

Stochastic Background Search Correlating ALLEGRO with LIGO Engineering Data

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Abstract. We describe the role of correlation measurements between the LIGO interferometer in Livingston, LA, and the ALLEGRO resonant bar detector in Baton Rouge, LA, in searches for a stochastic background of gravitational waves. Such measurements provide a valuable complement to correlations between interferometers at the two LIGO sites, since they are sensitive in a different, higher, frequency band. Additionally, the variable orientation of the ALLEGRO detector provides a means to distinguish gravitational wave correlations from correlated environmental noise. We describe the analysis underway to set a limit on the strength of a stochastic background at frequencies near 900 Hz using ALLEGRO data and data from LIGO's E7 Engineering Run.

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1. Introduction

One of the gravitational wave (GW) sources targeted by the current generation of ground-based interferometric and resonant detectors is a stochastic background of gravitational waves (SBGW), produced by an unresolved superposition of signals of astrophysical or cosmological origin [1, 2, 3]. Direct observational limits can be set on a SBGW by looking for correlations in the outputs of a pair of GW detectors. This has been done using two resonant-bar detectors [4], two “prototype” interferometers [5], and is currently being done with the two kilometer-scale LIGO interferometers [6, 7] (IFOs). In this paper we describe the first known effort to set a limit on a SBGW with correlations between an IFO and a bar. This pair of detectors—the LIGO Livingston Observatory (LLO) [8] in Livingston, LA and the ALLEGRO resonant bar detector [9] in Baton Rouge, LA—is separated by only 40 km, the closest pair among the ten modern ground-based GW detector sites. [10] This makes it an attractive pair for probing the stochastic GW spectrum around 900 Hz. In addition, the ALLEGRO bar can be rotated, changing the response of the correlated data streams to stochastic GWs and thus providing a means to distinguish correlations due to a SBGW from those due to correlated environmental noise.

In Sec. 2 we review the standard measure of stochastic background strength and the cross-correlation technique used to search for a SBGW. In Sec. 3 we describe the previous limits set on a SBGW with ground-based GW detectors. In Sec. 4 we describe the key features of the LLO-ALLEGRO correlation experiment in general. Section 5 describes the data taken during LIGO’s E7 engineering run and the analysis currently underway using those data. Finally, Sec. 6 describes the prospects for future correlation experiments using ALLEGRO and LIGO science data.

2. Stochastic Background Measurements

A SBGW is assumed for simplicity to be isotropic, unpolarized, Gaussian, and stationary. Subject to these assumptions, the stochastic GW background is completely described by its power spectrum. It is conventional to express this spectrum in terms of the GW contribution to the cosmological parameter $\Omega = \rho/\rho_{\text{crit}}$:

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_{\text{crit}}} \frac{d\rho_{\text{GW}}}{d \ln f} = \frac{f}{\rho_{\text{crit}}} \frac{d\rho_{\text{GW}}}{df} . \quad (1)$$

Note that $\Omega_{\text{GW}}(f)$ has been constructed to be dimensionless, and represents the contribution to the overall Ω_{GW} per *logarithmic* frequency interval. In particular, it is *not* equivalent to $d\Omega_{\text{GW}}/df$. Note also that since the critical density ρ_{crit} , which is used in the normalization of $\Omega_{\text{GW}}(f)$, is proportional to the square of the Hubble constant H_0 [11], it is convenient to work with $h_{100}^2 \Omega_{\text{GW}}(f)$, which is independent of the observationally determined value of $h_{100} = \frac{H_0}{100 \text{ km/s/Mpc}}$.

