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Black holes in Gödel-type universes with a cosmological constant

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Abstract

We discuss supersymmetric black holes embedded in a Gödel-type universe with cosmological constant in five dimensions. The spacetime is a fibration over a four-dimensional Kähler base manifold, and generically has closed timelike curves. Asymptotically the space approaches a deformation of AdS_5, which suggests that the appearance of closed timelike curves should have an interpretation in some deformation of $D = 4$, $\mathcal{N} = 4$ super-Yang-Mills theory.

Finally, a Gödel-de Sitter universe is also presented and its causal structure is discussed.

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1 Introduction

For specific cases we have already a fairly good understanding of (vacuum) geometries in which string theory can be embedded without breaking supersymmetry. A general picture is however still missing, but one step in this direction is the classification of supersymmetric bosonic field configurations of lower-dimensional supergravity, obtained by compactification of string theory (or M-theory). For the minimal supergravity in five dimensions this was recently done in [1, 2]*, where the BPS solutions were classified by a Killing vector field, which is always present due to supersymmetry. In fact, unbroken supersymmetry requires the existence of at least one Killing spinor, which in turn implies the existence of a Killing vector. This Killing vector is constructed as fermionic bi-linear, and can be null or timelike, but not spacelike. The null case describes pp-wave-type solutions, whereas examples with a timelike Killing vector are the BPS black holes [4].

There is another class of BPS solutions with a timelike Killing vector, that are however neither asymptotically flat nor anti-de Sitter as in the case of gauged supergravity. These are the Gödel-type solutions of [1], which are pathological in the sense that they exhibit closed timelike curves (CTCs), which are not shielded by any horizon. Many attempts have been made to understand or to cut-off the regions with CTCs by holographic screens or by appropriate probes [5, 6, 7, 8, 9, 10], but so far a deeper understanding of this phenomenon is still missing. One interesting observation is the link of the ungauged case to an integrable model and the appearance of a pole in the partition function indicating a phase transition [11, 12]. In gauged supergravity on the other hand, all CTCs disappear if the cosmological constant is sufficiently large [13]. The situation here is reminiscent of the rotating BMPV black hole [14] in asymptotically flat spacetime, where CTCs are present only in the over-rotating case, but disappear if the mass (for fixed angular momentum) becomes large enough [15, 16].

In this paper, we want to discuss in more detail the interplay between (rotating) black holes and Gödel solutions. We are particularly interested in black holes embedded in Gödel universes with cosmological constant, that approach asymptotically

*For a systematic classification of BPS supergravity solutions in other dimensions cf. [3].

a deformation of AdS_5. The AdS/CFT correspondence opens then the possibility to relate the appearance of closed timelike curves in the bulk to properties (like loss of unitarity) of a dual field theory residing on the deformed boundary of the five-dimensional spacetime.

In detail, the remainder of this paper is organized as follows: In section 2, we construct various BPS black hole solutions embedded in a Gödel-AdS spacetime. We discuss their causal structure and point out a possible holographic interpretation of the appearance of closed timelike curves in the AdS/CFT correspondence. In section 3 a more general class of BPS solutions is constructed. These solutions are given in terms of a fibration over a four-dimensional Kähler base manifold which is a complex line bundle over a two-dimensional surface of constant curvature. They include a Gödel-type deformation of the rotating AdS black holes obtained recently in [17] as a special subcase. Finally, in section 4 we present a Gödel-de Sitter universe and analyze some of its physical properties.

2 Supersymmetric Gödel-AdS black holes

Gauntlett and Gutowski [2] classified all supersymmetric solutions of minimal gauged supergravity in five dimensions, with bosonic action

$$S = \frac{1}{4\pi G}\int\left(-\frac{1}{4}[{}^5R - 2\Lambda] * 1 - \frac{1}{2}F\wedge *F - \frac{2}{3\sqrt{3}}F\wedge F\wedge A\right) . \qquad (2.1)$$

The solutions fall into two classes, depending on whether the Killing vector constructed from the Killing spinor is timelike or null. Let us consider the former class. In order to make our paper self-contained, we briefly review the results of [2] for the timelike case. The line element can be written as

$$ds^2 = f^2(dt+\omega)^2 - f^{-1}h_{mn}dx^m dx^n , \qquad (2.2)$$

where h_{mn} denotes the metric on a four-dimensional Kähler base manifold $\mathcal{B}$ and

$$f = -\frac{2\chi^2}{R} , \qquad (2.3)$$

where R is the scalar curvature of $\mathcal{B}$ and χ is related to the cosmological constant by $\chi^2 = 2\Lambda^\dagger$. The one-form ω is determined by

$$f d\omega = G^+ + G^- \tag{2.4}$$

where G^+ is a self-dual two-form on the base manifold, given by

$$G^+_{mn} = -\frac{\sqrt{3}}{\chi}(\mathcal{R}_{mn} - \frac{1}{4} R X^{(1)}_{mn}) \,, \tag{2.5}$$

with $\mathcal{R}$ the Ricci form and $X^{(1)}$ the Kähler form of $\mathcal{B}$. On the other hand G^- is an anti-self-dual two-form and is decomposed as

$$G^- = \lambda^1 X^{(1)} + \lambda^2 X^{(2)} + \lambda^3 X^{(3)} \,, \tag{2.6}$$

where $X^{(2)}$ and $X^{(3)}$ are additional anti-self-dual two-forms on $\mathcal{B}$, that, together with $X^{(1)}$, satisfy the algebra of unit quaternions,

$$X^{(i)}{}_m{}^p X^{(j)}{}_p{}^n = -\delta^{ij}\delta_m{}^n + \epsilon^{ijk} X^{(k)}{}_m{}^n \,. \tag{2.7}$$

