Swift Pointing and the Association Between Gamma-Ray Bursts and Gravitational-Wave Bursts

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ABSTRACT

The currently accepted model for gamma-ray burst phenomena involves the violent formation of a rapidly rotating solar mass black hole. Gravitational waves should be associated with the black-hole formation, and their detection would permit this model to be tested, the black hole progenitor (e.g., coalescing binary or collapsing stellar core) identified, and the origin of the gamma rays (within the expanding relativistic fireball or at the point of impact on the interstellar medium) located. Even upper limits on the gravitational-wave strength associated with gamma-ray bursts could constrain the gamma-ray burst model. To do any of these requires joint observations of gamma-ray burst events with gravitational and gamma-ray detectors. Here we examine how the quality of an upper limit on the gravitational-wave strength associated with gamma-ray burst observations depends on the relative orientation of the gamma-ray-burst and gravitational-wave detectors, and apply our results to the particular case of the Swift Burst-Alert Telescope (BAT) and the LIGO gravitational-wave detectors. A result of this investigation is a science-based "figure of merit" that can be used, together with other mission constraints, to optimize the pointing of the Swift telescope for the detection of gravitational waves associated with gamma-ray bursts.

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1. Introduction

The currently accepted model for gamma-ray burst phenomena involves the violent formation of an approximately solar-mass black hole surrounded by a similarly massive debris torus. The gamma-ray burst is powered by the release of binding energy as the debris torus is accreted onto the black hole. The formation of the black hole and debris torus may take place through the coalescence of a compact binary or the collapse of a quickly rotating massive stellar core; the gamma-ray burst emission may take place at the site of crossing internal shock waves within the expanding relativistic fireball powered by the accretion onto the black hole, or as the fireball is decelerated by the interstellar medium (Meszaros & Rees 1993; Rees & Meszaros 1994).

A gravitational-wave burst is likely to be associated with the formation of the rapidly rotating, central black-hole engine of a gamma-ray burst. The character of the gravitational-wave burst (e.g., energy, spectrum, polarization) will depend on the degree of non-axisymmetry associated with the collapse, and thus its progenitor (Kobayashi & Meszaros 2002, 2003). Similarly, the relative time of arrival of the gravitational and gamma-ray bursts will depend on whether the gamma-ray burst is generated by internal shocks in the exploding fireball or external shocks when the fireball is decelerated by the interstellar medium.[4] Observation of gravitational-wave bursts associated with gamma-ray bursts thus may reveal details of the gamma-ray burst mechanism that cannot be revealed through observations of the gamma rays alone. In this paper we examine how gravitational-wave observations using the LIGO detectors (Sigg 2001) can be made in conjunction with gamma-ray burst observations by the Swift satellite to determine whether there is an association between gamma-ray and gravitational-wave bursts, a first step in the ambitious program of developing gravitational-wave observations into a tool of astronomical discovery.

Finn et al. (1999) have described how the cross-correlated output of two gravitational-wave detectors, taken in coincidence with gamma-ray burst (GRB) events, can be used to detect or place upper limits on the emission of gravitational-wave bursts (GWBs) by GRBs. This procedure has already been used in the analysis of data from the EXPLORER and NAUTILUS gravitational-wave detectors at times associated with 47 GRBs detected by the BeppoSAX satellite to bound, at 95% confidence, the root-mean-square gravitational-wave strain h_{RMS} associated with these GRBs to less than 6.5×10^{-19} in the gravitational-wave-detector waveband (approximately 0.5 Hz about 900 Hz), assuming that the gamma rays originate in internal shocks (Astone et al. 2002). Finn et al. (1999) estimate that 1000 GRB observations combined with observations from the (broad-band) initial LIGO detectors could produce an upper limit on the gravitational-wave strain associated with GRBs of approximately $h_{\mathrm{RMS}} \leq 1.7 \times 10^{-22}$ at 95% confidence.

An important consequence of the cosmological origin of GRBs is their isotropic distribution on

[4]Though the fireball is highly relativistic, it travels subluminally; hence, the further the gamma rays are produced from the central engine the longer the time lag between the arrival of the gravitational waves and the gamma rays. This time lag can range from 1 second to 100 seconds. On the other hand, the time lag between the black-hole formation (which produces the gravitational-wave burst) and the launch of the relativistic jet (which produces the GRB) should be on the order of the dynamical timescale of milliseconds, and is a much smaller effect.

the sky. In their original work Finn et al. (1999) assumed that GRBs would be detected isotropically as well – i.e., that the GRB detector had an isotropic antenna pattern. They did note, however, that the Swift satellite, a next-generation multi-wavelength satellite dedicated to the study of GRBs, does not have an isotropic antenna pattern and that this has potentially important consequences for the ability of the combined gamma-ray burst/gravitational-wave detector array to detect or limit the gravitational-wave flux on Earth owing to GRBs. Here we study this question specifically in the context of the Swift satellite and the LIGO gravitational-wave detectors; i.e., we determine, as a function of Swift's pointing, the sensitivity of the Swift/LIGO detector array to gravitational waves from GRBs, and propose a figure-of-merit that can be used in Swift mission scheduling to optimize the sensitivity of the Swift/LIGO array to the gravitational-wave flux from GRBs. We find that the upper limit that can be placed on h_{RMS} differs by a factor of 2 between best and worst orientations of the satellite.