The standard method to search for such a background is to cross-correlate the outputs of two gravitational wave detectors [1]. If the noise in the two detectors is uncorrelated, the only non-zero contribution to the average cross-correlation will come from the stochastic GW background. In the optimally-filtered cross-correlation method (described in more detail in [2, 12, 13]), one calculates a cross-correlation statistic

$$Y = \int dt_1 dt_2 h_1(t_1) Q(t_1 - t_2) h_2(t_2) = \int df \tilde{h}_1^*(f) \tilde{Q}(f) \tilde{h}_2(f) \quad (2)$$

where $h_{1,2}(t)$ are the data streams from the two detectors, $\tilde{h}_{1,2}(f)$ are their Fourier transforms, and $Q(t_1 - t_2)$ (with Fourier transform $\tilde{Q}(f)$) is a suitably-chosen optimal filter. It is sensitive[12] to backgrounds on the order of

$$\Omega^{\text{UL}} \sim \left(T \int df \frac{\gamma_{12}^2(f)}{f^6 P_1(f) P_2(f)} \right)^{-1/2}. \quad (3)$$

This sensitivity improves with time and is limited by the power spectral densities $P_{1,2}(f)$ of the noise in the two detectors. The factor

$$\gamma_{12}(f) = d_{1ab} d_2^{cd} \frac{5}{4\pi} \int_{S^2} d^2\hat{\Omega} e^{i2\pi f \hat{\Omega} \cdot \Delta \vec{x}/c} P_{cd}^{ab}(\hat{\Omega}) \quad (4)$$

in the numerator of the integral is the *overlap reduction function* [14], which describes the observing geometry. Here $P_{cd}^{ab}(\hat{\Omega})$ is a projector onto symmetric traceless tensors transverse to a direction $\hat{\Omega}$ and $d_{1,2}^{ab}$ are the *detector response tensors* for the two detectors. These are the tensors with which the metric perturbation h_{ab} at the detector should be contracted to obtain the *gravitational wave strain* $h = d^{ab} h_{ab}$. If u_a and v_a are unit vectors pointing in the directions of an IFO's two arms, its response tensor is

$$d_{ab} = \frac{1}{2}(u_a u_b - v_a v_b) \quad (5)$$

while the response tensor for a resonant bar whose long axis is parallel to the unit vector w_a is

$$d_{ab} = w_a w_b. \quad (6)$$

The overlap reduction function is equal to unity for the case of a pair of IFOs (or an IFO and a bar) at the same location with their arms aligned, and is suppressed as the detectors are rotated out of alignment or separated from one another. The frequency dependence comes about for the following reason: if the wavelength is comparable to or smaller than the separation between two detectors, the detectors will see different phases of the wave at the same time, and this phase difference will depend on the direction of propagation of the wave. Since the stochastic GW background is assumed to be isotropic, averaging over different propagation directions suppresses the sensitivity of a pair of detectors to high-frequency waves. For example, a wave whose wavelength is twice the distance between the two detectors will drive them 180° out of phase if it travels along the line separating them, but *in* phase if its direction of propagation is perpendicular to this line. Figure 1 shows the overlap reduction functions for several detector pairs of interest.

3. Previous Results

The current best upper limit on a SBGW from direct observation with GW detectors is $h_{100}^2 \Omega_{\text{GW}}(900 \text{ Hz}) \leq 60$ [4], set by correlating the resonant bar detectors Explorer[15] (in Geneva, Switzerland) and Nautilus[16] (near Rome, Italy).[‡] A broad-band limit of $h_{100}^2 \Omega_{\text{GW}}(f) \leq 3 \times 10^5$ was set using a pair of “prototype interferometers” [5], and more recently an analysis of E7 engineering data from the LLO and LHO sites [6] set a limit of $h_{100}^2 \Omega_{\text{GW}}(40\text{--}215 \text{ Hz}) \leq 7.7 \times 10^4$. Analysis [7] of LLO and LHO data from LIGO's first science run (S1) is expected to improve substantially upon this limit.

[‡] The same group had previously[17] set a limit of $h_{100}^2 \Omega_{\text{GW}}(900 \text{ Hz}) \leq 100$ using a single bar detector.

