ of the full quantum space-time.

In this paper, we will focus on 4 dimensions where the qualitative picture is similar but the arguments are based on a number of assumptions. We will spell these out at various steps in the discussion. As we will see, specific calculations need to be performed to test if the assumptions are valid and the scenario is viable also in 4 dimensions. Our hope is that the proposed paradigm will provide direction and impetus for the necessary detailed analysis which will deepen our understanding of the evaporation process, irrespective of whether or not the paradigm is realized.

The paper is organized as follows. In section 2, we summarize the resolution of the Schwarzschild singularity by effects associated with the quantum nature of geometry. The new paradigm for black hole evaporation is presented in section 3. Section 4 contains some concluding remarks.

II. QUANTUM GEOMETRY AND THE SCHWARZSCHILD INTERIOR

Since the key issues involve the final black hole singularity and since we expect this singularity to be generically space-like, one can first focus just on the interior of the Schwarzschild horizon. This region is naturally foliated by 3-manifolds which are spatially homogeneous with the Kantowski-Sachs isometry group. Therefore, we can begin with the Kantowski-Sachs 'mini-superspace' of vacuum, spatially homogeneous space-times. Using quantum geometry, we can go to the exact quantum theory [1]. The situation is similar but technically

[4] There is a qualitative similarity with the theory of ferromagnetism. The (inverse) metric is analogous to the magnetization vector. If you have a large ferromagnet (such as the earth) a small, central portion of which is heated beyond the Curie temperature, the mean field approximation will hold far away from this central region and the magnetization operator will have a well-defined mean value there. That region is analogous to the part of the full, quantum space-time where there is a well-defined classical metric. The analysis of the Hawking effect is analogous to that of spin-waves on this part of the ferromagent, where the mean field approximation holds. While the mean field approximation fails in the central region where the expectation value of magnetization vanishes, quantum theory provides a good description of the entire magnet, including the central region, in terms of microscopic spins.

more complicated than that encountered in the rigorous treatment of spatially homogeneous and isotropic cosmologies [16]. (See also [15] where the same kind of representation is used, based on ADM variables.)

The first result is that, although the co-triad and curvature diverge at the singularity in the classical theory, the corresponding quantum operators are in fact bounded on the full kinematic Hilbert space. This analysis is analogous to that which showed that the quantum operator representing the inverse scale factor is bounded above in the spatially homogeneous, isotropic quantum cosmology [16, 17]. As in that analysis, the co-triad operator has various nice properties one expects of it and departures from the classical behavior appear only in the deep Planck regime (i.e. very near what was classical singularity). This finiteness results from the fact that the 'polymer representation' of the Weyl relations underlying our quantum description is inequivalent to the 'standard representation' used in quantum geometrodynamics (for details, see, e.g., [18]). It is analogous to the finiteness of matter Hamiltonians in the full theory [19]. This result already indicates that dynamics would be singularity-free.

Using quantum geometry, one can write down a well-defined Hamiltonian constraint. In the mini-superspace under consideration, there are only two degrees of freedom. One can be interpreted as the radius of any (round) 2-sphere in the slice and the other (the norm of the translational Killing field) is a measure of the anisotropy. It is natural to use the first as an intrinsic 'clock' and analyze how anisotropy 'evolves' with passage of this 'time'. In quantum theory, one can expand out the state $|\Psi\rangle$ as $|\Psi\rangle = \sum_{\phi,\tau} \psi(\phi,\tau)|\phi,\tau\rangle$ where ϕ are eigenvalues of the anisotropy operator and τ of the radius operator.The Hamiltonian constraint is of the form:

$$f_+(\tau)\,\hat{O}_+\,\psi(\phi,\tau+2\delta) + f_o(\tau)\,\hat{O}_o\,\psi(\phi,\tau) + f_-(\tau)\,\hat{O}_-\,\psi(\phi,\tau-2\delta) = 0 \qquad (1)$$