We begin in Section 2 with a review of how Finn et al. (1999) proposed using Student's t-test to detect a GRB–GWB association and place an upper limit on the gravitational-wave strength associated with GRBs. We extract the direction dependence of this upper limit in Section 3, and apply to the case of Swift in Section 4. We conclude with some brief remarks in Section 5.

2. Observing a GRB–GWB association

Finn et al. (1999) described how the signal from two independent gravitational-wave detectors can be analyzed to identify the gravitational-wave signal associated with gamma-ray bursts and either bound or measure the population average of the gravitational-wave flux on Earth from this potential source. In this section we review their analysis methodology in anticipation of using it to determine the sensitivity of joint LIGO/Swift observations to the detectors' relative orientation.

2.1. Detecting a GRB–GWB association

Consider a set of N GRB detections. Assume that, as a result of each detection, we know the direction to the source $\widehat{\Omega}_k$ and the arrival time τ_k of the burst at Earth's barycenter. For our purposes, each GRB observation is completely characterized by the pair $(\widehat{\Omega}_k, \tau_k)$.[5] Focus attention on a pair of gravitational-wave detectors located at positions $\mathcal{D}_i$ $(i = 1, 2)$ relative to Earth's barycenter. The arrival time of GRB k at detector i is

$$t_k^{(i)} = \tau_k - \widehat{\Omega}_k \cdot \mathcal{D}_i \tag{1}$$

in units where the speed of light c is unity.

Finn et al. (1999) note that the two LIGO detectors are very nearly co-planar and co-aligned. Consequently, a plane gravitational wave incident on the detector pair from the direction $\widehat{\Omega}_k$ will

[5] Afterglow observations will also give the distance to the GRB source (Metzger et al. 1997).

lead to correlated detector responses with a time lag equal to

$$\Delta t_k = t_k^{(2)} - t_k^{(1)}. \tag{2}$$

To identify the presence of GWBs associated with GRBs, Finn et al. (1999) focus attention on the correlated energy in the detector outputs corresponding to plane GWBs incident on the detectors from the direction of the corresponding GRBs: i.e., the correlation of the detector outputs at times that differ by Δt_k.

Let $s_i(t)$ be the output of gravitational wave detector $\mathcal{D}_i$, which we assume to consist of detector noise $n_i(t)$ and a possible gravitational wave signal $h_i(t)$ *produced by the GRB source*:[6]

$$s_i(t) = n_i(t) + h_i(t)\,. \tag{3}$$

Finn et al. (1999) define

$$S(\widehat{\Omega}_k, \tau_k) = \int_0^T dt \int_0^T dt'\, s_1(t_k^{(1)} - t)\, Q(t-t')\, s_2(t_k^{(2)} - t')\,, \tag{4}$$

as the energy in the cross-correlation of the two detectors corresponding to the GRB characterized by $(\widehat{\Omega}_k, \tau_k)$. Here Q is a freely specifiable, symmetric filter function (Finn et al. 1999), and T is chosen large enough to encompass the range of possible times by which the gravitational waves from a GRB event may precede the gamma rays, which is typically thought to be of order 1 s for GRBs produced by internal shocks and 100 s for GRBs produced by external shocks (Sari & Piran 1997; Kobayashi, Piran, & Sari 1997).

Writing the detector output s_i as the sum of the detector noise n_i and the gravitational-wave signal h_i associated with the GRB we can, in turn, write

$$S_k \equiv S(\widehat{\Omega}_k, \tau_k) = \langle n_1, n_2\rangle + \langle n_1, h_2\rangle + \langle h_1, n_2\rangle + \langle h_1, h_2\rangle\,, \tag{5}$$

where

$$\langle f, g\rangle = \int_0^T dt \int_0^T dt'\, f(t_k^1 - t)\, Q(t-t')\, g(t_k^2 - t')\,. \tag{6}$$

The terms $\langle n_i, h_j\rangle$ in equation (5) vanish in the mean over an ensemble of noise since the noise in our gravitational wave detector is uncorrelated with any gravitational wave signal. The term $\langle n_1, n_2\rangle$ is, in the noise ensemble mean, a constant, which will be zero if the noise in the two detectors is uncorrelated.

