

On Scherk-Schwarz mechanism in gauged five-dimensional supergravity and on its relation to bigravity

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Abstract

We demonstrate the relation between the Scherk-Schwarz mechanism and flipped gauged brane-bulk supergravities in five dimensions. We discuss the form of supersymmetry violating Scherk-Schwarz terms in pure supergravity and in supergravity coupled to matter. We point out that brane-induced supersymmetry breakdown in 5d Horava-Witten model is not of the Scherk-Schwarz type. We discuss in detail flipped super-bigravity, which is the locally supersymmetric extension of the $(++)$ bigravity.

1 Introduction

The issue of hierarchical supersymmetry breakdown in supersymmetric brane worlds is one of the central issues in the quest for a phenomenologically viable extra-dimensional extension of the Standard Model. Many attempts towards formulating scenarios of supersymmetry breakdown that use new features offered by extra-dimensional setup have been made [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12],[13]. One of them is supersymmetry breakdown triggered by imposing nontrivial boundary conditions on field configurations along the compact transverse dimensions usually referred to as the Scherk-Schwarz mechanism. In the work published so far the investigation concentrated on flat Minkowski-type geometries of the branes and bulk, neglecting the backreaction of the gauge sectors on the gravitational background (in particular assuming a fixed radii of the extra dimensions), and in general the interactions of various fields with (super)gravity (but see [14], [10]). On the other hand, it is precisely partial ‘unification’ of the Standard Model with gravity that makes the Brane World scenarios so intriguing and appealing. In this note we would like to clarify the status of the Scherk-Schwarz approach to supersymmetry breaking in the nontrivial gravity backgrounds using the class of simple warped gauged supergravities with flipped boundary conditions described in [15].

In particular, we find out that the simple flipped supergravity forms the locally supersymmetric extension of the $(++)$ bigravity model of Kogan et. al. [16],[17],[18]. In such a setup one circumvents the van Dam-Veltman-Zakharov observation about the nondecoupling of the additional polarization states of the massive graviton [19],[20],[21]. The size of the residual four-dimensional cosmological constant can be tuned to arbitrarily small values by taking the distance between branes suitably large. We discuss the mass spectrum of gravitons and gravitini in the resulting super-bigravity.

To begin with let us notice that the brane-world version of the Scherk-Schwarz mechanism contains a number of ingredients. Assuming for simplicity that the extra dimension is just a circle S^1 , and that the higher-dimensional theory has a group G of global symmetries, one can impose on various fields periodicity with a twist, that is instead of standard periodicity conditions $\psi(y + 2\pi\rho) = \psi(y)$ it is consistent to demand that after following the circle a configuration goes back to itself up to a symmetry transformation U_β : $\psi(y + 2\pi\rho) = U_\beta\psi(y)$. In particular, if the twist matrix is different for bosons and fermions, such boundary conditions lead to supersymmetry breakdown. If the extra dimension is an orbifold, say S^1/Γ where Γ is a discrete group, for instance Z_2 or $Z_2 \times Z'_2$, then Γ may be embedded nontrivially into the group of global and local symmetries of the model, and its combined geometrical and internal action has to be taken into account. Finally, the matter and gauge fields located on the branes have in general nontrivial interaction with bulk fields, and bulk fields must be allowed to have selfinteractions. In the context of a locally supersymmetric theory bulk selfinteractions imply that one has a gauged supergravity in the bulk, with a number of hypermultiplets and vector and tensor multiplets. The coupling of gravity and bulk matter to branes implies that branes act as nontrivial sources for bulk configurations, and modify the behaviour of fields at the fixed points. We shall try to determine how the Scherk-Schwarz mechanism works in the presence of all these ingredients and how does it affect stability of the extra dimensions.

2 Flipped and detuned supergravity in five dimensions

The simple N=2 d=5 supergravity multiplet contains metric tensor (represented by the vielbein e_α^m), two gravitini Ψ_α^A and one vector field A_α – the graviphoton. We shall consider gauging of a ($U(1)$) subgroup of the global $SU(2)_R$ symmetry of the 5d Lagrangian. In general, coupling of bulk fields to branes turns out to be related to the gauging, and the bulk-brane couplings will preserve only a subgroup of the $SU(2)_R$. Purely gravitational 5d action describing such a setup reads $S = \int_{M_5} e_5 \mathcal{L}_{grav}$, where

$$\begin{aligned} \mathcal{L}_{grav} = & \frac{1}{2}R - \frac{3}{4}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta} - \frac{1}{2\sqrt{2}}A_\alpha\mathcal{F}_{\beta\gamma}\mathcal{F}_{\delta\epsilon}\epsilon^{\alpha\beta\gamma\delta\epsilon} + \\ & -\frac{1}{2}\bar{\Psi}_\alpha^A\gamma^{\alpha\beta\gamma}D_\beta\Psi_{\gamma A} + \frac{3i}{8\sqrt{2}}\left(\bar{\Psi}_\gamma^A\gamma^{\alpha\beta\gamma\delta}\Psi_{\delta A} + 2\bar{\Psi}^{\alpha A}\Psi_\beta^B\right)\mathcal{F}_{\alpha\beta} + \\ & -\frac{i}{\sqrt{2}}\mathcal{P}_{AB}\bar{\Psi}_\alpha^A\gamma^{\alpha\beta}\Psi_\beta^B - \frac{8}{3}\text{Tr}(\mathcal{P}^2). \end{aligned} \quad (1)$$

Covariant derivative contains both gravitational and gauge connections:

$$D_\alpha\Psi_\beta^A = \nabla_\alpha\Psi_\beta^A + A_\alpha\mathcal{P}_B^A\Psi_\beta^B, \quad (2)$$

where ∇_α denotes covariant derivative with respect to gravitational transformations and $\mathcal{P} = \mathcal{P}_i i \sigma^i$ is the gauge prepotential. The pair of gravitini satisfies symplectic Majorana condition $\bar{\Psi}^A \equiv \Psi_A^\dagger \gamma_0 = (\epsilon^{AB}\Psi_B)^T C$ where C is the charge conjugation matrix and ϵ^{AB} is antisymmetric $SU(2)_R$ metric (we use $\epsilon_{12} = \epsilon^{12} = 1$ convention). Supersymmetry transformations are also modified by the gauging

$$\delta e_\alpha^m = \frac{1}{2}\bar{\epsilon}^A\gamma^m\Psi_{\alpha A}, \quad \delta A_\alpha = \frac{i}{2\sqrt{2}}\Psi_\alpha^A\epsilon_A, \quad (3)$$

$$\delta\Psi_\alpha^A = D_\alpha\epsilon^A - \frac{i}{6\sqrt{2}}(\gamma_\alpha^{\beta\gamma} - 4\delta_\alpha^\beta\gamma^\gamma)\mathcal{F}_{\beta\gamma}\epsilon^A + \frac{\sqrt{2}i}{3}\mathcal{P}^{AB}\gamma_\alpha\epsilon_B. \quad (4)$$

If one puts $\mathcal{P} = 0$ and stays on the circle, then as the twist matrix one may take any $SU(2)$ matrix acting on the symplectic indices $a = 1, 2$. On a circle the $U(1)$ prepotential takes the form $P = g_S s_a i \sigma^a$ and the twist matrix is $U_\beta = e^{i\beta s_a \sigma^a}$. However, in this case the unbroken symmetry is a local one, and the Scherk-Schwarz condition is equivalent to putting in a nontrivial Wilson line, [13], we shall come back to this issue later in the paper.

When one moves over to an orbifold S^1/Γ , one needs to define in addition to the gauging the action of the space group Γ on the fields. Let us take $\Gamma = Z_2$ first. Then we have two fixed points at $y = 0, \pi$, and we can define the action of Z_2 in terms of two independent boundary conditions (Ψ stands here for a doublet of symplectic-Majorana spinors or for a doublet of scalars, like two complex scalars from the hypermultiplet)

$$\Psi(-y) = \hat{Q}_0\Psi(y), \quad \Psi(\pi r_c - y) = \hat{Q}_\pi\Psi(\pi r_c + y), \quad (5)$$

where \hat{Q}_0, \hat{Q}_π are some arbitrary matrices, independent of the space-time coordinates, such that $\hat{Q}_0^2 = \hat{Q}_\pi^2 = 1$. Conditions (5) imply:

$$\Psi(y + 2\pi r_c) = \hat{Q}_\pi\hat{Q}_0\Psi(y). \quad (6)$$

Hence, if the boundary conditions at $y = 0$ and $y = \pi r_c$ are different, one obtains twisted boundary conditions with $U_\beta = \hat{Q}_\pi \hat{Q}_0$. It is easy to see that $U_\beta \hat{Q}_{0,\pi} U_\beta = \hat{Q}_{0,\pi}$, which is the consistency condition considered in [12],[22],[23]. This is immediately generalized to $S^1/(Z_2 \times Z'_2)$ with two fixed points for each of the Z_2 s, $y = 0, \frac{1}{2}\pi r_c, \pi r_c, \frac{3}{2}\pi r_c$, and independent \hat{Q}_y at each of the fixed points.

