5

The Light Cone Open Supermembrane

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Abstract

We discuss aspects of the light-cone quantisation of the eleven dimensional open supermembrane. The vertex operators for the massless states which couple to the open membrane boundaries are derived. Our results have direct applications to Matrix theory by appropriate regularisations.

1 Introduction

More than five years after the advent of M-theory its microscopic degrees of freedom remain elusive. To some extent the "natural" candidate to such a fundamental description is the eleven dimensional supermembrane. The supermembrane is a $2+1$ dimensional object moving in 11 dimensional space, with a world volume theory [1] which when quantised should give us a glimpse of the fundamental degrees of freedom of M-theory. Attempts to quantise the world volume theory in analogy to 10 dimensional world sheet string theory have revealed many interesting features which distinguish it from the 10 dimensional string theory, and also make it difficult to solve. The main source of difficulties lies in the fact that, unlike the string, the $2+1$ dimensional world volume theory is interacting. Moreover it does not have conformal invariance. Certain features of the supermembrane are understood, namely, it can be regularized to yield a supersymmetric Matrix theory [2], and it's spectrum is continuous [3,4]. These lead to a multiparticle interpretation of the spectrum [5], and the need for a second quantised description of the membrane. We still do not have a complete understanding of this, nor is the existence of a normalisable ground state confirmed. However, there is evidence from Matrix theory that such a state indeed exists and it contains massless states corresponding to the massless multiplet of 11 dimensional supergravity [6]. Reviews on supermembranes discuss many of these issues in details [7].

The interactions of the massless sector is a interesting avenue to explore. As we know, in the case of the superstring [8] as well as the superparticle [9,10] scattering amplitudes are evaluated by determining the vertex operators and inserting them into path-integral amplitudes. The vertex operators for the supermembrane derived in [11], precisely seek to achieve the same for supermembrane scattering amplitudes [12,13]. The operators are determined uniquely and provide a step towards understanding massless interactions.

In this article we discuss the vertex operators for the open membrane. Unlike the closed supermembrane, the open membrane ends on 1,5 or 9 dimensional hypersurfaces inducing super string theories on the boundary. For the 9 dimensional case, one expects the heterotic string [14] though the membrane origins of the $E_8 \times E_8$ gauge fields remains obscure. For the 5 dimensional case, little string theories with a world volume theory of the self dual five brane is induced. In recent times, it has acquired importance due to its relation to non-commutative theories in presence of a background constant three form gauge field strength [15]. It shall be interesting to investigate the string theory induced on the 1 dimensional hypersurface.

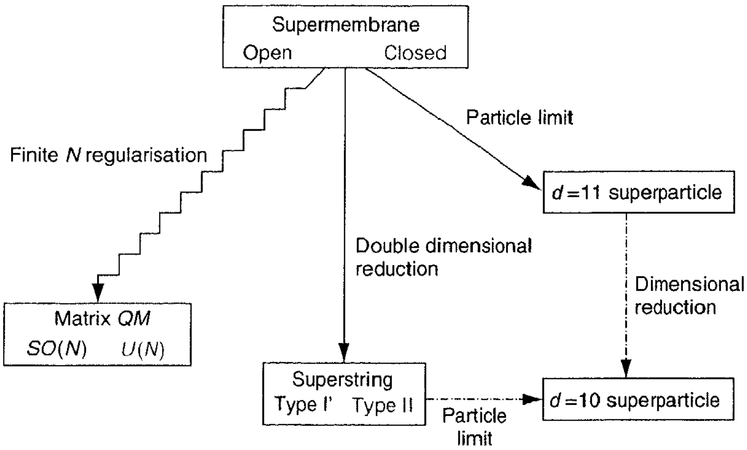


FIGURE 1

Various limits of the supermembrane model

As in the case of the closed supermembrane discussed in [11], the open supermembrane vertex operators are determined from considerations of supersymmetry. The only difference is in the presence of boundaries which reduces the number of supersymmetries. However for consistency checks, when we perform a double dimensional reduction [16] to get superstring vertex operators, there is a subtlety involved in the dimensional reduction of the open supermembrane. Since the open membrane can end only on different dimensional (9,5, or 1) hypersurfaces, we get different string theories. Also, depending on which direction Neumann or Dirichlet we choose to wrap the membrane, we get string theories in lower dimensions which are dual to each other. In this article, we shall discuss only the case where the open membrane ends on a 9dimensional hypersurface to yield a heterotic string theory on the boundary. On dimensionally reducing along one of the Dirichlet directions, one gets purely heterotic string theory. However, wrapping one of the Neumann directions, one gets a open string theory stretched between hypersurfaces, which can be identified to give Type I' string theory, dual to the heterotic string theory.

