Preprint typeset in JHEP style - HYPER VERSION

hep-th/0311110
AEI-2003-092

Anti-de-Sitter vacua require fermionic brane charges

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ABSTRACT: We argue that a modification of the super-AdS algebras which accounts for the presence of D-branes requires not only the inclusion of *bosonic* brane charges, but also the inclusion of new *fermionic* ones. We show that such fermionic brane charges are indeed present in the matrix model and the supermembrane in the pp-wave limit of the corresponding backgrounds. We briefly comment on an AdS version of Sezgin's M-algebra inspired by this observation.

KEYWORDS: M-theory, superalgebras, D-branes.

Contents

1. Introduction

Rather surprisingly, a modification of the superalgebra of anti-de-Sitter backgrounds[1] which accounts for the presence of D-branes in the string spectrum is still unknown. At an algebraic level, D-branes manifest themselves through non-zero expectation values of bosonic tensorial charges. There exists a widespread, but incorrect, belief that the inclusion of these brane charges into the anti-de-Sitter superalgebras follows the well-known flat-space pattern. In flat space, the inclusion of brane charges leads to a rather minimal modification of the super-Poincaré algebra: the bosonic tensorial charges appear on the right-hand side of the anti-commutator of supercharges, transform as tensors under the Lorentz boosts and rotations, while they commute with all other generators [1]. The brane charges are therefore often loosely called "central", and the resulting algebra is referred to as the maximal bosonic "central" extension of the super-Poincaré algebra. However, despite several attempts to construct a similar modification of anti-de-Sitter superalgebras, a *physically satisfactory* solution is as of yet unknown.

There are two basic physical requirements which have to be satisfied by an anti-de-Sitter algebra which is modified to include brane charges. The algebra has to include at least the brane charges which correspond to all D-branes that are already known to exist, and it also has to admit at least the supergraviton multiplet in its spectrum. Mathematically consistent modifications of anti-de-Sitter superalgebras can be constructed, but all existing proposals fail to satisfy one or both of these physical criteria. We refer the reader to [2] for an extensive discussion of this problem and the existing literature on this topic. In the present letter we show that there is a simple reason why previous attempts to extend anti-de-Sitter superalgebras with brane charges have failed: such extensions are *only* physically acceptable when one adds new *fermionic* brane charges as well.

[1] When we talk about anti-de-Sitter backgrounds in this letter, we always mean the maximally supersymmetric $\mathrm{AdS}_p \times S^q$ backgrounds. Similarly, we use the phrase "anti-de-Sitter (super)algebras" as a shorthand to refer to the (super)isometry algebras of these product backgrounds.

The necessity of including new fermionic brane charges into the modified algebra can be understood from a very simple argument based on Jacobi identities, in combination with the two physical requirements just mentioned. Consider an anti-de-Sitter superisometry algebra, or a pp-wave contraction of it. The bracket of supercharges can, very symbolically, be written in the form

$$\{Q_\alpha, Q_\beta\} = (\Gamma^{AB})_{\alpha\beta} M_{AB}\,, \tag{1.1}$$

where Q and M are the supercharges and rotation generators respectively (we have grouped together momentum and rotation generators by using a notation in the embedding space). Suppose now that we add a bosonic tensorial brane charge Z on the right-hand side of this bracket. This extension has to be made consistently with the Jacobi identities. Consider the (Q, Q, Z) identity, which takes the symbolic form

$$\begin{aligned}(Q_\alpha, Q_\beta, Z) &= [\{Q_\alpha, Q_\beta\}, Z] - \big[[Q_\alpha, Z], Q_\beta\big] - \big[[Q_\beta, Z], Q_\alpha\big] \\ &= (\Gamma^{AB})_{\alpha\beta}[M_{AB}, Z] - 2\,\big[[Q_{(\alpha}, Z], Q_{\beta)}\big]\,.\end{aligned} \tag{1.2}$$