If one writes $\hat{Q}_\pi \hat{Q}_0 = \exp(i\beta_a \sigma^a)$, the condition (5) is solved by

$$\Psi = e^{i\beta_a \sigma^a f(y)} \hat{\Psi} , \quad (7)$$

where $\hat{\Psi}$ is periodic on the circle and $f(y)$ obeys the conditions

$$f(y + 2\pi r_c) = f(y) + 1 , \quad f(-y) = -f(y) . \quad (8)$$

When one expresses the initial fields Ψ through $\hat{\Psi}$, the kinetic term in the Lagrangian generates mass terms for *periodic* fields $\hat{\Psi}$:

$$\bar{\Psi} \gamma^M \partial_M \Psi \supset i f' \hat{\Psi} \gamma^5 \beta_a \sigma^a \hat{\Psi} . \quad (9)$$

2.1 Scherk-Schwarz mechanism in the $SU(2)$ R-symmetry of 5d gauged supergravity

Let us now move on to the specific case of a 5d supergravity with a gauged $U(1)$ subgroup of the $SU(2)$ R symmetry. The Z_2 action on the gravitino is defined as follows:

$$\begin{aligned} \Psi_\mu^A(-y) &= \gamma_5(Q_0)_B^A \Psi_\mu^B(y) , & \Psi_5^A(-y) &= -\gamma_5(Q_0)_B^A \Psi_5^B(y) , \\ \Psi_\mu^A(\pi r_c - y) &= \gamma_5(Q_\pi)_B^A \Psi_\mu^B(\pi r_c + y) , & \Psi_5^A(\pi r_c - y) &= -\gamma_5(Q_\pi)_B^A \Psi_5^B(\pi r_c + y) , \end{aligned} \quad (10)$$

and the parameters ϵ^A of the supersymmetry transformations obey the same boundary conditions as the 4d components of gravitini. Symplectic Majorana condition ($(Q_{0,\pi})^C = \sigma_2(Q_{0,\pi})^* \sigma_2 = -Q_{0,\pi}$) and normalization $(Q_{0,\pi})^2 = 1$ imply $Q_{0,\pi} = (q_{0,\pi})_a \sigma^a$, where $(q_{0,\pi})_a$ are real parameters [24]. We would like to gauge a $U(1)$ subgroup of the global $SU(2)$. In the general case [15] we can choose the prepotential of the form

$$P = g_R \epsilon(y) R + g_S S, \quad (11)$$

where $R = r_a i \sigma^a$ and $S = s_a i \sigma^a$. On an orbifold $S^1/(Z_2 \times Z'_2)$ the expression $\epsilon(y) R$ gets replaced by $\bar{R}(y)$ which is a piece-wise constant matrix with discontinuities (jumps) at the positions of the four branes. The basic relation between the boundary conditions and the prepotential comes from the requirement, that under supersymmetry variations the transformed gravitino $\Psi_\alpha^A + \delta \Psi_\alpha^A$ should obey the same boundary conditions as Ψ_α^A . Taking into account that the gauge field present in the supersymmetry transformation of the gravitini is that graviphoton, whose 4d part we choose to take Z_2 -odd with respect to each brane (we need only $N = 1$ supersymmetry on the branes), and the fifth component is always even, we obtain the relations valid for any segment containing a pair of neighbouring fixed points

$$[Q_{0,\pi}, R] = 0, \quad \{Q_{0,\pi}, S\} = 0. \quad (12)$$

For nonzero R this implies Q_y proportional to R , i.e. $Q_y = \alpha (i\sqrt{R^2})^{-1} R$ with $\alpha = \pm 1$. The simplest case of interest corresponds to $Q_0 = -Q_\pi$. As shown in [15], in this case the closure of supersymmetry transformations requires putting on the branes equal tensions whose magnitude is determined by R (we quote only the bosonic gravity part of the action):

$$M^{-3}S = \int d^5x \sqrt{-g_5} \left(\frac{1}{2}R + 6k^2 \right) - 6 \int d^5x \sqrt{-g_4} k T (\delta(x^5) + \delta(x^5 - \pi r_c)) \quad (13)$$

where

$$k = \sqrt{\frac{8}{9}(g_R^2 R^2 + g_S^2 S^2)} \text{ and } T = \frac{g_1 \sqrt{\bar{R}^2}}{\sqrt{(g_1^2 R^2 + g_2^2 S^2)}}. \quad (14)$$

This is easily generalized. If on a $S^1/(\Pi Z_2)$ one takes boundary conditions given by pairs of Q and $-Q$ one after another, then this implies that all branes on S^1/Z_2 , $S^1/(Z_2 \times Z'_2)$, $S^1/(\Pi Z_2)$ have the same brane tension. Assuming also $S \neq 0$ such a system gives a static vacuum with AdS_4 foliation and fixed radius of the orbifold¹. In the case of Z_2 the overall twist matrix is given by $U_\beta = -1$ and in the case of $Z_2 \times Z'_2$ there is no overall twist: $U_\beta = +1$. This may be generalized again. From the analysis of [15] it follows that if in the boundary conditions Q is followed by $+Q$ (and not $-Q$) on the next brane, then the brane tension on the second brane must be equal in magnitude but of opposite sign to that on the first brane. Together with the previous findings this leads to quasi-quiver diagrams where branes with brane tensions $\pm\lambda$ and boundary conditions $(\pm Q), (\pm Q) \dots$ follow each other respecting ΠZ_2 symmetry. At first glance possible strings of boundary conditions could be for instance $(Q), (Q), (-Q), (-Q)$ or $(Q), (-Q), (-Q), (Q), (-Q), (-Q)$. However, the stretches between $(Q), (Q)$ branes collapse to a point. This is easily seen from the fact, that to have a finite-length distance between $(-Q), (+Q)$ branes the brane tensions must be smaller than the critical value $\lambda < \lambda_{cr}$, equivalently $T < 1$, where the critical tension is that of the Randall–Sundrum brane. This implies, that the branes at the ends of a $(Q), (Q)$ segment have brane tensions of opposite sign and of the same magnitude smaller than the critical value. Solving boundary conditions on such a segment, with the maximally symmetric foliation ansatz, leads to the result $|y_{b1} - y_{b2}| = 0$, i.e. the branes coincide. Hence the possible static quasi-quivers correspond to the chains $(Q), (-Q), (Q), \dots, (-Q), (Q), (-Q)$. These are locally supersymmetric backgrounds corresponding to the models of the type discussed in [11].

Another extension of the picture is possible. Instead of cancellation of the supersymmetry variation of the brane tension by means of a ‘jumping’ prepotential in the bulk, one can consider, [25],[26], adding 4d gravitino mass terms on branes, and modifying the variations of the fifth component of the gravitino. Also in this case one can impose different boundary conditions on different walls. The general case of such an extension shall be discussed elsewhere, here we quote a simple example with $R = 0$ for illustration. In this case one has a brane action of the form

$$S_b = \int d^5x e_4 \delta(y - y_b) (-\lambda_0 + M_{AB} \bar{\Psi}_\mu^A \gamma^{\mu\nu} \Psi_\nu^B) \quad (15)$$

with gravitini components Ψ_ν^B satisfying local boundary conditions given by a matrix Q_{y_b} . The necessary modification of the transformation law is $\delta\Psi_\mu^A = 2\delta(y - y_b)\epsilon^{AC} M_{(CB)}\gamma_5\epsilon^B$ where the

¹For two equal and negative brane tensions the maximally symmetric solution doesn't exist.

hat denotes a flat space index and $(..)$ is the symmetrization with the weight $1/2$. Cancellation of susy variations requires that the brane tension and the matrix M_{AB} satisfy the equation

$$\lambda_0 \epsilon_{AB} + 4\sqrt{2}i (M_{(AC)}P_B^C + P_{(AC)}\epsilon^{CD}M_{(DB)}) = 0. \quad (16)$$

In the above suitable projections are made with the help of the projective operators $\Pi_{\pm} = 1/2(\delta_B^A \pm \gamma_5 Q_{y_b}^A)$, remembering that Ψ_5^A and Ψ_{μ}^B have opposite parities, see (10). In particular, the simple choice $\lambda_0 = M_{AB} = 0$ is sufficient to illustrate the way the twisting works. In this case the relevant condition on the operators Q_{y_b} is $\{Q_{y_b}, P\} = 0$.

Let us discuss the generation of the Scherk-Schwarz (nonsupersymmetric) mass terms in the two limiting cases. First, let us take $\Gamma = Z_2$ and Q_0 and Q_{π} orthogonal to each other. In this case one is free to take $Q_0 = \sigma_3$ and $Q_{\pi} = \sigma_1$. Then $Q_{\pi}Q_0 = -i\sigma_2$. Hence, the only possibility for the prepotential is $gP = ig\sigma_2$, because other directions do not commute or anticommute with both Q_0 and Q_{π} . In such case the only unbroken $U(1)$ subgroup is generated by the σ_2 direction of the global $SU(2)$ symmetry. Notice that we can write $Q_{\pi}Q_0 = -i\sigma_2 = \exp(i\beta_2\sigma^2)$, where $\beta_2 = \frac{3}{2}\pi + 2k\pi$ and $k \in Z$. Then the solution which satisfies the boundary conditions, expressed in terms of periodic fields, is:

$$\Psi = e^{i\beta_2\sigma^2 f(y)} \hat{\Psi}. \quad (17)$$

Hence in the action expressed in terms of periodic fields, one obtains supersymmetry violating mass terms:

$$-e_5 \frac{i}{2} \delta_{AB} \bar{\Psi}_{\mu}^A \gamma^{\mu\nu} \gamma^5 \Psi_{\nu}^B \beta_2 f'(y). \quad (18)$$

Let us take as an example the function $f = y/(2\pi r_c)$. This function leads to the mass term $-\frac{i}{2} \delta_{AB} \bar{\Psi}_{\mu}^A \gamma^{\mu\nu} \gamma^5 \Psi_{\nu}^B \frac{\beta_2}{2\pi r_c}$. Already here one can conclude that the bulk Lagrangian after redefinition cannot be put into the form compatible with linearly realized supersymmetry. To see this, one should note that the only mass terms compatible with supersymmetry are given by a prepotential. Supersymmetry requires, that the same prepotential determines the bulk scalar potential. In our case, we have redefined the gravitini only, hence the bulk potential term stays unchanged, independent of β_2 . At the same time any prepotential that should describe the Scherk-Schwarz mass terms shall depend on β_2 , hence the supersymmetric relation between mass terms and the scalar potential is necessarily violated. This is what we mean when we call the Scherk-Schwarz mass terms explicitly non-supersymmetric. On the other hand it is obvious, that the Scherk-Schwarz picture is equivalent indeed to a spontaneously broken flipped supergravity.

