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The one-loop vacuum energy and RG flow induced by double-trace operators in AdS/CFT and dS/CFT correspondence

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ABSTRACT

It has been shown that the renormalization group (RG) flow connecting different vacua may occur due to the multi-trace operators in four-dimensional CFT. In the AdS/CFT set-up, the double trace operators correspond to the tachyonic scalar field living in the bulk five-dimensional AdS. In this paper, we consider the five dimensional black hole as the bulk space-time instead of the pure AdS. In frames of the AdS/CFT correspondence, the AdS black hole is believed to describe dual CFT at finite temperature. From the AdS side, we consider the difference of energies between the two vacua by one-loop calculation for the scalar field with tachyonic mass in five-dimensional AdS black hole. In order to check the dS/CFT correspondence, the corresponding calculations are done in five-dimensional deSitter space. The difference between the two vacua is specified by the difference of the boundary conditions of the scalar field. We show that for AdS black hole, there might occur instability which could be the manifestation of the Hawking-Page phase transition. For stable phase of AdS black hole as well as for deSitter bulk, proposed c -function found beyond the leading order approximation shows the monotonic behaviour consistent with c -theorem.

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1 Introduction

The holographic principle realized in string theory in the form of celebrated AdS/CFT correspondence (for a general review, see [1]) brought to our attention not only the new concepts but also better understanding of well-known phenomena in high energy physics. In particular, well-known RG theory has been enriched by the holographic RG (for a general introduction and list of references, see [2, 3].) In frames of holographic RG it turned out to be easier to study the number of questions, like the check of so-called c-theorem conjectured in four dimensions in ref.[4] by analogy with the two-dimensional Zamolodchikov's c-theorem [5]. In fact, it has been shown [6, 7] (for earlier studies, see [8, 9]) that four-dimensional c-theorem follows from classical supergravity side of AdS/CFT. The very important check of c-theorem in one-loop approximation (beyond classical supergravity approximation) has been recently done in ref.[10].

The consideration of Gubser-Mitra [10] is very much related with Witten's discussion of multi-trace operators [11] (see also [12]) and their dual gauge theory description in (asymptotically) AdS space. We may add a term given by the product of the two single trace operators \mathcal{O} , in the form of $\frac{f}{2}\mathcal{O}^2$, to the gauge theory Lagrangian. If the coefficient f has the dimension of mass, the operator $\frac{f}{2}\mathcal{O}^2$ is relevant and the RG flow is generated. In the RG flow, the UV fixed point corresponds to $f = 0$ and the IR one to $f = \infty$. In the AdS side, the two fixed points can be specified by the boundary conditions for the massive scalar field ϕ corresponding to the operator \mathcal{O} . In [10], the one-loop vacuum energy of the scalar field ϕ was discussed. The obtained vacuum energy itself diverges but the difference between the two vacuum energies corresponding to the different boundary conditions of ϕ is finite. The difference between two vacuum energies corresponds to the difference of the values of c-functions in the UV and IR fixed points. Then we find the conformal invariance is broken by the one-loop effects, which lead to non-trivial RG flow. Since the one-loop effect gives the sub-leading correction in the $1/N$ -expansion, the difference between the values of c-function in the two vacuum is the sub-leading.

The present work is motivated by the study done in ref.[10]. Using the methods of above paper we search for the description of RG flow when double-trace operator for tachyon field is incorporated in the situation when bulk space is not pure AdS. Specifically, we start from the tachyon field living

in 5d AdS black hole (BH). (It is not difficult to extend the calculation for other dimensions). It is known [13] that AdS/CFT correspondence is applied well to situation when bulk space is AdS BH so all discussion of ref.[10] may be directly used. Since the black hole has a temperature, the corresponding dual CFT is thermal theory. It is known that Hawking-Page phase transitions occur between AdS BH and global AdS vacuum. These phase transitions have the AdS/CFT interpretation [13] as confinement-deconfinement transitions in dual gauge theory. Then, it is expected that RG flow induced by double-trace operator when bulk space is AdS BH which is dual to thermal QFT should be qualitatively more complicated. In particular, some instability (divergence) corresponding to the description of phase transition should appear at the one-loop order. Indeed, such instability is explicitly found in our calculation of difference of vacuum energies at IR and UV.

Another interesting aspect of such RG flow investigation is related with the fact that it may be applied to another duality. To be specific, we considered also five-dimensional dS bulk space. This space is expected to be crucial in proposed dS/CFT correspondence[14, 15]. Despite the fact that explicit reasonable dual theory in dS_5/CFT_4 correspondence is not constructed yet, we found the corresponding RG flow and conjectured c-function in direct analogy with AdS/CFT. In other words, the check of four-dimensional c-theorem as it follows from dS/CFT correspondence (for a general introduction, see [16]) is done beyond the classical supergravity approximation.

The paper is organized as follows. In the next section we propose c-function from AdS black hole (no deformation case). Section three is devoted to the study of massive scalar field in five-dimensional AdS black hole bulk. Using simplified solution of Klein-Gordon equation which correctly reproduces the asymptotics of the scalar field in the bulk, the corresponding propagator is constructed. It is used in the calculation of difference of vacuum energies for boundary conditions specified by double-trace operator. The interpretation of instability in such difference as indication to Hawking-Page phase transition is given. Our investigation permits to propose c-function in AdS black hole phase using AdS/CFT, while usually it is not easy to define c-function in thermal QFT. Section four is devoted to the study of RG flow found in the similar way but for dS bulk. This discussion may be relevant for dS/CFT correspondence as it gives the information about conjectured c-function in the situation when dual CFT is not found yet. Some outlook is presented in final section.

2 c-function from AdS black hole: no deformation

Let us start from the discussion of c-function as it may appear from AdS BH. In [6], a candidate of the c-function from 5d AdS space has been proposed in terms of the metric as follows:

$$c_{\text{GPPZ}} = \left(\frac{dA}{dz} \right)^{-3} , \quad (1)$$

where the metric is taken in the warped form:

$$ds^2 = dz^2 + \sum_{\mu=1}^4 e^{2A} dx_{\mu} dx^{\mu} . \quad (2)$$

Then for the pure anti-deSitter (AdS) spacetime, where $A = \frac{z}{l}$, the c-function becomes a constant:

$$c_{\text{GPPZ}} = l^3 . \quad (3)$$

The c-function (1) is positive as it should be. Note that the c-function is monotonically increasing function of the energy scale z .

One may propose a similar c-function for the Schwarzschild-Anti-de Sitter (SAdS) space with the flat horizon:

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = -e^{2\rho} dt^2 + e^{-2\rho} dr^2 + r^2 \sum_{i=1,2,3} (dx^i)^2 , \quad e^{2\rho} = \frac{r^2}{l^2} - \frac{\mu}{r^2} . \quad (4)$$

If we redefine the coordinates r and x^i by new ones η and \hat{x}^i as

$$r^2 = l\mu^{\frac{1}{2}} \cosh \frac{2\eta}{l} , \quad x^i = l^{-\frac{1}{2}} \mu^{-\frac{1}{4}} \hat{x}^i , \quad (5)$$

and furthermore Wick-rotate the time coordinate t to θ by

$$t = \frac{il^{\frac{3}{2}}}{2\mu^{\frac{1}{4}}} \theta , \quad (6)$$

the metric (4) can be rewritten as

$$ds^2 = \frac{l^2 \sinh^2 \frac{2\eta}{l}}{4 \cosh \frac{2\eta}{l}} d\theta^2 + d\eta^2 + l^2 \cosh \frac{2\eta}{l} \sum_{i=1,2,3} (d\hat{x}^i)^2 . \quad (7)$$

A little bit surprisingly, the obtained metric does not depend on μ , which parametrizes the black hole (BH). As is clear from (5), the pure AdS ($\mu \rightarrow 0$) corresponds to $\eta \rightarrow \infty$ limit and the large black hole corresponds to $\eta \rightarrow 0$ limit. The horizon exists at $\eta = 0$ or $r^2 = l\mu^{\frac{1}{2}}$. In order to avoid the orbifold singularity at the horizon, the Euclid time coordinate θ has a period of 2π .