The first section is an introduction to supermembrane basics, mainly to fix notations. We also review the relation of supermembrane to Matrix theory, and then the multiparticle interpretation of the supermembrane spectrum. In the second section we give the vertex operators for the massless sector of the theory.

2 Supermembranes

In analogy with the particle and the string, the action for the membrane is given by the $2+1$ dimensional world volume swept out in the target space. The supermembrane is a extension obtained by adding fermionic target space 'coordinates'. A supersymmetric action involving the bosonic and the fermionic coordinates can be written down consistently only in special dimensions, of which $d=11$ is the maximal value. This action in flat space in $d=11$ takes the form

$$S = -\int d^3\xi \Big\{ \sqrt{-g(X,\theta)} - \epsilon^{ijk}\bar{\theta}\Gamma_{\mu\nu}\partial_i\theta \left[\frac{1}{2}\partial_j X^\mu(\partial_k X^\nu + \bar{\theta}\Gamma^\nu\partial_k\theta) + \frac{1}{6}\bar{\theta}\Gamma^\mu\partial_j\theta\bar{\theta}\Gamma^\nu\partial_k\theta\right]\Big\}. \tag{2.1}$$

The reparametrisation of the world volume can be used to fix the bosonic degrees of freedom to eight. (E.g. one can go to static gauge by identifying $X^{0,1,2} = \xi^{0,1,2}$ leaving 8 transverse degrees). The κ symmetry of the action is then used to reduce the number of fermionic degrees of freedom. The most efficient way of dealing with the above action and its symmetries however is to go to the lightcone gauge [1,2]. The light cone directions $X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^1)$ are singled out, and X^+ is identified with the time direction of the world volume, which we denote by τ here (hereafter, we denote the spatial directions $\xi^{1,2} \equiv \sigma^{1,2}$). The κ symmetry also allows to set 1/2 of the fermionic degrees of freedom to zero. The gauge conditions thus read:

$$X^+ = X_0^+ + \tau \tag{2.2}$$

$$\Gamma^+\theta = 0, \tag{2.3}$$

where $\Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^0 \pm ^1)$. One is left with 9 transverse degrees of freedom and the fermionic coordinate θ is reduced to having 16 degrees of freedom in this gauge. In addition one has to solve for X^- which gives an extra constraint reducing the total number of bosonic degrees of freedom to 8. The details of this derivation can be found in [2,1].

The light cone Hamiltonian is still invariant under a class of diffeomorphisms, called area-preserving diffeomorphisms (APD), acting as [17,2]:

$$\delta X^\mu = -\xi^r\partial_r X^\mu \qquad \delta\theta = -\xi^r\partial_r\theta \tag{2.4}$$

with $\xi^r = \epsilon^{rs}\partial_s\xi$, such that $\partial_r\xi^r = 0$ (ξ is a scalar parameter), and hence area of the two surface is preserved under this transformation.

The area preserving diffeomorphisms as defined above can be attributed with the following Lie bracket structure:

$$\delta A = \{\xi, A\} = \epsilon^{rs}\partial_r\xi\partial_s A. \tag{2.5}$$

Thus $\{A,B\} = \epsilon^{rs}\partial_r A\partial_s B$ defines a Lie bracket for any two functions A,B. It shares all the requisite properties of Lie bracket, namely antisymmetry, associativity and satisfies the Jacobi identity. The Hamiltonian density can thus be rewritten as:

$$\mathcal{H} = \left(\vec{P}\cdot\vec{P} - \frac{1}{2}\{X^a, X^b\}^2 + \theta\gamma^a\{X^a,\theta\}\right) \tag{2.6}$$

In fact, with the above knowledge, one can start with a lagrangian invariant under APD, to yield (2.6) as the Hamiltonian. The APD invariant Lagrangian can be written in a compact form by introducing an auxiliary field ω, which transforms as a gauge field under APD transformations:

$$\delta\omega = \partial_\tau\omega + \{\xi,\omega\} \tag{2.7}$$

By defining a covariant derivative,

$$DX^a = \partial_\tau X^a - \{\omega, X^a\}, \tag{2.8}$$

the lagrangian becomes [2]:

$$\mathcal{L} = \frac{1}{2}(DX^a)^2 - \frac{1}{4}\{X^a,X^b\}^2 - i\theta D\theta - i\theta\gamma^a\{X^a,\theta\} \tag{2.9}$$