Overlap Reduction Function

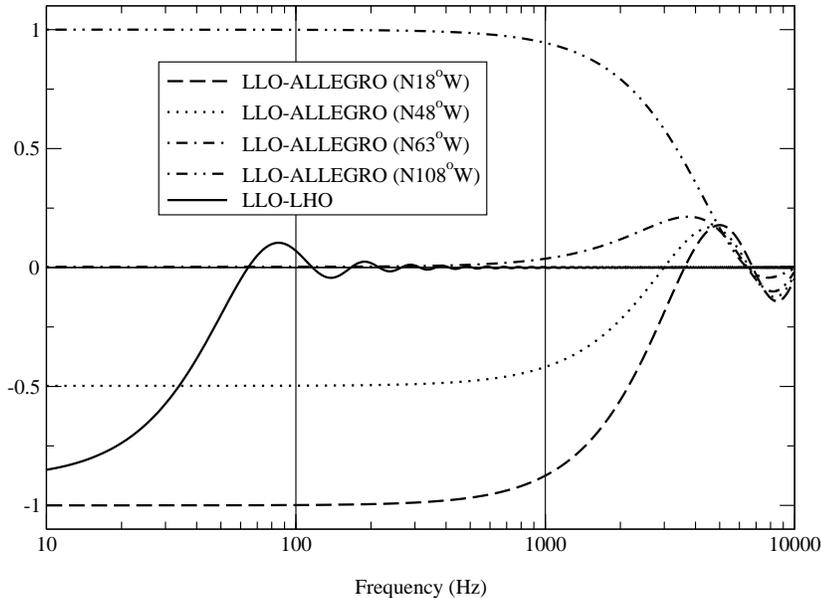


Figure 1. The overlap reduction function for LIGO Livingston Observatory (LLO) with ALLEGRO and with LIGO Hanford Observatory (LHO). The four LLO-ALLEGRO curves correspond to four orientations: “N18°W” is nearly parallel to the y-arm of LLO (“anti-aligned”); “N108°W” is nearly parallel to the x-arm of LLO (“aligned”); “N63°W” is halfway in between these two orientations (a “null alignment” midway between the two LLO arms); “N48°W” is an intermediate alignment in which ALLEGRO was also operated during the E7 run.

More stringent upper limits can be set on astrophysical grounds. They are detailed elsewhere [2, 3, 7], but we mention the bound from big-bang nucleosynthesis[11, 3], which states that a cosmological SBGW is limited by

$$\int_{10^{-8} \text{ Hz}}^{\infty} \frac{df}{f} h_{100}^2 \Omega_{\text{GW}}(f) \leq 10^{-5}. \quad (7)$$

This broad-band limit tells us that any cosmologically interesting SBGW almost certainly lies several orders of magnitude below the existing limits.

4. ALLEGRO-LLO Correlations

Due to the 3000 km distance between the two LIGO detectors, the overlap reduction function (see Fig. 1) limits the range of frequencies at which they are sensitive to a stochastic background. The separation distance is half of a GW wavelength at a frequency of 50 Hz, so the upper end of the frequency range is a few times this. (The lower end of the frequency range is set by the seismic noise in the two detectors.) The ALLEGRO bar detector is far closer to the LIGO Livingston site than the LIGO Hanford site is, with only about 40 km separating the two and a “half-wavelength”

frequency of 3750 Hz. Thus the observing geometry for ALLEGRO-LLO allows for observations of correlations out to much higher frequencies. On the other hand, the sensitivity of ALLEGRO (see Fig. 2) is concentrated in two narrow frequency bands in the vicinity of 900 Hz, so correlations between ALLEGRO and LIGO Livingston probe a different part of the frequency domain than correlations between the two LIGO detectors. The major challenge in detecting a SBGW is accounting for the effects of noise which may be correlated between the two detectors. Even in widely separated detectors such as LLO and LHO there may exist correlations (the most obvious example being the due to the power line frequency and its harmonics) which could mimic or mask a real SBGW. With detectors in much closer proximity, such as ALLEGRO and LLO, the problem could potentially be more severe. A method proposed by Lazzarini and Finn [18] is to rotate the ALLEGRO bar about a vertical axis and measure the cross-correlation for different alignments of the ALLEGRO-LLO pair. Changing the alignment of the bar from one of the interferometer arms to the other changes the sign of the expected correlation due to a SBGW. Thus any correlated noise which is independent of alignment can be removed. Another possibility is to record data with the bar aligned at 45° from the arms where no correlation due to a SBGW is expected. This “off source” measurement gives an uncontaminated estimate of the cross-correlated noise.