where $f_\pm, f_o$ are rather simple functions of τ, $\hat{O}_\pm, \hat{O}_o$ are rather simple operators on functions of ϕ alone and δ is a number whose value is determined by the smallest area eigenvalue in Planck units. Being a constraint, it simply restricts the physically allowed states. However, one can also regard it as providing 'time-evolution' of the quantum state through discrete time steps of magnitude 2δ (in Planck units). The functions f and the operators $\hat{O}$ are such that this evolution does not break down at $\tau = 0$ (which corresponds to the classical singularity). Thus, as in quantum cosmology [16, 20], one finds that the *quantum* evolution does not stop at the singularity; one can evolve right through it [1]. The state remains pure. However, in the *deep* Planck regime around the singularity, the notion of a classical space-time geometry fails to make even an approximate sense. Nonetheless, since there is no longer a final boundary in the interior, the full quantum evolution is quite different from the classical one.

This calculation was done [1] in the Kantowski-Sachs mini-superspace and $|\Psi\rangle$ represents the state of the Schwarzschild black hole interior in loop quantum gravity. This black hole can not evaporate: there is no matter and because of the restriction to spherical symmetry there can not be Hawking radiation of gravitons either. However, since the generic singularity is expected to be space-like, one may hope that the general intuition about the resolution of the Schwarzschild singularity it provides can be taken over to models in which gravity is coupled to scalar fields, where the evaporation does occur. We will assume that the *overall, qualitative* features of our singularity resolution will continue to be valid in these models.

III. EVAPORATION PROCESS

The physical situation we wish to analyze is the following: some radiation field on $\mathcal{I}^-$ collapses and forms a large, macroscopic black hole which then evaporates. For simplicity, we will restrict ourselves to the *spherically symmetric sector of Einstein gravity coupled to a massless Klein-Gordon field*. The incoming state on $\mathcal{I}^-$ will be assumed to be a coherent state peaked at a classical scalar field representing a large 'pulse', i.e., a field which is large over a compact region of $\mathcal{I}^-$ and vanishes (or become negligible) outside this region. Note that there is a single scalar field, coupled to gravity, whose collapse from $\mathcal{I}^-$ leads to the formation of the black hole and whose quanta are radiated to $\mathcal{I}^+$ during the evaporation process. There are no test fields; the system is 'closed'.

In this setting, conclusions drawn from classical general relativity should be valid to an excellent approximation until we are in the Planck regime near the singularity. Thus, marginally trapped surfaces would emerge and their area would first grow. In this phase the world tube of marginally trapped surfaces would be space-like [22, 23] and constitute a *dynamical horizon* [3, 4]. During Hawking evaporation, it would be time-like and constitute a *time-like membrane* [11]. In the spherical symmetric case now under consideration, this scenario was discussed already in the eighties (see, in particular [12, 24]). However, constructions were tailored just to spherical symmetry and made use of some heuristic considerations involving an 'ergo-region of an approximate Killing field.' Therefore, although well-motivated, the discussions remained heuristic. Laws governing the growth of the area of dynamical horizons and shrinkage of area of time-like membranes are now available in a general and mathematically precise setting [4]. Furthermore, laws of black hole mechanics have been extended to these dynamical situations. These results strengthen the older arguments considerably and reenforce the idea that what evaporates is the dynamical horizon and the time-like membrane [11].

Let us now combine this semi-classical picture with the discussion of section II on the resolution of the singularity to draw qualitative conclusions on what the black hole evaporation process would look like in full loop quantum gravity. Once this regime is reached, a priori there are two possibilities:

• a) States which start out semi-classical on $\mathcal{I}^-$ never become semi-classical on the 'other side' (in the sense discussed in [21]). Then a space-time description is not possible for the entire process. However, one *can* look at the problem quantum mechanically and conclude that pure states remain pure. If we restricted them only to the classical part of the space-time and measure observables which refer only to this part, we would get a density matrix but this is not surprising; it happens even in laboratory physics when one ignores a part of the system.

• b) As in the CGHS model [13], after evolving through the deep Planck regime, the state becomes semi-classical on the 'other side' so we can again use a classical space-time description.