Let us now consider again the case where Q_0 and Q_{π} are parallel. Let us take for simplicity $Q_0 = \sigma_3$. Then $Q_{\pi} = \alpha\sigma_3$, where $\alpha = \pm 1$, and the twisted boundary conditions take the form:

$$\Psi(y + 2\pi r_c) = \alpha \Psi(y). \quad (19)$$

For $\alpha = 1$ we have usual case with periodic field. For $\alpha = -1$ we obtain ‘flipped’ supersymmetry of [15]. Let us take a nonzero S -part of (11). Assume the prepotential of the form

$$P = \frac{g}{\sqrt{2}} (\epsilon(y)\sigma_3 + \sigma_1). \quad (20)$$

For $\alpha = -1$ we can write:

$$Q_\pi Q_0 = -1 = e^{i\beta(\epsilon(y)\sigma_3 + \sigma_1)}, \quad (21)$$

where $\beta = \pi + 2k\pi$ and $k \in \mathbb{Z}$. Similarly to the case of the previous paragraph one obtains the following solution

$$\Psi = e^{i\beta(\epsilon(y)\sigma_3 + \sigma_1)f(y)} \hat{\Psi}, \quad (22)$$

and supersymmetry violating mass terms

$$-e_5 \frac{1}{2} \epsilon_{AB} \bar{\Psi}_\mu^A \gamma^{\mu\nu} \gamma^5 \left(\int_0^1 ds e^{is\beta(\epsilon(y)\sigma_3 + \sigma_1)} (i\beta(\epsilon f)' \sigma_3 + i\beta f' \sigma_1) e^{i(1-s)\beta(\epsilon(y)\sigma_3 + \sigma_1)} \right)_C^B \Psi_\nu^C. \quad (23)$$

Let us take again $f = y/(2\pi r_c)$. This gives

$$\begin{aligned} & -e_5 \frac{1}{2} \bar{\Psi}_\mu^A \gamma^{\mu\nu} \gamma^5 \frac{\beta}{2\pi r_c} (\sigma_{AB}^3 - \epsilon(y)\sigma_{AB}^1) \Psi_\nu^B - e_4 \delta(y - \pi r_c) \bar{\Psi}_\mu^A \gamma^{\mu\nu} \gamma^5 \sin(\beta/2) \sigma_{AB}^1 \Psi_\nu^B \\ & = -e_5 \frac{1}{2} \bar{\Psi}_\mu^A \gamma^{\mu\nu} \gamma^5 \frac{1}{2r_c} (\sigma_{AB}^3 - \epsilon(y)\sigma_{AB}^1) \Psi_\nu^B - e_4 \delta(y - \pi r_c) \bar{\Psi}_\mu^A \gamma^{\mu\nu} \gamma^5 \sigma_{AB}^1 \Psi_\nu^B. \end{aligned} \quad (24)$$

This example is more involved, but the same comments as in the previous case apply. The generalization to a quasi-quiver setup is obvious. One may notice, that only the bulk terms are proportional to the naive KK scale $1/r_c$. The scale of boundary terms is set by the 5d Planck scale.

Let us note already here, that even though the symmetry that we are using to implement the Scherk-Schwarz mechanism may be a local one, the Scherk-Schwarz masses cannot be removed, as one may naively think, by means of a gauge transformation. Such a transformation would have to be a ‘large’ one, leading from a periodic to an antiperiodic configuration. However, the definition of the model involves not only couplings in the Lagrangian but also the choice of specific boundary conditions. Hence such large gauge transformations connect two different (although physically equivalent) Hilbert spaces, and do not belong to the group of internal symmetries of our models.

We would like to note, that in a curved gravitational background different mass spectra for the ‘would be’ superpartners, like graviton and gravitino, is not an unambiguous sign of broken supersymmetry. In particular, in flipped supersymmetry models the background is of the AdS_4 -foliation form, and one knows [27],[28] that the AdS_4 supermultiplets contain in general particles with different mass terms. For instance, massive higher spin representations ($E_0 > s + 1$, $s \geq 1/2$) are of the form

$$D(E_0, s) \oplus D(E_0 + 1/2, s + 1/2) \oplus D(E_0 + 1/2, s - 1/2) \oplus D(E_0 + 1, s), \quad (25)$$

with the mass-squared operator $m^2 = E_0(E_0 - 3) - (s + 1)(s - 2)$. However, if one is able to obtain an approximate formula for a mass spectrum as a function of the quantization parameter, then one can compare its shape to the towers of supersymmetric masses. Since in any case the mass terms are certainly of phenomenological interest, we shall compute them and come back to the issue of seeing supersymmetry breaking through the spectrum later in the paper. The unambiguous sign of supersymmetry breakdown are nonzero vacuum values of the variations of fermions, or the absence of global Killing spinors, see [15].

2.2 Another picture of the Scherk-Schwarz mechanism in the presence of gauge symmetries

Let us consider again the 5d supergravity with $U(1)$ gauge symmetry and charged gravitino fields (i.e. $U(1)$ subgroup of $SU(2)_R$). The covariant derivative takes the form $D_M \Psi_N^A = \nabla_M \Psi_N^A + g A_M P_B^A \Psi_N^B$. In the case of unbroken supersymmetry all components of gauge fields should have vacuum expectation values equal to zero. To see this, one may have a look at the gravitino supersymmetry transformation, $\delta_\epsilon \Psi_M^A \supset \partial_M \epsilon^A + g A_M P_B^A \epsilon^B$. In the previous section we have assumed the expectation value of the gauge field to vanish, $\langle A_M \rangle = 0$, but the gravitino fields were chosen to satisfy twisted boundary conditions. On the other hand, one may try to eliminate twisted boundary conditions by a non-periodic (large) gauge transformation. Let us take for the sake of definiteness the first of the examples of the previous section, equations (17),(18), $P = ig\sigma^2$. The necessary gauge transformation is given by

$$\Psi_M \rightarrow \Psi'_M = e^{-i\beta_2 \sigma^2 f(y)} \Psi_M, \quad (26)$$

and obviously the primed gravitino field is periodic. However in such a case one has to transform the gauge field as well

$$\tilde{A}_5 \rightarrow \tilde{A}'_5 = \tilde{A}_5 + i\beta_2 \sigma^2 f'(y), \quad (27)$$

where the tilde denotes matrix gauge field. This implies in turn a non-zero vacuum expectation value of the transformed $U(1)$ gauge field $\langle A'_5 \rangle = 1/g f'(y) \beta_2$. At present the bulk non-supersymmetric gravitino masses arises in the theory as a result of a non-zero expectation value of the gauge part of the covariant derivative in the kinetic term. This is the Wilson line breaking, see [13]. Let us note that if one takes our standard choice $f(y) = y/(2\pi r_c)$, then the integrated Wilson line does not vanish, $\oint dx^5 \langle A_5 \rangle = i\beta_2 \sigma^2 \neq 0$, which is a sign that supersymmetry is broken globally.

3 The role of boundary couplings

The universal hypermultiplet $\{\zeta^a, q^i\}$ consists of a doublet of fermions and of four real scalars $q_i \in \{V, \sigma, x, y\}$ (we also use $\xi = x + iy$). Scalars parametrize a quaternionic manifold, whose global symmetry group is $Sp(2) \times SU(2)_R$. The kinetic term of the bosonic part reads:

$$\mathcal{L}_{kinetic} = -h_{ij} D_M q^i D^M q^j, \quad h_{ij} = V_i^{Aa} V_j^{Bb} \epsilon_{AB} \Omega_{ab}, \quad (28)$$

where h_{ij} is a quaternionic metric. We can write the quaternionic metric in the explicit form:

$$h_{ij} dq^i dq^j = \frac{1}{4V^2} dV^2 + \frac{1}{4V^2} [d\sigma + i(\bar{\xi} d\xi - \xi d\bar{\xi})]^2 + \frac{1}{V} d\xi d\bar{\xi}. \quad (29)$$

As explained in [29],[30], when some of the global symmetries are gauged, supersymmetry requires additional boundary terms involving bulk fields to be present in the model. This is the way the five-dimensional version of the Horava-Witten model, and brane-bulk supergravities of [29],[24] work. For simplicity let us consider again the Z_2 orbifold (generalizations follow

the same lines as in the previous chapters). In the case of flipped Horava-Witten and Randall-Sundrum models (with the hypermultiplet) the additional boundary terms in the action are

$$S_b = - \int d^5x e_4 6k \left(\theta - \frac{\alpha}{V} - \theta \frac{|\xi|^2}{V} \right) (\delta(y) + \delta(y - \pi r_c)), \quad (30)$$

where $\theta = 0$ gives the flipped Horava-Witten model, and $\alpha = 0$ corresponds to the flipped Randall-Sundrum case. These terms, like the gauging in the bulk, preserve only a $U(1)$ subgroup of the product of the $SU(2)_R$ and the symmetry group of the quaternionic manifold. Moreover, these boundary terms break explicitly the $N = 2$ bulk supersymmetry down to the local $N = 1$. It has been shown in [5] that if one would have at ones disposal the exact $SU(2)_R$ symmetry and embed the twist of the boundary conditions into this group to break $N = 2$ supersymmetry by the Scherk-Schwarz mechanism in the Horava-Witten model, and in addition project-out the Z_2 -odd mode, then in four dimensions this breaking would be seen as a spontaneous breaking of the effective $N = 1$ supergravity. However, the complete model contains also brane terms that break $N = 2$ explicitly. Second, as we have seen having a net twist of the boundary conditions requires opposite boundary conditions on each wall, and the same sign of boundary terms on both. This makes a physical difference with respect to the original Horava-Witten model, where the boundary terms had to have opposite signs. These observations imply in particular, that the five-dimensional picture of the Scherk-Schwarz supersymmetry breaking in Horava-Witten and Randall-Sundrum type models is physically different from the breaking by gaugino or superpotential (or flux) condensation in these models, see for instance [7],[29],[9].

