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QUANTUM GRAVITY AND THE BIG BANG

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Abstract. Quantum gravity has matured over the last decade to a theory which can tell in a precise and explicit way how cosmological singularities of general relativity are removed. A branch of the universe "before" the classical big bang is obtained which is connected to ours by quantum evolution through a region around the singularity where the classical space-time dissolves. We discuss the basic mechanism as well as applications ranging to new phenomenological scenarios of the early universe expansion, such as an inflationary period.

1 Introduction

When the big bang is approached, the volume becomes smaller and smaller and one enters a regime of large energy densities. Classically, those conditions will become so severe that a singularity is reached; the theory simply breaks down. For a long time, the expectation has been that somewhere along the way quantum gravity takes over and introduces new effects, e.g. a discrete structure, which prevent the singularity to develop. This presumably happens at scales the size of the Planck length ℓ_P, i.e. when the universe has about a volume ℓ_P^3.

Since at the classical singularity space itself becomes singular and gravitational interactions are huge, such a quantum theory of gravity must be background independent and non-perturbative. A theory satisfying these conditions is in fact available in the form of loop quantum gravity/quantum geometry (see [1, 2] for reviews). One of its early successes was the derivation of discrete spectra of geometric operators like area and volume [3, 4, 5]. Thus, the spatial geometry is discrete in a precise sense. Furthermore, matter Hamiltonians exist as well-defined operators in the theory which implies that ultraviolet divergences are cured in the fundamental formulation [6, 7].

Both properties must be expected to have important consequences for cosmology. The discreteness leads to a new basic formulation valid at small volume, and since gravity couples to the matter Hamiltonian, its source term is modified at small scales when the good ultraviolet behavior is taken into account. It is possible to introduce both effects into a cosmological model in a systematic way, which allows us to test the cosmological consequences of quantum gravity (reviewed in [8, 9]).

2 Cosmological evolution equations

Classically, the dynamics of a flat isotropic universe is described by the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{16\pi G}{3}\rho(a) \tag{1}$$

where we can choose the energy density of a single scalar,

$$\rho(a) = a^{-3}H(a) = a^{-3}\left(\frac{1}{2}\frac{p_\phi^2}{a^3} + a^3V(\phi)\right) \tag{2}$$

with its potential $V(\phi)$ and momentum p_ϕ. It can be quantized by turning the momentum of a into a derivative operator acting on a wave function $\psi(a,\phi)$, resulting in the Wheeler–DeWitt equation

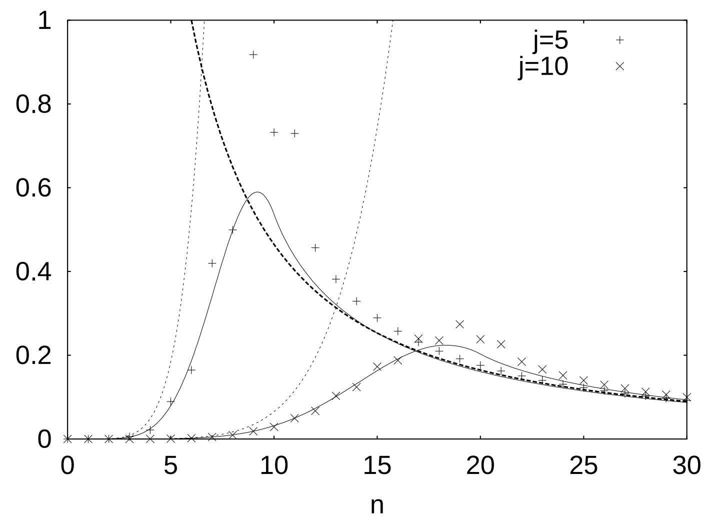

Figure 1: Eigenvalues of the density operator for two choices of the ambiguity parameter, compared to the classical expectation a^{-3} (thick, dashed). Also shown are continuous approximations to the discrete eigenvalues (solid), and small-volume approximations.

[10, 11]

$$-\frac{1}{6}\ell_P^4 a^{-1}\frac{\partial}{\partial a}a^{-1}\frac{\partial}{\partial a}a\psi(a,\phi) = 8\pi G\hat{H}(a)\psi(a,\phi)\,. \tag{3}$$

Here, $\hat{H}(a)$ is the matter Hamiltonian acting on the scalar dependence of ψ. It also depends on a via the volume.