The coefficient λ^1 is fixed in terms of the base space geometry,

$$\lambda^1 = \frac{\sqrt{3}}{\chi R}\Big(\frac{1}{2}\nabla^m \nabla_m R + \frac{2}{3}\mathcal{R}_{mn}\mathcal{R}^{mn} - \frac{1}{3}R^2 \Big) \,, \tag{2.8}$$

with ∇ denoting the Levi-Cività connection on the base manifold with respect to h. If we adopt complex coordinates $z^j, z^{\bar{\jmath}}$ on $\mathcal{B}$ with respect to $X^{(1)}$ (i. e. $X^{(1)}{}^j{}_k = i\delta^j{}_k$, $X^{(1)}{}^{\bar{\jmath}}{}_{\bar{k}} = -i\delta^{\bar{\jmath}}{}_{\bar{k}}$), then λ^2 and λ^3 are determined by the differential equation

$$\Theta_j = -(\partial_j - iP_j)[R(\lambda^2 - i\lambda^3)] \,, \tag{2.9}$$

which implies that $\lambda^2 - i\lambda^3$ is fixed up to an arbitrary antiholomorphic function on the base. In (2.9), P and Θ are given by

$$P_m = \frac{1}{8}(X^{(3)np}\nabla_m X^{(2)}{}_{np} - X^{(2)np}\nabla_m X^{(3)}{}_{np}) \,, \tag{2.10}$$

$$\Theta_m = X^{(2)}{}_m{}^n (*_4 T)_n \,, \tag{2.11}$$

with

$$T = \frac{\sqrt{3}}{\chi}\left(-dR \wedge \mathcal{R} + d\left[\frac{1}{2}\nabla^m \nabla_m R + \frac{2}{3}\mathcal{R}^{mn}\mathcal{R}_{mn} - \frac{1}{12}R^2 \right] \wedge X^{(1)} \right) \,.$$

†With mostly minus signature, positive Λ corresponds to AdS and negative Λ to dS.

In summary, f and $G^{\pm}$ are fixed by the geometry of the base manifold (up to an antiholomorphic function); then, ω is given by (2.4) and finally the gauge potential reads

$$A_m = \chi^{-1} P_m + \frac{\sqrt{3}}{2} f \omega_m \,, \qquad A_t = \frac{\sqrt{3}}{2} f \,. \tag{2.12}$$

Note that all fields are independent of t.

In order to obtain black hole solutions immersed in a Gödel-type universe, we choose as metric on the base manifold

$$h_{mn} dx^m dx^n = H^{-2} dr^2 + \frac{r^2}{4} H^2 (\sigma_3^L)^2 + \frac{r^2}{4} [(\sigma_1^L)^2 + (\sigma_2^L)^2] \,, \tag{2.13}$$

where

$$H(r) = \sqrt{1 + \frac{\chi^2}{12} r^2 (1 + \frac{\mu}{r^2})^3} \,. \tag{2.14}$$

For $\mu = 0$, this metric reduces to the Bergmann metric, which is Einstein-Kähler. The right-invariant (or "left") one-forms on SU(2) are given by

$$\begin{aligned} \sigma_1^L &= \sin\phi d\theta - \cos\phi \sin\theta d\psi \,, \\ \sigma_2^L &= \cos\phi d\theta + \sin\phi \sin\theta d\psi \,, \\ \sigma_3^L &= d\phi + \cos\theta d\psi \,, \end{aligned} \tag{2.15}$$

with the Euler angles $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$, $0 \leq \psi \leq 4\pi$. By introducing the complex coordinates

$$z^1 = h(r) \cos\frac{\theta}{2} e^{\frac{i}{2}(\phi+\psi)} \,, \qquad z^2 = h(r) \sin\frac{\theta}{2} e^{\frac{i}{2}(\phi-\psi)} \,, \tag{2.16}$$

where

$$h(r) = \exp \int \frac{dr}{H^2 r} \,, \tag{2.17}$$

one can verify that (2.13) is Kähler, with Kähler potential given by

$$K(r) = \int \frac{r\, dr}{H^2} \,. \tag{2.18}$$

For the metric (2.13) one finds [2]: $\Theta = 0$, $R = -2\chi^2(1 + \mu/r^2)$ and

$$\begin{aligned} X^{(1)} &= d(\frac{r^2}{4} \sigma_3^L) \,, \\ X^{(2)} &= H^{-1} \frac{r}{2} dr \wedge \sigma_1^L + \frac{r^2}{4} H d\sigma_1^L \,, \\ X^{(3)} &= H^{-1} \frac{r}{2} dr \wedge \sigma_2^L + \frac{r^2}{4} H d\sigma_2^L \,, \end{aligned} \tag{2.19}$$

as well as

$$P_{z^1} = \frac{i\chi^2}{8r^2h(r)^2}(r^2+\mu)^2\bar{z}^1\,, \qquad P_{z^2} = \frac{i\chi^2}{8r^2h(r)^2}(r^2+\mu)^2\bar{z}^2\,. \tag{2.20}$$