The obtained metric (7) does not have the warped form as in (2). Since $\sqrt{g} = e^{4A}$ in (2), identifying η with z in (2), one may propose a c-function c_1 as

$$c_1 = \left(\frac{1}{8} \frac{d \ln g}{d\eta} \right)^{-3}. \quad (8)$$

When the metric can be written in the warped form (2), identifying η with z , (8) reproduces (1). When the metric has a warped form (2), the scale transformation $x^\mu \rightarrow e^\lambda x^\mu$ with a constant parameter λ of the transformation can be absorbed into the shift of A like $A \rightarrow A - \lambda$. If we identify the coordinate z as a parameter of the scale, the scale invariance requires $\frac{d^2 A}{dz^2} = 0$. Similarly for the metric (7), the scale transformation $\theta \rightarrow e^\lambda \theta$ and $x^i \rightarrow e^\lambda x^i$ with a constant λ can be absorbed into the scale transformation of the metric $\frac{l^2 \sinh^2 \frac{2\eta}{l}}{4 \cosh \frac{2\eta}{l}} \rightarrow e^{-2\lambda} \frac{l^2 \sinh^2 \frac{2\eta}{l}}{4 \cosh \frac{2\eta}{l}}$ and $l^2 \cosh \frac{2\eta}{l} \rightarrow e^{-2\lambda} l^2 \cosh \frac{2\eta}{l}$. Then if η corresponds to the parameter of the scale transformation, the scale invariance requires $\frac{d^2}{d\eta^2} \left(\frac{l^2 \sinh^2 \frac{2\eta}{l}}{4 \cosh \frac{2\eta}{l}} \right) = \frac{d^2}{d\eta^2} \left(l^2 \cosh \frac{2\eta}{l} \right) = 0$. Then one may define two kinds of c-function corresponding to (θ, θ) - and (i, j) -components of the metric. The c-function (8) is a kind of the average of the two c-functions. At least, at the RG fixed point, if exists, where there is a scale invariant, we find $\frac{dc_1}{d\eta} = 0$.

By using (7), one gets

$$g = \frac{l^8 \sinh^2 \frac{2\eta}{l} \cosh \frac{2\eta}{l}}{4} = \frac{l^8 \sinh^2 \frac{4\eta}{l}}{16}. \quad (9)$$

Then

$$c_1 = l^3 \tanh^3 \frac{4\eta}{l} \quad (10)$$

This expression reproduces (3) in the pure AdS limit ($\eta \rightarrow \infty$). We also note that c_1 is monotonically increasing function of η and therefore the energy scale. It is interesting that c-function vanishes at the horizon $\eta = 0$, which can be identified with the large black hole limit. The central charge, in general, measures the massless degrees of freedom. Then the vanishing central

charge means that there are no massless modes. At the horizon, the time coordinate becomes degenerate (as well-known, anything near the horizon moves slowly for the observer far from the horizon). Then effectively, any mass or energy goes to infinity and only constant modes can survive there.

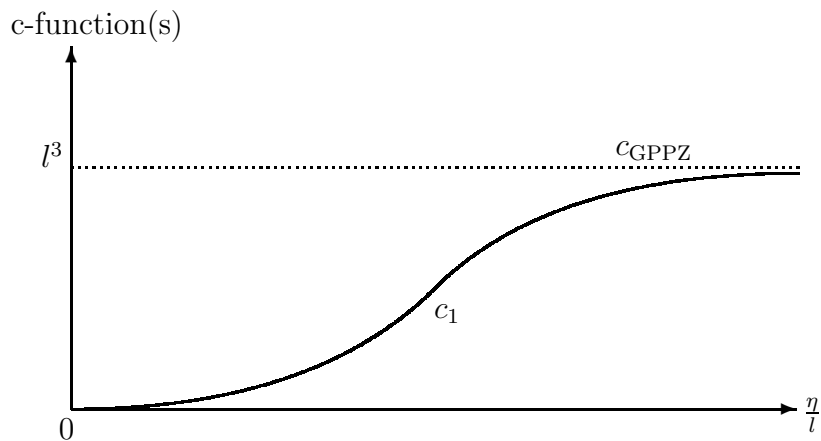


Figure 1: c_{GPPZ} (3) for the pure AdS is given by the dotted line and c_1 (10) by the solid line.

3 The scalar field in AdS black hole and one-loop vacuum energy for tachyon

In AdS/CFT correspondence, RG flow can be generated by the double trace operator. On supergravity side, the double trace operator corresponds to the scalar field with tachyonic mass. We now consider the propagation of the

free scalar field ϕ with tachyonic mass m ($m^2 < 0$):

$$S_\phi = \int d^5x \sqrt{-g^{(5)}} \left\{ -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 \right\} \quad (11)$$

in five-dimensional AdS BH. In the black hole spacetime (7), which has been Wick-rotated into the Euclidean signature, the Klein-Gordon equation has the following form:

$$\begin{aligned} & \frac{4 \cosh \frac{2\eta}{l}}{l^2 \sinh^2 \frac{2\eta}{l}} \partial_\theta^2 \phi + \frac{1}{\sinh \frac{2\eta}{l} \cosh \frac{2\eta}{l}} \partial_\eta \left(\sinh \frac{2\eta}{l} \cosh \frac{2\eta}{l} \partial_\eta \phi \right) \\ & + \frac{1}{\cosh \frac{2\eta}{l}} \sum_{i=1,2,3} \frac{\partial^2 \phi}{\partial \tilde{x}^i{}^2} - m^2 \phi = 0 . \end{aligned} \quad (12)$$

Taking the plane wave, one replaces ∂_θ^2 and $\frac{\partial^2}{\partial \tilde{x}^i{}^2}$ by

$$\partial_\theta^2 \rightarrow -n^2 , \quad \sum_{i=1,2,3} \frac{\partial^2}{\partial \tilde{x}^i{}^2} \rightarrow -k^2 . \quad (13)$$

Here n is an integer. With redefined scalar field ϕ

$$\phi = \frac{\tilde{\phi}}{\sinh^{\frac{1}{2}} \frac{2\eta}{l} \cosh^{\frac{1}{2}} \frac{2\eta}{l}} , \quad (14)$$

the Schrödinger-like equation looks like:

$$-\partial_\eta^2 \tilde{\phi} + \left\{ -\frac{1}{l^2} \left(\frac{1}{\sinh^2 \frac{2\eta}{l} \cosh^2 \frac{2\eta}{l}} - 4 \right) + \frac{4n^2 \cosh \frac{2\eta}{l}}{l^2 \sinh^2 \frac{2\eta}{l}} + \frac{k^2}{\cosh^2 \frac{2\eta}{l}} \right\} \tilde{\phi} = m^2 \tilde{\phi} . \quad (15)$$