This calculation is yet to be undertaken in 4 space-time dimensions.[5] If it turns out that the possibility a) holds, it would be impossible to speak of a scattering matrix since there would not be an adequate $\mathcal{I}^+$ or a space-like surface in the distant future for the 'final' states to live on. Hence, it would be quite difficult to say anything beyond the statement that pure states remain pure. If b) holds, one can compare various scenarios. Therefore, in the rest of the article, we will focus on this scenario.

A space-time diagram that could result in scenario b) is depicted in figure 2. Here, the

[5] However, a kinematical setting for the gravitational sector of this midi-superspace has been developed [26]. It should be relatively straightforward to write down the analog of (1), although one would most likely have to solve it numerically.

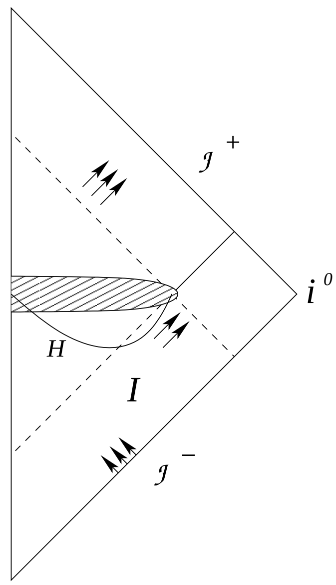

FIG. 2: Space-time diagram of black hole evaporation where the classical singularity is resolved by quantum geometry effects. The shaded region lies in the 'deep Planck regime' where geometry is genuinely quantum mechanical. H is the dynamical horizon which is first space-like and grows because of infalling matter and then becomes time-like and shrinks because of Hawking evaporation. In region I, there is a well-defined semi-classical geometry.

extended, 'quantum space-time' has a single asymptotic region in the future, i.e., there are no 'baby universes'. This is an *assumption*. It is motivated by two considerations: i) the situation in the CGHS model where detailed calculations are possible and show that the quantum space-time has this property; and ii) experience with the action of the Hamiltonian constraint in the spherically symmetric midi-superspace in 4 dimensions. However, only detailed calculations can decide whether this assumption is correct. We will refer to figure 2 as a 'Penrose diagram' where the inverted commas will serve as a reminder that we are not dealing with a purely classical space-time. Throughout the quantum evolution, the pure state remains pure and so we again have a pure state on $\mathcal{I}^+$. In this sense there is no information loss. Noteworthy features of this 'Penrose diagram' are the following.

i) *Effect of the resolution of the classical singularity:* Region marked I is well-approximated by a classical geometry. Modulo small quantum fluctuations, this geometry is determined via Einstein's equations by the classical data on $\mathcal{I}^-$ at which the incoming quantum state is peaked. The key difference between figures 1 and 2 is that while space-time ‘ends’ at the singularity in figure 1 it does not end in figure 2. But there is not even an approximate classical space-time in the shaded region representing the ‘deep Planck regime’.

ii) *Event horizon:* Since the shaded region does not have a classical metric, it is not meaningful to ask questions about causal relations between this region and the rest. Therefore, although it is meaningful to analyze the causal structure (to an excellent approximation) within each local semi-classical region, due care must be exercised to address *global* issues which require knowledge of the metric on the entire space-time. This is in particular the case for the notion of the event horizon, the future boundary of the causal past of $\mathcal{I}^+$. Because there is no classical metric in the shaded region, while one can unambiguously find some space-time regions which are in the past of $\mathcal{I}^+$, we can not determine what the *entire* past of $\mathscr{I}^+$ is. If we simply cut out this region and look at the remaining classical space-time, we will find that the past is not all of this space-time. But this procedure can not be justified especially for purposes of quantum dynamics. Thus, because the geometry in the deep Planck regime is genuinely quantum mechanical, the global notion of an event horizon ceases to be useful. It may well be that there is a well-defined, new notion of quantum causality and using it one may be able to reanalyze this issue. However, the standard classical notion of the event horizon is ‘transcended’ because of absence of a useful classical metric in the deep Planck region.