As the brane charge Z is a tensor charge, it will transform non-trivially under the rotation generators. This implies that the first term of (1.2) will not vanish. The Jacobi identity can then only hold if Z *also* transforms non-trivially under the action of the supersymmetry generators! (In flat space, only the vanishing bracket $[P, Z]$ appears in the first term of the Jacobi identity (1.2), because in that case the $\{Q, Q\}$ anti-commutator closes on the translation generators). The simplest option is to assume that *no new fermionic charges* should be introduced, and that therefore symbolically

$$[Q_\alpha, Z] = Q_\alpha\,. \tag{1.3}$$

Although it is possible to construct an algebra based on (1.3) which satisfies all Jacobi identities, it is physically unsatisfactory [2]. The essential reason is that brackets like (1.3) are incompatible with multiplets on which the brane charge is zero (the left-hand side would vanish on all states in the multiplet, while the right-hand side is not zero). In other words, one cannot "turn off" the brane charges.[2] The only other way out is to add *new fermionic charges* Q'_α to the algebra, such that (1.3) is replaced with

$$[Q_\alpha, Z] = Q'_\alpha\,. \tag{1.4}$$

In this case it becomes possible to find representations in which both Z and the new charge Q'_α are realised trivially, as expected for e.g. the supergraviton multiplet, while still allowing for multiplets with non-zero brane charges.

This formal argument based on Jacobi identities may come as a surprise, and one would perhaps find it more convincing to see new fermionic brane charges appear in *explicit* models. In the present letter, we will show that such charges indeed do appear. In order to

[2]The precise situation is a slightly complicated since there is more than one bosonic brane charge; it could in principle be that a subtle interplay between these charges resolves the problem mentioned above. The full analysis of [2] shows that this mechanism can, however, not be realised. Similarly, we have shown that it is not possible to close the Jacobi identities by e.g. assuming non-standard transformation behaviour of the brane charges under the rotation generators. [2] contains a full list of (failing) alternatives.

show this, we will analyse the world-volume superalgebras of the supermatrix model and the supermembrane in a pp-wave limit of the anti-de-Sitter background. These models exhibit, in the absence of brane charges, a world-volume version of the superisometry algebra of the background geometry. When bosonic winding charges are included, the algebra automatically exhibits fermionic winding charges as well. Moreover, configurations on which these charges are non-zero can be found explicitly, or can alternatively be generated from configurations on which the fermionic winding charges are zero. On the basis of these results we will briefly discuss a D-brane extension of the osp*(8|4) superisometry algebra with bosonic as well as fermionic brane charges, which avoids the problems with purely bosonic modifications as first observed in [2].

For historical completeness, we should mention that the extension of world-volume superalgebras with fermionic central charges is not a new idea. Green [3] has suggested a string world-volume algebra in which a fermionic central charge is introduced as the fermionic partner of the momentum generator. Generalisations of this algebra to other brane world-volume superalgebras were made by Bergshoeff and Sezgin [4] and several other authors. A similar idea was implemented at the level of the super-Poincaré isometry algebra of the Minkowski vacuum [5, 6]. In all of these constructions, however, the extra fermionic brane or winding charges were *optional* rather than required for physical consistency.

2. Fermionic brane charges

2.1 Matrix-model charges in a pp-wave

The supersymmetry algebra of the matrix model in a pp-wave background contains brane charges. These charges (but not the full algebra!) have been determined by Hyun and Shin [7] by computing the Dirac bracket of the supercharges (see also the work of Sugiyama and Yoshida [8] in which a similar calculation was done for the supermembrane). In this section we will show that there are further, fermionic brane charges, as expected from the Jacobi identity argument sketched in the introduction.

Let us first briefly review the existing calculation as given in [7]. In the matrix model, brane charges are "traces of commutators", which identically vanish for finite N. These correspond to topological or winding charges (total derivatives) in the supermembrane model. For example, the supermembrane has a two-form winding charge, which is related to a charge in the matrix model according to

$$Z^{IJ} = \int \mathrm{d}^2\sigma\, \epsilon^{0rs} \partial_r X^I \partial_s X^J \quad \leftrightarrow \quad Z^{IJ} = \mathrm{Tr}[X^I, X^J]\,. \tag{2.1}$$

The "integration over the world-volume" in the supermembrane corresponds to "taking traces over $SU(N)$ indices" in the matrix model, and the trace over the commutator clearly only makes sense in the $N \to \infty$ limit. In practise, one computes the algebra of charge densities, or in matrix-model language, quantities obtained before taking traces. See [7] for more details, as well as the papers by Banks et al. [9] and Ezawa et al. [10], where central charges were first computed in matrix model context (though in flat space-time).