When η is small, Eq.(15) behaves as

$$-\partial_\eta^2 \tilde{\phi} + \frac{1}{\eta^2} \left\{ -\frac{1}{4} + n^2 \right\} \tilde{\phi} = 0 . \quad (16)$$

Then

$$\tilde{\phi} \sim \eta^{\frac{1}{2} \pm n} , \quad (17)$$

or by using (14), we find

$$\phi \sim \eta^{\pm n} . \quad (18)$$

On the other hand, when η is large, Eq.(15) behaves as

$$-\partial_\eta^2 \tilde{\phi} + \frac{4}{l^2} \phi = m^2 \tilde{\phi} , \quad (19)$$

and one gets

$$\tilde{\phi} \sim e^{\pm \eta \sqrt{\frac{4}{l^2} + m^2}} , \quad (20)$$

or

$$\phi \sim e^{\eta \left(-\frac{2}{l} \pm \sqrt{\frac{4}{l^2} + m^2} \right)} . \quad (21)$$

Using the coordinate r , we have

$$\phi \sim r^{-\Delta_\pm} , \quad \Delta_\pm \equiv 2 \pm \sqrt{4 + m^2 l^2} , \quad (22)$$

which reproduces the usual AdS/CFT results, that is, the power law behaviour corresponds to the operator in CFT with conformal weight Δ_\pm .

Since it is difficult to solve the Klein-Gordon equation exactly in the Schwarzschild-AdS background, we now consider a simplified model. From Eqs.(16) and (19), we can find that the \tilde{x}^i or k -dependence is not essential for both of small η and large η cases. Then one may neglect the k -dependence for the qualitative argument. We now approximate the Klein-Gordon equation (15) by

$$\partial_\eta^2 \tilde{\phi} + \left\{ -\frac{4}{l^2} + \left(\frac{1}{4} - n^2 \right) \frac{1}{\eta^2} \right\} \tilde{\phi} = m^2 \tilde{\phi} , \quad (23)$$

which reproduces (16) for small η and (19) for large η . We also assume, instead of (14),

$$\phi = \sqrt{\frac{l}{2\eta}} e^{-\frac{2\eta}{l}} \tilde{\phi} , \quad (24)$$

which coincides with (14) in the large η (except $\eta^{-\frac{1}{2}}$ and small η limits. The solution of (23) is given by the modified Bessel functions I_n and K_n :

$$\tilde{\phi} = \eta^{\frac{1}{2}} (\alpha I_n(\gamma\eta) + \beta K_n(\gamma\eta)) . \quad (25)$$

Here

$$\gamma \equiv \sqrt{\frac{4}{l^2} + m^2} . \quad (26)$$

Then one gets

$$\phi = \sqrt{\frac{l}{2}} e^{-\frac{2\eta}{l}} (\alpha I_n(\gamma\eta) + \beta K_n(\gamma\eta)) . \quad (27)$$

Since $I_n(z) \sim \frac{e^z}{\sqrt{2\pi z}}$ and $K_n(z) \sim e^{-z} \sqrt{\frac{\pi}{2z}}$ for large z , when η is large, ϕ given by combining (24) and (25) behaves as

$$\phi \sim \alpha \frac{e^{\left(-\frac{2}{l} + \sqrt{\frac{4}{l^2} + m^2}\right)\eta}}{\sqrt{2\pi}} + \beta e^{\left(-\frac{2}{l} - \sqrt{\frac{4}{l^2} + m^2}\right)\eta} \sqrt{\frac{2}{\pi}} . \quad (28)$$

In [10], the deformation of the CFT by the double trace operator

$$\frac{f}{2} \int d^4x \mathcal{O}^2 \quad (29)$$

has been discussed from the AdS side. The parameter f corresponds to the boundary condition for the bulk scalar field. In the present case, the boundary condition looks like

$$f = \frac{\alpha}{\pi\beta} . \quad (30)$$

With two independent solutions for $\tilde{\phi}$

$$\tilde{\phi}_a = \eta^{\frac{1}{2}} (\alpha_a I_n(\gamma\eta) + \beta_a K_n(\gamma\eta)) , \quad a = 1, 2 , \quad (31)$$

the propagator $\tilde{G}(\eta, \xi)$ of $\tilde{\phi}$ can be constructed as

$$\tilde{G}(\eta, \xi) = \frac{\phi_1(\eta)\phi_2(\xi)\theta(\eta - \xi) + \phi_1(\xi)\phi_2(\eta)\theta(\xi - \eta)}{\gamma(\alpha_1\beta_2 - \beta_1\alpha_2)} . \quad (32)$$

In fact $\tilde{G}(\eta, \xi)$ satisfies the following equation, corresponding to (23):

$$-\partial_\eta^2 \tilde{G}(\eta, \xi) + \left\{ -\frac{4}{l^2} - m^2 + \left(\frac{1}{4} - n^2 \right) \frac{1}{\eta^2} \right\} \tilde{G}(\eta, \xi) = \delta(\eta - \xi) . \quad (33)$$

Then from (24) the propagator $G(\eta, \xi)$ for ϕ is given by

$$G(\eta, \xi; m^2) = \sqrt{\frac{l}{4\eta\xi}} e^{-\frac{2(\eta+\xi)}{l}} \tilde{G}(\eta, \xi) . \quad (34)$$

Then the vacuum energy is expressed as [10]

$$\begin{aligned}
V(\eta) &= -\frac{1}{2} \int_{-\frac{4}{l^2}}^{m^2} d\tilde{m}^2 G(\eta, \eta; \tilde{m}^2) \\
&= -\frac{1}{2} \int_0^{\gamma^2} d\tilde{\gamma}^2 G(\eta, \eta; \tilde{m}^2(\tilde{\gamma})) \\
&= -\frac{1}{2} \int_0^{\gamma^2} d\tilde{\gamma}^2 \frac{le^{-\frac{4\eta}{l}}}{2\tilde{\gamma} \sin(\varphi_1 - \varphi_2)} \sum_{n=-\infty}^{\infty} \left\{ \cos \varphi_1 \cos \varphi_2 I_n(\tilde{\gamma}\eta)^2 \right. \\
&\quad \left. + \sin(\varphi_1 + \varphi_2) I_n(\tilde{\gamma}\eta) K_n(\tilde{\gamma}\eta) + \sin \varphi_1 \sin \varphi_2 K_n(\tilde{\gamma}\eta)^2 \right\} . \quad (35)
\end{aligned}$$

Here we denote

$$\alpha_a = \cos \varphi_a , \quad \beta_a = \sin \varphi_a , \quad (36)$$

since the overall factor of α_a and β_a is irrelevant for the propagator. By comparing (30), (32) and (36), we find

$$f = \frac{1}{\pi \tan \varphi_1} . \quad (37)$$

On the other hand, the parameter φ_2 could be determined from the boundary condition at the horizon $\eta = 0$. It is natural that the scalar field is regular at the horizon. Then

$$\varphi_2 = 0 . \quad (38)$$

Since

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} I_n(z)^2 &= I_0(2z) , \\
\sum_{n=-\infty}^{\infty} I_n(z) K_n(z) &= K_0(2z) , \quad (39)
\end{aligned}$$

one gets

$$V(\eta) = -\frac{1}{2} \int_0^{\gamma^2} d\tilde{\gamma}^2 \frac{le^{-\frac{4\eta}{l}}}{2\tilde{\gamma} \sin \varphi_1} \left\{ \cos \varphi_1 I_0(2\tilde{\gamma}\eta) + \sin \varphi_1 K_0(2\tilde{\gamma}\eta) \right\} . \quad (40)$$

