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First upper limit analysis and results from LIGO science data: stochastic background

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Abstract
I describe analysis of correlations in the outputs of the three LIGO interferometers from LIGO's first science run, held over 17 days in August and September of 2002, and the resulting upper limit set on a stochastic background of gravitational waves. By searching for cross-correlations between the LIGO detectors in Livingston, LA and Hanford, WA, we are able to set a 90% confidence level upper limit of $h_{100}^2\Omega_0 < 23 \pm 4.6$.

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1. Introduction

The LIGO interferometric gravitational wave (GW) detector held its first science run (S1) in 2002, from 23 August to 9 September [1]. LIGO consists of an interferometer (IFO) with 4 km arms in Livingston, LA, USA (the LIGO Livingston Observatory, or LLO), called L1 for short, and two IFOs, with arms of 4 km and 2 km, in Hanford, WA, USA (the LIGO Hanford Observatory, or LHO), called H1 and H2, respectively. The data were analysed to search for GW bursts [2], signals from inspiralling neutron star binaries [3], periodic signals from a rotating neutron star [4] and stochastic backgrounds [5]. This paper summarizes the analysis method and results of the search for a stochastic background of gravitational waves (SBGW), which are explained in more detail in [5].

2. Fundamentals of analysis method

A SBGW is assumed for simplicity to be isotropic, unpolarized, Gaussian and stationary. Subject to these assumptions, the SBGW is completely described by its power spectrum. It

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[1] The members of the LIGO Scientific Collaboration are listed in the paper by Allen and Woan for the LIGO Scientific Collaboration, in these proceedings: stacks.iop.org/CQG/21/S671.

is conventional to express this spectrum in terms of the GW contribution to the cosmological parameter $\Omega = \rho/\rho_{\mathrm{crit}}$:

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{crit}}}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln f} = \frac{f}{\rho_{\mathrm{crit}}}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}f}. \tag{1}$$

Note that $\Omega_{\mathrm{GW}}(f)$ has been constructed to be dimensionless, and represents the contribution to the overall Ω_{GW} per *logarithmic* frequency interval. In particular, it is *not* equivalent to $\mathrm{d}\Omega_{\mathrm{GW}}/\mathrm{d}f$. Note also that since the critical density ρ_{crit}, which is used in the normalization of $\Omega_{\mathrm{GW}}(f)$, is proportional to the square of the Hubble constant H_0 [6], it is convenient to work with $h_{100}^2\Omega_{\mathrm{GW}}(f)$, which is independent of the observationally determined value of $h_{100} = \frac{H_0}{100\,\mathrm{km/s/Mpc}}$.[2]

The standard method of searching for such a background is to cross-correlate the outputs of two GW detectors [7]. If the noise in the two detectors is uncorrelated, the only non-zero contribution to the average cross-correlation (CC) will come from the SBGW. In the optimally filtered CC method (described in more detail in [8–10]), one calculates a CC statistic

$$Y = \int \mathrm{d}t_1\,\mathrm{d}t_2\, h_1(t_1)Q(t_1 - t_2)h_2(t_2) = \int \mathrm{d}f\, \tilde{h}_1^*(f)\tilde{Q}(f)\tilde{h}_2(f), \tag{2}$$

where $h_{1,2}(t)$ are the data streams from the two detectors, $\tilde{h}_{1,2}(f)$ are their Fourier transforms and $Q(t_1 - t_2)$ (with Fourier transform $\tilde{Q}(f)$) is a suitably chosen optimal filter. The choice which optimizes the signal-to-noise ratio for a constant-$\Omega_{\mathrm{GW}}(f)$ background is [9]

$$\tilde{Q}(f) \propto \frac{\gamma(f)}{f^3 P_1(f)P_2(f)}. \tag{3}$$

The normalization of the optimal filter is conventionally chosen so that in the presence of a SBGW of strength $\Omega_{\mathrm{GW}}(f) = \Omega_0$, the expected mean value of the CC statistic is

$$\langle Y\rangle = h_{100}^2\Omega_0 T, \tag{4}$$

where T is the duration of the analysed datasets. The expected variance of the CC statistic is

$$\sigma_{\mathrm{theor}}^2 = \frac{T}{4}\int \mathrm{d}f\, P_1(f)|\tilde{Q}(f)|^2 P_2(f) \propto \left(\int \frac{\mathrm{d}f}{f^6}\frac{[\gamma(f)]^2}{P_1(f)P_2(f)}\right)^{-1}. \tag{5}$$