The action of M(atrix) theory in a pp-wave background is given by [3]

$$S = \mathrm{Tr}\left(\frac{1}{2R}D_0X^ID_0X^I - \frac{1}{2R}\left(\frac{\mu}{3}\right)^2(X^i)^2 - \frac{1}{2R}\left(\frac{\mu}{6}\right)^2(X^{i'})^2 + \frac{i}{2}\theta^\alpha D_0\theta^\alpha \right.$$
$$\left. - \frac{i\mu}{8}\theta^\alpha\Pi_{\alpha\beta}\theta^\beta + \frac{R}{4}[X^I,X^J]^2 + \frac{R}{2}\theta^\alpha\gamma^I_{\alpha\beta}[X^I,\theta^\beta] - \frac{i\mu}{3}\epsilon_{ijk}X^iX^jX^k\right) \quad (2.2)$$

Note that all fields X, P and θ are $SU(N)$ matrix valued and therefore the ordering is important. The parameter μ is the pp-wave mass parameter and R corresponds to the DLCQ radius; flat space is recovered by taking $\mu \to 0$. The derivative D_0 includes the usual coupling to the world-line gauge field $D_0X^I = \partial_0 - [\omega, X^I]$; see [11] for further details.

The Hamiltonian, rotation generators, supercharges and several bosonic non-perturbative charges have been computed in [7] and we will not comment any further on their derivation. For reference, an overview of these charges (or rather their "un-integrated" forms, i.e. the expressions obtained before taking the trace) is given in table 1. We refer to [12] for a discussion of the z^I charge. The algebra can be determined by systematically applying the Dirac brackets

$$\big\{(X^I)_a{}^b, (P^J)_c{}^d\big\}_{\mathrm{DB}} = \delta^{IJ}\delta_a{}^d\delta_c{}^b\,, \quad \big\{(\theta^\alpha)_a{}^b, (\theta^\beta)_c{}^d\big\}_{\mathrm{DB}} = -i\delta^{\alpha\beta}\delta_a{}^d\delta_c{}^b\,. \quad (2.4)$$

Here the indices $a, b, \ldots$ are in the fundamental of SU(N). We will keep the subscript "DB" on the Dirac brackets to distinguish them from SU(N) anti-commutators. The bosonic brane charges appear in the various brackets of the two supercharges Q and $\tilde{Q}$, as computed by Hyun and Shin [7] (again, see footnote 3 for our conventions and table 1 for the explicit form of the generators),

$$\begin{aligned}\big\{q_{(\alpha\,a}{}^b, Q_{\beta)}\big\}_{\mathrm{DB}} =& -4iR\,\mathcal{H}_a{}^b\delta_{\alpha\beta} + i\frac{2\mu}{3}(\Pi\gamma^{ij})_{\alpha\beta}(j^{ij})_a{}^b - i\frac{\mu}{3}(\Pi\gamma^{i'j'})_{\alpha\beta}(j^{i'j'})_a{}^b \\ &- 2i\,\gamma^I_{\alpha\beta}(z^I)_a{}^b - 2i\,\gamma^{IJKL}_{\alpha\beta}(z^{IJKL})_a{}^b \\ &- \frac{\mu}{3}(\Pi\gamma^{j'})_{\alpha\beta}\big[2(X^i)^2 - (X^{i'})^2, X^{j'}\big]_a{}^b \\ &- \frac{\mu}{6}\epsilon_{ijk}\gamma^{iji'j'}_{\alpha\beta}\big[X^{i'},\{X^k,X^{j'}\}\big]_a{}^b\,, \\ \big\{q_{\alpha\,a}{}^b, \tilde{Q}_\beta\big\}_{\mathrm{DB}} =& -2i\left(P^I\gamma^I_{\alpha\beta} - \gamma^{IJ}_{\alpha\beta}z^{IJ} - \frac{\mu}{3R}X^i(\Pi\gamma^i)_{\alpha\beta} + \frac{\mu}{6R}X^{i'}(\Pi\gamma^{i'})_{\alpha\beta}\right)_a{}^b\,.\end{aligned} \quad (2.5)$$