The lagrangian is also invariant under the target space supersymmetries, 16 of which are linearly realised with parameter η, and due to the gauge fixing, another 16 of which are non-linearly realised with parameter ϵ. They are of the form:

$$\delta X^a = -2\epsilon\gamma^a\theta \qquad \delta\omega = -2\epsilon\theta$$
$$\delta\theta = iDX^a\gamma_a\epsilon - \frac{i}{2}\{X^a,X^b\}\gamma_{ab}\epsilon + \eta \tag{2.10}$$

Note that the action is invariant under the above transformations up to total derivatives, and for closed supermembranes the total derivatives do not make any contribution (The fields are assumed to vanish at $\tau = \pm\infty$). However, for open supermembranes, where there are boundaries at the end of spatial directions of the world volume, Dirichlet or Neumann boundary conditions have to be imposed to ensure that the boundary terms vanish. One finds that to ensure invariance under supersymmetry, it is necessary that the supermembrane ends only on 1, 5 or 9-dimensional hypersurfaces. This set of conditions have been derived earlier in [19,18] and by demanding invariance under κ symmetry of the covariant action [20].

To impose the boundary conditions on the ends of the supermembrane, we define the normal and tangential derivatives on the boundary:

$$\partial_n X^\mu \equiv n^r \partial_r X^\mu \tag{2.11}$$

$$\partial_t X^\mu \equiv \epsilon^{rs} n_r \partial_s X^\mu \tag{2.12}$$

where n^r is the unit normal on the boundary and $\epsilon^{rs} n_r$ the unit tangential vector to the boundary. For the supermembrane ending on a p dimensional hypersurface the following boundary conditions are required:

$$\partial_n X^M = 0 \qquad \text{for } M = 2, \ldots, p \qquad \text{(Neumann)} \tag{2.13}$$

$$\partial_t X^m = 0 \qquad \text{for } m = p+1, .., 10 \qquad \text{(Dirichlet)} \tag{2.14}$$

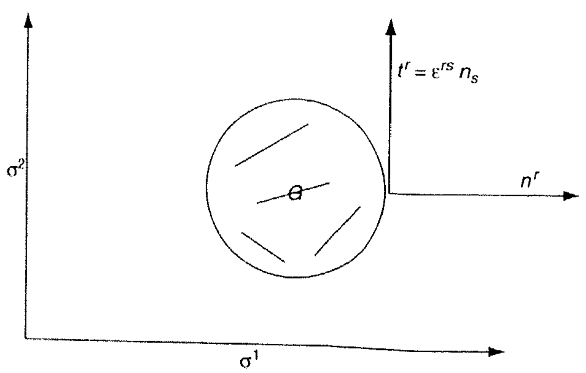
To check for invariance under supersymmetry transformations (2.10), we vary the action and find the following boundary terms:

$$\begin{aligned} \delta S = & -\int d\tau \int_{\partial G} d\sigma \eta \gamma^a \theta \partial_t X^a + \\ & + \int d\tau \int_{\partial G} d\sigma \epsilon \gamma^d \left(\gamma \cdot DX - \frac{1}{2} \gamma^{ab} \{X^a, X^b\} \right) \theta \partial_t X^d . \end{aligned} \tag{2.15}$$

On imposing the (2.14), we find to get the terms to vanish along the Neumann directions, additional conditions on θ must be imposed. These translate as

FIGURE 2

Open Membrane boundary



$$\eta\gamma^M\theta = \gamma^M\gamma^N\theta = \gamma^M\gamma^m\gamma^N\theta = 0. \qquad (2.16)$$

Defining $P_\pm = 1/2(1 \pm \gamma^{p+1}\dots\gamma^{10})$, which act as projection operators for dimensions $p = 1, 2, 5, 6, 9$, we find that the following conditions:

$$P_-\theta = 0 \qquad P_+\epsilon = 0 \qquad P_-\eta = 0 \qquad (2.17)$$

are required so that (2.16) is obeyed, which restrict $p = 1, 5, 9$. It is interesting to see that (2.17) results in the fermionic degrees of freedom being reduced to 8 on the boundary. For $p = 9$, P_- coincides with the chiral operator for the boundary theory. This is the first sign that the boundary theory, which is essentially a string theory induced by the membrane has a heterotic structure. In fact, by looking at the equations of motion obeyed by the membrane on the boundary, we find that they are 'free' equation of motion for a string. For simplicity, we discuss the $p = 9$ case, and its relation to heterotic Matrix theory [21]. The $p = 5$ case also has many interesting applications [22], especially in the light of non-commutative open membrane theories proposed and discussed in [15]. The $p = 1$ case is yet to be investigated.