To illustrate how the Scherk-Schwarz breaking would work in a model with a hypermultiplet, let us put $\theta = 0$. One can check that in this case the full action (including boundary terms) is invariant under the following transformation:

$$\zeta^a \rightarrow e^{-i\beta\sigma_3} \zeta^a, \quad \Psi^A \rightarrow e^{-i\beta\sigma_3} \Psi^A, \quad \xi \rightarrow e^{i2\beta} \xi, \quad (31)$$

where β is the transformation parameter and ζ^a is a hyperino. Now, one can impose associated boundary conditions:

$$\zeta^a(y + 2\pi r_c) = e^{-i\beta\sigma_3} \zeta^a(y), \quad \Psi^A(y + 2\pi r_c) = e^{-i\beta\sigma_3} \Psi^A(y), \quad \xi(y + 2\pi r_c) = e^{i2\beta} \xi(y). \quad (32)$$

It is obvious that fields

$$\zeta^a = e^{-i\beta\sigma_3 f(y)} \hat{\zeta}^a, \quad \Psi^A = e^{-i\beta\sigma_3 f(y)} \hat{\Psi}^A, \quad \xi = e^{i2\beta f(y)} \hat{\xi} \quad (33)$$

expressed in terms of new periodic (hatted) fields satisfy the conditions (32). As the consequence the Scherk-Schwarz non-supersymmetric mass terms are generated for hyperini

$$\frac{1}{2} e_4 (i\beta) \bar{\zeta}^a \gamma^{\hat{5}} \sigma_{ab}^1 \zeta^b f'(y), \quad (34)$$

gravitini, and for the complex scalars ξ . In fact, one generates also new quartic terms in the scalar potential, in addition to the supersymmetric scalar potential $V_{susy} = g^2 8/3 \text{tr}(P^2) + g^2/2 h_{ij} k^i k^j$, where here $k^\xi = 2i\xi$. Assuming vanishing expectation values of brane sources for

the scalar σ the terms in the non-supersymmetric part of the potential that do not contain $\partial_5 \xi$ are

$$V(\hat{\xi}) = \frac{\beta^2 (f')^2}{V^2} |\hat{\xi}|^2 (4V + 3|\hat{\xi}|^2), \quad (35)$$

hence the mass-squared parameter for the normalized scalar is $4\beta^2 (f')^2$. One can see that performing the Scherk-Schwarz redefinition of scalar fields, which may be identified for instance with the higgs-like field in the observable sector, one can create a complicated scalar potential. However, it will always contain the same physics as the locally supersymmetric lagrangian in the spontaneously broken phase, which is usually much simpler to analyse. The same comment concerns the fermionic sector. The Scherk-Schwarz masses for matter fermions superficially look like terms breaking supersymmetry in a hard way (like quartic terms in the potential), but the equivalence to the spontaneously broken phase guarantees cancellation of dangerous divergencies. The fact that the Scherk-Schwarz masses for chiral fermions do not belong to a linearly realized 5d supersymmetry may be seen from the observation, that supersymmetric masses are defined by the geometry of the quaternionic manifold and by the Killing vectors k^i , none of which had changed under the Scherk-Schwarz redefinition. To summarize, the redefinitions have broken linear supersymmetry both in hypermultiplet and in gravity sectors.

Another issue concerning boundary couplings of bulk fields is a statement, that they may lead to dangerous singular terms in the equations of motion of the bulk fields, and therefore one should use various redefinitions of fields to redefine them away. In fact, in consistent theories such as bulk-branes supergravities these singularities are harmless, see [9],[31]. To see how the cancellation works, let us take the example of a Z_2 -odd bulk field coupled derivatively to brane operators, which is of particular importance in Horava-Witten and Randall-Sundrum type models. In this case the Lagrangian includes a coupling to sources on the hidden wall and to operators consisting of observable fields on the visible wall

$$S(\Phi) = \int d^5x \left(\frac{1}{2} \partial \Phi \partial \Phi + \partial_5 \Phi \mathcal{S} \delta(x^5 - \pi\rho) + \partial_5 \Phi \mathcal{O} \delta(x^5) \right) \quad (36)$$

The bulk equation of motion is

$$\square_4 \Phi + \partial_5^2 \Phi = \mathcal{S}(x) \partial_5 \delta(x^5 - \pi\rho) + \mathcal{O}(x) \partial_5 \delta(x^5) \quad (37)$$

The equation of motion for the non-zero mode $\psi(x; x^5)$ coincides then with the full equation of motion, the proper boundary conditions on the half-circle being

$$\begin{aligned} \lim_{x^5 \rightarrow \pi\rho} \psi &= \frac{1}{2} \mathcal{S} \\ \lim_{x^5 \rightarrow 0} \psi &= -\frac{1}{2} \mathcal{O} \end{aligned} \quad (38)$$

One easily finds the solution in the form

$$\psi = \frac{\mathcal{S} + \mathcal{O}}{2\pi\rho} x^5 - \frac{\mathcal{O}}{2} + \psi_1 + \psi_2 + \dots \quad (39)$$

where the higher terms in the series are vanishing on the branes and can be computed from the recursive relation $\square_4 \psi_{n-1} + \partial_5^2 \psi_n = 0$. One can check, that the singular terms cancel out from

the equations of motion of all fields ψ_n , $n = 1, 2, \dots$. For instance, the equation of motion for ψ_1 is

$$\square_4 \psi_1 + \partial_5^2 \psi_1 = \frac{-\square_4(\mathcal{O} + \mathcal{S})}{2\pi r_c} y + \frac{\square_4 \mathcal{O}}{2}. \quad (40)$$

Hence the field ψ_1 has no discontinuities, and no singularities in the equation of motion. It is interesting to notice, that derivatives of boundary operators act effectively as bulk sources for ψ_1 .

4 Wave functions and mass quantization in flipped supergravity

In this chapter we would like to have a closer look at the localization of wave functions and mass quantization in simple models with twisted supersymmetry, and compare this to the well known Randall-Sundrum case with supersymmetry and to the same model with supersymmetry explicitly broken through the ‘wrong’ sign of the brane tension on one of the walls. The specific twisted model we shall discuss here is the locally supersymmetric generalization of the $(++)$ bigravity model of [17].

4.1 Naive RS model with flipped boundary conditions

Let us focus on the supergravity action with the prepotential of the form $P = g\epsilon(y)i\sigma_3 R$. We define Z_2 action on the gravitino sector as:

$$\begin{aligned} \Psi_\mu^A(-y) &= \gamma_5(\sigma_3)_B^A \Psi_\mu^B(y), & \Psi_5^A(-y) &= -\gamma_5(\sigma_3)_B^A \Psi_5^B(y), \\ \Psi_\mu^A(\pi r_c - y) &= -\gamma_5(\sigma_3)_B^A \Psi_\mu^B(\pi r_c + y), & \Psi_5^A(\pi r_c - y) &= \gamma_5(\sigma_3)_B^A \Psi_5^B(\pi r_c + y) \end{aligned}, \quad (41)$$

which implies flipped ($\Psi_\alpha^A(y + 2\pi r_c) = -\Psi_\alpha^A(y)$) boundary conditions. Cosmological constant that arises from the prepotential is given by $\Lambda_5 = -\frac{16}{3}g^2 R^2$. To obtain the Randall-Sundrum exponential warp-factor, we assume the following brane action:

$$S_{brane} = -6 \int d^5x \sqrt{-e_4} k (\delta(y) - \delta(y - \pi r_c)), \quad (42)$$

where we have defined $k = \frac{2\sqrt{2}}{3}g|R|$. Notice, that in this case we have broken supersymmetry on the second brane [32],[15].

Let us investigate the spectrum of the effective theory. Consider small fluctuations of the 4d components of the metric tensor around RS vacuum solution: $g_{\mu\nu}(x^\rho, y) = e^{-2k|y|}\eta_{\mu\nu} + \phi_h(y)h_{\mu\nu}(x^\rho)$, where $h_{\mu\nu}(x^\rho)$ is a 4d wave function ($\partial^\rho \partial_\rho h_{\mu\nu} = m^2 h_{\mu\nu}$) in the gauge $\partial^\mu h_{\mu\nu} = h_\mu^\mu = 0$. Linearized Einstein equations reduce to:

$$\frac{1}{2}\phi_h'' - 2k^2\phi_h + 2k(\delta(y) - \delta(y - \pi r_c))\phi_h + \frac{1}{2}e^{2k|y|}m^2\phi_h = 0. \quad (43)$$

Massless and massive solution are $\phi_h = A_0 e^{-2k|y|}$ and $\phi_h = A_m J_2(\frac{m}{k} e^{k|y|}) + B_m Y_2(\frac{m}{k} e^{k|y|})$ respectively. J_2 and Y_2 are Bessel and Newman function of the second kind. Matching delta function at the fixed points determines quantization mass condition:

$$J_1\left(\frac{m}{k}\right)Y_1\left(\frac{m}{k}e^{k\pi r_c}\right) - Y_1\left(\frac{m}{k}\right)J_1\left(\frac{m}{k}e^{k\pi r_c}\right) = 0. \quad (44)$$

To compare KK masses in bosonic and fermionic sectors, let us focus on the gravitino equation of motion:

$$\gamma^{\alpha\beta\gamma} D_\beta \Psi_\gamma^A + i\sqrt{2}g P_B^A \gamma^{\alpha\gamma} \Psi_\gamma^B = 0, \quad (45)$$

where $D_\beta \Psi_\gamma^A = (\partial_\beta + \frac{1}{4}(\omega_\beta)_{mn} \gamma^{mn}) \Psi_\gamma^A$ are covariant derivative and $(\omega_\beta)_{mn}$ is a spinor connection. In the flat background and in gauge $\Psi_5 = 0$ we can write:

$$\gamma^{\mu\rho\nu} \partial_\rho \Psi_\nu^A - \gamma^{\mu\nu} \gamma^5 \partial_5 \Psi_\nu^A + k\epsilon \gamma^{\mu\nu} \gamma^5 \Psi_\nu^A - \sqrt{2}g R \epsilon(\sigma_3)_B^A \gamma^{\mu\nu} \Psi_\nu^B = 0, \quad (46)$$

where a hat denotes a flat space Dirac matrix. In the next step we factorize the gravitino wave function as follows:

$$\begin{aligned} (\Psi_\mu^1)_R &= \phi_\psi^+(y) (\psi_\mu^+)_R(x^\rho), & (\Psi_\mu^1)_L &= \phi_\psi^-(y) (\psi_\mu^+)_L(x^\rho), \\ (\Psi_\mu^2)_L &= -\phi_\psi^+(y) (\psi_\mu^-)_L(x^\rho), & (\Psi_\mu^2)_R &= \phi_\psi^-(y) (\psi_\mu^-)_R(x^\rho), \end{aligned} \quad (47)$$

where 4d gravitini satisfy the Rarita-Schwinger equations with the mass parameter m

$$\begin{aligned} \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho^+ &= m \gamma^{\mu\rho} \psi_\rho^+ \\ \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho^- &= -m \gamma^{\mu\rho} \psi_\rho^- \end{aligned} \quad (48)$$