Here $\mathcal{H}_a{}^b$ denotes the Hamiltonian; we will not need its explicit form but it can be found in [7] along with the $\{\tilde{q}, \tilde{Q}\}$ bracket (which we will also not need). One can verify, using a similar calculation, that the brackets of the brane charges with themselves and with

[3] We are using the conventions of Banks et al. [9] and Hyun and Shin [7]. For the SO(9) gamma matrices this means that products satisfy the symmetry properties $\delta_{(\alpha\beta)}, \gamma^I_{(\alpha\beta)}, \gamma^{IK}_{[\alpha\beta]}, \gamma^{IKL}_{[\alpha\beta]}$ and so on. The 9-dimensional indices I, K, L split in SO(3) indices i, j, k and SO(6) indices i', j', k'. We also use the standard pp-wave symbol $\Pi = \gamma^{123}$.

Supercharges:

$$q_\alpha = \sqrt{\frac{R}{2}}\Big\{P^I\gamma^I_{\alpha\beta} - \frac{i}{2}[X^I,X^J]\gamma^{IJ}_{\alpha\beta} - \frac{\mu}{3R}X^i(\Pi\gamma^i)_{\alpha\beta} + \frac{\mu}{6R}X^{i'}(\Pi\gamma^{i'})_{\alpha\beta},\, \theta^\beta\Big\}\,, \tag{2.3a}$$

$$\tilde{q}_\alpha = \sqrt{\frac{2}{R}}\theta_\alpha\,, \tag{2.3b}$$

Rotation generators:

$$j^{ij} = X^iP^j - P^iX^j - \tfrac{i}{4}\theta\gamma^{ij}\theta\,, \tag{2.3c}$$

$$j^{i'j'} = X^{i'}P^{j'} - P^{i'}X^{j'} - \tfrac{i}{4}\theta\gamma^{i'j'}\theta\,, \tag{2.3d}$$

Bosonic brane charges:

$$z^I = iR\Big\{P^J,\big[X^J,X^I\big]\Big\} - \frac{R}{2}\Big[\theta^{\alpha'},\big[\theta^{\alpha'},X^I\big]\Big]\,, \tag{2.3e}$$

$$z^{IJ} = \frac{i}{2}\big[X^I,X^J\big]\,, \tag{2.3f}$$

$$z^{IJKL} = R\,X^{[I}X^JX^KX^{L]}\,. \tag{2.3g}$$

Table 1: "Standard" charges densities of the M(atrix) model in a pp-wave, as derived in [7]. The charges are obtained by tracing over the SU(N) indices, i.e. $Q_\alpha = \mathrm{Tr}\, q_\alpha$. For brevity we have suppressed an exponential involving the time coordinate; see (7)–(9) of [7]. This exponential is related to the time-dependence of the Killing spinors, which enters crucially in the construction of the supersymmetry transformation rules [13]. Note that the anti-commutator in the first line is an anti-commutator of SU(N) matrices, not a Dirac bracket.

each other vanish identically (this is true despite the fact that there is a momentum factor appearing in the z^I charge).

Our main aim of this letter is to show that new fermionic brane charges appear when one acts with a supersymmetry charge on the bosonic brane charges. We will only show this for the two-form charge Z^{IJ} as the story is very similar for the other brane charges. By straightforward application of the basic Dirac brackets, we find the key result

$$\big\{Q_\alpha,(z^{KL})_c{}^d\big\}_{\mathrm{DB}} = (-i)\sqrt{\frac{R}{2}}\big[X^K,(\gamma^L\theta)_\alpha\big]_c{}^d - (K\leftrightarrow L)\,, \tag{2.6}$$

where z^{KL} is the two-form brane charge density. This calculation shows that, indeed, the matrix model presents us with a new *fermionic brane charge*:

$$Q^I_\alpha := i\sqrt{\frac{R}{2}}\,\mathrm{Tr}\,\Big(\big[X^I,\theta_\alpha\big]\Big)\,. \tag{2.7}$$

We should emphasise that this calculation is completely identical to the one in flat space. In a flat background, the new fermionic operator Q^I_α also follows from the algebra (as can be seen from the fact that (2.6) does not depend on μ). However, in flat space one can (and typically does) consider representations for which $Q^I_\alpha|\psi\rangle \equiv 0$. The remaining issue is therefore to show that such representations are *impossible* in the pp-wave background, because they would violate the Jacobi identity $(Q, Q, Z)|\psi\rangle = 0$.