All four of the contributions to S in equation (5) are generally unknown for any particular GRB. Correspondingly, Finn et al. (1999) consider the collection $\widehat{\mathcal{S}}_{\text{on}}$ of S_k,

$$\widehat{\mathcal{S}}_{\text{on}} = \{S_k : k = 1 \ldots N\}\,, \tag{7}$$

[6]There may be a flux of gravitational waves from other sources incident coincidentally on the detector at the same time; however, since this radiation is not correlated with the gamma-ray burst it is, for our purpose, noise and we lump its contribution to s in with n.

and a second collection $\widehat{\mathcal{S}}_{\rm off}$

$$\widehat{\mathcal{S}}_{\rm off} = \left\{ S'_m : m = 1 \ldots M \right\} \,, \tag{8}$$

where each S'_m is constructed as in equation (4) but with a $(\widehat{\Omega}, \tau)$ pair chosen randomly and not associated with a GRB. The collections $\widehat{\mathcal{S}}_{\rm on}$ and $\widehat{\mathcal{S}}_{\rm off}$ are samples drawn from *populations* $\mathcal{S}_{\rm on}$ and $\mathcal{S}_{\rm off}$. The sample means $\widehat{\mu}_{\rm on}$ and $\widehat{\mu}_{\rm off}$ and variances $\widehat{\sigma}^2_{\rm on}$ and $\widehat{\sigma}^2_{\rm off}$ are estimates of the *population means* $\mu_{\rm on}$ and $\mu_{\rm off}$ and variances $\sigma^2_{\rm on}$ and $\sigma^2_{\rm off}$. These are, in turn, related by

$$\begin{aligned} \mu_{\rm on} - \mu_{\rm off} &= \overline{\langle h_1, h_2 \rangle} \,, & (9) \\ \sigma^2_{\rm on} - \sigma^2_{\rm off} &= \overline{\langle n_1, h_2 \rangle^2} + \overline{\langle n_2, h_1 \rangle^2} + \mathcal{O}(h^3) \,, & (10) \end{aligned}$$

where the overbar represents a mean over the population of GRBs. Comparing the two sample sets $\widehat{\mathcal{S}}_{\rm on}$ and $\widehat{\mathcal{S}}_{\rm off}$ thus provides a sensitive measure of the presence or absence of gravitational waves associated with GRBs.

When the detector noise is sufficiently well-behaved that terms like $\langle n_1, h_2 \rangle$, $\langle n_2, h_1 \rangle$, and $\langle n_1, n_2 \rangle$ are normally distributed the Student t-test (Snedecor and Cochran 1967) can be used to distinguish between the two sample sets; in other cases a non-parametric test such as the Mann-Whitney test (Siegal & Castellan 1988) can be used.

In the Student t-test the difference between the two distributions represented by the sample sets $\widehat{S}_{\rm on}$ and $\widehat{S}_{\rm off}$ is characterized by the t-statistic:

$$\begin{aligned} \widehat{t} &= \frac{\widehat{\mu}_{\rm on} - \widehat{\mu}_{\rm off}}{\widehat{\Sigma}} \sqrt{\frac{N_{\rm on} N_{\rm off}}{N_{\rm on} + N_{\rm off}}} \,, & (11) \\ \widehat{\Sigma}^2 &= \frac{\widehat{\sigma}^2_{\rm on}(N_{\rm on} - 1) + \widehat{\sigma}^2_{\rm off}(N_{\rm off} - 1)}{N_{\rm on} + N_{\rm off} - 2} \,, & (12) \end{aligned}$$

If both $\widehat{\mathcal{S}}_{\rm on}$ and $\widehat{\mathcal{S}}_{\rm off}$ are drawn from same normal distribution then the distribution of $\widehat{t}$ is given by Student's distribution with $N_{\rm on} + N_{\rm off} - 2$ degrees of freedom (Cramer 1999). This distribution itself tends toward a normal distribution with unit variance when $N_{\rm on} + N_{\rm off}$ is large.

Now suppose that there is no GWB–GRB association. In this event $\widehat{S}_{\rm on}$ and $\widehat{S}_{\rm off}$ are drawn from the same distribution and there is a number $t_0(p)$ such that $\widehat{t}$ will be less than $t_0(p)$ in a fraction p of all observations of sample sets $\widehat{S}_{\rm on}$ and $\widehat{S}_{\rm off}$.[7] If, in our particular observation of $\widehat{S}_{\rm on}$ and $\widehat{S}_{\rm off}$ $\widehat{t}$ is less that $t_0(p)$ then we accept the hypothesis that there are *no* gravitational-waves associated with gamma-ray bursts. If, on the other hand, we find $\widehat{t}$ greater than $t_0(p)$ then we reject, with confidence p, this hypothesis; i.e., we assert that there is an association of gravitational-waves with gamma-ray bursts.

[7] On physical grounds the expectation value of $\widehat{t}$ will be positive semi-definite for the LIGO detector pair if gravitational waves are associated with gamma-ray bursts.