Once we have ensured that the boundary terms vanish, the bulk equation of motion for the fields are:

$$D^2X^a - \{\{X^a, X^b\}, X^b\} - i\{\theta, \gamma^a\theta\} = 0 \qquad (2.18)$$

$$D\theta + \{\gamma\cdot X, \theta\} = 0 \qquad (2.19)$$

$$\{DX^a, X^a\} - i\{\theta, \theta\} = 0 \qquad (2.20)$$

The last of these, the equation of motion for the auxiliary gauge field ω is same as the constraint. What is interesting is how these equations reduce on the boundary. Taking $a = m$ in (2.18) on the boundary and if we set $\partial_n^2 X^M|_{\partial G} = 0$, (we can also set $X^M = X^M(\tau, \sigma^t)$) it follows that:

$$\partial_n X^m = \text{constant} \qquad (2.21)$$

Using this, we define $\bar\gamma = \sum_{p+1}^{10} \partial_n X^m \gamma_m =$ constant matrix. The equations obtained by putting $a = M$ in (2.18, 2.19) reduce to the following linear wave equations on the boundary:

$$(\partial^2 - \partial_t^2)X^M = 0 \qquad (2.22)$$

$$(\partial - \bar\gamma\partial_t)\theta = 0 \qquad (2.23)$$

(recall that ∂_t is the tangential derivative along the spatial boundary of the membrane). In this way we see that the supermembrane equations of motion with the above boundary conditions induce a superstring theory on its boundary. The restrictions on the value of p can then be easily understood, because only for these values is it possible to match bosonic and fermionic degrees of freedom on the boundary. In particular, for $p = 9$, we obtain the world sheet equations for the heterotic string, and $\bar{\gamma} = \gamma^{10}$ becomes a chirality matrix on the $(9+1)$-dimensional brane. By (2.17), only one of the chiralities of θ survives on the boundary, leaving an appropriate equation of motion for the chiral coordinate. We had to put $\partial_n \theta = 0$ to obtain the free heterotic string equations of motion.

After we have obtained the gauge fixed lagrangian, with the appropriate equation of motion for the world volume fields, what remains is the quantisation of the theory. Some attempts in this direction have been reviewed in [7]. Here we confine ourselves to brief comments and show how Matrix regularisation of the membrane leads to a interpretation of the spectrum. We describe the latter first.

2.1 Relation to Matrix Theory

We now concentrate on the relation of the APD diffeomorphism transformations to SU(N) or SO(N) for the open membrane gauge transformations under suitable regularisations [2]. It was first discussed in [2], and led to the supermembranes' relation to Matrix theory, and M theory [5]. In the case of open membranes, it leads to the relation to heterotic Matrix theory for membranes ending on 9-branes [14].

Since this discussion appears in previous reviews [7], we just briefly give the relation here, with an emphasis on the regularisation of the open membrane. As stated earlier, the APD bracket has a Lie bracket structure. It is also easy to check that the commutator of two APDs leads to a third APD:

$$\{\xi_2, \xi_1\} = \xi_3 \tag{2.24}$$

Given a basis of orthogonal functions $\{Y^A\}$ on the spatial manifold, we can expand the coordinates as $X^a(\sigma) = X_0^a + \sum_A X^{aA} Y_A(\sigma)$. The Lie bracket then assumes the following form:

$$\{Y_A, Y_B\} = g_{ABC} Y^C \qquad Y^A = \eta^{AB} Y_B \qquad \eta^{AB}\eta_{BC} = \delta_C^A \tag{2.25}$$

where

$$g_{ABC} = \int d^2\sigma \epsilon^{rs} Y_A \partial_r Y_B \partial_s Y_C \qquad \eta_{AB} = \int d^2\sigma Y_A Y_B \tag{2.26}$$

Given the above, the basis which is infinite dimensional for a continuum manifold, can be restricted to some finite $A = 1, .., \Lambda$ such that

$$\lim_{\Lambda\to\infty} f_\Lambda^{ABC} = g^{ABC} \tag{2.27}$$

with f_Λ^{ABC} the structure constant of some finite dimensional group labeled by Λ. For closed membranes of arbitrary topology [2,23] this finite dimensional group is SU(N), and the Hamiltonian of the regulated membrane turns out to precisely coincide with the Hamiltonian of dimensionally reduced SU(N) super Yang-Mill's theory. This can easily be seen by substituting the regularized coordinates in (2.6), and replacing the APDs by appropriate commutators. The same Hamiltonian was used in [5] to describe N $D0$-branes in the infinite momentum frame, with their momenta along the 11th direction of $d = 11$ space. Hence supermembranes through Matrix theory are intimately related to D-branes and M-theory.