As $\Theta = 0$, Eq. (2.9) admits the trivial solution $\lambda^2 = \lambda^3 = 0$ giving the supersymmetric electrically charged AdS black holes[‡] first constructed in [18][§]. It is however possible to solve (2.9) in general. To this end, we note that

$$P_i = \partial_i L(r)\,, \tag{2.21}$$

where

$$L(r) = \int \frac{i\chi^2}{4H^2r^3}(r^2+\mu)^2\,dr\,. \tag{2.22}$$

This leads to the general solution

$$\lambda^2 - i\lambda^3 = \frac{\mathcal{F}(\bar{z}^1,\bar{z}^2)}{R}e^{iL}\,, \tag{2.23}$$

with $\mathcal{F}$ denoting an arbitrary antiholomorphic function. If we choose $\mathcal{F}$ to be constant, we get the supersymmetric solution

$$\begin{aligned} ds^2 &= f^2(dt+\omega)^2 - f^{-1}(H^{-2}dr^2 + \frac{r^2}{4}H^2(\sigma_3^L)^2 + \frac{r^2}{4}[(\sigma_1^L)^2 + (\sigma_2^L)^2])\,, \\ A &= \frac{\sqrt{3}}{2}f[dt - \mathcal{F}_1h^2(r)\sigma_1^L + \mathcal{F}_2h^2(r)\sigma_2^L]\,, \end{aligned} \tag{2.24}$$

with

$$\begin{aligned} f^{-1} &= 1 + \frac{\mu}{r^2}\,, \\ \omega &= \frac{\chi r^2}{4\sqrt{3}}\left(1+\frac{\mu}{r^2}\right)^3\sigma_3^L - \mathcal{F}_1h^2(r)\sigma_1^L + \mathcal{F}_2h^2(r)\sigma_2^L\,, \end{aligned} \tag{2.25}$$

where $\mathcal{F}_1$ and $\mathcal{F}_2$ are arbitrary constants related to the real and imaginary part of $\mathcal{F}$ respectively and the function $h(r)$ is given by (2.17).

For $\mu = \chi = 0$, the solution (2.24) reduces to the maximally supersymmetric Gödel-type universe found in [1]. Turning on the parameter μ while keeping $\chi = 0$

‡Actually these solutions describe naked singularities. In a slight abuse of notation, we shall nevertheless refer to them as black holes.

§For generalizations to the case of gauged supergravity coupled to vector multiplets see [19].

yields the one half supersymmetric Gödel black hole studied in detail in [20]. For $\mathcal{F}_1 = \mathcal{F}_2 = 0$, $\chi \neq 0$, we recover the AdS black holes of [18]. In the case $\mu = 0$, $\chi \neq 0$ (2.24) describes a generalization of the Gödel-type universe of [1] to include a cosmological constant. This solution was first given in [2], and its chronological structure was studied in [13]. Although the geometry (2.24) has a naked singularity at $r^2 + \mu = 0$ ($R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} \sim \frac{f^{11}}{H^6}$) not hidden by an event horizon, we shall refer to it as a black hole immersed in a Gödel-type universe with cosmological constant¶. In section 3 we shall construct Gödel-type deformations of AdS black holes with genuine horizons.

Physical discussion

In what follows, we will discuss some physical properties of (2.24). First of all, let us consider its chronological structure. One finds that the induced metric on hypersurfaces of constant t and r is always spacelike iff

$$g(r) \equiv \mathcal{F}_1^2 + \mathcal{F}_2^2 + \sqrt{(\mathcal{F}_1^2 + \mathcal{F}_2^2)^2 + \frac{\chi^2 r^4}{12 f^6 h^4}(\mathcal{F}_1^2 + \mathcal{F}_2^2)} - \frac{r^2}{2 f^3 h^4} < 0\,. \tag{2.26}$$

For $g(r) > 0$ it becomes timelike and thus closed timelike curves (CTCs) appear. When we approach the naked singularity at $r^2 = -\mu$ (where $f \to \infty$), $g(r)$ goes to $2(\mathcal{F}_1^2 + \mathcal{F}_2^2)$ and thus, as long as $\mathcal{F}_1$ and $\mathcal{F}_2$ are nonvanishing, we have always CTCs near the singularity. On the other hand, for $r \to \infty$, $g(r)$ is negative provided

$$\frac{\chi^2}{3} h^4(\infty)(\mathcal{F}_1^2 + \mathcal{F}_2^2) < 1\,, \tag{2.27}$$

where $h(\infty)$ indicates the value of h at infinity, which is easily shown to be a constant. If (2.27) is satisfied, there are *no* CTCs if r is sufficiently large. However, this conclusion is only valid on constant time slices, i.e. $dt = 0$. If we instead allow t to vary, one can construct through every point in spacetime a CTC. Namely, by going inside the future light cone towards the black hole singularity, constructing there a time machine as discussed in [13] and finally coming back to the starting point. Note that, as $\mathcal{F}_1^2 + \mathcal{F}_2^2$ measures the angular momentum/magnetic flux, (2.27) can be viewed as a bound on the angular momentum of the solution (2.24).