4.1. LLO-ALLEGRO Coincidence Operation

The ALLEGRO detector took data during LIGO “E7” Engineering Run that took place from December 2001 to January 2002. In the following section we discuss the initial investigations underway. Following the the E7 run ALLEGRO went off-line for a major upgrade. Beginning in the summer of 2002 a new transducer [19] was installed which should substantially improve the sensitive bandwidth of the bar detector. Figure 2 shows the projected improvement in bandwidth and sensitivity. The installation was ongoing during the time of the LIGO S1 science run in August-September 2002. Following the installation, a series of cool-down attempts were made. A succession of repairs were done on the vacuum system and a successful cool-down was finally made starting in February 2003. This allowed data to be taken with ALLEGRO during approximately the second half of the LIGO S2 run.

5. Investigations with LIGO Engineering Data

From December 28, 2001 to January 14, 2002, LIGO held its seventh Engineering Run (E7), the last engineering run before the start of scientific operation in August 2002. The ALLEGRO detector was in operation for the end of that run. Table 1 summarizes the ALLEGRO data which overlap with science-quality LLO data.

If no excess correlations are found, and we assume that all of the ALLEGRO data have a level of noise given by the solid curve in Fig. 2, and all of the LLO data are of a quality described by [21], (3) tells us to expect to set an upper limit of around 1.2×10^4 .§

Analysis of the data described in Table 1 is currently in progress. In the remainder of this section, we outline some of the technical challenges involved in this analysis.

§ This would actually be better than the limit of 7.7×10^4 set in [6], which is understandable because it involves only one engineering-quality LIGO data stream, while the LLO-LHO analysis involves two. The sensitivity of at both LIGO sites improved markedly between E7 and the science runs. [22]

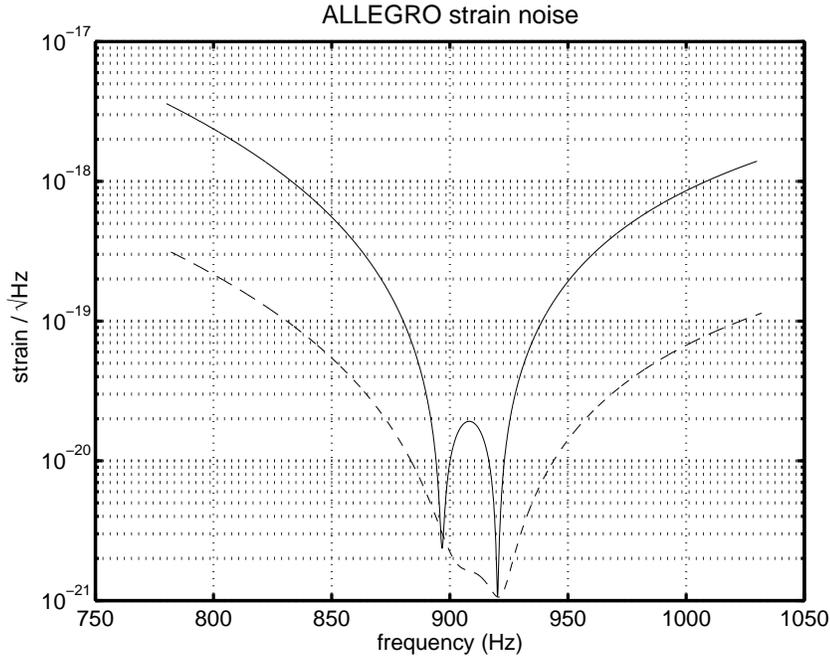


Figure 2. The figure shows the ALLEGRO strain noise curve for operation during E7 (solid) [20] as well as the projected improvement for operation with the new transducer during S2 (dashed)[19]

Azimuth	$\gamma(907 \text{ Hz})$	Clean LLO Segments
N48°W	-0.43	134 × 15 min =33:30
N18°W	-0.90	82 × 15 min =20:30
N63°W	-0.03	110 × 15 min =27:30

Table 1. Summary of ALLEGRO data coincident with LLO operation during E7. The table lists ALLEGRO’s alignment in degrees West of North, the resulting value of the overlap reduction function $\gamma(f)$ (see Fig. 1) in ALLEGRO’s sensitivity band, and the number of 15-minute segments of clean LLO data which coincide with ALLEGRO data in each alignment.