iii) *Dynamical horizon:* Nonetheless, we can trust classical theory in region I and this region will admit marginally trapped surfaces. It is reasonable to expect that a spherical dynamical horizon will be formed. The precise nature of the dynamical horizon during black hole formation will not play a significant role in our main discussion. Numerical simulations [22, 23] indicate that in the formation phase, the situation is similar to that with the Vaidya metrics [4, 11], the dynamical horizon would be space-like and the area will grow during collapse. In the classical theory, the dynamical horizon will eventually settle down to a null, isolated horizon which will coincide with (the late portion of) the event horizon. However, in quantum theory the horizon will shrink because of Hawking radiation. While the black hole is large, the process will be very slow. Semi-classical calculations indicate that there is a positive flux of energy out of the black hole. The dynamical horizon H will now ‘evolve’ into a time-like membrane and its area loss will be dictated by the balance law [4]

$$\frac{dR}{dt} = -2GT_{ab}\hat{\tau}^a\hat{r}^b \tag{2}$$

where R is the area radius of cross-sections of marginally trapped 2-spheres in H, $\hat{r}^a$ is the unit radial normal to H and $\hat{\tau}^a$ the unit normal within H to the marginally trapped 2-spheres in H. This process is depicted in figure 2. Thus, although we no longer have a well-defined notion of an event horizon, we can still meaningfully discuss formation and evaporation of the black hole using dynamical horizons because most of this process occurs in the semi-classical region and, more importantly, because *the notion of a dynamical horizon is quasi-local*. When the black hole is large, the evaporation process is extremely slow. Therefore, it seems reasonable to assume that the intuition developed from the quantum geometry of isolated horizons [28] will continue to be valid. If so, the quantum geometry of this time-like (weakly) dynamical horizon will be described by the $U(1)$ Chern-Simons theory on a punctured S^2, where the punctures result because the polymer excitations of the bulk geometry pierce the dynamical horizon, endowing it with certain area quanta. During the evaporation process, the punctures slowly disappear, the horizon shrinks and quanta of area are converted into quanta of the scalar field, seen as Hawking radiation at infinity.[6] The existence, in the classical theory, of a meaningful generalization of the first law of black hole mechanics to dynamical horizons [3, 4] supports the view that the process can be interpreted as evaporation of the dynamical horizon.

iv) *Reconciliation with the semi-classical information loss:* Consider observers restricted to lie in region I (see figure 3). For a macroscopic black hole this semi-classical region is very large. These observers would see the radiation resulting from the evaporation of the horizon. This would be approximately thermal, only approximately because, among other things, the space-time geometry is not fixed as in Hawking's original calculation [9], but evolves slowly. Although the full quantum state is 'pure', there is no contradiction because these observers look at only part I of the system and trace over the rest which includes a purely quantum part. In effect, for them space-time has a future boundary where information is lost. Since the black hole is assumed to be initially large, the evaporation time is long (about 10^{70} years for a solar mass black hole). Suppose we were to work with an approximation that the black hole takes *infinite* time to evaporate. Then, the space-time diagram will be figure 4 because the horizon area would shrink to zero only at i^+. In this case, there would be an event

[6] While Equation (2) relates the change in the area of the time-like dynamical horizon with the flux of the (negative) energy falling into it, because of the dynamical nature of geometry, there is no simple relation between this ingoing flux at the horizon and the energy carried by the outgoing quanta on $\mathcal{I}^+$.