In this quantization we are not able to see any discrete picture or other modification at small volume. In fact, this equation, though quantized, cannot be shown to remove the singularity. One can see that these problems are related to the fact that we just used quantum mechanical techniques in going from the simple Friedmann equation to the Wheeler–DeWitt equation. The same techniques cannot be applied to more complicated systems, let alone the full theory. It is then very likely that consistency conditions, which would arise only in the complicated systems, are overlooked in the quantization of the simple model. It would be more reliable if we used a full quantum theory of gravity, such as loop quantum gravity, and introduced the symmetries there. This is in fact possible [12], and leads us to loop quantum cosmology where the basic evolution equation for the isotropic case is [13]

$$\begin{aligned}-\frac{1}{2\sqrt{6}}\ell_P\Big[\left(|n+2|^{3/2}-|n|^{3/2}\right)\psi_{n+1}(\phi)-2\left(|n+1|^{3/2}-|n-1|^{3/2}\right)\psi_n(\phi)\\+\left(|n|^{3/2}-|n-2|^{3/2}\right)\psi_{n-1}(\phi)\Big]=8\pi G\hat{H}(n)\psi_n(\phi)\,.\end{aligned} \tag{4}$$

It is immediately clear that the formulation is now discrete since we have a difference equation in the integer n replacing a as a label of the wave function, with the relation $|n| = 6a^2/l_P^2$. Another difference is that, unlike a, n can also take negative values, the sign corresponding to the orientation of space. Most importantly, the equation is *non-singular*! Starting from initial values for ψ at large positive n, we can evolve backwards up to and right through the classical singularity at $n = 0$ [14]. The evolution does not stop, and we obtain a collapsing branch at negative n preceding the classical singularity. One must keep in mind, however, that the classical space-time picture dissolves around $a = 0$ and is replaced by a discrete structure. A smooth transition, as sometimes presumed, is impossible since smoothness would not even be defined.

So far we only commented on the left hand side of Eq. (4) which is obviously different from that of Eq. (3). The right hand side, however, is also changed because as matter Hamiltonian we have to use one which is related to that of the full theory. Since ultraviolet divergences are cut off there, also the divergence of a^{-3} in the kinetic term of (2) is cut off at small scales, which has consequences regarding the evolution of the early universe. (In fact, some aspects of these modifications are already important for the removal of the singularity [14].)

3 Inflation

Quantum gravity is expected to provide a cut-off for curvatures which would otherwise diverge when a cosmological singularity is approached. In the isotropic context, curvature components are proportional to inverse powers of the scale factor a, for instance the density a^{-3} which also appears in the kinetic term of a matter Hamiltonian (2). A natural cut-off is in fact realized in loop quantum gravity, where quantization methods of the full theory [15] imply a peak in the eigenvalues of an operator quantizing a^{-3} [16, 17]; see Fig. 1. Since this operator is not a basic one of the quantum theory, it is subject to quantization ambiguities. In particular the position of the peak changes when values parameterizing the ambiguities are changed. This can easily be seen in Fig. 1, where the eigenvalues are plotted for two choices of a parameter j (a half-integer). Also the peak in the eigenvalues (at a scale factor $a \sim \sqrt{j}\ell_P$) is obvious, as well as the fact that for even smaller volume the eigenvalues decrease rather than showing the classical divergence. This demonstrates the expected curvature cut-off by quantum gravity effects.

It is possible to approximate the discrete eigenvalues by a continuous curve, which does not diverge at $a = 0$,

$$(a^{-3})^{(j)} = a^{-3} p(3a^2/j\ell_P^2)^6 \tag{5}$$

depending on the parameter j (the approximation becomes better for larger j, as can be seen in Fig. 1). The function

$$\begin{aligned} p(q) \;=\;& \tfrac{8}{77} q^{1/4} \Big[7 \left((q+1)^{11/4} - |q-1|^{11/4} \right) \\ & - 11q \left((q+1)^{7/4} - \mathrm{sgn}(q-1)|q-1|^{7/4} \right) \Big] \,, \end{aligned} \tag{6}$$

is derived from the quantum theory [17] (but also subject to minor ambiguities) and provides the *interface to cosmological phenomenology* in the following way: When the matter Hamiltonian is derived from the quantum theory, the density a^{-3} in its kinetic part must show the cut-off. We can explicitly realize that by replacing a^{-3} with the modified $(a^{-3})^{(j)}$ for some half-integer j. (Only the factor a^{-3} in the kinetic term is changed, not the pre-factor of the Hamiltonian in the density (2), since the Hamiltonian is the primary object for the quantization. Dividing by a^3 to obtain the density is done at the classical level which cannot receive quantum modifications.) In this way, we obtain the *effective Friedmann equation*

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{16\pi}{3} G a^{-3} \left(\frac{1}{2} a^{-3} p(3a^2/j\ell_P^2)^6 p_\phi^2 + a^3 V(\phi) \right) . \tag{7}$$

Since the right hand side now depends differently on a for small a compared to the classical behavior, the dynamics is clearly modified. In particular, since the function p in (6) is increasing as a function of its argument when it is small, the matter Hamiltonian at the right hand side is an increasing function of the volume at small volume. Thermodynamically, this implies *negative pressure* and therefore *inflation* [18]. In fact, simple numerical solutions of the effective Friedmann equation (7) clearly show an early phase of accelerated expansion (Fig. 2).