2.2. Setting an upper limit on the gravitational-wave strength associated with GRBs

As described in the previous section, Student's t-test tells us only if there is a link between GWBs and GRBs. An alternative analysis, also described by Finn et al. (1999), uses the t statistic to derive a confidence interval or upper limit on the population-averaged gravitational-wave flux associated with GRBs from a measured value $\widehat{t}$ of the t statistic. In this analysis we derive the classical confidence interval or upper limit from the probability distribution $P(t|h, I)$ of the t statistic assuming that GRBs radiate GWBs with intrinsic amplitude described by h and other model parameters (gravitational-wave burst luminosity function, burst characteristic, etc.) described by I.

In the limit that the GWBs are weak relative to the sensitivity of the individual gravitational-wave detectors and the numbers of on- and off-source observations are separately large, Finn et al. (1999) showed that

$$P(t|h, I) = N\left(t \middle| \mu, \frac{\sigma^2}{2}\right), \tag{13}$$

where

$$\mu = \overline{\langle h_1, h_2 \rangle} \tag{14}$$

$$\sigma^2 = \frac{T}{4}\int_{-\infty}^{\infty} df\, P_1(f) P_2(f) |\widetilde{Q}(f)|^2 . \tag{15}$$

Here $P_i(f)$ is the one-sided power spectral density of the $i^{\rm th}$ detector, defined as

$$P_i(|f|) = 2\int_{-\infty}^{\infty} dt\, e^{i2\pi f t} n_i(\tau) n_i(\tau + t), \tag{16}$$

$\overline{\langle h_1, h_2 \rangle}$ is the average of $\langle h_1, h_2 \rangle$ over the GRB population and the associated GWB luminosity function described by I, $N(t|\mu, \nu)$ is the normal distribution with mean μ and variance ν, and $\widetilde{Q}(f)$ is the Fourier transform of $Q(\tau)$. For larger-amplitude GRBs or different sample sizes (e.g., smaller number of GRB observations) the distribution can be determined via Monte Carlo simulations. An observation of t thus allows us to find a confidence limit on $\overline{\langle h_1, h_2 \rangle}$ (Feldman & Cousins 1998), which describes the flux of gravitational-waves on Earth owing to GRBs.

3. Direction-dependence of the upper limit

The analysis described by Finn et al. (1999) assumed that the arms of the two LIGO gravitational wave detectors all resided in the same plane, that pairs of arms were parallel to each other, and that the antenna pattern of the detectors was isotropic on the sky. In this section we relax all of these approximations: i.e., we properly account for the position and orientation of the two LIGO detectors on the Earth and the dependence of their sensitivity to the direction to the GRB source. Our result is an expression for the dependence of the upper limit on population-averaged gravitational-wave strength $\overline{\langle h_1, h_2 \rangle}$ as a function of the distribution of *detected* GRBs on the sky. In section 4 we combine this result with the directional sensitivity of the Swift detector to determine the dependence on Swift pointing of the upper limit on $\overline{\langle h_1, h_2 \rangle}$ that can be set by joint LIGO/Swift observations.

The gravitational-wave component h_i of the LIGO detector output is, in the small antenna limit,[8] a linear function of the physical gravitational-wave strain $h_{ab}(t, \vec{x})$,

$$h_i(t) = h_{ab}(t, \vec{x}_i)\, d_i^{ab}\,, \tag{17}$$

where $\vec{x}_i$ is the gravitational-wave detector's location. For interferometer i with arms pointing in the directions $\widehat{X}_i$, $\widehat{Y}_i$,

$$d_i^{ab} = \frac{1}{2}(\widehat{X}_i^a \widehat{X}_i^b - \widehat{Y}_i^a \widehat{Y}_i^b)\,. \tag{18}$$

(With this normalization $h_i(t)$ is equal to the fractional change in differential arm length.) For gravitational wave bursts incident on the Earth from direction $\widehat{\Omega}$,

$$h_{ab}(t_k^{(1)} - t, \vec{x}_1) = h_{ab}(t_k^{(2)} - t, \vec{x}_2) = h_{ab}(\tau_k - t, \vec{0}) \tag{19}$$

where $t_k^{(i)}$ is defined in terms of the direction to the source $\widehat{\Omega}$ by equation (1). We can thus ignore the physical separation of the detectors when computing the cross-correlation statistic (cf. equation (4)). Finally, it is convenient to resolve h_{ab} on the two polarization tensors ϵ^+ and $\epsilon^\times$,

$$h_{ab}(t, \widehat{\Omega}) = h_+(t)\epsilon_{ab}^+(\widehat{\Omega}) + h_\times(t)\epsilon_{ab}^\times(\widehat{\Omega})\,, \tag{20}$$

where

$$\epsilon^+ : \epsilon^+ \;=\; \epsilon^\times : \epsilon^\times \;=\; 2\,, \tag{21}$$

$$\epsilon^+ : \epsilon^\times = 0\,, \tag{22}$$

and

$$\epsilon^+ \cdot \widehat{\Omega} = \epsilon^\times \cdot \widehat{\Omega} = 0\,. \tag{23}$$

Lacking any detailed model for the gravitational waves that may be produced in a GRB event, we make the following assumptions about h_+ and $h_\times$:

- The waves have equal power in the two polarizations:

$$\overline{h_+(t)h_+(t')} = \overline{h_\times(t)h_\times(t')}\,. \tag{24}$$

- The two polarizations are uncorrelated:

$$\overline{h_+(t)h_\times(t')} = 0\,. \tag{25}$$

[8] This is appropriate for gravitational wave frequencies in the LIGO detector band.