Gravitino equation of motion yields

$$\begin{aligned} \phi_\psi^{+'} + \frac{1}{2}k\epsilon\phi_\psi^+ - m e^{k|y|}\phi_\psi^- &= 0 \\ \phi_\psi^{-'} - \frac{5}{2}k\epsilon\phi_\psi^- + m e^{k|y|}\phi_\psi^+ &= 0 \end{aligned} \quad (49)$$

For $m = 0$ the solutions are $\phi_\psi^+ = A_0^+ \epsilon_\pi e^{-\frac{k}{2}|y|}$ and $\phi_\psi^- = A_0^- \epsilon_0 e^{\frac{5k}{2}|y|}$, where the ‘flipped’ step functions $\epsilon_0(y) = \epsilon(\frac{y}{2})$ and $\epsilon_\pi(y) = \epsilon(\frac{y+\pi r_c}{2})$ are needed to satisfy conditions (41). In this case both $\partial_5 \phi_\psi^+$ and $\partial_5 \phi_\psi^-$ contain delta functions at points $y = \pi r_c$ and $y = 0$ respectively, and those cannot be cancelled in the equation of motion (49). Hence, the massless modes do not exist in the model. Solutions for nonzero modes are

$$\begin{aligned} \phi_\psi^+ &= e^{\frac{3}{2}k|y|} \epsilon_\pi \left(A_m J_2\left(\frac{m}{k} e^{k|y|}\right) + B_m Y_2\left(\frac{m}{k} e^{k|y|}\right) \right) \\ \phi_\psi^- &= e^{\frac{3}{2}k|y|} \epsilon_0 \left(A_m J_1\left(\frac{m}{k} e^{k|y|}\right) + B_m Y_1\left(\frac{m}{k} e^{k|y|}\right) \right) \end{aligned} \quad (50)$$

Matching conditions imply vanishing of the functions ϕ_ψ^+ and ϕ_ψ^- at the points $y = \pi r_c$ and $y = 0$ respectively. Associated mass quantization condition reads:

$$J_1\left(\frac{m}{k}\right)Y_2\left(\frac{m}{k}e^{k\pi r_c}\right) - Y_1\left(\frac{m}{k}\right)J_2\left(\frac{m}{k}e^{k\pi r_c}\right) = 0. \quad (51)$$

The quantization of the mass parameters for gravitons and gravitini can be read from the figures 1 and 2. The spectra, together with the absence of the zero modes for gravitini imply broken supersymmetry (the background is flat here).

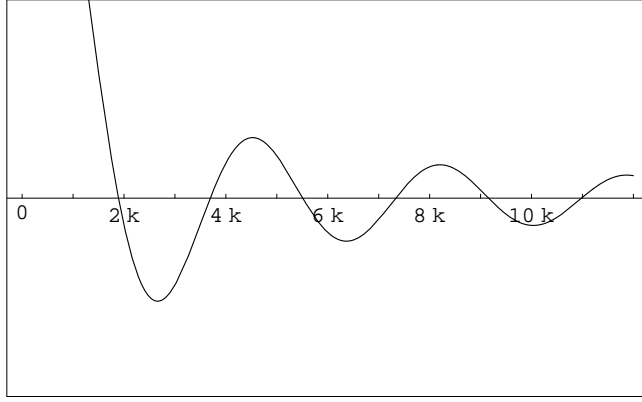


Figure 1: LHS of the equation (44) as a function of the mass parameter m . Zeros denote mass spectrum of the graviton ($k\pi r_c = 1$).

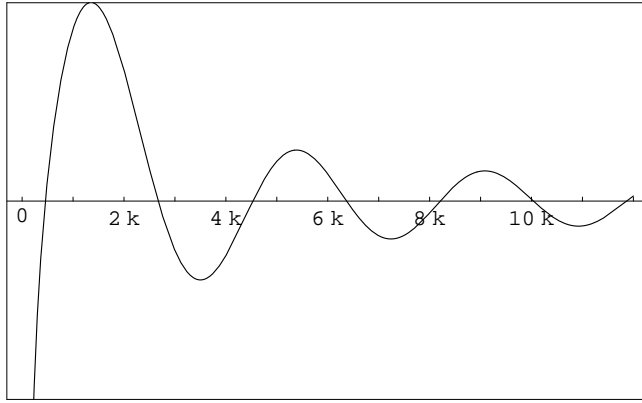


Figure 2: LHS of the equation (51) as a function of the mass parameter m . Zeros denote mass spectrum of the gravitino ($k\pi r_c = 1$).

4.2 AdS_4 compactification of the pure supergravity with flipped boundary conditions (super-bigravity)

In section 4.1 we saw, that in the Randall-Sundrum model with flipped boundary conditions, supersymmetry is broken in the effective 4d theory. The reason for this fact is twofold: flipped boundary conditions and explicitly broken supersymmetry at the point $y = \pi r_c$. To proceed let us go on to the locally supersymmetric model with a flip along the fifth dimension. The price for local supersymmetry and the trouble one encounters is the nonzero curvature in 4d sections. Let us take the supergravity action with the prepotential of the form: $P = g_R \epsilon(y) i \sigma_3 R + g_S i \sigma_1 S$, and the brane action required by supersymmetry:

$$S_{brane} = -6 \int d^5 x \sqrt{-e_4} k T (\delta(y) + \delta(y - \pi r_c)), \quad (52)$$

where $k = \frac{2\sqrt{2}}{3} \sqrt{g_R^2 R^2 + g_S^2 S^2}$, and $T = g_R |R| / \sqrt{g_R^2 R^2 + g_S^2 S^2}$. One should notice that brane tensions have the same sign. As a consequence gravitational background has no flat 4d

Minkowski foliation, and the consistent solution is that of AdS_4 branes:

$$ds^2 = a^2(y)\bar{g}_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad (53)$$

where

$$a(y) = \frac{\sqrt{-\bar{\Lambda}}}{k} \cosh\left(k|y| - \frac{k\pi r_c}{2}\right), \quad (54)$$

and $\bar{g}_{\mu\nu}dx^\mu dx^\nu = \exp(-2\sqrt{-\bar{\Lambda}}x_3)(-dt^2 + dx_1^2 + dx_2^2) + dx_3^2$ is the four dimensional AdS metric.

The radius of the fifth dimension is determined by brane tensions:

$$k\pi r_c = \ln\left(\frac{1+T}{1-T}\right). \quad (55)$$

Normalization $a(0) = 1$ leads to the fine tuning relation $\bar{\Lambda} = (T^2 - 1)k^2 < 0$. As in the previous paragraph we look at small fluctuations around vacuum metric: $g_{\mu\nu}(x^\rho, y) = a^2(y)\bar{g}_{\mu\nu} + \phi_h(y)h_{\mu\nu}(x^\rho)$, where $h_{\mu\nu}(x^\rho)$ is a 4d wave function in AdS_4 background ($(\square_{AdS} + 2\bar{\Lambda})h_{\mu\nu} = m^2 h_{\mu\nu}$ [14],[33]). The analog of the equation (43) reads:

$$\frac{1}{2}\phi_h'' - 2k^2\phi_h + 2k \tanh\left(\frac{k\pi r_c}{2}\right)(\delta(y) + \delta(y - \pi r_c))\phi_h + \frac{1}{2}a^2(y)(m^2 - 2\bar{\Lambda})\phi_h = 0. \quad (56)$$

It is easy to check, that the massless mode $\phi_h = A_0 \cosh^2(k|y| - k\pi r_c/2)$ satisfies the equation of motion in the bulk and the boundary conditions. Massive modes can be written as

$$\begin{aligned} \phi_h = & A_m \text{LP}\left(\frac{1}{2}(-1 + \sqrt{1 + 4\bar{m}^2}), 2, \tanh\left(k|y| - \frac{k\pi r_c}{2}\right)\right) + \\ & + B_m \text{LQ}\left(\frac{1}{2}(-1 + \sqrt{1 + 4\bar{m}^2}), 2, \tanh\left(k|y| - \frac{k\pi r_c}{2}\right)\right), \end{aligned} \quad (57)$$

where $\text{LP}(m, n, x)$ and $\text{LQ}(m, n, x)$ are associated Legendre functions of the first and second kind respectively. We have introduced the new symbol $\bar{m} = \sqrt{-m^2/\bar{\Lambda} + 2}$. Matching delta functions at fixed points leads to the following mass quantization condition

$$\begin{aligned} 0 = & (2t\text{LQ}(\cdot, -t) + c\text{LQ}'(\cdot, -t))(-2t\text{LP}(\cdot, t) + c\text{LP}'(\cdot, t)) + \\ & - (2t\text{LP}(\cdot, -t) + c\text{LP}'(\cdot, -t))(-2t\text{LQ}(\cdot, t) + c\text{LQ}'(\cdot, t)), \end{aligned} \quad (58)$$

where we have introduced notation $t = \tanh(k\pi r_c/2)$ and $c = \cosh^{-2}(k\pi r_c/2)$.