¶For black holes in Gödel spacetimes without cosmological constant cf. [20, 21].

Asymptotically for $r \to \infty$ the metric (2.24) does not approach AdS_5, but a deformation thereof. The induced metric on hypersurfaces of constant r is, for large r, conformal to

$$ds^2 = \frac{\chi}{2\sqrt{3}}\sigma_3^L(dt - \mathcal{F}_1 h(\infty)^2\sigma_1^L + \mathcal{F}_2 h(\infty)^2\sigma_2^L) - \frac{1}{4}[(\sigma_1^L)^2 + (\sigma_2^L)^2 + (\sigma_3^L)^2]\,, \quad (2.28)$$

which is always nondegenerate. If $\mathcal{F}_1$ and $\mathcal{F}_2$ were zero, (2.28) would be the standard metric on $\mathbb{R} \times S^3$ (after setting $\phi = \phi' + \chi t/\sqrt{3}$), but for $\mathcal{F}_1$ or $\mathcal{F}_2$ different from zero, (2.28) describes a deformation of this standard metric. According to the AdS/CFT correspondence, the bulk solution (2.24) should have a dual description in terms of (a deformation of) $D = 4$, $\mathcal{N} = 4$ super-Yang-Mills theory defined on the curved manifold (2.28). A massless vector field A in AdS_5, which naturally couples to a CFT R-current J, typically falls off like r^{-2} or like r^0 for $r \to \infty$. The latter behaviour is the non-normalizable mode corresponding to the insertion of the dual operator. From (2.24) we see that in our case the dual operator is inserted[‖], i. e. , the CFT is deformed by the term

$$\int d^4x\sqrt{-h}A_\mu^a J_a^\mu\,, \quad (2.29)$$

where h denotes the determinant of the metric (2.28) and a is an SO(6) R-symmetry index. (In our case A^a takes values in the Cartan subgroup SO(2) × SO(2) × SO(2) of SO(6), with all three components equal). J has dimension $\Delta = 3$, and thus our bulk solution is described by a relevant deformation of $\mathcal{N} = 4$ super-Yang-Mills theory residing on the curved manifold (2.28). Of course this deformation preserves only part of the original supersymmetry. The situation encountered here is somewhat similar to that for the Gödel black hole without cosmological constant studied in [20]. Both this spacetime and the BMPV black hole are described (after uplifting to ten dimensions) by a deformation of the D1-D5-pp-wave system, but the BMPV perturbation is normalizable whereas the Gödel perturbation is non-normalizable and corresponds to the insertion of an operator in the dual two-dimensional CFT [20]. For the BMPV black hole, the rotation corresponds to a VEV of the CFT R-currents. Now the classification of unitary representations of superconformal algebras typically yields inequalities on the conformal weights and R-charges. Generically unitarity is violated

[‖]E.g. for $\mu = 0$ one has $h^2(r) = C(1 - 12\chi^{-2}r^{-2} + \mathcal{O}(r^{-4}))$ for $r \to \infty$, where C denotes an arbitrary integration constant that can be absorbed into $\mathcal{F}_{1,2}$.

if the R-charges become too large. It has been shown in [22] that the threshold where CTCs develop in the bulk of the BMPV black hole (when the angular momentum becomes too large, overrotating case) corresponds exactly to a unitarity bound in the dual CFT.

It would be interesting to see whether a similar holographic interpretation can be given to the bound (2.27), i. e. , if the appearance of closed timelike curves in the bulk is related to loss of unitarity in the dual field theory.

But this is just a rephrasing of the pathological situation and what we need is a mechanism which avoids or forbids these deformations. An interesting way could be to construct the non-extreme solutions and to investigate the thermodynamical stability, but we leave a more detailed discussion of this for future work.