Focus attention now on the mean gravitational-wave contribution $\overline{\langle h_1, h_2\rangle}$ to the t statistic. This contribution depends on both the gravitational wave detector sensitivity to GWBs arriving from different directions as well as the gamma-ray bursts detector sensitivity to GRBs from different directions (which determines the relative number of bursts that will be observed from that direction). For GWBs arriving from the direction $\widehat{\Omega}$,

$$\langle h_1, h_2\rangle = \rho_{\rm GWB}(\widehat{\Omega}|d_1, d_2) \int_0^T dt \int_0^T dt'\, Q(t-t')\, h_+(t)\, h_+(t')\,, \tag{26}$$

where

$$\rho_{\rm GWB}(\widehat{\Omega}|d_1, d_2) \equiv \sum_{A=+,\times} \left(d_1 : \epsilon^A(\widehat{\Omega})\right) \left(d_2 : \epsilon^A(\widehat{\Omega})\right) \tag{27}$$

describes the direction-dependence of the sensitivity of the gravitational-wave detector pair to the GWB (cf. equations (4), (24), and (25)).

To complete the evaluation of $\overline{\langle h_1, h_2\rangle}$ turn to the fraction of GRB detections that arise from different patches on the sky. Since the intrinsic GRB population is isotropic, the distribution of detection on the sky depends entirely on the directional sensitivity of the GRB detector. Let the fraction of GRB detections in a sky patch of area $d^2\Omega$ centered at $\widehat{\Omega}$ be given by

$$\rho_{\rm GRB}(\widehat{\Omega}|\widehat{\Omega}', \widehat{n}) d^2\widehat{\Omega}\,, \tag{28}$$

where the GRB detector orientation is given by $\widehat{\Omega}'$, the direction in which the detector is pointed, and $\widehat{n}$, which describes the rotation of the satellite about its pointing direction.

In terms of $\rho_{\rm GWB}$ and $\rho_{\rm GRB}$ the mean gravitational wave contribution $\overline{\langle h_1, h_2\rangle}$ to t for a gamma-ray burst detector with fixed orientation $(\widehat{\Omega}', \widehat{n})$ is thus

$$\overline{\langle h_1, h_2\rangle} = \left[\int d^2\Omega\ \rho_{\rm GWB}(\widehat{\Omega}|d_1, d_2)\rho_{\rm GRB}(\widehat{\Omega}|\widehat{\Omega}', \widehat{n'})\right] \left[\int_0^T dt' \int_0^T dt\, Q(t-t') h_+(t) h_+(t')\right] \tag{29}$$

The first bracketed term contains all the direction and orientation dependence of the gravitational wave and gamma-ray burst detectors, while the second term is strictly a property of the gravitational waves without reference to the orientation of the detectors. Correspondingly, the sensitivity of the upper limit on gravitational wave strength averaged over the observed GRB population when the orientation of GWB and the GRB detectors are given by $(d_1, d_2, \widehat{\Omega}, \widehat{n})$ is proportional to

$$\zeta(\widehat{\Omega}, \widehat{n}, d_1, d_2) = \int d^2\widehat{\Omega}'\, \rho_{\rm GRB}(\widehat{\Omega}'|\widehat{\Omega}, \widehat{n})\, \rho_{\rm GWB}(\widehat{\Omega}'|d_1, d_2)\,. \tag{30}$$

Satellite GRB detectors orbit the Earth and so their orientation is constantly changing; similarly, the orientation of Earth-based gravitational wave detectors are constantly changing as the Earth rotates about its axis. The quantity $\overline{\langle h_1, h_2\rangle}$ will, in the end, involve the time average of ζ over all these motions. Since our principal purpose here is to evaluate the sensitivity of the GRB/GWB detector array to GWBs from GRBs as a function of the relative orientations of the detectors we focus on ζ.

Clearly ζ can be regarded as a figure of merit that describes how capable the gravitational-wave/gamma-ray burst detector combination is at identifying GWBs associated with GRBs as a function of the detector orientations. This figure of merit may be normalized to have a maximum of unity; however, regardless of the normalization

$$\zeta(\widehat{\Omega}', \widehat{n}', d_1, d_2)/\zeta(\widehat{\Omega}, \widehat{n}, d_1, d_2) \tag{31}$$

is the ratio of the upper limits on the squared gravitational-wave amplitude that can be attained by orienting the GRB satellite as $(\widehat{\Omega}, \widehat{n})$ versus $(\widehat{\Omega}', \widehat{n}')$. To the extent that, e.g., the GRB detector orientation $(\widehat{\Omega}, \widehat{n})$ can be manipulated on orbit, choosing orientations that maximize ζ will lead to larger signal contributions to t and thus more sensitive measurements of the gravitational wave strength associated with GRBs.