For open supermembranes, the matrix regularisation yields different finite dimensional groups, dependent on the topology of the continuum membrane: for the disc D^2, the cylinder, and the Möbius strip, we get the groups $SO(N)$, whereas for the projective plane we get $USp(2N)$ [24]. Moreover, depending whether the X^a are Neumann or Dirichlet, they either transform in the symmetric or adjoint representation of the gauge group. We shall illustrate here with the example of the disc, the regularisation of the open membrane. Consider $Y_A = Y_{lm}(\theta,\phi)$, or spherical harmonics, with $m \leq |l|$. The restriction that $l \leq N-1$ for the spherical membrane leads to a basis with $\sum_l^{N-1}(2l+1) = N^2 - 1$ independent components, and the $\mathbf{X}$ transform in the adjoint of the SU(N) gauge group. However, when the membrane is stuck on a hypersurface, it essentially corresponds to a disc topology, with boundary conditions imposed on the X^m $(m = p+1, ..10)$, this would translate as $K_{lm}^- = Y_{lm} - (-)^{(l+m)} Y_{lm}$ for $l+m$ odd being the correct basis. This gives $\sum_l^{N-1} l = N(N-1)/2$ as the number of generators of the finite group. This as we know is the number of generators of SO(N). For the Neumann directions (2.12), we get these directions to transform as *symmetric* tensor representations of SO(N) [18,19]. In fact, the regularised action has the following form (in 11 dimensions):

$$S = \int Tr\Big(D\mathbf{X}^2 + DA_{10} + [\mathbf{A}_{10}, \mathbf{X}^M]^2 + [\mathbf{X}^M, \mathbf{X}^N]^2 \\ -i\Theta^+ D\Theta^+ - i\Theta^- D\Theta^- + 2i\Theta^+\Gamma^M[\mathbf{X}^M, \Theta^-]\Big) \tag{2.28}$$

Where we have distinguished $X_{10} = A_{10}$ as it transforms in the adjoint of the SO(N) group. Also the fermions are broken up as Θ^+, Θ^- which transform in the adjoint and symmetric representation of the gauge group. The $\mathbf{X}^\mathbf{M}$ transform in the symmetric traceless representation of

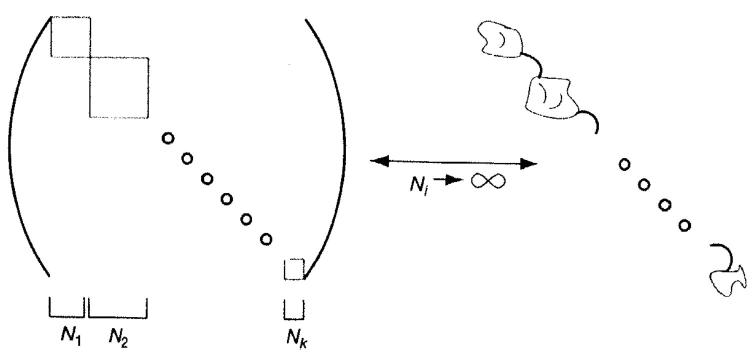
SO(N). The above matrix regularisation is the heterotic Matrix theory [21], but without the twisted sector fields expected to yield the additional $E_8 \otimes E_8$ degrees of freedom [14]. These twisted fields appear at the boundaries of the membranes, and live only on the 9-branes. A proper membrane origin of these fields is yet to be determined. For the case of the membrane ending on a five brane, there are many interesting possibilities [22], but much remains to be done.

The resultant theory is yet to be quantised fully. However, from the nature of the Hamiltonian, it can be seen that the supermembrane spectrum is continuous and there is no mass gap. This points towards a multiparticle interpretation of the spectrum, and hence a second quantised picture of the supermembrane.

