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Vortex Unpinning in Precessing Neutron Stars

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ABSTRACT
The neutron vortices thought to exist in the inner crust of a neutron star interact with nuclei and are expected to pin to the nuclear lattice. Evidence for long-period precession in pulsars, however, requires that pinning be negligible. We estimate the strength of vortex pinning and show that hydrodynamic forces present in a precessing star are likely sufficient to unpin all of the vortices of the inner crust. In the absence of precession, however, vortices could pin to the lattice with sufficient strength to explain the giant glitches observed in many radio pulsars.

Key words: stars: interiors — stars: neutron — stars: evolution — stars: rotation — superfluid — dense matter

1 INTRODUCTION

Observations of precession of neutron stars probe the manner in which the stellar crust is coupled to the liquid interior. Stairs, Lyne & Shemar (2000) recently presented strong evidence for precession of an isolated neutron star at a period of $\simeq$ 1000 d, 500 d or 250 d, with correlated variations in the pulse duration. The wobble angle of the precession is inferred to be $\simeq 3^\circ$ (Link & Epstein 2001). Less compelling evidence for precession is seen from other neutron stars. Quasi-periodic timing residuals are exhibited by PSR B1642-03 (Shabanova, Lyne & Urama 2001), in association with roughly cyclical changes in pulse duration (Cordes 1993). Quasi-periodic timing residuals are seen in the Crab (Lyne, Pritchard & Smith 1988; Čadež *et al.* 2001) and Vela pulsars (Deshpande & McCulloch 1996), though associated changes in pulse duration are not seen. The 35-d periodicity seen in the accreting system Her X-1 (Tannanbaum *et al.* 1972) has been interpreted as precession by many authors (*e.g.*, Brecher 1972; Trümper *et al.* 1986; Čadež, Galičič & Calvani, 1997; Shakura, Postnov & Prokhorov 1998).

The fluid of the neutron star inner crust is expected to form a neutron superfluid threaded by vortices. Above a fluid density of $\sim 10^{13}$ g cm^{-3}, an attractive vortex-nucleus interaction could pin vortices to the lattice. Vortex pinning in the crust has been a key ingredient in models of the spin jumps, *glitches*, observed in many radio pulsars (*e.g.*, Lyne, Shemar & Smith 2000). In all candidate precessing neutron stars, the putative precession is of long period, months to years, which is in dramatic conflict with the notion of vortex pinning in the neutron star's inner crust. As demonstrated by Shaham (1977), vortex pinning exerts a large torque on the crust that causes the star to precess with a period $(I_c/I_p)p$, where I_p is the moment of inertia of the pinned superfluid, p is the spin period and I_c is the moment of inertia of the solid crust and any component coupled to it over a timescale $\ll p$. If pinning occurs through most of the inner crust (as assumed in many models of glitches), I_p constitutes about 1% of the entire star. Though the coupling of the solid to the core is not well-known, I_c cannot exceed the star's total moment of inertia, so the precession period is *at most* $\sim$ 100 spin periods. Hence, if the long-period periodic behavior seen in the precession candidates really represents precession, *the crustal vortices cannot be pinned.* In this paper we present a resolution to this puzzle. We show that vortices initially pinned to the inner crust would probably be unpinned by the forces exerted on them by a crust set into precession. In the absence of precession, vortices could still pin to the inner crust with sufficient strength to account for giant pulsar glitches.

The outline of the paper is as follows. In Section 2 we review the dynamics of precession with pinning. In Section 3 we calculate the Magnus forces on a pinned vortex in a precessing neutron star, and show that the Magnus force per unit length is $\sim 10^{17}$ dyne cm^{-1} throughout most of the crust of PSR B1828-11, nearly two orders of magnitude larger than the minimum force on pinned vortices in Vela just prior to a giant glitch. Thus, if we assume that the force/length f_p required to unpin a superfluid vortex is in the range $10^{15} \lesssim f_p \lesssim 4\times 10^{16}$ dyne cm^{-1}, a consistent picture emerges for PSR B1828-11 (and the other long-period precession candidates mentioned above): *the precessional motion itself unpins the vortices and keeps them unpinned.* In Section 4 we show that this interpretation makes sense theoretically; we estimate the force/length required to unpin a vortex in the crust, and find $f_p \sim 10^{16}$ dyne cm^{-1}. We summarize our findings in Section 5.

2 FREE PRECESSION WITH PINNING

The problem of precession of the neutron star crust with a pinned superfluid has been studied by Shaham (1977), Alpar & Pines (1985), Sedrakian, Wasserman & Cordes (1999) and Jones & Andersson (2001). We revisit the problem here to emphasize key results for later use.