The remark is in order. The obtained vacuum energy is generally divergent quantity. The calculation of vacuum energy as finite quantity requires the application of some regularization. Using mainly zeta-regularization, the

calculation of above vacuum energy for bulk scalars and spinors on five-dimensional AdS space was performed in refs.[17, 18, 19, 20, 21, 22]. Instead of repeating such complicated calculations, we follow to the prescription developed in [10] and calculate the difference between vacuum energies corresponding to different boundary conditions specified by the double-trace operator. For pure AdS space such difference corresponds to difference between IR and UV points (end-points of RG flow) and is finite.

Then the difference of vacuum energies specified by the two boundary conditions corresponding to $\varphi_1 = \Theta_{(1)}$ and $\varphi_1 = \Theta_{(2)}$ is given by

$$\begin{aligned}\delta V &= V(\eta, \varphi_1 = \Theta_{(1)}) - V(\eta, \varphi_1 = \Theta_{(2)}) \\ &= -(\cot \Theta_{(1)} - \cot \Theta_{(2)}) \frac{1}{2} \int_0^\gamma d\gamma e^{-\frac{4\eta}{l}\gamma} I_0(2\gamma\eta) \\ &= -(\cot \Theta_{(1)} - \cot \Theta_{(2)}) \frac{e^{-\frac{4\eta}{l}\gamma}}{2} {}_1F_2\left(\frac{1}{2}; 1, \frac{3}{2}; \gamma^2\eta^2\right). \quad (41)\end{aligned}$$

Here ${}_1F_2(a; b, c; z)$ is a hypergeometric function. When $\gamma = 0$, $\delta V = 0$. This is consistent with the corresponding CFT. Since $\gamma = 0$ corresponds to $\Delta_- = 2$, the deformation of the CFT is marginal and the central charge does not change, that is, $\delta V = 0$. Since $f \rightarrow \infty$ corresponds to $\varphi \rightarrow 0$ and $f \rightarrow 0$ to $\varphi \rightarrow \frac{\pi}{2}$ and, if we consider the difference between $f \rightarrow \infty$ and $f \rightarrow 0$, that is $\Theta_1 \rightarrow 0$ and $\Theta_2 \rightarrow \frac{\pi}{2}$, δV diverges. This is different from the result in pure AdS case and is presumably related with phase transitions as we argue below. For the pure AdS case, one gets[10]

$$\delta V = \frac{1}{12\pi^2 l^5} \left\{ \frac{(\Delta_- - 2)^3}{3} - \frac{(\Delta_- - 2)^5}{5} \right\} = -\frac{1}{12\pi^2 l^5} \left\{ \frac{\gamma^3 l^3}{3} - \frac{\gamma^5 l^5}{5} \right\}. \quad (42)$$

When η is large, since $I_0(z) \sim \frac{e^z}{\sqrt{2\pi z}}$, we find δV (41) behaves as

$$\delta V \sim -\frac{(\cot \Theta_{(1)} - \cot \Theta_{(2)})}{4} \int_0^\gamma d\gamma \frac{e^{-(\frac{4}{l}-2\gamma)\eta}}{\sqrt{\gamma\pi\eta}}. \quad (43)$$

Therefore if $l\gamma < 2$ ($l\gamma > 2$), δV exponentially decreases (increases). From the viewpoint of CFT, $l\gamma = 2$ corresponds to $\Delta_- = 0$.

Let us define $F(\gamma; \eta)$ by

$$F(\gamma; \eta) \equiv \frac{1}{2} \int_0^\gamma d\gamma e^{-\frac{4\eta}{l}\gamma} I_0(2\gamma\eta) = \frac{e^{-\frac{4\eta}{l}\gamma}}{2} {}_1F_2\left(\frac{1}{2}; 1, \frac{3}{2}; \gamma^2\eta^2\right), \quad (44)$$

which appears in (41). Some plots have been given in Figure 2 for $l\gamma = 1$, Figure 3 for $l\gamma = 2$, and Figure 4 for $l\gamma = 2.1$ as functions of $\frac{\eta}{l}$, which corresponds to the horizontal axis. When $l\gamma \leq 2$, $F(\gamma; \eta)$ is a monotonically decreasing function of $\frac{\eta}{l}$. When $l\gamma > 2$, $F(\gamma; \eta)$ increases exponentially for large $\frac{\eta}{l}$ and there appears a minimum. The minimum appears at $\frac{\eta}{l} = 7.6068 \dots$ and the minimal value of F is $0.0108396 \dots$. When $l\gamma = 4$, there appears a minimum at $\frac{\eta}{l} = 0.4176 \dots$ and the minimum value of F is $0.910144 \dots$. In the language of the holographic renormalization group, the coordinate η corresponds to the parameter of the scaling. If δV corresponds to the difference of the central charges, the minimum, which appears for $l\gamma > 2$, should correspond to the renormalization fixed point. On the other hand, as explained before, large η corresponds to the pure AdS and small η to the large AdS black hole. Then the minimum might divide two phases corresponding to the pure AdS or small black hole phase and the large black hole phase. Since the Hawking temperature is given by $T_H = \frac{r_H}{\pi l^2}$, the larger black hole has the higher temperature. Here r_H is the radius of the horizon. Then the phase transition, if exists, might correspond to the thermal transition of the CFT.

Indeed, it has been suggested by Hawking and Page [23], there is a phase transition between AdS BH spacetime and global AdS vacuum. BH is stable at high temperature but it becomes unstable at low temperature, which can be understood from the free energy F :

$$F = -\frac{V_3}{\kappa^2} r_H^2 \left(\frac{r_H^2}{l^2} - \frac{k}{2} \right) \quad (45)$$

Here if $k > 0$ the boundary can be three dimensional sphere, if $k < 0$, hyperboloid, or if $k = 0$, flat space. Properly normalizing the coordinates, one can choose $k = 2, 0$, or -2 . Then if $k = 2$, the free energy F vanishes at $r_H = l$, which is the critical point of the phase transition. From the point of view of the AdS/CFT correspondence [1], this phase transition corresponds to the confinement-deconfinement transition in dual gauge theory [13]. Eq.(45) seems to tell that there is no phase transition when $k = 0$ or $k = -2$, which is not always true. There is an argument that it is easier to subtract the action of AdS soliton [24] instead of the vacuum AdS [25]. In this case, besides the temperature, the area of the horizon becomes an independent parameter on which the thermodynamical quantities depend.