3 More general supersymmetric solutions

The solution (2.24) is actually a special case of a more general class of BPS solutions, that are given in terms of a base space $\mathcal{B}$ which is a complex line bundle over a two-dimensional surface Σ. For the metric on $\mathcal{B}$ we choose

$$h_{mn}dx^m dx^n = \frac{dr^2}{V(r)} + V(r)(d\phi + \mathcal{A})^2 + F^2(r)d\Sigma^2 \,, \tag{3.1}$$

where the one-form $\mathcal{A}$ on Σ and the functions $V(r), F(r)$ will be determined below by requiring that (3.1) be the metric on a Kähler manifold. Although arbitrary surfaces Σ might be possible, we shall consider only the case where Σ is a space of constant curvature k, where without loss of generality $k = 0, \pm 1$. For the line element $d\Sigma^2$ on Σ we can take

$$d\Sigma^2 = d\theta^2 + S^2(\theta)d\psi^2 \,, \tag{3.2}$$

with

$$S(\theta) = \begin{cases} \sin\theta & , \quad k = 1\,, \\ \sinh\theta & , \quad k = -1\,, \\ 1 & , \quad k = 0\,. \end{cases} \tag{3.3}$$

The anti-self-dual two-forms $X^{(i)}$ on the base manifold $\mathcal{B}$ can be chosen as

$$\begin{aligned} X^{(1)} &= e^1 \wedge e^2 - e^3 \wedge e^4 \,, \\ X^{(2)} &= e^1 \wedge e^3 + e^2 \wedge e^4 \,, \\ X^{(3)} &= e^1 \wedge e^4 - e^2 \wedge e^3 \,, \end{aligned}$$

where the vierbein is given by

$$e^1 = \frac{dr}{\sqrt{V(r)}} \,, \quad e^2 = \sqrt{V(r)}\sigma_3 \,, \quad e^3 = F(r)\sigma_1 \,, \quad e^4 = F(r)\sigma_2 \,,$$

and we defined

$$\begin{aligned} \sigma_1 &= \sin\alpha\phi \, d\theta - S(\theta)\cos\alpha\phi \, d\psi \,, \\ \sigma_2 &= \cos\alpha\phi \, d\theta + S(\theta)\sin\alpha\phi \, d\psi \,, \\ \sigma_3 &= d\phi + \mathcal{A} \,, \end{aligned} \tag{3.4}$$

with α to be determined below. $X^{(1)}$ is then closed provided

$$\mathcal{A} = \begin{cases} n\cos\theta \, d\psi & , \quad k = 1 \,, \\ -n\cosh\theta \, d\psi & , \quad k = -1 \,, \\ \frac{n}{2}(\psi \, d\theta - \theta \, d\psi) & , \quad k = 0 \,, \end{cases} \tag{3.5}$$

$\alpha = k/n$ and

$$F^2(r) = nr \,, \tag{3.6}$$

where n is an arbitrary constant. Eq. (3.5) means that $d\mathcal{A}$ is proportional to the Kähler form on Σ. It can then be checked that for arbitrary $V(r)$, the two-forms $X^{(2)}$ and $X^{(3)}$ satisfy

$$\begin{aligned} \nabla_m X^{(2)}_{np} &= P_m X^{(3)}_{np} \,, \\ \nabla_m X^{(3)}_{np} &= -P_m X^{(2)}_{np} \,, \end{aligned} \tag{3.7}$$

with P given by

$$P = \left(\frac{k}{n} - \frac{V'(r)}{2} - \frac{V(r)}{2r} \right) \sigma_3 \,. \tag{3.8}$$

Note that (3.7) implies (2.10).

In conclusion, (3.1) is a Kähler metric for arbitrary function $V(r)$, provided $\mathcal{A}$ and $F(r)$ satisfy (3.5) and (3.6) respectively. The Kähler form on $\mathcal{B}$ is given by $X^{(1)}$.

This general base manifold $\mathcal{B}$ can be used as a starting point for the construction of a variety of new supersymmetric solutions of minimal gauged supergravity in five dimensions. Note that for general $V(r)$, the one-form Θ defined in (2.11) does not vanish, which makes it rather difficult to solve equation (2.9). If we choose $n = 1$ and

$$V(r) = r\left[k + \frac{\chi^2}{3} r \left(1 + \frac{\mu}{r}\right)^3\right], \tag{3.9}$$

where μ denotes an arbitrary parameter, Θ vanishes. The base manifold has then the scalar curvature

$$R = -2\chi^2 \left(1 + \frac{\mu}{r}\right), \tag{3.10}$$

which yields for the function f

$$f^{-1} = 1 + \frac{\mu}{r}. \tag{3.11}$$

The spherical case $k = 1$ leads (after the coordinate transformation $r \to r^2/4$) to the supersymmetric Gödel black hole (2.24) already discussed above. Let us therfore focus our attention to the cases $k = -1$ and $k = 0$. For $k = -1$, Σ is a hyperbolic space (or a quotient thereof). One can choose the complex coordinates

$$z^1 = h(r) \cosh \frac{\theta}{2} e^{-\frac{i}{2}(\phi - \psi)}, \qquad z^2 = h(r) \sinh \frac{\theta}{2} e^{-\frac{i}{2}(\phi + \psi)}, \tag{3.12}$$

on $\mathcal{B}$ with respect to $X^{(1)}$, where

$$h(r) = \exp\left[-\int \frac{dr}{2V(r)}\right]. \tag{3.13}$$

This leads to

$$P_{z^1} = -\frac{i\chi^2}{2rh(r)^2}(r + \mu)^2 \bar{z}^1, \qquad P_{z^2} = \frac{i\chi^2}{2rh(r)^2}(r + \mu)^2 \bar{z}^2, \tag{3.14}$$