4. LIGO and Swift

Let us now consider the special case of the Burst-Alert Telescope (BAT) on the Swift satellite[9] and the LIGO gravitational wave detectors (Sigg 2001). The BAT is a wide field-of-view coded-aperture gamma-ray imager that will detect and locate GRBs with arc-minute positional accuracy. Its sensitivity to GRBs depends on the the angle λ between the line of sight to the GRB and the BAT axis, as well as the rotational orientation of the satellite about the BAT axis. Averaged over this azimuthal angle, the BAT sensitivity as a function of λ is approximately[10]

$$\rho_{\text{GRB}} = \begin{cases} 2\cos\lambda - 1 + 0.077\sin\left[13(1-\cos\lambda)\right] & \lambda \in [0, \pi/3]\,, \\ 0 & \text{otherwise}\,. \end{cases} \tag{32}$$

For the purposes of illustration we will use this azimuthal-angle averaged expression for the BAT sensitivity.

The second component of ζ is the function ρ_{GWB}, which depends on the projection of the gravitational-wave strain h_{ab} on to the gravitational-wave detector (cf. equations (17) and (20)). To evaluate ρ_{GWB}, introduce an Earth-centered Cartesian coordinate system, described by unit basis vectors $\widehat{x}$, $\widehat{y}$, and $\widehat{z}$, with

- $\widehat{z}$ pointing parallel to the Earth's polar axis and in the direction of the north celestial pole,
- $\widehat{x}$ parallel to the line that runs in the equatorial plane from Earth's center to the intersection of the equator with the prime meridian at Greenwich, and
- $\widehat{y}$ chosen to form a right-handed coordinate system.

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[9] http://swift.gsfc.nasa.gov/

[10] See the BAT section of the Swift homepage, http://swift.gsfc.nasa.gov/science/instruments/bat.html

Similarly, we introduce the usual spherical-polar coordinate system,

$$r^2 = x^2 + y^2 + z^2 , \tag{33}$$
$$\cos\theta = z/r , \tag{34}$$
$$\tan\phi = y/x . \tag{35}$$

In these coordinates we write the gravitational wave polarization vectors as

$$\epsilon^+_{ab} = \widehat{m}_a\widehat{m}_b - \widehat{n}_a\widehat{n}_b , \tag{36}$$
$$\epsilon^\times_{ab} = \widehat{m}_a\widehat{n}_b + \widehat{n}_a\widehat{m}_b , \tag{37}$$

where[11]

$$\widehat{m} = -\widehat{x}\sin\phi + \widehat{y}\cos\theta , \tag{38}$$
$$\widehat{n} = \widehat{x}\cos\phi\cos\theta + \widehat{y}\sin\phi\cos\theta - \widehat{z}\sin\theta , \tag{39}$$
$$\widehat{\Omega} = \widehat{x}\cos\phi\sin\theta + \widehat{y}\sin\phi\sin\theta + \widehat{z}\cos\theta . \tag{40}$$

Similarly, denoting the detector projection tensor (cf. equation (18)) for the LIGO Hanford (LIGO Livingston) Observatory detector by $d^{ab}_{\rm LHO}$ ($d^{ab}_{\rm LLO}$) we have (Althouse et al. 2000)

$$d^{ab}_{\rm LHO} = \begin{pmatrix} -0.3926 & -0.0776 & -0.2474 \\ -0.0776 & 0.3195 & 0.2280 \\ -0.2474 & 0.2280 & 0.0731 \end{pmatrix} , \tag{41}$$

$$d^{ab}_{\rm LLO} = \begin{pmatrix} 0.4113 & 0.1402 & 0.2473 \\ 0.1402 & -0.1090 & -0.1816 \\ 0.2473 & -0.1816 & -0.3022 \end{pmatrix} . \tag{42}$$

Figure 1 shows the antenna pattern $\rho_{\rm GWB}$ (27). Since the two LIGO detectors share nearly the same plane and have arms nearly aligned with each other their combined antenna pattern is very similar to that for a single interferometer. In particular, the LHO/LLO detector combination is most sensitive to radiation arriving from a direction orthogonal to the (nearly common) plane of the detector arms (corresponding to the two peaks in Figure 1) and least sensitive to radiation arriving in the detector arm plane and parallel to the (nearly common) arm bisector (producing the four wells of low sensitivity in Figure 1). It is also precisely zero in certain directions.