When $k = 0$, the metric of SAdS BH (4) can be written as

$$\begin{aligned}
ds_{\text{BH}}^2 &= -e^{2\rho_{\text{BH}}(r)} dt_{\text{BH}}^2 + e^{-2\rho_{\text{BH}}(r)} dr^2 + r^2 \left(d\phi_{\text{BH}}^2 + \sum_{i=1,2} (dx^i)^2 \right), \\
e^{2\rho_{\text{BH}}(r)} &= \frac{1}{r^2} \left\{ -\mu_{\text{BH}} + \frac{r^4}{l^2} \right\}.
\end{aligned} \tag{46}$$

In (46), we choose a torus for the $k = 0$ Einstein manifold for simplicity. The coordinates of the torus are ϕ_{BH} and $\{x^1, x^2\}$. One assumes ϕ_{BH} has a period of η_{BH} : $\phi_{\text{BH}} \sim \phi_{\text{BH}} + \eta_{\text{BH}}$. The AdS soliton solution can be obtained by exchanging the signature of t_{BH} and ϕ_{BH} as $t_{\text{BH}} \rightarrow i\phi_s$ and $\phi_{\text{BH}} \rightarrow it_s$. Then the metric of the AdS soliton is given by

$$\begin{aligned}
ds_s^2 &= -r^2 dt_s + e^{-2\rho_s(r)} dr^2 + e^{2\rho_s(r)} d\phi_s^2 + r^2 \sum_{i=1,2} (dx^i)^2, \\
e^{2\rho_s(r)} &= \frac{1}{r^2} \left\{ -\mu_s + \frac{r^4}{l^2} \right\}.
\end{aligned} \tag{47}$$

The free energy F is obtained as follows:

$$F = -\frac{\eta_{\text{BH}} V_2 l^6}{\kappa^2} \left\{ (\pi T_H)^4 - \left(\frac{\pi}{l\eta_{\text{BH}}} \right)^4 \right\}. \tag{48}$$

Eq.(48) tells that there is a phase transition at

$$T_{\text{BH}} = \frac{1}{l\eta_{\text{BH}}}. \tag{49}$$

Hence, when $T_H > \frac{1}{l\eta_{\text{BH}}}$, the black hole is stable but when $T_H < \frac{1}{l\eta_{\text{BH}}}$, the black hole becomes unstable and the AdS soliton is preferred. Although the AdS soliton solution is not connected with the SAdS solution by any continuous parameter, the maximum of δV (41), which appears when $l\gamma > 2$, might correspond to the above phase transition.

Note that if one defines a c-function by

$$\begin{aligned}
c_2 &= \delta V - \delta V(\eta = 0) \\
&= -\left(\cot \Theta_{(1)} - \cot \Theta_{(2)} \right) (F(\gamma; \eta) - F(\gamma; \eta = 0)),
\end{aligned} \tag{50}$$

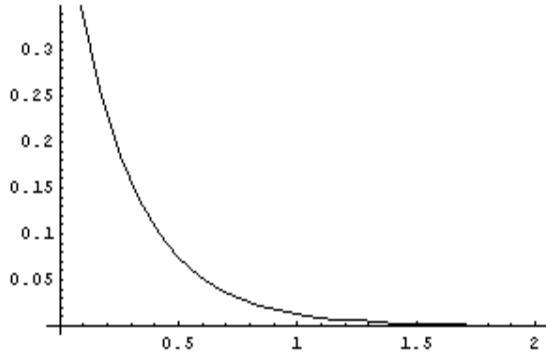


Figure 2: $F(\gamma; \eta)$ versus $\frac{\eta}{l}$ for $l\gamma = 1$. $F(\gamma; \eta)$ decreases exponentially.

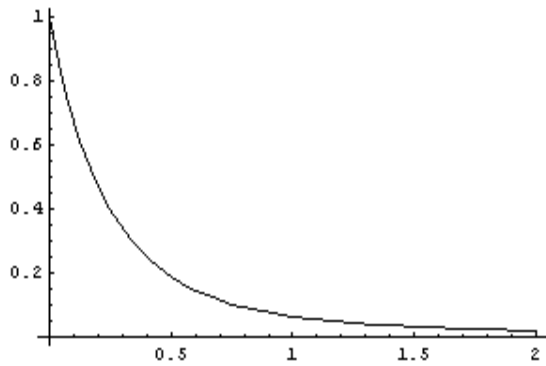


Figure 3: $F(\gamma; \eta)$ versus $\frac{\eta}{l}$ for $l\gamma = 2$. $F(\gamma; \eta)$ decreases exponentially.

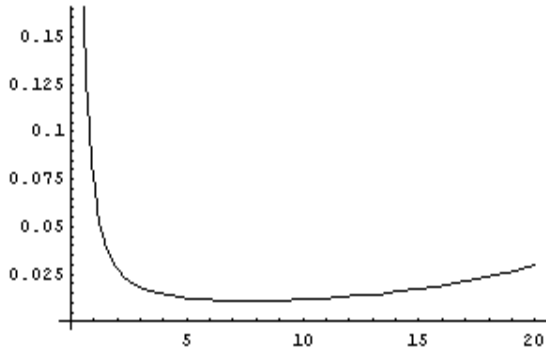


Figure 4: $F(\gamma; \eta)$ versus $\frac{\eta}{l}$ for $l\gamma = 2.1$. $F(\gamma; \eta)$ increases exponentially for large $\frac{\eta}{l}$ and there appears a minimum.

the c-function c_2 is a monotonically increasing function of η if $0 < l\gamma \leq 2$ and $\cot \Theta_{(1)} - \cot \Theta_{(2)} > 0$. We also note that c_2 vanishes by construction, at $\eta = 0$ or large black hole limit as c_1 (10).

We now consider the difference between two black hole solutions. Let a radial coordinate of one of the black hole solutions is associated with a mass parameter μ_1 as η_1 and another with μ_2 as η_2 . Then Eq.(5) shows

$$\xi \equiv \left(\frac{\mu_1}{\mu_2} \right)^{-\frac{1}{2}} = \frac{\cosh \frac{2\eta_1}{l}}{\cosh \frac{2\eta_2}{l}}, \quad (51)$$

We now assume $\mu_2 > \mu_1$ and therefore $\eta_1 > \eta_2$ and $\xi > 1$. In the UV region, $\eta_{1,2}$ becomes large

$$\xi = \left(\frac{\mu_1}{\mu_2} \right)^{-\frac{1}{2}} \rightarrow e^{\frac{2(\eta_1 - \eta_2)}{l}}. \quad (52)$$

As an IR region, we consider a limit $\xi_2 \rightarrow 0$. Then in the limit

$$e^{\frac{2\eta_1}{l}} \sim \xi + \sqrt{\xi^2 - 1}. \quad (53)$$

Here the sign in front of $\sqrt{\xi^2 - 1}$ can be fixed since $\eta_1 > \eta_2 = 0$ $e^{\frac{2\eta_1}{l}} > 1$.