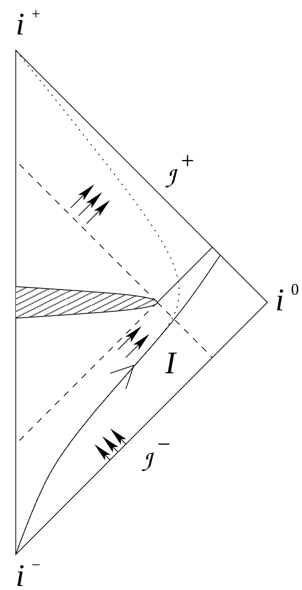

FIG. 3: The solid line with an arrow represents the world-line of an observer restricted to lie in region I. While these observers must eventually accelerate to reach $\mathcal{I}^+$, if they are sufficiently far away, they can move along an asymptotic time translation for a long time. The dotted continuation of the world line represents an observer who is not restricted to lie in region I. These observers can follow an asymptotic time translation all the way to i^+.

horizon and information would be genuinely lost for any observer in the initial space-time; it would go to a second asymptotic region which is inaccessible to observers in the initial space-time. Of course this does not happen because the black hole evaporates only in a finite time.

v) *'Recovery' of the 'apparently lost' information:* Since the black hole evaporates only in a finite amount of time, the point at which the black hole shrinks to zero size is *not* i^+ and the space-time diagram looks like figure 3 rather than figure 4. Now, i^+ lies to the 'future' of the 'deep Planck' region and there are observers lying entirely in the asymptotic region going from i^- to i^+ (represented by the dotted continuation of the solid line in figure 3). This family of observers will recover the apparently lost correlations. Note that these observers always remain in the asymptotic region where there is a classical metric to an excellent approximation; they never go near the deep Planck region. The total quantum state on $\mathcal{I}^+$ will be pure and will have the complete information about the initial state on $\mathcal{I}^-$. It looked approximately thermal at early times, i.e., to observers represented by the

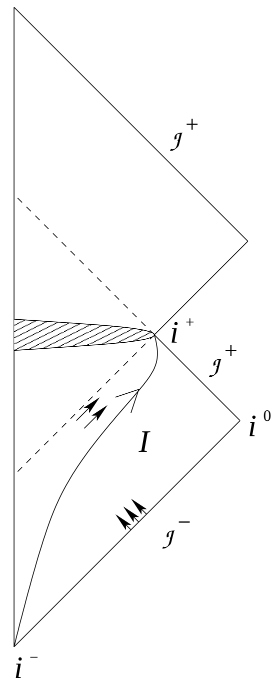

FIG. 4: The 'would be' space-time if the black hole were to take an infinite time to evaporate.

solid line, only because they ignore a part of space-time. The situation has some similarity with the EPR experiment in which the two subsystems are first widely separated and then brought together (see also [29]).

v) *Entropy:* Since the true state is always pure, one might wonder what happens to black hole entropy. It is only the observers in region I that 'sense' the presence of a black hole. In the quantum geometry approach to black hole entropy, entropy is not an absolute concept associated objectively with a space-time. Rather, it is associated with a family of observers who have access to only a part of space-time. Indeed, the entropy of an isolated horizon calculated in [27, 28] referred to the family of observers for whom the isolated horizon serves as the internal boundary of accessible space-time. So, for observers restricted to region I, that entropy calculation is still meaningful, at least so long as the black hole is macroscopic (i.e., the area of marginally trapped surfaces on H is much larger than Planck area). And it is these observers who see the (approximate) Hawking radiation. More precisely, since these observers have access only to observables of the type $A_{\rm I} \otimes 1$, they trace over the part of the system not in I, getting a density matrix $\rho_{\rm I}$ on the Hilbert space $\mathcal{H}_{\rm I}$. Entropy for them is simply $Tr_{\rm I}\rho_{\rm I} \ln \rho_{\rm I}$. Had there been a true singularity 'ending' the space-time, this entropy would have become objective in the sense that it would be associated with *all* observers who do not fall into the singularity.