The gravitino equation of motion in the AdS background reads

$$\gamma^{\mu\rho\nu}\nabla_\rho\Psi_\nu^A - \gamma^{\mu\nu}\gamma^5\partial_5\Psi_\nu^A + k\epsilon\gamma^{\mu\nu}\gamma^5\Psi_\nu^A - \sqrt{2}(g_R R\epsilon(\sigma_3)_B^A + g_S S(\sigma_1)_B^A)\gamma^{\mu\nu}\Psi_\nu^B = 0, \quad (59)$$

where ∇_μ denotes AdS_4 covariant derivative. Notice that the prepotential mixes Ψ_μ^1 and Ψ_μ^2 fields. To eliminate this mixing, let us define the following functions

$$\begin{aligned} \Psi_\mu^+ &= g_S|S|\Psi_\mu^1 + \epsilon(\sqrt{g_R^2 R^2 + g_S^2 S^2} - g_R|R|)\Psi_\mu^2, \\ \Psi_\mu^- &= g_S|S|\Psi_\mu^2 - \epsilon(\sqrt{g_R^2 R^2 + g_S^2 S^2} - g_R|R|)\Psi_\mu^1. \end{aligned} \quad (60)$$

The factorization

$$\begin{aligned}(\Psi_\mu^+)_R &= \phi_\psi^+(y)(\psi_\mu^+)_R(x^\rho), & (\Psi_\mu^+)_L &= \phi_\psi^-(y)(\psi_\mu^+)_L(x^\rho), \\(\Psi_\mu^-)_L &= -\phi_\psi^+(y)(\psi_\mu^-)_L(x^\rho), & (\Psi_\mu^-)_R &= \phi_\psi^-(y)(\psi_\mu^-)_R(x^\rho),\end{aligned}\tag{61}$$

where 4d gravitini satisfy the Rarita-Schwinger equations in AdS_4 :

$$\begin{aligned}\gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho^+ &= (m - \sqrt{-\Lambda})\gamma^{\mu\rho}\psi_\rho^+, \\ \gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho^- &= -(m - \sqrt{-\Lambda})\gamma^{\mu\rho}\psi_\rho^-\end{aligned}\tag{62}$$

leads to the equation

$$\begin{aligned}\phi_\psi^{+'} + k\epsilon\left(\tanh(k|y| - \frac{k\pi r_c}{2}) + \frac{3}{2}\right)\phi_\psi^+ - a^{-1}(y)(m - \sqrt{-\Lambda})\phi_\psi^- &= 0 \\ \phi_\psi^{-'} + k\epsilon\left(\tanh(k|y| - \frac{k\pi r_c}{2}) - \frac{3}{2}\right)\phi_\psi^- + a^{-1}(y)(m - \sqrt{-\Lambda})\phi_\psi^+ &= 0.\end{aligned}\tag{63}$$

The boundary conditions are imposed by the action of the Z_2 in the fermionic sector (41). One needs to demand that the fields $(\Psi_\mu^2)_R$ and $(\Psi_\mu^1)_R$ vanish at the points $y = 0$ and $y = \pi r_c$ respectively. This implies

$$\phi_\psi^+(0) = -e^{\frac{k\pi r_c}{2}}\phi_\psi^-(0), \quad \phi_\psi^-(\pi r_c) = e^{\frac{k\pi r_c}{2}}\phi_\psi^+(\pi r_c).\tag{64}$$

This condition removes the mode $m = 0$ from the spectrum. Solutions of the equation (63) with a nonzero mass can be written down as follows

$$\begin{aligned}\phi_\psi^+ &= e^{-\frac{k}{2}(|y| - \frac{\pi r_c}{2})}\left(A_m(1-z) {}_2F_1[-M, M, 2, \frac{1-z}{2}] + B_m\frac{M}{4}(1+z)^2 {}_2F_1[1-M, 1+M, 3, \frac{1+z}{2}]\right) \\ \phi_\psi^- &= e^{\frac{k}{2}(|y| - \frac{\pi r_c}{2})}\left(A_m\frac{M}{4}(1-z)^2 {}_2F_1[1-M, 1+M, 3, \frac{1-z}{2}] + B_m(1+z) {}_2F_1[-M, M, 2, \frac{1+z}{2}]\right),\end{aligned}\tag{65}$$

where we have introduced $z = \tanh(k|y| - \frac{k\pi r_c}{2})$ and $M = (m/\sqrt{-\Lambda}) - 1$. The symbol ${}_2F_1[a, b, c, x]$ denotes a hypergeometric function. The condition (64) takes on the following form

$$\begin{aligned}\left(\frac{M}{4}(1+t)^2 F\left[3, \frac{1+t}{2}\right] + (1+t)F\left[2, \frac{1+t}{2}\right]\right)\left((1+t)F\left[2, \frac{1+t}{2}\right] - \frac{M}{4}(1+t)^2 F\left[3, \frac{1+t}{2}\right]\right) + \\ + \left((1-t)F\left[2, \frac{1-t}{2}\right] - \frac{M}{4}(1-t)^2 F\left[3, \frac{1-t}{2}\right]\right)\left((1-t)F\left[2, \frac{1-t}{2}\right] + \frac{M}{4}(1-t)^2 F\left[3, \frac{1-t}{2}\right]\right) = \\ = 0,\end{aligned}\tag{66}$$

where for simplicity we have introduced the notation $F\left[2, \frac{1\pm t}{2}\right] = {}_2F_1[-M, M, 2, \frac{1\pm t}{2}]$ and $F\left[3, \frac{1\pm t}{2}\right] = {}_2F_1[1-M, 1+M, 3, \frac{1\pm t}{2}]$. In the figures we have shown the mass quantization that follows from the conditions (58) and (66). The spectrum of the gravitino mass parameter m is shifted by approximately half a distance between consecutive zeros with respect to the mass spectrum of the graviton. It turns out that one can compute analytically the graviton and gravitini mass spectra in the limiting cases of a large extra dimension ($kr_c \gg 1$) and in the case of a small extra dimension ($kr_c \ll 1$) (see Appendix). In the regime $kr_c \gg 1$ we obtain the ultra-light graviton mode

$$m_{light}^2 \approx 12k^2 e^{-k\pi r_c} \cosh^{-2}(k\pi r_c/2),\tag{67}$$

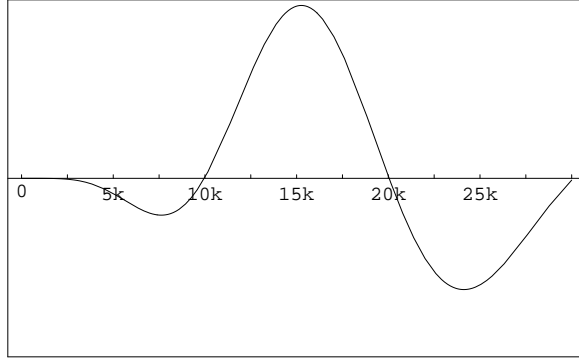


Figure 3: The zeros corresponds to the mass spectrum of the graviton, the quantization condition is plotted as the function of the variable $m \cosh(\pi r_c/2)$ ($kr_c = 0.1$).

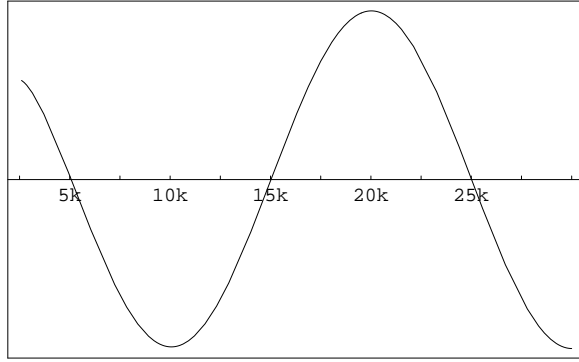


Figure 4: The zeros correspond to the mass spectrum of the gravitino (condition(66)), the quantization condition is plotted as the function of the variable M ($kr_c = 0.1$).

and heavy modes

$$m_h^2 \approx k^2(-2 + n + n^2) \cosh^{-2}(k\pi r_c/2) = (-2 + n + n^2)|\bar{\Lambda}|, \quad (68)$$

for $n > 1$. For gravitini we obtain:

$$m_f^2 \approx k^2(n + 1)^2 \cosh^{-2}(k\pi r_c/2) = (n + 1)^2|\bar{\Lambda}|. \quad (69)$$

For large n we can write for gravitons: $m \approx k(1/2 + n) \cosh^{-1}(k\pi r_c/2)$.

In the limit $kr_c \ll 1$, the equations (56) and (63) together with the assumptions $kr_c \ll 1$, $\bar{m} \approx \sqrt{m^2/k^2 + 2} \gg 1$ and $M \approx m/k \gg 1$ give the following mass quantization

$$m_h^2 = \frac{n^2}{r_c^2}, \quad (70)$$

and

$$m_\psi^2 = \frac{1}{r_c^2} \left(\frac{1}{2} + n \right)^2. \quad (71)$$

The approximate spectra for the gravitini masses that we have just obtained can be compared to the spectra of the massive spin-2 states belonging to the AdS_4 supermultiplets discussed earlier given the AdS_4 mass formula $m^2 = C_2(E_0, s) - C_2(s+1, s) = E_0(E_0 - 3) - (s+1)(s-2)$ for representations $D(E_0, s)$. In the limit of dimensional reduction [33] this implies the spin-2 and spin-3/2 spectra $m_{2,n}^2 = (E_0 + 1/2 + n)(E_0 - 5/2 + n)$ and $m_{3/2,n}^2 = (E_0 + n)(E_0 + n - 3) + 5/4$, $m_{3/2,n}'^2 = (E_0 + n + 1)(E_0 + n - 2) + 5/4$, for some E_0 and $n = 0, 1, 2, \dots$ (in units of $\sqrt{-\Lambda}$). The above mass formula fits the limiting ($kr_c \gg 1$) spectra of graviton (except the first massive mode) and gravitino masses (68) and (69) if $E_0 = 3/2$, but this value does not correspond to a unitary supermultiplet, since the necessary condition $E_0 > s + 1$ [27] is not fulfilled for $s = 3/2$ and $E_0 = 3/2$. The natural value for dimensional reduction $5d \rightarrow 4d$ would be $E_0 = 3$. This gives $m_{2,n}^2 = n^2 + 4n + 7/4$, $m_{3/2,n}^2 = (n + 3/2)^2 - 1$ and $m_{3/2,n}'^2 = (n + 5/2)^2 - 5$, again in clear mismatch with (68) and (69). It is also clear that the graviton mass spectrum for a finite kr_c differs from the supersymmetric one. In the case where the r_c is much smaller than the curvature radius, the spectrum of gravitons and gravitini approaches the usual, flat space, KK form with gravitini masses shifted with respect to these of the gravitons. Also in this limit the spectrum is clearly nonsupersymmetric, and the shift is due solely to the twisted boundary conditions.