or

$$P_i = \partial_i L(r), \qquad L(r) = \int \frac{i\chi^2}{2Vr}(r + \mu)^2 \, dr \tag{3.15}$$

for the holomorphic components of the one-form P. Using this as well as $\Theta = 0$, one can solve Eq. (2.9) to obtain λ^2 and λ^3 up to an arbitrary antiholomorphic function $\mathcal{F}(\bar{z}^1, \bar{z}^2)$, which we will take to be constant. One arrives then finally at the BPS solution

$$\begin{aligned} ds^2 &= f^2(dt + \omega)^2 - f^{-1}(V^{-1}dr^2 + V(\sigma_3)^2 + r[(\sigma_1)^2 + (\sigma_2)^2]), \\ A &= \frac{\sqrt{3}}{2} f[dt - \mathcal{F}_1 h^2(r)\sigma_1 + \mathcal{F}_2 h^2(r)\sigma_2], \end{aligned} \tag{3.16}$$

with

$$\omega = \frac{\chi r}{\sqrt{3}} f^{-3}\sigma_3 - \mathcal{F}_1 h^2(r)\sigma_1 + \mathcal{F}_2 h^2(r)\sigma_2\,, \tag{3.17}$$

where $\mathcal{F}_1$ and $\mathcal{F}_2$ are arbitrary constants related to the real and imaginary part of the antiholomorphic function $\mathcal{F}$. The one-forms σ_i and $h(r)$ are given by (3.4) and (3.13) respectively and for $\mu = 0$, (3.16) reduces to the solution found in [13]. For $\mathcal{F}_1 = \mathcal{F}_2 = 0$, we recover the hyperbolic black holes of [23].

For $k = 0$, Σ is flat and as in [24], we choose as complex coordinates

$$\zeta = \frac{1}{2}(\theta - i\psi)\,, \qquad S = -\int \frac{dr}{V(r)} - i\phi + \frac{1}{4}(\theta^2 + \psi^2)\,, \tag{3.18}$$

in terms of which the base space metric reads

$$h_{mn}dx^m dx^n = V(dS - 2\bar{\zeta}\,d\zeta)(d\bar{S} - 2\zeta\,d\bar{\zeta}) + 4r\,d\zeta\,d\bar{\zeta}\,. \tag{3.19}$$

This yields

$$P_S = -\frac{i\chi^2}{4r}(r+\mu)^2\,, \qquad P_\zeta = \frac{i\chi^2}{2r}(r+\mu)^2\bar{\zeta}\,, \tag{3.20}$$

which implies again (3.15). Proceeding like in the cases $k = \pm 1$, we get then the supersymmetric solution

$$\begin{aligned} ds^2 &= f^2(dt+\omega)^2 - f^{-1}(V^{-1}dr^2 + V(\sigma_3)^2 + r[(\sigma_1)^2 + (\sigma_2)^2])\,, \\ A &= \frac{\sqrt{3}}{2} f[dt - \mathcal{F}_1(-h^2(r)\,d\psi + \phi\,d\theta + \frac{1}{4}\theta^2\,d\psi) \\ &\qquad + \mathcal{F}_2(h^2(r)\,d\theta + \phi\,d\psi - \frac{1}{4}\psi^2\,d\theta)]\,, \end{aligned} \tag{3.21}$$

with

$$\begin{aligned} \omega &= \frac{\chi r}{\sqrt{3}} f^{-3}\sigma_3 - \mathcal{F}_1(-h^2(r)\,d\psi + \phi\,d\theta + \frac{1}{4}\theta^2\,d\psi) \\ &\qquad + \mathcal{F}_2(h^2(r)\,d\theta + \phi\,d\psi - \frac{1}{4}\psi^2\,d\theta)\,, \end{aligned}$$

where again $\mathcal{F}_1$, $\mathcal{F}_2$ are constants,

$$h^2(r) = \int \frac{dr}{V(r)}\,,$$

and the one-forms σ_i are given by (3.4). For $\mu = 0$, (3.21) reduces to the solution found in [13] whereas for $\mathcal{F}_1 = \mathcal{F}_2 = 0$, we obtain the supersymmetric black holes of [23]. It is interesting to note that these black holes were recovered in [2] by taking a different base manifold. This means that different base geometries can lead to the same BPS solution of gauged supergravity.