This crucial Jacobi identity takes, in matrix model variables, the more explicit form

$$0 = \Big\{\{Q_\alpha, Q_\beta\}_{\rm DB}, (z^{KL})_c{}^d\Big\}_{\rm DB} - 2\Big\{\{Q_{(\alpha}, (z^{KL})_c{}^d\}_{\rm DB}, Q_{\beta)}\Big\}_{\rm DB}\,. \tag{2.8}$$

We first compute the intermediate result

$$\big\{J^{ij}, (z^{KL})_c{}^d\big\}_{\rm DB} = 2\,(z^{iK})_c{}^d\delta^{Lj} - 2\,(z^{iL})_c{}^d\delta^{Kj}\,, \tag{2.9a}$$

$$\big\{J^{i'j'}, (z^{KL})_c{}^d\big\}_{\rm DB} = 2\,(z^{i'K})_c{}^d\delta^{Lj'} - 2\,(z^{i'L})_c{}^d\delta^{Kj'}\,, \tag{2.9b}$$

where anti-symmetry with unit weight in i, j and i', j' is implicitly assumed on the right-hand side. Using this result we can compute the first term in (2.8). One obtains a "rotated" bosonic brane charge, simply because this charge carries space-time vector indices:

$$\begin{aligned}\Big\{\{Q_{(\alpha}, Q_{\beta)}\}_{\rm DB}, (z^{KL})_c{}^d\Big\}_{\rm DB}&\\ = i\frac{2\mu}{3}\Big(2\,(\Pi\gamma^{iL})_{\alpha\beta}(z^{iK})_c{}^d - (\Pi\gamma^{i'L})_{\alpha\beta}(z^{i'K})_c{}^d\Big) - (K \leftrightarrow L)\,.\end{aligned} \tag{2.10}$$

Both sides are non-trivial when acting on a state $|\psi\rangle$ which carries the bosonic brane charge. Using (2.6) as well as the symmetry properties of the gamma matrices as listed in footnote 3, the second term in (2.8) (including the "-2") is found to be

$$\begin{aligned}-\,\big\{Q_\alpha, (\gamma^K q^L)_{\beta c}{}^d\big\}_{\rm DB} - (\alpha \leftrightarrow \beta)&\\ = (-i)\frac{2\mu}{3}\Big(2\,(z^{iK})_c{}^d(\Pi\gamma_i{}^L)_{\alpha\beta} - (z^{i'K})_c{}^d(\Pi\gamma_{i'}{}^L)_{\alpha\beta}\Big) - (K \leftrightarrow L)\,.\end{aligned} \tag{2.11}$$

Here $(q^I_\alpha)_a{}^b$ denotes the charge density of the new fermionic brane charge (2.7). The μ-independent terms in this bracket are double commutators or commutators involving the momentum variable, which should be set to zero.

The crucial point is now that in a pp-wave both sides of equation (2.10) act non-trivially on any state $|\psi\rangle$ which carries the bosonic brane charge. Hence, in order to satisfy the Jacobi identity, one has to make both sides of (2.11) non-vanishing as well. That is, the new fermionic charge has to act non-trivially on the state $|\psi\rangle$ (i.e. $q^I_\alpha|\psi\rangle \neq 0$). In that case we find that the sum of (2.10) and (2.11), when acting on $|\psi\rangle$, indeed vanishes. Contrast this with the situation in flat space, where the right-hand side of (2.10) and (2.11) are zero because $\mu = 0$. In this case it is consistent with the Jacobi identities to have a state with non-vanishing z^{KL} charge but vanishing q^I_α charge. These are indeed the representations which one usually considers in flat space.