The method is sensitive of backgrounds of the order of

$$\Omega^{\mathrm{UL}} \sim \left(T\int \mathrm{d}f\,\frac{[\gamma(f)]^2}{f^6 P_1(f)P_2(f)}\right)^{-1/2}. \tag{6}$$

The sensitivity of this method improves with time and is limited by the power spectral densities $P_{1,2}(f)$ of the noise in the two detectors. The factor

$$\gamma(f) = d_{1ab}d_2^{cd}\frac{5}{4\pi}\int_{S^2}\mathrm{d}^2\Omega\,\exp(\mathrm{i}2\pi f\mathbf{n}\cdot\mathbf{\Delta x}/c)P^{ab}_{cd}(\mathbf{n}), \tag{7}$$

in the numerator of the integral is the *overlap reduction function* [11], which describes the observing geometry. Here $P^{ab}_{cd}(\mathbf{n})$ is a projector onto symmetric traceless tensors transverse to a direction $\mathbf{n}$ and $d_{1,2}^{ab}$ are the *detector response tensors* for the two detectors. These are the tensors with which the metric perturbation h_{ab} at the detector should be contracted to obtain the *GW strain* $h = d^{ab}h_{ab}$. If u_a and v_a are unit vectors pointing in the directions of an IFO's two arms, its response tensor is

$$d_{ab} = \tfrac{1}{2}(u_a u_b - v_a v_b). \tag{8}$$

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[2] Although h_{100} is now much more accurately known than it once was, we still work in terms of $h_{100}^2\Omega_{\mathrm{GW}}(f)$ to facilitate comparisons with prior results.

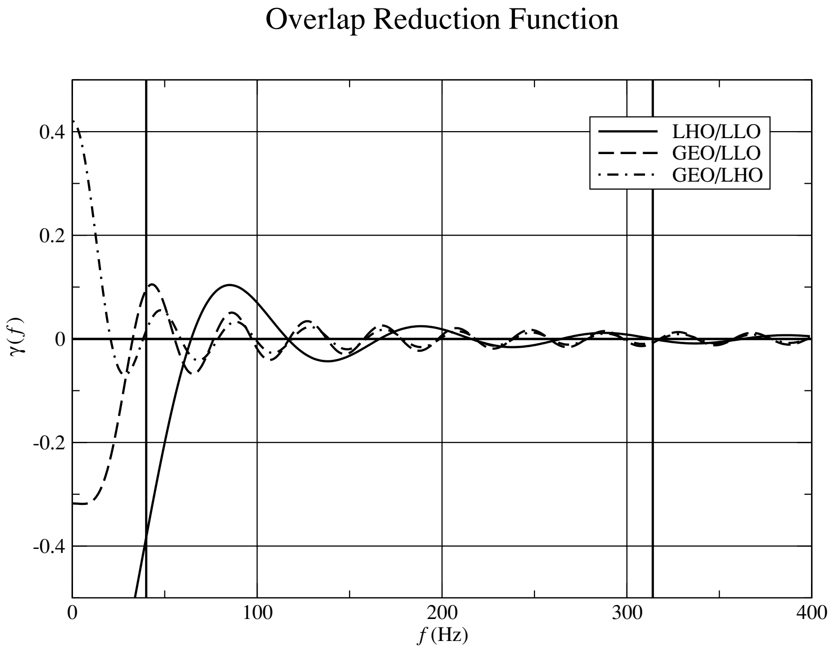

Figure 1. The overlap reduction function $\gamma(f)$ for combinations of the LIGO Livingston Observatory (LLO) and LIGO Hanford Observatory (LHO) with each other and with the GEO600 site. (GEO600 was also operational during S1, but has not been included in this analysis because it was considerably less sensitive than the LIGO detectors.) Note that the overlap reduction function for correlations between the two detectors at LHO (H1 and H2) is identically equal to unity. The solid lines (at $f = 40$ Hz and 314 Hz) show the range of frequencies used in our analysis.

The overlap reduction function is equal to unity for the case of a pair of IFOs at the same location with their arms aligned, and is suppressed as the detectors are rotated out of alignment or separated from one another. It also oscillates with frequency as correlations are suppressed for detectors whose separation is comparable to or greater than the corresponding GW wavelength. Figure 1 shows the overlap reduction functions for combinations of detectors which were operational during S1.