First, the nature of the data recorded is different: LIGO data consists of a real time series $h_L(t)$, sampled at 16384 Hz, which is related by a linear transfer function to the gravitational wave strain in the detector. The Fourier transform $\tilde{h}_L(f)$ of this data stream contains frequencies from -8912 Hz to 8912 Hz , although the negative frequency components are all determined by the positive frequency ones. Because the ALLEGRO detector’s sensitivity is confined to a band with $\Delta f < f$, its data stream $h_A(t)$ is heterodyned (mixed with an oscillating signal at frequency $f_H = 907 \text{ Hz}$) and low-pass filtered (before it is digitized) and then sampled at the reduced rate of 250 Hz. The result is to produce a complex signal $h_A^H(t) = e^{i2\pi(-f_H t + \phi_A)} h_A(t)$ whose Fourier transform $\tilde{h}_A^H(f) \approx \tilde{h}_A(f_H + f) e^{i2\pi\phi_A}$ contains completely independent data at frequencies $-125 \text{ Hz} < f < 125 \text{ Hz}$, which are approximations of the Fourier transforms of the data in the original time series between $(907 - 125) \text{ Hz}$

and $(907 + 125)$ Hz.

The initial approach we are taking to correlating these data streams with different sampling frequencies is to heterodyne and resample the LIGO data in software (producing an $h_L^H(t) = e^{i2\pi(-f_H t + \phi_L)} h_L(t)$ whose Fourier transform is $\tilde{h}_L^H(f) \approx \tilde{h}_L(f_H + f)e^{i2\pi\phi_L}$) and then correlate it with the ALLEGRO data as though they were any two heterodyned data streams. This requires a certain amount of care, because the phase ϕ_L at the start of a stretch of data of the reference oscillator for the LIGO heterodyning is specified (because it's done in software), while the phase ϕ_A of the ALLEGRO reference oscillator is only known if it is experimentally determined. In principle, it is not difficult to keep track of this, but it is also possible to post-process the cross-correlation statistics to account for ignorance of the phase difference $\phi_A - \phi_L$ between the two oscillators.[23]

Additional technical challenges include converting ALLEGRO data into the frame format used by LIGO[24] so that it can be processed by the LIGO Data Analysis System[25]. Fortunately, the LIGO stochastic search codes[13, 26, 27] have been designed with an eye to working on both heterodyned and non-heterodyned data, and the data structures describing site geometry|| allow for either interferometric or resonant bar detector geometries.

6. Future Outlook

The upper limit we expect to set from E7 data is several orders of magnitude above the current direct observational limit[4] at the frequencies in question. But the practical application of the LLO-ALLEGRO cross-correlation method to engineering data will pave the way for future such observations, whose sensitivity is expected to improve for several reasons:

- (i) Most substantially, the sensitivity of the LLO IFO has already improved markedly[22] and will continue to improve as the instrument approaches its design sensitivity[21].
- (ii) ALLEGRO's new transducer will increase the bandwidth of the bar detector (see the dashed curve in Fig. 2).
- (iii) Future observing runs will provide more coincident data than the 80 hours or so taken during E7.

Assuming a full year of coincident data in optimal alignment, with LLO operating at design sensitivity and ALLEGRO operating at the sensitivity given by the dashed curve in Fig. 2, we estimate using (3) that an upper limit could be set in the range $890 < f < 930$ of $h_{100}^2 \Omega_{\text{GW}}(f) \lesssim 10^{-2}$.

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|| LAL Software Documentation, version 2.0, Section 9.2; available from [26]

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