Thus, quantum geometry provides a new mechanism for inflation. It is a consequence of a kinetic term modified by non-perturbative quantum effects and is quite independent of the particular potential: even a zero potential implies inflation. Furthermore, it is not necessary to introduce an inflaton field, since any matter component will show the modification and therefore lead to inflation via its kinetic term. The details of the potential are, however, important for the observational viability of an inflationary scenario.

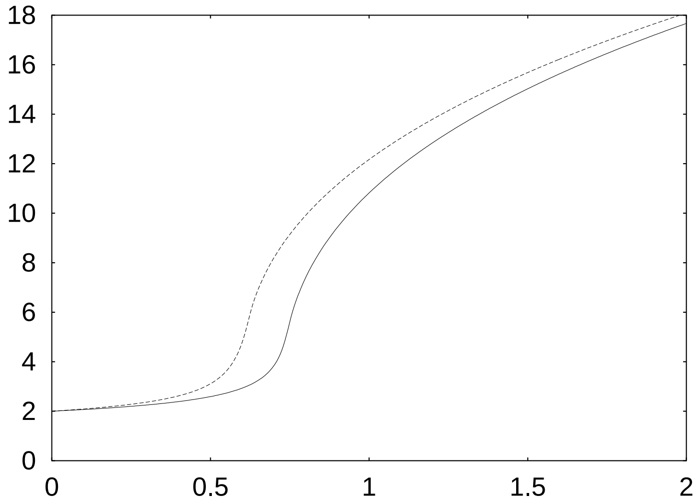

Figure 2: A numerical solution $a(t)$ of the effective Friedmann equation with vanishing potential (solid) and a small quadratic potential (dashed). The ambiguity parameter is $j = 100$.

Further possibilities for model building arise from the fact that matter fields are driven away from their potential minima during quantum geometry inflation because the usual friction term changes sign [19]: From the effective matter Hamiltonian

$$H^{\rm eff}(a) = \frac{1}{2}a^{-3}p(3a^2/j\ell_P^2)^6 p_\phi^2 + a^3V(\phi)$$

at the right hand side of (7) we obtain the Hamiltonian equations of motion

$$\dot\phi = \{\phi, H^{\rm eff}(a)\} = a^{-3}p(3a^2/j\ell_P^2)^6 p_\phi$$

and

$$\dot p_\phi = \{p_\phi, H^{\rm eff}(a)\} = -a^3V'(\phi)$$

for the scalar and its momentum. Both equations yield a second order equation

$$\ddot\phi = p_\phi \frac{{\rm d}[a^{-3}p(3a^2/j\ell_P^2)^6]}{{\rm d}t} + a^{-3}p(3a^2/j\ell_P^2)^6\dot p_\phi = a\frac{{\rm d}\log[a^{-3}p(3a^2/j\ell_P^2)^6]}{{\rm d}a}H\dot\phi - p(3a^2/j\ell_P^2)^6\,V'(\phi)$$

for ϕ. For large a, the function p is close to one and we obtain the usual friction term $-3H\dot\phi$ which dampens the evolution of ϕ. For small a, however, we noted repeatedly that the effective density $a^{-3}p(3a^2/j\ell_P^2)^6$ increases as a function of a. Thus, the derivative in the friction term has the opposite sign and ϕ is driven up its potential. This mechanism can be used to drive a subsequent phase of slow-roll inflation, or may have consequences for structure generation or reheating. These possibilities are currently being investigated.

4 Conclusions

With new developments in quantum geometry, quantum gravity has become a theory which can make concrete predictions about the very early stages of the universe. Results include possible solutions of old conceptual problems, as the singularity problem [14] and the problem of initial conditions [20], and also new phenomenological proposals which can be confronted with cosmological observations

[18]. The models currently available are most likely too simple, but more complicated ones with less symmetries (e.g., [21]) and more realistic matter content are being developped. An advantage of the formalism is that the relation between models and the full theory of loop quantum gravity is known so that lessons learned for models can be taken over to the full theory. In this way we will be able to guide developments in quantum gravity by cosmological observations.

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