The matrix regularisation of the supermembrane proves very useful, as one can use matrix quantum mechanics and interpret the $N \to \infty$ as the quantum supermembrane. However, the $N \to \infty$ limit is very subtle (for closed supermembranes with different topology and hence different APD are approximated by the same SU(N) regularisation). But the multiparticle interpretation of the supermembrane spectrum comes entirely from its relation to Matrix theory. It was shown in [3] that the supermembrane spectrum is continuous and there is no-mass gap. The continuous spectrum of the supermembrane was interpreted initially as a signature of instability. However, now due to its relation to matrix theory, this is attributed to the presence of multi-particle states. In the original conjecture of [5], the diagonal elements of $SU(N)$ matrices are positions of $D0$ branes, and in the case of block diagonal matrices, each block corresponds to N_i coincident $D0$-branes, where N_i is the dimension of the ith block. Each of these can be thought as separate entities, and in the infinite N limit, as separate membranes linked by thin tubes. This multiparticle interpretation of the supermembrane spectrum implies, that we should essentially treat the supermembrane world volume as a second quantised theory.

FIGURE 3

:-membrane correspondence



However, one crucial question still remains: can one find a normalisable ground state for the theory? And if so, does the spectrum have massless states? By just looking at the zero mode sector of the theory, one can build states which transform as $\mathbf{44} \oplus \mathbf{84}$ of SO(9) in the bosonic sector and $\mathbf{128}$ of SO(9) in the fermionic sector. Since the Hamiltonian and hence mass does not depend on the zero modes, this should give the massless sector of the theory, provided the groundstate corresponding to the non-zero modes transforms as a SO(9) singlet. Attempts to prove this have not seen much success up to now. However, in Matrix theory, considerable progress has been made in efforts to prove the existence of normalisable ground states [6] in the case of SU(2) or SU(3). Further information can be found in the review [7] and we refrain from giving the details here. In the next section, we discuss vertex operators for the supermembrane, which is one of the directions which can shed more light on this side of M-theory.

3 Vertex Operators

Vertex operators by definition are of the following form:

$$V_h = \int d^3\xi h \cdot O_h[X, \theta] e^{ik\cdot x} \qquad (3.1)$$

where h denotes the polarisation of the state, and O the local operator corresponding to that state which has a momentum $\vec{k}$. Given the fact that supermembrane world volume theory does not have conformal invariance we have to look elsewhere to determine O. However, the requirement that the vertex operators transform into one another under supersymmetry transformation completely determines their structure. In addition the technique of double dimensional reduction to give the superstring in lower dimensions gives an additional check for the supermembrane vertex operators. The superparticle vertex operators for $d = 11$ were determined in [9], and also serve as a useful guide in our calculations.

Thus for the closed supermembrane the vertex operator for the graviton should be of the form (3.1)

$$V_h = h_{ab} \int d\tau d^2\sigma O_h^{ab}[X^a(\tau, \sigma_i), \theta(\tau, \sigma_i)] e^{ik\cdot x} \qquad (3.2)$$

h_{ab} denotes the polarisation tensor of the graviton, and O^{ab} is a local operator of the supermembrane light cone coordinates. $\vec{k}$ denotes the momentum of the graviton, and this operator in the superstring case creates a graviton state when acting on the string ground state.

For the open supermembrane the string vertex operators shall couple only at the boundary, and hence shall be of the following form:

$$V_h|_{\partial G} = h_{MN} \int_{\partial G} d\tau d\sigma^t O^{MN} e^{ik\cdot x} \tag{3.3}$$

(recall σ^t denotes the tangential direction on the boundary, and M,N run over Neumann directions.) Using the fact that under supersymmetry transformations

$$\delta V_h = V_{\delta\psi(h)} \tag{3.4}$$

where $\delta\psi_h$ is the transformed corresponding fermion polarisation vector, we can solve for V_h. And V_ψ, V_C which have the following transformations under supersymmetry:

$$\delta V_C = V_{\delta\psi(h)} \qquad \delta V_\psi = V_{\delta h} + V_{\delta C} \tag{3.5}$$

in the bulk. However on the boundary, as explained in the earlier section, the fields are that of the Heterotic string, which are obtained by projecting the bulk fields onto the boundary also (except for the gauge fields). Hence, in principle we check for the supersymmetry of the bulk operators and then take appropriate projections unto the boundary after ensuring that the boundary terms which arise in the supersymmetry transformations vanish.