When η_1 and η_2 are large, using (52) one gets

$$\begin{aligned} F(\gamma; \eta_1) &\sim \frac{1}{4\sqrt{\pi}} \int_0^\gamma d\tilde{\gamma} \frac{e^{-\frac{4\eta_1}{t} + 2\tilde{\gamma}\eta_1}}{\sqrt{\tilde{\gamma}\eta_1}} \\ &= \frac{1}{4\sqrt{\pi}} \sqrt{\frac{\eta_2}{\eta_1}} e^{-\frac{4(\eta_1 - \eta_2)}{t}} \int_0^\gamma d\tilde{\gamma} \frac{e^{-\frac{4\eta_2}{t} + 2\tilde{\gamma}\eta_2 + \frac{\tilde{\gamma}}{t} \ln \xi}}{\sqrt{\tilde{\gamma}\eta_1}}, \end{aligned} \quad (54)$$

and

$$e^{\frac{4\eta_1}{t}} F(\gamma; \eta_2) < e^{\frac{4\eta_2}{t}} F(\gamma; \eta_1) < e^{\frac{4\eta_1}{t}} F(\gamma; \eta_2) e^{\frac{\gamma}{t} \ln \xi}, \quad (55)$$

since $\frac{\eta_1}{\eta_2} \rightarrow 1$. Let denote δV (41) or (43) corresponding to η_1 as δV_1 and that to η_2 as δV_2 . Then if $(\cot \Theta_{(1)} - \cot \Theta_{(2)}) > 0$, Eq.(55) indicates that

$$e^{-\frac{4}{t} \ln \xi} \delta V_2 < \delta V_1 < \delta V_2 e^{-\frac{4-2\gamma}{t} \ln \xi}, \quad (56)$$

in the limit that η_1 and η_2 become large by keeping the difference finite : $\eta_1 - \eta_2 = \frac{l}{2} \ln \xi$. On the other hand, in the limit $\eta_2 \rightarrow 0$, one has

$$\delta V_2 - \delta V_1 = - \left(\cot \Theta_{(1)} - \cot \Theta_{(2)} \right) \left\{ \frac{\gamma}{2} - F(\gamma; \eta_1) \right\}. \quad (57)$$

Here η_1 is given by (53). The right hand side of (57) vanishes when $\eta_1 = 0$. When $\gamma \leq 2$, since $F(\gamma; \eta_1)$ is monotonically decreasing function as in Figures 2 and 3, we have if $(\cot \Theta_{(1)} - \cot \Theta_{(2)}) > 0$

$$\delta V_2 < \delta V_1. \quad (58)$$

Even if $\gamma > 2$, $F(\gamma; \eta_1)$ is monotonically decreasing function for small η_1 as in Figure 4 and we obtain (58) but if η_1 becomes larger, we have

$$\delta V_2 > \delta V_1. \quad (59)$$

Eqs.(58) and (59) indicate that there might be a phase transition (the Hawking-Page phase transition) if the black hole radius becomes large. The point where $\delta V_2 = \delta V_1$, of course, depends on the value of $\eta_{1,2}$. The point can be read off from Figure 4 by shifting η . The line of the graph obtained by the shift always crosses the line of the original graph. The crossing point corresponds to $\delta V_2 = \delta V_1$. Thus, we demonstrated that away from

phase transition, in AdS BH phase the conjectured c-function³ deformed by double-trace operator shows the monotonically increasing behaviour in accord with c-theorem. Note also that above consideration may be generalized to other odd dimensions.

The confinement-deconfinement transition at finite temperature occurs even without the double trace operator. Using the propagator (34) of the tachyonic scalar field ϕ , we may consider the correlation function of the operators \mathcal{O} corresponding to ϕ . From the AdS/CFT correspondence, the correlation function of two \mathcal{O} is proportional to the propagator:

$$\langle \mathcal{O}\mathcal{O} \rangle \propto G(\eta, \eta; m^2) \Big|_{\eta \rightarrow \text{boundary}} . \quad (60)$$

The connected part of the two double trace operators is

$$\left\langle \frac{f}{2} \mathcal{O}^2 \frac{f}{2} \mathcal{O}^2 \right\rangle \propto \left(G(\eta, \eta; m^2) \Big|_{\eta \rightarrow \text{boundary}} \right)^2 . \quad (61)$$

We now consider the correlation functions in the coordinate space and use the Euclidean time τ , which is defined by

$$\tau = \frac{l^{\frac{3}{2}}}{2\mu^{\frac{1}{4}}} \theta = \frac{\theta}{2\pi T_H} , \quad (62)$$

as in (6). Then by summing up the Fourier expansion, we obtain

$$\begin{aligned} G_4(\eta, \tau) &= \sum_{n=-\infty}^{\infty} G(\eta, \eta; m^2) e^{i2\pi n T_H \tau} \\ &= \sum_{n=-\infty}^{\infty} e^{i2\pi n T_H \tau} \frac{l e^{-\frac{4\eta}{l}}}{2\gamma \sin \varphi_1} \left\{ \cos \varphi_1 I_n(\gamma\eta)^2 + \sin \varphi_1 I_n(\gamma\eta) K_n(\gamma\eta) \right\} \\ &= \frac{l e^{-\frac{4\eta}{l}}}{2\gamma \sin \varphi_1} \left\{ \cos \varphi_1 I_0 \left(\gamma\eta \sqrt{2(1 + \cos(2\pi T_H \tau))} \right) \right. \\ &\quad \left. + \sin \varphi_1 K_0 \left(\gamma\eta \sqrt{2(1 + \cos(2\pi T_H \tau))} \right) \right\} . \end{aligned} \quad (63)$$

³It is known that formulation of RG in thermal QFT is ambiguous as not only high energy but also high temperature limit may be considered. Then, the construction of c-function in thermal QFT is not so trivial. It seems that AdS/CFT correspondence as discussed above for AdS BH may provide an idea about correct c-function in dual thermal field theory.

Here $\varphi_2 = 0$ is assumed as in (38) and the following formulae are used:

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} \sin n\theta I_n(w)I_n(z) &= 0 , \\
\sum_{n=-\infty}^{\infty} \cos n\theta I_n(w)I_n(z) &= I_0\left(\sqrt{w^2 + z^2 + 2zw \cos \theta}\right) , \\
\sum_{n=-\infty}^{\infty} \sin n\theta I_n(w)K_n(z) &= 0 , \\
\sum_{n=-\infty}^{\infty} \cos n\theta I_n(w)K_n(z) &= K_0\left(\sqrt{w^2 + z^2 + 2zw \cos \theta}\right) . \quad (64)
\end{aligned}$$

As

$$r^2 = \pi^2 l^4 T_H \cosh \frac{2\eta}{l} , \quad (65)$$

from (5), at low temperature limit $T_H \rightarrow 0$ keeping τ and r to be finite, we find

$$G_4(\eta, \tau) \sim \frac{l e^{-\frac{4\eta}{l}}}{2\gamma \sin \varphi_1} \left\{ \cos \varphi_1 \frac{2\gamma\eta}{\sqrt{4\pi\gamma\eta}} + \sin \varphi_1 \ln(2\pi\gamma\eta T_H \tau) \right\} . \quad (66)$$

The last logarithmic term depending on τ is similar to the Green function for the massless scalar in two dimensions, which may occur since we neglected the k -dependence and substantially reduced the spacetime dimensions. On the other hand, at high temperature limit, which could correspond to the limit of $\eta \rightarrow 0$, we find

$$G_4(\eta, \tau) \sim \frac{l}{2\gamma \sin \varphi_1} \left\{ \cos \varphi_1 + \sin \ln\left(\gamma\eta\sqrt{2(1 + \cos(2\pi T_H \tau))}\right) \right\} . \quad (67)$$

The periodicity of the Euclidean time τ is typical for thermal field theory. From Eq.(37), in the UV limit, where $f = 0$, the first terms in (66) and (67) vanish. On the other hand, in the IR limit, where $f \rightarrow \infty$, the first terms become large and the second terms can be neglected. Then in the IR limit, the τ -dependences vanish, which means that the operator \mathcal{O} becomes non-dynamical or decouples. The decoupling of the operator \mathcal{O} in the IR limit can be naturally expected from the RG flow.