Convolving $\rho_{\rm GWB}$ with the Swift sensitivity function $\rho_{\rm Swift}$ as in equation (30) gives the figure of merit ζ for the Swift pointing, which is shown in Figure 2. This plot is a smeared version of the LIGO antenna pattern of Figure 1. In particular, the four minima of the LIGO sensitivity are smeared into two minima which are wider but not as deep. The figure of merit is nowhere zero, varying by a factor of approximately 4 between best (near zenith of detectors) and worst (near planes of detectors at 45 degrees from arms) orientations of Swift. The all-sky average of the figure of merit is 0.56 times the maximum value.

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[11] Our conventions follow those of Allen & Romano (1999) except that we use $\widehat{\Omega}$ to denote the direction *to* the GRB/GWB source on the sky, rather than the propagation direction of the GWB. The net effect is $\widehat{m} \to -\widehat{m}$ in (38).

5. Discussion

The currently accepted model for gamma-ray burst phenomena involves the violent formation of a rapidly rotating approximately solar mass black hole. Gravitational waves should be associated with the black-hole formation, and their detection would permit this model to be tested, the black-hole progenitor (e.g., coalescing binary or collapsing stellar core) to be identified, and the origin of the gamma rays (within the expanding relativistic fireball or at the point of impact on the interstellar medium) to be located. Even upper limits on the gravitational-wave strength associated with gamma-ray bursts could constrain the gamma-ray burst model.

We have evaluated how the quality of an upper limit on the gravitational-wave strength associated with gamma-ray burst observations depends on the relative orientation of the gamma-ray burst and gravitational-wave detectors, with particular application to the Swift Burst-Alert Telescope (BAT) and the LIGO gravitational wave detectors. Setting aside other physical and science constraints on the Swift mission, careful choice of BAT pointing leads to an upper limit on the observed GRB population-averaged mean-square gravitational-wave strength a factor of two lower than the upper limit resulting from pointing that does not take this science into account.

There are, of course, numerous science and technical constraints that determine the pointing profile of a satellite like Swift; correspondingly, even in the most optimistic case the ratio of the best possible upper limit to the best attainable upper limit will be reduced from this factor of two. Nevertheless, when it can be done without jeopardizing other mission objectives there is an advantage to optimizing the pointing of Swift to maximize the joint LIGO/Swift sensitivity to gamma-ray burst systems. The pointing dependent part (cf. equation (30)) of the anticipated upper limit that can be set with these joint observations, suitably normalized, is closely related to the science enabled by joint LIGO/Swift observations and constitutes an excellent "figure-of-merit" that can be used to incorporate this objective in Swift mission planning.

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REFERENCES

Allen, B., & Romano, J. D. 1999, Phys. Rev. D, 59, 102001

Althouse, W. E. et al., LIGO-P000006-C-E.

Astone, P. et al. 2002, Phys. Rev. D, 66, 102002

Cramer, H. 1999, Mathematical Methods of Statistics, Princeton: Princeton University Press

Feldman, G. J., & Cousins, R. D. 1998, Phys. Rev. D, 57, 3873

Finn, L. S., Mohanty, S. D., & Romano, J. D. 1999, Phys. Rev. D, 60, 121101

Kobayashi, S., & Meszaros, P. 2002, submitted to ApJ, astro-ph/0210211

Kobayashi, S., & Meszaros, P. 2003, to appear in ApJL, astro-ph/0212539

Kobayashi, S., Piran, T., & Sari, R. 1997, ApJ, 490, 92

Mészáros, P., & Rees, M. J. 1993, ApJ, 405, 278

Metzger, M. R. et al. 1997, Nature, 387, 878

Piran, T. 2002, gr-qc/0205045

Rees, M. J., & Mészáros, P. 1994, ApJ, 430, L93

Sari, R., & Piran, T. 1997, ApJ, 485, 270

Sigg, D. 2001, Class. Quant. Grav., 19, 1429

Siegal, S., & Castellan Jr., N. J. 1988, Nonparametric statistics for the Behavioral Sciences, New York: McGraw Hill Book Company

Snedecor, G. W., & Cochran, W. G. 1967, Statistical Methods, Ames, Iowa: Iowa State University Press