The vertex operators, for closed membranes were derived in [11]. We explicitly state them below: ($R^{abc} = \frac{1}{12}\theta\gamma^{abc}\theta$, $R^{ab} = \frac{1}{4}\theta\gamma^{ab}\theta$).

$$V_h = h_{ab}\big[DX^a DX^b - \{X^a, X^c\}\{X^b, X^c\} - i\theta\gamma^a\{X^b, \theta\}$$
$$-2DX^a R^{bc} k_c - 6\{X^a, X^c\} R^{bcd} k_d + 2R^{ac} R^{bd} k_c k_d\big] e^{-ik\cdot X} \tag{3.6}$$

$$V_{h_+} = -2h_{a+}(DX^a - R^{ab} k_b) e^{-ik\cdot X} \tag{3.7}$$

$$V_{h_{++}} = h_{++} e^{-ik\cdot X} \tag{3.8}$$

$$V_C = -C_{abc} DX^a \{X^b, X^c\} e^{-ik\cdot X} + F_{abcd}\Big[(DX^a - \tfrac{2}{3} R^{ae} k_e) R^{bcd}$$
$$-\tfrac{1}{2}\{X^a, X^b\} R^{cd} - \tfrac{1}{96}\{X^e, X^f\}\theta\gamma^{abcdef}\theta\Big] e^{-ik\cdot X} \tag{3.9}$$

$$V_{C_+} = C_{ab+}(\{X^a, X^b\} + 3R^{abc} k_c) e^{-ik\cdot X} \tag{3.10}$$

$$V_\Psi = \psi_a\big[(DX^a - 2R^{ab}k_b + \gamma_c\{X^c,X^a\})\theta\big]e^{-ik\cdot X} \quad (3.11)$$
$$+\tilde\psi_a\Big[\gamma\cdot DX(DX^a - 2R^{ab}k_b + \gamma_c\{X^c,X^a\})\theta$$
$$+\tfrac{1}{2}\gamma_{bc}\{X^b,X^c\}(DX^a - \{X^a,X^d\}\gamma^d)\theta + 8\gamma_b\theta\{X^b,X^c\}R^{cad}k_d$$
$$+\tfrac{5}{3}\gamma_{bc}\theta\{X^b,X^c\}R^{ad}k_d + \tfrac{4}{3}\gamma_{bc}\theta(\{X^a,X^b\}R^{cd} + \{X^c,X^d\}R^{ab})k_d$$
$$+\tfrac{2}{3}i(\gamma_b\theta\{X^a,\theta\}\gamma^b\theta - \theta\{X^a,\theta\}\theta) + \tfrac{8}{9}\gamma^b\theta R^{ac}R^{bd}k_ck_d\Big]e^{-ik\cdot X}$$
$$V_{\Psi_+} = -\Big[\psi_+\theta + \tilde\psi_+(\gamma^a DX^a + \tfrac{1}{2}\gamma^{ab}\{X^a,X^b\})\theta\Big]e^{-ik\cdot X} \quad (3.12)$$

The above operators were derived by checking for consistency under supersymmetries, and we see how the linear supersymmetry is realised. We implement the following transformation in the graviton vertex (3.6): $\delta X^a = \delta\omega = 0$, $\delta\theta = \eta$. In otherwords only the terms proportional to θ shall contribute to the variation, which is written thus:

$$\delta V_h = k_b h_{ca}\eta\gamma^{bc}\big[DX^a - 2R^{ad}k_d - \gamma^d ad\theta\big]e^{-ik\cdot x}$$
$$- h_{ab}\big[\{X^a, k\cdot X\}\eta\gamma^b\theta + i\eta\gamma^a\{X^b,\theta\}\big]e^{ik\cdot X} \quad (3.13)$$

The terms in the first line can be grouped together to yield the gravitino polarisation. The terms in the second line have to vanish clearly, and by a partial integration cancel each other. Note that, in the case of boundaries, we have to be careful in order to ensure the vanishing of the above. In fact, we find that the additional condition of $\partial_t\omega\big|_{\partial G} = 0, h_{mM}\big|_{\partial G} = 0$ has to be imposed to ensure the vanishing of the boundary terms. The condition can be understood easily as the residual symmetry of the string worldsheet is just a constant shift in the coordinates and there is no analogous APD gauge transformation. The second one implies that there are no $R\otimes R$ one forms on the boundary string theory. In case of the membrane ending on 9-branes this is easy to understand as the boundary string theory is heterotic string theory.