As a final choice, which includes both (3.9) and the supersymmetric AdS_5 black holes obtained recently in [17], we take $k = 1$, $n = 1$ and

$$V(r) = a_2 r^2 + a_1 r + a_0 + \frac{a_{-1}}{r}\,, \tag{3.22}$$

which behaves for large r as the Bergmann metric and for small r as the black hole discussed before. But depending on the parameters there can be a horizon for some finite r; see below. As the only restriction on the parameters, we impose $\Theta = 0$ yielding

$$3a_{-1}(a_1 - 1) = a_0^2\,, \tag{3.23}$$

and hence the base space is now parameterized by three parameters. The scalar curvature of the base space becomes then

$$R = -2\left(3a_2 + \frac{a_1 - 1}{r}\right)\,, \tag{3.24}$$

which implies

$$f^{-1} = \frac{3a_2}{\chi^2} + \frac{a_1 - 1}{\chi^2 r}\,. \tag{3.25}$$

In order to solve (2.9), we introduce complex coordinates as in (2.16), with $h(r)$ given by

$$h(r) = \exp \int \frac{dr}{2V(r)}\,. \tag{3.26}$$

This leads to

$$P_{z^1} = -\frac{i}{h^2}\left(1 - \frac{V'}{2} - \frac{V}{2r}\right)\bar{z}^1\,, \qquad P_{z^2} = -\frac{i}{h^2}\left(1 - \frac{V'}{2} - \frac{V}{2r}\right)\bar{z}^2\,, \tag{3.27}$$

or

$$P_i = \partial_i L(r)\,, \qquad L(r) = -i\int \left(1 - \frac{V'}{2} - \frac{V}{2r}\right)\frac{dr}{V} \tag{3.28}$$

for the holomorphic components of the one-form P. Using this as well as $\Theta = 0$, one can again solve Eq. (2.9) to obtain λ^2 and λ^3 up to an arbitrary antiholomorphic function $\mathcal{F}(\bar{z}^1, \bar{z}^2)$, which we take as usual to be constant. One obtains then the supersymmetric solution

$$\begin{aligned}
ds^2 &= f^2(dt+\omega)^2 - f^{-1}(V^{-1}dr^2 + V(\sigma_3^L)^2 + r[(\sigma_1^L)^2 + (\sigma_2^L)^2])\,, \\
A &= \frac{\sqrt{3}}{2} f[dt - \mathcal{F}_1 h^2(r)\sigma_1^L + \mathcal{F}_2 h^2(r)\sigma_2^L] \\
&\quad + \frac{3a_0a_2 - (a_1-1)^2}{4\chi^3 r} f\sigma_3^L\,,
\end{aligned} \tag{3.29}$$

with

$$\begin{aligned}
\omega &= \frac{3r(a_1-1)^2 + (18a_2r^2 + 2a_0)(a_1-1) + 18r^3a_2^2 + 9a_0a_2r}{2\sqrt{3}r^2\chi^3}\sigma_3^L \\
&\quad - \mathcal{F}_1 h^2(r)\sigma_1^L + \mathcal{F}_2 h^2(r)\sigma_2^L\,.
\end{aligned}$$

Note that by rescaling

$$t \to \gamma^{-1}t\,, \quad r \to \gamma r\,, \quad a_0 \to \gamma a_0\,, \quad a_2 \to \gamma^{-1}a_2\,, \quad \mathcal{F}_{1,2} \to \gamma^{-1}\mathcal{F}_{1,2}\,,$$

we can set $a_2 = \chi^2/3$.

We recover our solution (2.24) if $a_{-1} = \mu^3\chi^2/3$, $a_0 = \mu^2\chi^2$, $a_1 = 1 + \mu\chi^2$, whereas the choice $a_{-1} = a_0 = 0$, $a_1 = 4a^2$, $\mathcal{F}_{1,2} = 0$ yields the rotating supersymmetric black holes with regular event horizon obtained recently in [17]. (The rotation parameter a corresponds to their α; the radial coordinate ρ used in [17] is related to r by $r = 12a^2\chi^{-2}\sinh^2\frac{\chi\rho}{2\sqrt{3}}$.) As before, the Gödel deformation for this black hole corresponds to non-vanishing values of $\mathcal{F}_{1,2}$, which gives

$$\begin{aligned}
ds^2 &= f^2(dt+\omega)^2 - f^{-1}\Big(V^{-1}dr^2 + V(\sigma_3^L)^2 + r\big[(\sigma_1^L)^2 + (\sigma_2^L)^2\big]\Big)\,, \\
A &= \frac{\sqrt{3}}{2} f\Big[dt + \Big(1 + \frac{12a^2}{\chi^2 r}\Big)^{-\frac{1}{4a^2}}\Big(-\mathcal{F}_1\sigma_1^L + \mathcal{F}_2\sigma_2^L\Big)\Big] - \frac{(4a^2-1)^2}{4\chi^3 r} f\sigma_3^L\,,
\end{aligned}$$

with

$$\omega = \frac{3(4a^2-1)^2 + 6\chi^2 r(4a^2-1) + 2r^2\chi^4}{2\sqrt{3}r\,\chi^3}\sigma_3^L + \Big(1 + \frac{12a^2}{\chi^2 r}\Big)^{-\frac{1}{4a^2}}\Big(-\mathcal{F}_1\sigma_1^L + \mathcal{F}_2\sigma_2^L\Big)\,,$$

$$V = \frac{\chi^2}{3}r^2 + 4a^2 r\,, \qquad f^{-1} = 1 + \frac{4a^2-1}{\chi^2 r}\,.$$