4 One-loop vacuum energy for scalar field in deSitter space

In this section, we consider the renormalization group flow in the context of the dS/CFT correspondence[15, 14]. The D -dimensional deSitter space can be realized by embedding the dS into $D + 1$ -dimensional flat space, whose metric is given by

$$ds^2 = (dX^1)^2 + (dX^2)^2 + \dots (dX^{D-1})^2 + (dX^D)^2 - (dX^{D+1})^2 \quad (68)$$

by the constraint,

$$l^2 = (X^1)^2 + (X^2)^2 + \dots (X^{D-1})^2 + (X^D)^2 - (X^{D+1})^2 . \quad (69)$$

The metric (68) and (69) is invariant under the $SO(D, 1)$ transformation. Then deSitter space has an isometry of $SO(D, 1)$, which is identical with the group generated by the conformal transformation in the Euclidean $D - 1$ space. Solving the constraint (69) by choosing the independent coordinates as

$$U = X^D + X^{D+1} , \quad x^i = \frac{1}{U} X^i , \quad (i = 1, 2, \dots, D) , \quad (70)$$

we obtain the following metric of the dS:

$$ds_{\text{dS}}^2 = \sum_{\mu, \nu=0}^{D-1} g_{\text{dS}\mu\nu} dx^\mu dx^\nu = -\frac{l^2}{U^2} dU^2 + U^2 (dx^i)^2 . \quad (71)$$

Here $x^0 = U$. If we change the coordinate U by $U = e^{\frac{t}{l}}$, the metric in (71) can be rewritten as

$$ds_{\text{dS}}^2 = -dt^2 + e^{\frac{2t}{l}} (dx^i)^2 , \quad (72)$$

which has the form very similar to the metric of AdS:

$$ds_{\text{AdS}}^2 = dz^2 + e^{\frac{2z}{l}} dx^\mu ds_\mu . \quad (73)$$

In case of AdS, the conformal symmetry can be realized on the surface with $z \rightarrow +\infty$. Then it is very natural to expect that the (Euclidean) conformal symmetry can be realized on the spacelike surface with $t \rightarrow +\infty$. The coordinate system in (72) covers the half of the whole deSitter spacetime. Using

the coordinate system which covers the whole deSitter spacetime, the metric of the deSitter spacetime can be written as

$$ds_{\text{dS}}^2 = -d\tau^2 + l^2 \cosh^{\frac{2\tau}{l}} d\Omega_{(4)}^2 . \quad (74)$$

Here $d\Omega_{(4)}^2$ is the metric of 4d sphere with unit radius. The space-like surface $t \rightarrow +\infty$ in (72) corresponds to the surface $\tau \rightarrow +\infty$ in (74). Then even in the metric (74), one may expect that the conformal symmetry is realized on the the surface $\tau \rightarrow +\infty$. In the AdS/CFT set-up, the shift of z in (73) corresponds to the scale transformation of the CFT. Then we may assume that the shift of t in (72) should also correspond to the scale transformation. Therefore in a way parallel to [10], one may consider the RG flow and c-functions in the framework of the dS/CFT correspondence.

By using the expression of the metric (71), for the two points on the dS, $SO(D, 1)$ invariant distance can be defined by

$$\begin{aligned} L^2 &\equiv X_{(1)}^1 X_{(2)}^1 + \cdots + X_{(1)}^D X_{(2)}^D - X_{(1)}^{D+1} X_{(2)}^{D+1} \\ &= -\frac{1}{2} U_{(1)} U_{(2)} (x_{(1)} - x_{(2)})^2 + \frac{l^2}{2} \left(\frac{U_{(1)}}{U_{(2)}} + \frac{U_{(2)}}{U_{(1)}} \right) . \end{aligned} \quad (75)$$

Then when two points coincide with each other, one has $L^2 = l^2$. In the following, we concentrate on the case of $D = 5$. The propagator G of the real scalar field ϕ with mass m should only depend on L^2 : $G = G(L^2)$. The equation determining the propagator

$$\frac{1}{\sqrt{-g_{\text{dS}}}} \partial_\mu (g^{\text{dS}\mu\nu} \partial_\nu G) - m^2 G = \frac{1}{\sqrt{-g_{\text{dS}}}} \delta^D (x_{(1)}^\mu - x_{(2)}^\mu) \quad (76)$$

can be rewritten, when $x_{(1)}^\mu - x_{(2)}^\mu \neq 0$, as

$$0 = \left(-\frac{(L^2)^2}{l^2} + l^2 \right) \frac{d^2 G}{d(L^2)^2} - \frac{5}{l^2} \frac{dG}{d(L^2)} - m^2 G = 0 . \quad (77)$$

With the new coordinate z defined by

$$z = \frac{1 - \frac{L^2}{l^2}}{2} , \quad (78)$$

Eq.(77) has the following form:

$$0 = z(1-z)\frac{d^2F}{dz^2} + \left(\frac{5}{2} - 5z\right)\frac{dF}{dz} - M^2 . \quad (79)$$

Here $M^2 \equiv m^2 l^2$. The solutions of (79) are given by Gauss' hypergeometric functions $F(\alpha, \beta, \gamma; z)$:

$$G(z) = \frac{\Gamma(\alpha_+) \Gamma(\alpha_-)}{(4\pi)^{\frac{5}{2}} \Gamma\left(\frac{5}{2}\right) l^3} F\left(\alpha_+, \alpha_-, \frac{5}{2}; 1-z\right) + \frac{\xi}{l^3} F\left(\alpha_+, \alpha_-, \frac{5}{2}; z\right) ,$$

$$\alpha_{\pm} \equiv 2 \pm \sqrt{4 - M^2} . \quad (80)$$

Note the above expression (80) is well-known, for example, see [26]. $z \rightarrow 0$ corresponds to $x_{(1)}^\mu - x_{(2)}^\mu \rightarrow 0$. The coefficient of the first term is fixed to reproduce the δ -function in (76). On the other hand the coefficient ξ of the second term should be determined by the boundary condition. When z is large, $G(z)$ behaves as

$$G(z) \sim \frac{\Gamma(\alpha_- - \alpha_+)}{\Gamma(\alpha_-) \Gamma(5 - \alpha_+) l^3} \left(\frac{\Gamma(\alpha_+) \Gamma(\alpha_-)}{(4\pi)^{\frac{5}{2}} \Gamma\left(\frac{5}{2}\right) l^3} + \xi(-1)^{-\alpha_+} \right) z^{-\alpha_+}$$

$$+ \frac{\Gamma(\alpha_+ - \alpha_-)}{\Gamma(\alpha_+) \Gamma(5 - \alpha_-) l^3} \left(\frac{\Gamma(\alpha_+) \Gamma(\alpha_-)}{(4\pi)^{\frac{5}{2}} \Gamma\left(\frac{5}{2}\right) l^3} + \xi(-1)^{-\alpha_-} \right) z^{-\alpha_-} , \quad (81)$$

which corresponds to the asymptotic behavior of the scalar field ϕ :

$$\phi \sim \alpha z^{-\alpha_+} + \beta z^{-\alpha_-} ,$$

$$\frac{\alpha}{\beta} = \frac{\frac{\Gamma(\alpha_- - \alpha_+)}{\Gamma(\alpha_-) \Gamma(5 - \alpha_+)} \left(\frac{\Gamma(\alpha_+) \Gamma(\alpha_-)}{(4\pi)^{\frac{5}{2}} \Gamma\left(\frac{5}{2}\right) l^3} + \xi(-1)^{-\alpha_+} \right)}{\frac{\Gamma(\alpha_+ - \alpha_-)}{\Gamma(\alpha_+) \Gamma(5 - \alpha_-)} \left(\frac{\Gamma(\alpha_+) \Gamma(\alpha_-)}{(4\pi)^{\frac{5}{2}} \Gamma\left(\frac{5}{2}\right) l^3} + \xi(-1)^{-\alpha_-} \right)} . \quad (82)$$