Summarising, we have shown that the operator algebra of the matrix model in the pp-wave contains the new fermionic brane charge (2.7), and that no representations exist for which $Z^{IJ}|\psi\rangle \neq 0$ but $Q^I_\alpha|\psi\rangle = 0$. Along similar lines one can construct fermionic partners of the other bosonic brane charges in table 1. Details will appear elsewhere.[4]

2.2 The supermembrane analogy

Everything computed in the previous section has a direct analogue in the supermembrane model. To give just one example, consider for instance the supersymmetry transformation of the two-brane charge, in matrix form expressed in (2.7). Using the elementary Dirac bracket (see also footnote 5 below)

$$\big\{X^I(\sigma), P^J(\sigma')\big\}_{\rm DB} = \delta^{(2)}(\sigma-\sigma')\delta^{IJ}\,, \tag{2.13}$$

this transformation is now given by

$$\begin{aligned}
\big\{Q_\alpha, Z^{KL}\big\}_{\rm DB} &= \bigg\{ \int_\Sigma {\rm d}^2\sigma\, P^I(\sigma)(\gamma_I\theta)_\alpha(\sigma)\,,\ \int_\Sigma {\rm d}^2\sigma'\,\epsilon^{rs}\,\partial_r X^K(\sigma')\,\partial_s X^L(\sigma')\bigg\}_{\rm DB}\\
&= -2\int_\Sigma {\rm d}^2\sigma\int_\Sigma {\rm d}^2\sigma'\,\big(\gamma^K\theta(\sigma)\big)_\alpha\,\epsilon^{rs}\left(\frac{\partial}{\partial\sigma'_r}\delta^{(2)}(\sigma-\sigma')\right)\partial_s X^L(\sigma')\\
&= -2\int_\Sigma {\rm d}^2\sigma\int_{\partial\Sigma}{\rm d}\sigma'\,n_r\epsilon^{rs}\,\big(\gamma^K\theta(\sigma)\big)_\alpha\,\delta^{(2)}(\sigma-\sigma')\,\partial_s X^L(\sigma')\\
&= -2\int_{\partial\Sigma}{\rm d}\sigma\,n_r\epsilon^{rs}\,\big(\gamma^K\theta(\sigma)\big)_\alpha\,\partial_s X^L(\sigma)\\
&= -2\int {\rm d}^2\sigma\,\epsilon^{rs}\big(\gamma^K\partial_r\theta(\sigma)\big)_\alpha\,\partial_s X^L(\sigma) = 2\gamma^K Q^L_\alpha\,.
\end{aligned} \tag{2.14}$$

Here n_r denotes the vector normal to the integration boundary, and anti-symmetry in the K,L indices is again implicitly understood everywhere. This calculation[5] shows that the

[4] One might expect, by very similar logic, that a new supercharge is also required in order to satisfy the (Q,Q,P) Jacobi identity. This situation is, however, slightly different. The bracket of the supercharge with the momentum generator produces

$$\big\{Q_\alpha, P^I\big\}_{\rm DB} = \sqrt{\frac{R}{2}}\frac{\mu}{6R}\Big(2\,(\Pi\gamma^i\theta)_\alpha - (\Pi\gamma^{i'}\theta)_\alpha\Big) = \frac{\mu}{12}\Big(2\,(\Pi\gamma^i\tilde{Q})_\alpha - (\Pi\gamma^{i'}\tilde{Q})_\alpha\Big)\,. \tag{2.12}$$

The right-hand side is thus proportional to the “old” kinematic supercharge $\tilde{Q}$, which is the trace of the expression given in (2.3b), and we do not obtain a new fermionic charge.

[5] Strictly speaking, we have here used a Dirac bracket which does *not* preserve the boundary conditions. The true Dirac brackets, which incorporate the boundary conditions by treating them as constraints, lead to a dynamical evolution in which the winding charges are not dynamical variables [14]. What we have computed here is the world-volume version of the “spectrum generating” algebra, which relates physical states with different boundary conditions. Compare this with e.g. the action of rotation generators on bosonic winding charges,

$$\big\{M_{IJ}, Z^{KL}\big\} = 4\,\delta_{[I}{}^{[K} Z_{J]}{}^{L]}\,. \tag{2.15}$$

This action changes the boundary conditions and produces new configurations which are not related to the old ones by dynamical evolution.