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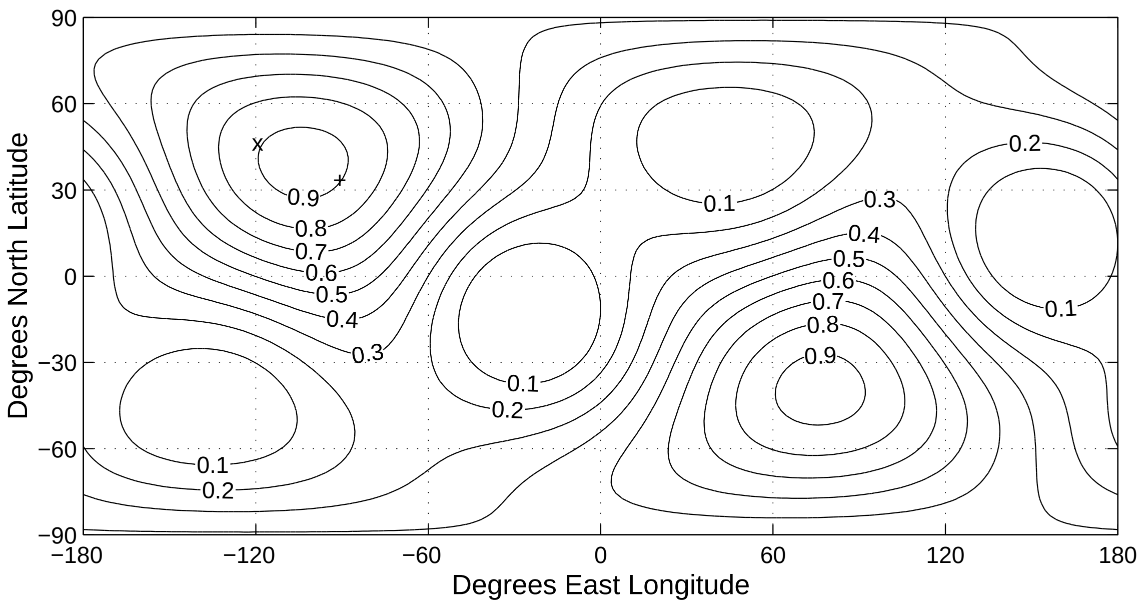

Fig. 1.— LIGO sensitivity pattern $\rho_{\rm GWB}$ (27) in Earth-based coordinates for gravitational-wave bursts satisfying (24), (25). The + and × mark the locations of the LLO and LHO detectors. The array is most sensitive in the directions orthogonal to the plane of the LIGO detectors, corresponding to the two peaks at upper left and lower right. The sensitivity is lowest in the plane of the detectors near the directions at 45 degrees to the arms and vanishes in certain directions, producing the four wells of low sensitivity. The sensitivity has been scaled to range from $[0, 1]$ in this plot.

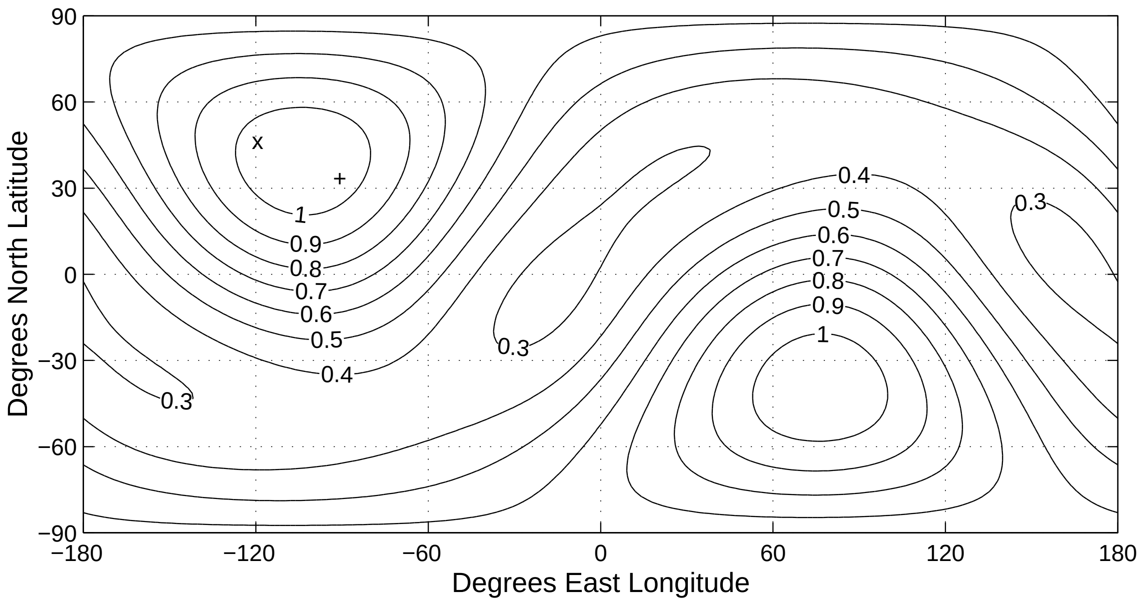

Fig. 2.— Figure of merit ζ (30) for Swift pointing in Earth-based coordinates, produced by convolving the LIGO sensitivity pattern $\rho_{\rm GWB}$ (27) with the Swift sensitivity function $\rho_{\rm Swift}$ (32). The four minima of the LIGO sensitivity pattern of Figure 1 are smeared into two minima which are not so deep. The figure of merit is nowhere zero, having a range of $[0.25, 1.00]$ and an all-sky average of 0.56. The $+$ and $\times$ mark the locations of the LLO and LHO detectors.