Thus taking the vertex operator induced by the above on the boundary should give us the open membrane vertex operator we are looking for. To determine the operator induced on the boundary, we take help of the following: The SO(9) spinors also decompose into two SO(8) spinors $(S_\alpha, S_{\dot\alpha})$. Thus $(M,N = 2,\ldots,9$ and Γ^M are SO(8) matrices):

$$R^{MN} = \tfrac{1}{4}S\Gamma^{MN}S + \tfrac{1}{4}\tilde S\Gamma^{MN}\tilde S \qquad R^{M10} = \tfrac{1}{2}\tilde S\Gamma^M S \quad (3.14)$$
$$R^{MNP} = \tfrac{1}{6}S\Gamma^{MNP}\tilde S \qquad R^{MN10} = \tfrac{1}{12}S\Gamma^{MN}S - \tfrac{1}{12}\tilde S\Gamma^{MN}\tilde S \quad (3.15)$$

On the boundary, due to the projection operator

$$(1-\gamma^{10})\Theta = 0, \tag{3.16}$$

one finds that only S_α or $S_{\dot\alpha}$ are retained. Imposing the Neumann and Dirichlet conditions on the boundary gives the boundary vertex operator as:

$$\begin{aligned}(V_h)_{\partial G} &= h_{MN}\Big[\partial_0 X^M \partial_0 X^N - \partial_t X^M \partial_1 X^N - \tfrac{1}{2}\partial_0 X^M (S\Gamma^{NP}S)k_P \\ &\quad + \tfrac{1}{2}\partial_t X^M (S\Gamma^{NP}S)k_P\Big]e^{-ik\cdot X} \\ &= h_{MN}\Big(\partial_+ X^M - \tfrac{1}{2}S\Gamma^{MP}Sk_P\Big)(\partial_- X^N)e^{-ik\cdot X}\end{aligned} \tag{3.17}$$

The same can be repeated for the three form vertex operator and the gravitino vertex operators to get the heterotic $N = 1$ supergravity multiplet on the boundary. Thus, we have the complete supermembrane vertex operators both for closed as well as open supermembranes. It should be mentioned that in the case of open membranes ending on 9-branes, heterotic string, the additional massless states of the $E_8 \times E_8$ gauge fields are not there. One has to be careful about the dimensional reduction to get the superstring vertex operators. Wrapping the supermembrane along one of the Neumann directions, which is subsequently taken to zero, one gets the Type I' vertex operators.

3.1 ● Matrix Theory Vertex Operators

Since the Matrix-regularisation of the supermembrane is a straightforward procedure described in section 2.2 the vertex operators can be easily applied to Matrix theory. The coordinates X^M, θ^+ transform in the symmetric representation of SO(N), while the Dirichlet direction transforms in the adjoint and is denoted by A_9. The coordinates are matrices, and the continuum integral is replaced by a Trace operation. Hence the graviton vertex (written in configuration space) shall be of the form:

$$\begin{aligned}\mathbf{V}_h = Tr\Bigg[\Bigg\{&\dot{\mathbf{X}}^M\dot{\mathbf{X}}^N - [\mathbf{X}^M,\mathbf{X}^N]^2 - [\mathbf{X}^M,\mathbf{A}_{10}][\mathbf{X}^N,\mathbf{A}_{10}] - \Theta^+\Gamma^M[\mathbf{X}^N,\Theta^-] \\ &- 2\dot{\mathbf{X}}^M(\mathbf{R}^{+MP}+\mathbf{R}^{-MP})\frac{\partial}{\partial\mathbf{X}^P} + 6[\mathbf{X}^M,\mathbf{X}^P]\Theta^{+MPQ}\Theta^-\frac{\partial}{\partial\mathbf{X}^Q} \\ &+ 2[\mathbf{X}^M,\mathbf{A}_{10}](\mathbf{R}^{+NQ}-\mathbf{R}^{-NQ})\frac{\partial}{\partial\mathbf{X}_Q} \\ &+(\mathbf{R}^{+MP}+\mathbf{R}^{-MP})(\mathbf{R}^{+NQ}+\mathbf{R}^{-MQ})\frac{\partial}{\partial\mathbf{X}^P}\frac{\partial}{\partial\mathbf{X}^Q}\Bigg\}h_{MN}(\mathbf{X})\Bigg]\end{aligned} \tag{3.18}$$

(with $\mathbf{R}^{\pm MN} = \frac{1}{4}\Theta^{\pm}\Gamma^{MN}\Theta^{\pm}$). Note that our vertex operators are known to all orders in θ and unlike as expected (*i.e.* terms up to θ^{32}), they contain terms only up to $O(\theta^5)$. It remains now to implement the above in a scattering amplitude calculation.

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