new fermionic charge Q^I_α is non-zero whenever $\theta(\sigma)$ is not single-valued on the membrane world-surface, or for open membranes, whenever $\theta(\sigma)$ takes different values at the two boundaries of the membrane.[6]

In the pp-wave background, such configurations with non-trivial $\theta(\sigma)$ behaviour indeed do exist! They are simplest to analyse for open strings, whose algebra can be shown to contain a topological fermionic charge similar to (2.7):

$$Q_w = \int_0^\pi \mathrm{d}\sigma \, (1 - P) \partial_\sigma \theta_1 \, . \tag{2.16}$$

Here P is the matrix which relates the two fermions θ_1 and θ_2 of the open string, implementing the boundary conditions. In a pp-wave background, the mode expansions of the fermions typically contain zero-modes which are independent of the world-sheet time τ but do depend on σ. The zero modes for a string with D1-brane boundary conditions are, for instance, given by [13]

$$\begin{aligned} \theta^1 &= (1 + \Gamma^{+-}\Pi)\, \theta^+ e^{\mu\sigma} + (1 - \Gamma^{+-}\Pi)\, \theta^- e^{-\mu\sigma} \, , \\ \theta^2 &= (\Gamma^{+-} + \Pi)\, \theta^+ e^{\mu\sigma} \;\; + (\Gamma^{+-} - \Pi)\, \theta^- e^{-\mu\sigma} \, , \end{aligned} \tag{2.17}$$

for arbitrary constant spinors θ^+, θ^-. When inserted in (2.16), these zero modes are responsible for a non-vanishing fermionic topological charge of the open string. Moreover, the fermionic zero modes are related, by a simple supersymmetry transformation, to bosonic zero-modes, which also depend non-trivially on σ (again, as explained in footnote 5, one acts with a broken supersymmetry transformation and thereby changes the boundary conditions). This analysis, which crucially differs from flat space because no zero-modes for the fermions exist in that case, can be extended to the supermembrane in a straightforward way.

3. Discussion and conclusions

We have shown that new fermionic brane or winding charges appear in the supermatrix model as well as the supermembrane. The presence of these charges was expected from a Jacobi-identity argument, but we have shown here that such charges are indeed present in explicit models. We have also shown that physical multiplets involving bosonic branes will always also contain states which carry these new fermionic brane charges.

This observation has important consequences for the construction of extensions of superisometry algebras of maximally supersymmetric backgrounds. In [2] we have shown that the $\mathrm{AdS}_7 \times S^4$ superisometry algebra osp$^*(8|4)$ cannot be extended (in a physically

[6] A similar calculation for the string in flat space-time was done by Hatsuda and Sakaguchi [15]. For historical completeness, we should also mention that a fermionic extension of the superalgebra of the string in an anti-de-Sitter background was considered by Hatsuda [16]. However, this paper constructs the new fermionic charge as the superpartner of the momentum generator (just like in Green's original construction [3]) and does not take into account winding charges. Another related paper is [17], which only considers particle world-line superalgebras and therefore misses the central charges as well.

acceptable way) with only bosonic brane charges. Taking into account additional fermionic brane charges leads to an algebra which resembles an "anti-de-Sitter version" of Sezgin's M-algebra [6]. One should note, however, that the algebras presented in the present letter are based solely on the non-trivial bracket $[Q, Z] = Q'$, while Sezgin's proposal, following Green [3], has in addition $[Q, P] = Q''$. In the supermatrix model or the supermembrane, the bracket $[Q, P]$ does not lead to a new supercharge, so it is unclear whether the M-algebra is indeed only consistent upon introduction of a new superpartner for the momentum generator. A more elaborate analysis of this problem will appear elsewhere.

It is clearly necessary to develop a better understanding of branes which carry the new fermionic charges and the supermultiplets in which they fit. One would perhaps also like to understand them in terms of supergravity solutions. Finally, we should mention that it would be very interesting to understand the implications of these new fermionic brane charges in the context of the AdS/CFT correspondence.

Acknowledgements

We thank Eric Bergshoeff, Ergin Sezgin, Paul Townsend and especially Bernard de Wit for comments on a preliminary version of this letter.

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