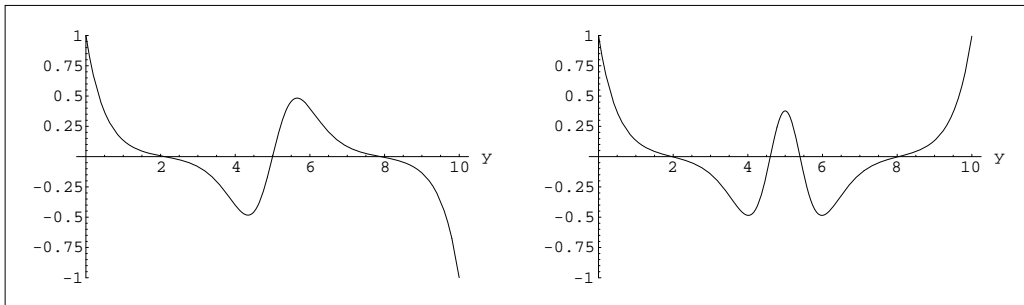


Figure 5: The third and the fourth graviton modes ($k\pi r_c = 10$, $m = 3.16k$ and $m = 4.24k$).

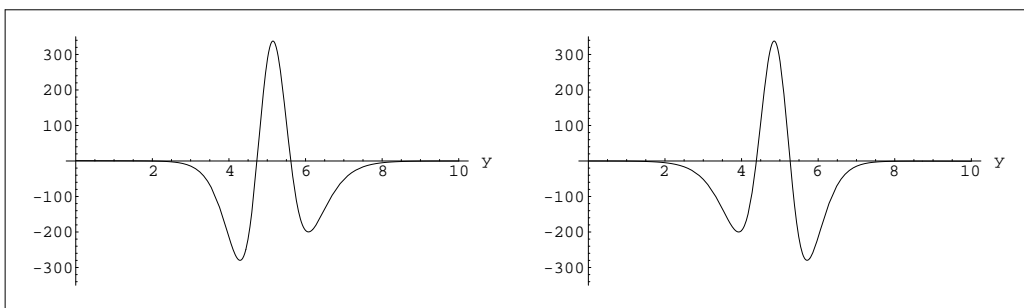


Figure 6: The fourth modes of the gravitini ϕ_ψ^+ and ϕ_ψ^- ($k\pi r_c = 10$, $m = 4k$).

One can see that even in the limit $r_c \gg 1/k$ supersymmetry is not restored, and the branes do not decouple like in the supersymmetric Randall-Sundrum case. The nondecoupling may also be seen from the shape of the wave functions of the massive modes (figure 5 and 6).

However, when kr_c goes to infinity, $m_{light} \rightarrow 0$. In such a regime we can take the following linear combinations of the ultra-light mode and the zero modes:

$$\phi_{left}(y) = \frac{1}{2} \left(\frac{\phi_0(y)}{\phi_0(0)} + \frac{\phi_{light}(y)}{\phi_{light}(0)} \right) , \quad \phi_{right}(y) = \frac{1}{2} \left(\frac{\phi_0(y)}{\phi_0(0)} - \frac{\phi_{light}(y)}{\phi_{light}(0)} \right) . \quad (72)$$

Then ϕ_{left} (ϕ_{right}) is localized on the brane at $y = 0$ ($y = \pi r_c$) and vanishes on the brane located at $y = \pi r_c$ ($y = 0$). Hence, effectively we have only one zero mode on each brane (figure 7).

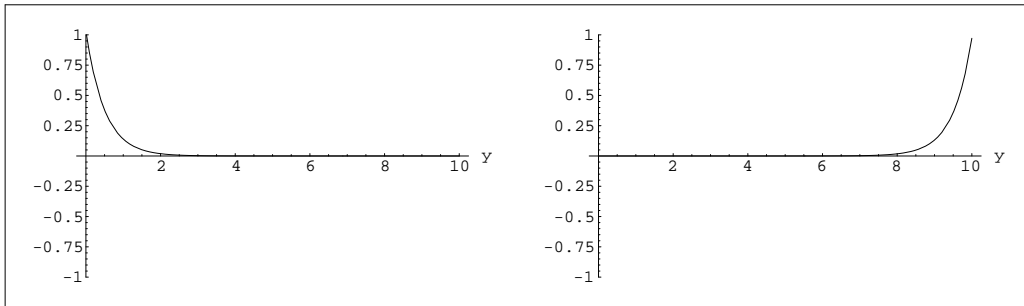


Figure 7: “Left” and “right” light graviton modes ($k\pi r_c = 10$).

To summarize the discussion of the supersymmetry breakdown in the case of the flipped supergravity let us inspect the equation for the Killing spinors:

$$\begin{aligned} \left(\frac{a'}{a} + \frac{2\sqrt{2}}{3}g_1\epsilon(y)|R| \right) \epsilon_+^A + \frac{2\sqrt{2}}{3}\gamma_5g_2|S|(\sigma^1)_B^A \epsilon_-^B &= 0 \\ \left(\frac{a'}{a} - \frac{2\sqrt{2}}{3}g_1\epsilon(y)|R| \right) \epsilon_-^A + \frac{2\sqrt{2}}{3}\gamma_5g_2|S|(\sigma^1)_B^A \epsilon_+^B &= 0 \quad , \end{aligned} \quad (73)$$

where $\epsilon_\pm^A = 1/2(\delta_B^A \pm \gamma_5 Q_B^A)\epsilon^B$. These equations result in the condition

$$\left(\left(\frac{a'^2}{a^2} - \frac{8}{9}g_1^2R^2 \right) - \frac{8}{9}g_2^2S^2 \right) \epsilon_\pm^A = 0 , \quad (74)$$

and together with Einstein equations this implies that for non-vanishing S there are no non-trivial solutions of the Killing equation.

5 Summary and conclusions

We have shown that flipped and gauged five-dimensional supergravity is closely related to the Scherk-Schwarz mechanism of symmetry breakdown. In this case the Scherk-Schwarz redefinition of fields connects two phases of the model. One phase is such that supersymmetry is broken spontaneously, in the sense that there do not exist vacua preserving some of the supercharges. In fact, one cannot undo this breakdown in a continuous way, since the choice of the projectors on both branes is a discrete one - one cannot deform continuously Q into $-Q$ within

the model. In particular, in the limit $r_c \rightarrow 0$ all gravitini (and all supercharges) get projected away. In the second, Scherk-Schwarz phase, linear supersymmetry is not realized explicitly in the Lagrangian, hence one finds susy breaking masses and potential terms in the bulk and/or on the branes. However, the physics of the two phases has to be the same, as they are related by a mere redefinition of variables.

We have found that the simple flipped 5d supergravity is a supersymmetrization of the $(++)$ bigravity with two positive tension branes. In the limit of the large interbrane separation there exists a ultra-light massive graviton mode in addition to the exactly massless mode (but there is no a nearly degenerate superpartner).

As an example the Scherk-Schwarz terms for gauged supergravity coupled to bulk matter have been worked out. In addition it has been shown that the five-dimensional Horava-Witten model is not of the Scherk-Schwarz type, since flipping of supersymmetry requires ‘wrong’ boundary terms on the branes.

In the class of models discussed here it is the AdS_4 background that appears naturally as a static solution of the equations of motion. However, firstly, there exist nearby time-dependent solutions leading to Robertson-Walker type cosmology on branes, and secondly, in more realistic models the gravitational background we have described shall be further perturbed by nontrivial gauge and matter sectors living on the branes.

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Appendix

In the regime $kr_c \gg 1$ we have used the following approximations:

$$\begin{aligned}
\text{LP}(, , -t) &\approx \frac{1}{\pi} \cos\left(\frac{1}{2}\sqrt{1+4\bar{m}^2}\pi\right) (e^{k\pi r_c} + \bar{m}^2 - 1) + O(e^{-k\pi r_c}) , \\
\text{cLP}'(, , -t) &\approx -\frac{2}{\pi} \cos\left(\frac{1}{2}\sqrt{1+4\bar{m}^2}\pi\right) e^{k\pi r_c} + O(e^{-k\pi r_c}) , \\
\text{LP}(, , t) &\approx \frac{1}{2}\bar{m}^2(\bar{m}^2 - 2)e^{-k\pi r_c} + O(e^{-2k\pi r_c}) , \\
\text{cLP}'(, , t) &\approx -\bar{m}^2(\bar{m}^2 - 2)e^{-k\pi r_c} + O(e^{-2k\pi r_c}) , \\
\text{LQ}(, , -t) &\approx -\frac{1}{2} \sin\left(\frac{1}{2}\sqrt{1+4\bar{m}^2}\pi\right) (e^{k\pi r_c} + \bar{m}^2 - 1) + O(e^{-k\pi r_c}) , \\
\text{cLQ}'(, , -t) &\approx \sin\left(\frac{1}{2}\sqrt{1+4\bar{m}^2}\pi\right) e^{k\pi r_c} + O(e^{-k\pi r_c}) , \\
\text{LQ}(, , t) &\approx \frac{1}{2} (e^{k\pi r_c} + \bar{m}^2 - 1) + O(e^{-k\pi r_c}) , \\
\text{cLQ}'(, , t) &\approx e^{k\pi r_c} + O(e^{-k\pi r_c}) , \\
t &\approx 1 - 2e^{-k\pi r_c} + O(e^{-2k\pi r_c}) , \tag{A.1}
\end{aligned}$$

and

$$\begin{aligned}
(1+t)F\left[2, \frac{1+t}{2}\right] &\approx -\frac{2}{(M^2-1)M\pi} \sin(M\pi) + O(e^{-k\pi r_c}) , \\
\frac{M}{4}(1+t)^2F\left[3, \frac{1+t}{2}\right] &\approx -\frac{2}{(M^2-1)\pi} \sin(M\pi) + O(e^{-k\pi r_c}) , \\
(1-t)F\left[2, \frac{1-t}{2}\right] &\approx 2e^{-k\pi r_c} + O(e^{-2k\pi r_c}) , \\
\frac{M}{4}(1-t)^2F\left[3, \frac{1-t}{2}\right] &\approx Me^{-2k\pi r_c} + O(e^{-3k\pi r_c}) . \tag{A.2}
\end{aligned}$$