3. Prior results

The previous best upper limit on a SBGW from direct observation with GW detectors was $h_{100}^2\Omega_{\rm GW}(900\,{\rm Hz}) \leqslant 60$ [12], set by correlating the resonant bar detectors Explorer (in Geneva, Switzerland) and Nautilus (near Rome, Italy). A broad-band limit of $h_{100}^2\Omega_{\rm GW}(f) \leqslant 3 \times 10^5$ was set using a pair of 'prototype' IFOs [13].

More stringent upper limits can be set on astrophysical grounds. They are detailed elsewhere [5, 8, 14], but we mention the bound from big-bang nucleosynthesis [6, 14], which states that a cosmological SBGW is limited by

$$\int_{10^{-8}\,{\rm Hz}}^{\infty} \frac{{\rm d}f}{f} h_{100}^2\Omega_{\rm GW}(f) \leqslant 10^{-5}. \tag{9}$$

This broad-band limit implies that any cosmologically interesting SBGW very likely lies several orders of magnitude below the existing limits.

4. Details of analysis method

Each of the three combinations of detectors (H1–L1, H2–L1 and H1–H2) was analysed separately for CCs. Since the power spectra $P_{1,2}(f)$ varied over the course of the S1 run, the coincident data for each pair of IFOs were divided into 15 min blocks, and an optimal filter constructed for each such block[3]. To maximize the overall signal-to-noise ratio [9], we combined the CC statistics from the different blocks using a weighting factor of $\sigma_{\mathrm{theor}}^{-2}$, where σ_{theor} is the theoretical standard deviation defined in (5). Note that this can be calculated from the individual power spectra, without cross-correlating the data. To avoid problems from noisy and presumably less Gaussian data, we discarded the 15 min blocks with the highest σ_{theor} values, corresponding to a 5% total contribution to $\sum \sigma_{\mathrm{theor}}^{-2}$.

For each block, an optimal filter was constructed with a frequency resolution of 0.25 Hz according to the discrete frequency-domain analogue of (3). The range of frequencies included in the calculation of the CC statistic was chosen to be 40–314 Hz for H1–L1 and H2–L1, and 40–300 Hz for H1–H2. Given the power spectra of the instruments and the expected spectrum of correlations associated with a constant $\Omega_{\mathrm{GW}}(f)$, frequencies outside that range were not expected to improve the sensitivity appreciably. Additionally, individual frequency bins associated with cross-correlated instrumental noise were omitted from the sum over frequencies (which means the optimal filter was effectively set to zero there). These were integer multiples of 16 Hz and 60 Hz, as well as a few frequencies which had a coherence over the entire run above 0.2, namely 250 Hz for L1–H2, and 168.25 Hz and 168.5 Hz for H1–H2.

Each 15 min block was divided into ten 90 s segments using a Tukey window which consisted of half second Hann transitions on either side of an 89 s flat top. The CC statistic was calculated for each segment, using the discrete analogue of the frequency-domain form of (2), and these were combined to give a CC statistic for the entire block. In this way, we were able to obtain, through the measured standard deviation of the ten CC statistics within a block, a statistical measure of the error associated with the CC statistic for the block. We also estimated the systematic error associated with the change in sensitivity and calibration over the course of each block[4]. We then combined all three, appropriately weighted over the whole run, to obtain a total estimated error $\widehat{\sigma}_{\mathrm{tot}}$ associated with the point estimate $h_{100}^2\widehat{\Omega}_0$ calculated from the weighted average of all the CC statistics using (4). In the absence of cross-correlated noise, the 90% confidence level upper limit on Ω_0, the constant value of $\Omega_{\mathrm{GW}}(f)$, is

$$h_{100}^2\Omega_0 \leqslant h_{100}^2\widehat{\Omega}_0 + 1.28\widehat{\sigma}_{\mathrm{tot}}. \tag{10}$$

5. Results

The results for the three IFO pairs are summarized in table 1. There is a statistically significant anti-correlation observed between H1 and H2, two IFOs which share the same vacuum envelope at the LHO site in Hanford, WA. Time-shift and χ^2 analyses show that this anti-correlation is inconsistent with a constant $\Omega_{\mathrm{GW}}(f)$ SBGW, so we conclude it is due to instrumental cross-correlations between the two colocated detectors[5]. For the inter-site

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[3] The power spectra were constructed using Welch's method, with 449 overlapping Hann-windowed periodograms averaged to produce a power spectrum estimate with a resolution of 0.25 Hz.