Generically this solution contains CTCs. This follows from the fact that asymptotically for $r \to \infty$ it approaches the Gödel-type deformation of AdS_5 studied in [13]. One can show (by expanding $g_{\psi\psi}$ for $r \to \infty$) that e. g. ∂_ψ can become timelike (at least for large r), provided

$$\mathcal{F}_1^2 + \mathcal{F}_2^2 > \frac{3}{\chi^2}\,. \tag{3.30}$$

It would be nice to see whether the spacetime contains no CTCs at all if $\mathcal{F}_1^2 + \mathcal{F}_2^2$ lies below this bound (as is the case for $a^2 = 1/4$ [13]). We will not attempt to do this here. Note that $r = 0$ is a Killing horizon of the Killing vector $\xi = \partial_t$. It is straightforward to show that the surface gravity

$$\kappa^2 = -\frac{1}{2}\nabla^\mu \xi^\nu \nabla_\mu \xi_\nu|_{\text{Hor.}}$$

vanishes, as it must be for supersymmetric black holes. The isometry group $\mathbb{R} \times U(1)_L \times SU(2)_R$ of the spacetime with $\mathcal{F}_{1,2} = 0$ [17] is broken down to $\mathbb{R} \times SU(2)_R$ by the Gödel deformation, i. e. , by nonvanishing $\mathcal{F}_1$ or $\mathcal{F}_2$. The $SU(2)_R$ is generated by the right vector fields

$$\begin{aligned}
\xi_1^R &= -\sin\psi\partial_\theta - \cot\theta\cos\psi\partial_\psi + \frac{\cos\psi}{\sin\theta}\partial_\phi\,, \\
\xi_2^R &= \cos\psi\partial_\theta - \cot\theta\sin\psi\partial_\psi + \frac{\sin\psi}{\sin\theta}\partial_\phi\,, \\
\xi_3^R &= \partial_\psi\,.
\end{aligned}$$

For $\mathcal{F}_{1,2} = 0$, one has an additional left Killing vector ∂_ϕ corresponding to $U(1)_L$.

The general solution (3.29) might have event horizons at $r = r_H > 0$, if $V(r_H) = 0$. These would not be Killing horizons of ∂_t, but probably of a linear combination $\partial_t + \Omega^i \xi_i^R$ for some suitably chosen constants Ω^i. It is straightforward to show that in this case the metric on the horizon would not be Euclidean, or, in other words, CTCs would develop outside the horizon. We leave a more detailed analysis of the general solution for the future.

Using our base manifold (3.1), constructing generalizations of the black holes of [17] to the case of flat ($k = 0$) or hyperbolic ($k = -1$) horizons should be straightforward. It would also be interesting to obtain the most general function $V(r)$ in the line bundle (3.1) that has $\Theta = 0$. Unfortunately this condition leads to a rather complicated fifth order differential equation for $V(r)$ that we were not able to solve in general. Another open question is whether the rotating black holes of [25] (which also have CTCs [26]) can be described by using the base (3.1), and if so, whether they still have $\Theta = 0$.

4 The Gödel-de Sitter universe

We close this paper by presenting a generalization of the Gödel-type universe of [1] to the case $\Lambda < 0$ (which, with our signature, corresponds to de Sitter). As it has been observed in [27, 18], one can embed asymptotically flat *supersymmetric* black holes into an asymptotically de Sitter space by adding an appropriate linear function in time to the harmonic function**. This breaks of course supersymmetry. It can also be done for the case at hand, and we find for the metric and gauge field (that solve the equations of motion),

$$
\begin{aligned}
ds^2 &= f^2(dt+\omega)^2 - f^{-1}(dr^2 + \frac{r^2}{4}[(\sigma_1^L)^2 + (\sigma_2^L)^2 + (\sigma_3^L)^2]) \,, \\
A &= \frac{\sqrt{3}}{2} f[dt - \mathcal{F}_1 r^2 \sigma_1^L + \mathcal{F}_2 r^2 \sigma_2^L] \,, \\
\omega &= -\mathcal{F}_1 r^2 \sigma_1^L + \mathcal{F}_2 r^2 \sigma_2^L \,, \\
f^{-1} &= \sqrt{\frac{-2\Lambda}{3}}\, t + \mathcal{H} \,,
\end{aligned} \tag{4.1}
$$

where $\mathcal{H}$ denotes an arbitrary harmonic function on the *flat* base manifold. For $\Lambda = 0$, it reduces to a solution found in [1] with the Gödel universe corresponding to $\mathcal{H} = 1$. Unlike the AdS case, for $t \to \infty$ the geometry (4.1) does approach de Sitter space and closed timelike curves occur whenever

$$
4f^3 r^2 (\mathcal{F}_1^2 + \mathcal{F}_2^2) > 1 \,. \tag{4.2}
$$

Since f is a time- and radial-dependent function, this relation defines a shell which moves through spacetime with increasing velocity towards larger values of r. In fact, as f goes to zero for $t \to \infty$, this means that on any slice given by $dt = 0$ the CTCs disappear for sufficiently large t.

**Or multiply the radial coordinate by an appropriate time exponential.

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