Then the conditions (58) and (66) reduce down to

$$\cot\left(\frac{1}{2}\sqrt{9+4m^2}\pi\right) = \pi m^2 \frac{m^2+2}{1-m^2} e^{-k\pi r_c} \approx 0 , \tag{A.3}$$

and

$$\sin^2(M\pi) = \pi^2 M^2 (M^2 - 1) e^{-2k\pi r_c} \approx 0 \tag{A.4}$$

respectively. Hence, we obtain the mass quantization for gravitons:

$$m_h^2 \approx k^2 (-2 + n + n^2) \cosh^{-2}(k\pi r_c/2), \tag{A.5}$$

and for gravitini:

$$m_f^2 \approx k^2 (n+1)^2 \cosh^{-2}(k\pi r_c/2). \tag{A.6}$$

One can also obtain the analytic form of the spectrum in the limit $kr_c \ll 1$. In this regime the equations (56) and (63) together with the assumptions $kr_c \ll 1$, $\bar{m} \approx \sqrt{m^2/k^2 + 2} \gg 1$ and $M \approx m/k \gg 1$ give

$$\phi_h'' + (m^2 - 2k^2)\phi_h = 0 , \quad (\text{A.7})$$

and

$$\begin{aligned} \phi_\psi^{+'} + \frac{3}{2}k\epsilon\phi_\psi^+ - m\phi_\psi^- &= 0 , \\ \phi_\psi^{-'} - \frac{3}{2}k\epsilon\phi_\psi^- + m\phi_\psi^+ &= 0 \end{aligned} \quad (\text{A.8})$$

respectively. The solutions are very simple and take the form

$$\phi_h = A_m \cos \left[\sqrt{(m^2 - 2k^2)} \left(|y| - \frac{\pi r_c}{2} \right) \right] + B_m \sin \left[\sqrt{(m^2 - 2k^2)} \left(|y| - \frac{\pi r_c}{2} \right) \right] , \quad (\text{A.9})$$

for gravitons, and

$$\begin{aligned} \phi_\psi^+ &= A_m \cos \left[\mu \left(|y| - \frac{\pi r_c}{2} \right) \right] + B_m \sin \left[\mu \left(|y| - \frac{\pi r_c}{2} \right) \right] , \\ \phi_\psi^- &= \left(\frac{3k}{2m}A_m + \frac{\mu}{m}B_m \right) \cos \left[\mu \left(|y| - \frac{\pi r_c}{2} \right) \right] + \left(\frac{3k}{2m}B_m - \frac{\mu}{m}A_m \right) \sin \left[\mu \left(|y| - \frac{\pi r_c}{2} \right) \right] , \end{aligned} \quad (\text{A.10})$$

for gravitini. We have introduced above the notation $\mu = \sqrt{m^2 - \frac{9}{4}k^2}$. Boundary conditions lead to the following mass quantizations

$$m_h^2 = \frac{n^2}{r_c^2} , \quad (\text{A.11})$$

and

$$m_\psi^2 = \frac{1}{r_c^2} \left(\frac{1}{2} + n \right)^2 . \quad (\text{A.12})$$

References

- [1] P. Horava and E. Witten, “Eleven-Dimensional Supergravity on a Manifold with Boundary,” Nucl. Phys. B **475** (1996) 94 [arXiv:hep-th/9603142].
- [2] P. Horava, “Gluino condensation in strongly coupled heterotic string theory,” Phys. Rev. D **54** (1996) 7561 [arXiv:hep-th/9608019].
- [3] I. Antoniadis and M. Quiros, “On the M-theory description of gaugino condensation,” Phys. Lett. B **416** (1998) 327 [arXiv:hep-th/9707208].
- [4] I. Antoniadis and M. Quiros, “Supersymmetry breaking in M-theory and gaugino condensation,” Nucl. Phys. B **505** (1997) 109 [arXiv:hep-th/9705037].
- [5] E. Dudas and C. Grojean, “Four-dimensional M-theory and supersymmetry breaking,” Nucl. Phys. B **507** (1997) 553 [arXiv:hep-th/9704177].

- [6] H. P. Nilles, M. Olechowski and M. Yamaguchi, “Supersymmetry breakdown at a hidden wall,” Nucl. Phys. B **530** (1998) 43 [arXiv:hep-th/9801030].
- [7] Z. Lalak and S. Thomas, “Gaugino condensation, moduli potential and supersymmetry breaking in M-theory models,” Nucl. Phys. B **515** (1998) 55 [arXiv:hep-th/9707223].
- [8] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, “Heterotic M-theory in five dimensions,” Nucl. Phys. B **552** (1999) 246 [arXiv:hep-th/9806051].
- [9] J. R. Ellis, Z. Lalak, S. Pokorski and W. Pokorski, “Five-dimensional aspects of M-theory dynamics and supersymmetry breaking,” Nucl. Phys. B **540** (1999) 149 [arXiv:hep-ph/9805377].
- [10] A. Falkowski, Z. Lalak and S. Pokorski, “Four dimensional supergravities from five dimensional brane worlds,” Nucl. Phys. B **613** (2001) 189 [arXiv:hep-th/0102145].
- [11] R. Barbieri, L. J. Hall and Y. Nomura, “A constrained standard model from a compact extra dimension,” Phys. Rev. D **63** (2001) 105007 [arXiv:hep-ph/0011311].
- [12] J. Bagger, F. Feruglio and F. Zwirner, “Brane induced supersymmetry breaking,” JHEP **0202** (2002) 010 [arXiv:hep-th/0108010].
- [13] G. von Gersdorff, M. Quiros and A. Riotto, “Radiative Scherk-Schwarz supersymmetry breaking,” Nucl. Phys. B **634** (2002) 90 [arXiv:hep-th/0204041].
- [14] N. Alonso-Alberca, P. Meessen and T. Ortin, “Supersymmetric brane-worlds,” Phys. Lett. B **482** (2000) 400 [arXiv:hep-th/0003248].
- [15] P. Brax, A. Falkowski and Z. Lalak, “Non-BPS branes of supersymmetric brane worlds,” Phys. Lett. B **521** (2001) 105 [arXiv:hep-th/0107257].
- [16] I. I. Kogan, S. Mouslopoulos, A. Papazoglou, G. G. Ross and J. Santiago, “A three three-brane universe: New phenomenology for the new millennium?,” Nucl. Phys. B **584** (2000) 313 [arXiv:hep-ph/9912552].
- [17] I. I. Kogan, S. Mouslopoulos and A. Papazoglou, “A new bigravity model with exclusively positive branes,” Phys. Lett. B **501** (2001) 140 [arXiv:hep-th/0011141].
- [18] I. I. Kogan, S. Mouslopoulos, A. Papazoglou and G. G. Ross, “Multi-brane worlds and modification of gravity at large scales,” Nucl. Phys. B **595** (2001) 225 [arXiv:hep-th/0006030].
- [19] I. I. Kogan, S. Mouslopoulos and A. Papazoglou, “The $m \rightarrow 0$ limit for massive graviton in dS(4) and AdS(4): How to circumvent the van Dam-Veltman-Zakharov discontinuity,” Phys. Lett. B **503** (2001) 173 [arXiv:hep-th/0011138].
- [20] M. Porrati, “No van Dam-Veltman-Zakharov discontinuity in AdS space,” Phys. Lett. B **498** (2001) 92 [arXiv:hep-th/0011152].

- [21] A. Karch, E. Katz and L. Randall, “Absence of a VVDZ discontinuity in AdS(AdS),” JHEP **0112** (2001) 016 [arXiv:hep-th/0106261].
- [22] C. Biggio, F. Feruglio, A. Wulzer and F. Zwirner, “Equivalent effective Lagrangians for Scherk-Schwarz compactifications,” arXiv:hep-th/0209046.
- [23] K. A. Meissner, H. P. Nilles and M. Olechowski, “Brane induced supersymmetry breakdown and restoration,” arXiv:hep-th/0205166.
- [24] E. Bergshoeff, R. Kallosh and A. Van Proeyen, “Supersymmetry in singular spaces,” JHEP **0010** (2000) 033 [arXiv:hep-th/0007044].
- [25] R. Altendorfer, J. Bagger and D. Nemeschansky, “Supersymmetric Randall-Sundrum scenario,” Phys. Rev. D **63** (2001) 125025 [arXiv:hep-th/0003117].
- [26] J. Bagger and D. V. Belyaev, “Supersymmetric branes with (almost) arbitrary tensions,” arXiv:hep-th/0206024.
- [27] W. Heidenreich, “All Linear Unitary Irreducible Representations Of De Sitter Supersymmetry With Positive Energy,” Phys. Lett. B **110** (1982) 461.
- [28] S. Ferrara, C. Fronsdal and A. Zaffaroni, “On $N = 8$ supergravity on AdS(5) and $N = 4$ superconformal Yang-Mills theory,” Nucl. Phys. B **532** (1998) 153 [arXiv:hep-th/9802203].
- [29] A. Falkowski, Z. Lalak and S. Pokorski, “Supersymmetrizing branes with bulk in five-dimensional supergravity,” Phys. Lett. B **491** (2000) 172 [arXiv:hep-th/0004093].
- [30] A. Falkowski, Z. Lalak and S. Pokorski, “Five-dimensional gauged supergravities with universal hypermultiplet and warped brane worlds,” Phys. Lett. B **509** (2001) 337 [arXiv:hep-th/0009167].
- [31] Z. Lalak and R. Matyszkiewicz, “Boundary terms in brane worlds,” JHEP **0111** (2001) 027 [arXiv:hep-th/0110141].
- [32] T. Gherghetta and A. Pomarol, “A Stueckelberg formalism for the gravitino from warped extra dimensions,” Phys. Lett. B **536** (2002) 277 [arXiv:hep-th/0203120].
- [33] A. Karch and L. Randall, “Locally localized gravity,” JHEP **0105** (2001) 008 [arXiv:hep-th/0011156].