IV. CONCLUDING REMARKS

In the last two sections we used a quantum gravity perspective to argue that information loss is not inevitable in the space-time description of black hole evaporation. The qualitative difference between figures 1 and 2 arises essentially from the fact that the singularity is resolved in quantum geometry, as per a general expectation that a satisfactory quantum theory of gravity should not have infinities. In this sense the paradigm shift is well-motivated. Furthermore, conclusions of the traditional paradigm drawn from the usual space-time diagram 1 are not simply discarded. For a large black hole, they continue to be approximately valid for a very long time. Figure 3 clarifies the approximation involved. However, from the *conceptual perspective of fundamental physics*, conclusions drawn from the *complete* space-time diagram 2 are qualitatively different from the standard ones. A pure state from $\mathcal{I}^-$ evolves to a pure state on $\mathcal{I}^+$ and there is no obstruction in quantum theory to evolving the final state on $\mathcal{I}^+$ backwards to recover full space-time. However the resulting geometry fails to be globally classical. In the shaded region, it is genuinely quantum mechanical and can be described only in terms of the quantum geometry states (i.e., in terms of spin-networks). However, in the region in which one can introduce classical geometry to an excellent approximation, it is meaningful to speak of marginally trapped surfaces, dynamical horizons and null infinity $\mathcal{I}^\pm$.[7] What 'evaporates' is the area of the dynamical horizon.

From the perspective of this paradigm, the conclusion that a pure state must evolve to a mixed state results if one takes the classical space-time diagram 1, *including the singular boundary in the future*, too seriously.[8] The viewpoint suggested by the CGHS model is that the classical singularity is only a reflection of the failure of the mean field approximation. Quantum geometry is defined on a larger manifold and when the analysis pays due respect

[7] Because of the presence of the purely quantum part, the space-time is not asymptotically simple [30]; the classical region admits null geodesics which do not end on $\mathcal{I}^\pm$. However, it is asymptotically flat and admits a global null infinity in the sense of [31].

[8] Perhaps an analogy from atomic physics would be to base the analysis of the ground state of the hydrogen atom on the zero angular momentum, classical electron trajectories, all of which pass through the 'singularity' at the origin.

to this extension, pure states can evolve to pure states, without any information loss.

The two dimensional analog of our paradigm is realized quite well by CGHS black holes [13]. However, 2 dimensional models have special features that are not shared by higher dimensional theories. To carry out the analogous analysis in 4 dimensions, one would have to complete several difficult steps: i) Discussion of quantum dynamics in the spherically symmetric midi-superspace [26]. To be directly useful, we would need to introduce a satisfactory generalization of the notion of 'time' used in [1]; ii) demonstration of the semi-classical behavior of the quantum state in regions where the dynamical horizon grows and the time-like membrane shrinks (in the regime where its area is large); iii) extension of the available theory [28] of quantum geometry from isolated to slowly evolving dynamical horizons; and iv) establishing that the quantum state becomes semi-classical again on the 'other side' of what was a classical singularity, with a single asymptotic region. To gain intuition on the last issue, numerical simulations of the 'past evolution' were recently performed [32] in the simplest mini-superspace models. It was found that, while the passage through the singularity does increase the quantum fluctuations somewhat, the state continues to be semi-classical after it crosses the deep Planck regime to the 'past' of what was the classical singularity. While this behavior is in accordance with the scenario used here, the support it provides is not so strong since the minisuperspace is highly restricted. Note, however, that any approach to quantum gravity will have to resolve similar issues if it is to provide a detailed 'space-time description' of the black hole evaporation in the Lorentzian framework. In particular, all discussions beyond the semi-classical approximation that we are aware of implicitly assume that there is a semi-classical regime in the future.

Finally, in this paradigm correlations are restored by part of the state that passes through the singularity and emerges on $\mathcal{I}^+$ to the future of region I of figure 2. Therefore, it is presumably necessary that this part should carry a non-trivial fraction of the total ADM mass of space-time (see, however, [29]). This seems physically plausible because one expects non-trivial space-time curvature also on the 'other side of the singularity'. However, whether this is realized in detailed calculations remains to be seen. Thus, the paradigm is based on pieces of calculations and analogy to the CGHS model, rather than a systematic detailed analysis. Recall, however, that the traditional reasoning that led to figure 1 was based on general considerations and plausibility arguments and a systematic analysis of the viability of approximations is still not available. Nonetheless, it led to a paradigm which proved to be valuable in focussing discussions. Our hope is that that the paradigm presented here will play a similar role.

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