[4] The calibration of the LIGO detectors was monitored by tracking the amplitude of a sinusoidal 'calibration line' signal injected into the IFO arm length. The output amplitude, recorded once per minute, allows construction of a frequency-dependent response function which accounts for IFO alignment drifts, as detailed in [1, 15].

[5] Of course, the fact that it is an anti-correlation rather than a correlation is another reason it cannot be due to a SBGW.

Table 1. Summary of the point estimate $h_{100}^2\widehat{\Omega}_0$ and total estimated error $\widehat{\sigma}_{\rm tot}$ for the three IFO pairs considered. Note that no upper limit is set from H1–H2, the two IFOs at the Hanford site, since there was evidence of cross-correlated noise. Associated with each of the values quoted is an additional 20% uncertainty arising from the calibration of the instruments.

IFO pair	Obs time (h:min)	$h_{100}^2\widehat{\Omega}_0$	$\widehat{\sigma}_{\rm tot}$	90% CL UL
H2–L1	51:15	0.2	18	23
H1–L1	64:00	32	18	55
H1–H2	100:15	−8.3	0.9	N/A

measurements (H1–L1 and H2–L1), the lack of statistically significant cross-correlations makes these checks trivial, and we proceed to set an upper limit from each pair[6]. The stronger upper limit is set by H2 and L1, and it is

$$h_{100}^2\Omega_0 \leqslant 23 \pm 4.6. \tag{11}$$

This represents a factor of 2–3 improvement over the previous direct upper limits described in section 3, and an improvement by a factor of over 1000 over the previous measurements with interferometric detectors.

For more details on the analysis, the reader is directed to [5].

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References

[1] Abbot B *et al* (LIGO Scientific Collaboration) 2003 Detector description and performance for the first coincidence observations between LIGO and GEO *Preprint* gr-qc/0308043 (*Nucl. Instrum. Methods* at press)

[2] Abbot B *et al* (LIGO Scientific Collaboration) 2003 First upper limits from LIGO on gravitational wave bursts *Preprint* gr-qc/0312056

[3] Abbot B *et al* (LIGO Scientific Collaboration) 2003 Analysis of LIGO data for gravitational waves from binary neutron stars *Preprint* gr-qc/0308069 (*Phys. Rev.* D submitted)

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[6] One might try to combine the different measurements into a single limit, but we choose to consider them individually, especially given the observed correlations between the two Hanford IFOs, which complicate the issue of combining the H1–L1 and H2–L1 as supposedly independent measurements.

[4] Abbot B *et al* (LIGO Scientific Collaboration) 2004 *Phys. Rev.* D **64** at press (*Preprint* gr-qc/0308050)

[5] Abbot B *et al* (LIGO Scientific Collaboration) 2003 Analysis of first LIGO science data for stochastic gravitational waves *Preprint* gr-qc/0312088 (*Phys. Rev.* D to be submitted)

[6] Kolb E W and Turner M S 1990 *The Early Universe* (Reading, MA: Addison-Wesley)

[7] Christensen N 1992 *Phys. Rev.* D **46** 5250

[8] Allen B 1997 *Proc. Les Houches School on Astrophysical Sources of Gravitational Waves* ed J A Marck and J P Lasota (Cambridge: Cambridge University Press) p 373
Allen B 1996 *Preprint* gr-qc/9604033

[9] Allen B and Romano J D 1999 *Phys. Rev.* D **59** 102001
Allen B and Romano J D 1997 *Preprint* gr-qc/9710117

[10] Whelan J T *et al* 2001 *Class. Quantum Grav.* **19** 1521
Whelan J T *et al* 2001 *Preprint* gr-qc/0110019

[11] Flanagan É É 1993 *Phys. Rev.* D **48** 2389
Flanagan É É 1993 *Preprint* astro-ph/9305029

[12] Astone P *et al* 1999 *Astron. Astrophys.* **351** 811
Astone P *et al* 1998 *Proc. 2nd Edoardo Amaldi Conf. on Gravitational Waves* ed E Coccia, G Pizzella and G Veneziano (Singapore: World Scientific) p 192

[13] Compton C, Nicholson D and Schutz B F 1994 *Proc. 7th Marcel Grossman Meeting on General Relativity* ed R T Jantzen and G M Keiser (Singapore: World Scientific) p 1078

[14] Maggiore M 2000 *Phys. Rep.* **331** 28
Maggiore M 1999 *Preprint* gr-qc/9909001

[15] Adhikari R, González G, Landry M and O'Reilly B (LIGO Scientific Collaboration) 2003 *Class. Quantum Grav.* **20** S903