Then the conformal weight Δ_{\pm} of the corresponding CFT primary fields in dS/CFT correspondence is given by

$$\Delta_{\pm} = 4 - \alpha_{\mp} = \alpha_{\pm} = 2 \pm \sqrt{4 - M^2} . \quad (83)$$

One may compare Eq.(83) with the expression for AdS, which is given by (22). Inside the square root, the sign of l^2 is changed. For AdS, if the Breitenlohner-Freedman bound $m^2 \geq -\frac{4}{l^2}$ is not fulfilled, the conformal weight is real but in case of dS, in order that the conformal weight is real, there is an upper bound for the mass $m^2 \leq \frac{4}{l^2}$.

As in Eq.(30), the parameters α and β (82) could be related with the coefficient f (29):

$$f = \frac{\alpha}{\pi\beta} . \quad (84)$$

Using (82), we find the values ξ_0 and ξ_∞ of ξ corresponding to $f \rightarrow 0$ and $f \rightarrow \infty$, respectively:

$$\begin{aligned} \xi_0 &= -(-1)^{\alpha_+} \frac{\Gamma(\alpha_+) \Gamma(\alpha_-)}{(4\pi)^{\frac{5}{2}} \Gamma\left(\frac{5}{2}\right)} \\ &= -\frac{(3-\alpha_+)(2-\alpha_+)(1-\alpha_+)}{\sin(\pi\alpha_+) \cdot 2^3 \cdot 3\pi^2} e^{i\pi\alpha_+} , \\ \xi_\infty &= -(-1)^{\alpha_-} \frac{\Gamma(\alpha_+) \Gamma(\alpha_-)}{(4\pi)^{\frac{5}{2}} \Gamma\left(\frac{5}{2}\right)} \\ &= -\frac{(3-\alpha_+)(2-\alpha_+)(1-\alpha_+)}{\sin(\pi\alpha_+) \cdot 2^3 \cdot 3\pi^2} e^{-i\pi\alpha_+} . \end{aligned} \quad (85)$$

Then one obtains

$$G(f = \infty, z = 0) - G(f = 0, z = 0) = -\frac{(3-\alpha_+)(2-\alpha_+)(1-\alpha_+)}{2^2 \cdot 3\pi^2 l^3} . \quad (86)$$

The new parameter γ may be defined:

$$\gamma \equiv \sqrt{4 - M^2} . \quad (87)$$

$M^2 = 4$ ($m^2 = \frac{4}{l^2}$) corresponds to $\gamma^2 = 0$. Note the mass is not always tachyonic. The difference δV of the vacuum energies corresponding to the two boundary conditions is given by

$$\begin{aligned} \delta V &= \frac{1}{2l^2} \int_0^{\gamma^2} d\tilde{\gamma}^2 \left\{ G(f = \infty, M^2 = 4 - \tilde{\gamma}^2, z = 0) \right. \\ &\quad \left. - G(f = 0, M^2 = 4 - \tilde{\gamma}^2, z = 0) \right\} \\ &= -\frac{1}{12\pi^2 l^5} \left(\frac{(2-\Delta_-)^3}{3} - \frac{(2-\Delta_-)^5}{5} \right) . \end{aligned} \quad (88)$$

The expression (88) is identical with that in the AdS bulk case in [10]. The only difference is the sign inside the square root in the expression of Δ_- (83). Δ_{\pm} in (83) becomes complex when $M^2 > 4$ or $m^2 > \frac{4}{l^2}$, which may be a sign that the corresponding CFT is not unitary. We should also note that δV (88) becomes purely imaginary when $M^2 > 4$.

One may now conjecture a c-function as

$$\begin{aligned} c_{\text{dS}} &= l^3 (1 + \delta V) \\ &= l^3 \left(1 - \frac{1}{12\pi^2} \left(\frac{(2 - \Delta_-)^3}{3} - \frac{(2 - \Delta_-)^5}{5} \right) \right). \end{aligned} \quad (89)$$

The c-function is, of course, less than l^3 , which is the value of the c-function before the deformation. Then it is consistent with the renormalization flow or c-theorem in four dimensions is fulfilled within dS/CFT. The situation is really funny, we have no the explicit example of dual CFT but the corresponding RG flow is known (and it actually coincides with AdS flow). We have observed that there should be RG flow even for dS/CFT in the same way as for AdS/CFT case [11]. In [27] from CFT side it has been explicitly verified that it is the same RG flow [10] interpolating between UV region, where the scalar operator has a dimension Δ_- , and IR region, where the dimension of the scalar operator is Δ_+ . Then the same arguments could be applied for dS/CFT correspondence where consistent dual CFT is not constructed yet. The geometry of dS will be determined by the leading order of corresponding ($1/N$?) approximation, as in the case of AdS, and the one-loop correction due to the scalar field will give next-to-leading order correction to the geometry. Thus, even in the absence of explicit example for dual CFT in dS_5/CFT_4 correspondence one can see that there is indication for existence of RG flow similar to AdS case.

5 Discussion

In summary, we found the difference of vacuum energies for boundary conditions specified by the double-trace operator. Five-dimensional AdS black hole which is dual to thermal QFT and deSitter space are considered. Using AdS/CFT (dS/CFT) correspondence such difference may be interpreted as difference between end-points of RG flow. As a result, the holographic c-theorem is verified beyond the leading order approximation, generalizing the

correspondent check done in ref.[10] for pure AdS space. The remarkable fact is about dS/CFT where we make predictions about c-function while explicit (probably , non-unitary) dual CFT is not constructed yet.

In Eq.(80), the propagator of the scalar field in the dS₅ background has been constructed. The corresponding propagator in the (Euclidean) AdS₅ background can be obtained by replacing the length parameter l^2 by $l^2 \rightarrow -l^2$ (or $M^2 = \frac{m^2}{l^2} \rightarrow -M^2$). In both of the dS and (Euclidean) AdS cases, the propagator only depends on $SO(5,1)$ invariant distance L^2 (75) or z (78). In case of AdS₅, one can choose the boundary, where dual CFT lives, to be flat 4d surface, 4d sphere, or 4d hyperboloid. Such a choice of the boundary can be transformed to each other by the $SO(5,1)$ transformation. Since the scalar propagator is invariant under the transformation, the results, say for δV in [10], do not depend on the choice of the boundary.

As a final remark let us note that it would be really interesting to extend this discussion for the situation when other bulk fields present (five-dimensional gauged supergravity).

Note that recently similar questions with account of RG flow for bulk masses were addressed in [28].

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