First, we briefly discuss the role that dissipative coupling between the core liquid and solid would play in precession. Precession creates time-dependent velocity differences between the crust and liquid that vary over the star's spin period. If the coupling time τ_{cc} between the crust and the core liquid is much longer than the crust's spin period p, the precession will damp over $\simeq 2\pi\tau_{cc}/p$ precession periods (Sedrakian, Wasserman & Cordes 1999). Coupling of the solid to the core liquid is not well understood. Magnetic stresses allow angular momentum exchange between the solid and the charged components of the core (Abney, Epstein & Olinto 1996; Mendell 1998), though this process is not by itself dissipative. If the core magnetic field, of average strength B, is confined to superconducting flux tubes, the crossing time for Alfvén type waves through the core is $t_A \sim 4B_{12}^{-1/2}$ s for a density of 10^{15} g cm^{-3}. The effective coupling time between the crust and core charges cannot be less than t_A. The coupling time between the neutron component of the core and the charges could exceed $\simeq 400$ rotation periods (Alpar & Sauls 1988). Hence, the timescale τ_{cc} for the *entire* core to achieve corotation with the solid could exceed many rotation periods. The core might therefore be effectively *decoupled* from the solid as the star precesses, with the crust precessing almost as if the core were not there. Given the uncertainties, we will consider two regimes: complete decoupling of the core liquid from the solid, and the opposite regime of perfect coupling.

To study the precessional dynamics, we approximate the crust's inertia tensor as the sum of a spherical piece, a centrifugal bulge that follows the instantaneous angular velocity of the crust and a deformation bulge aligned with the principal axis of the crust (Alpar & Pines 1985):

$$\boldsymbol{I}_c = I_{c,0}\boldsymbol{\delta} + \Delta I_\Omega\left(\boldsymbol{n_\Omega n_\Omega} - \frac{1}{3}\boldsymbol{\delta}\right) + \Delta I_d\left(\boldsymbol{n_d n_d} - \frac{1}{3}\boldsymbol{\delta}\right), \quad (1)$$

where $I_{c,0}$ is the moment of inertia of the crust (plus any components tightly coupled to it) when non-rotating and spherical, $\boldsymbol{\delta}$ is the unit tensor, $\boldsymbol{n_\Omega}$ is a unit vector along the crust angular velocity $\boldsymbol{\Omega}$, $\boldsymbol{n_d}$ is a unit vector along the crust's principal axis of inertia, ΔI_Ω is the increase in oblateness about $\boldsymbol{\Omega}$ due to rotation and ΔI_d is the *deformation* contribution due to rigidity of the crust. We assume $\Delta I_d \ll I_{c,0}$ and $\Delta I_\Omega \ll I_{c,0}$. For simplicity, we take the crust superfluid to be perfectly pinned along $\boldsymbol{n_d}$. The total angular momentum of the crust plus pinned superfluid is

$$\boldsymbol{J} = \boldsymbol{I}_c \cdot \boldsymbol{\Omega} + J_{sf}\boldsymbol{n_d}, \quad (2)$$

where J_{sf} is the magnitude of angular momentum in the pinned superfluid. For free precession $\boldsymbol{J}$ is conserved, with $\boldsymbol{J}$, $\boldsymbol{\Omega}$ and $\boldsymbol{n_d}$ all spanning a plane (see Fig. 1). The *wobble angle* θ between $\boldsymbol{n_d}$ and $\boldsymbol{J}$ is a constant of the motion. The precessional motion in the inertial frame can be seen by decomposing $\boldsymbol{\Omega}$ as (see, *e.g.*, Andersson & Jones 2001)

$$\boldsymbol{\Omega} = \dot{\phi}\boldsymbol{n_J} + \dot{\psi}\boldsymbol{n_d}, \quad (3)$$

where $\boldsymbol{n_J}$ is a unit vector along $\boldsymbol{J}$, ϕ and ψ are Euler angles and overdots denote time differentiation. For small wobble angle, the motion is given by $\dot{\phi} \simeq J/I_{c,0}$ and $\dot{\psi} \simeq -(\Delta I_d\Omega + J_{sf})/I_{c,0}$. The precession is a superposition of two motions: 1) a fast wobble of $\boldsymbol{n_d}$ about $\boldsymbol{n_J}$, with a constant angle θ between the axes, and, 2) a retrograde rotation about $\boldsymbol{n_d}$. In the body frame, both $\boldsymbol{J}$ and $\boldsymbol{\Omega}$ rotate about $\boldsymbol{n_d}$ at frequency $\dot{\psi}$, the body-frame precession frequency. For an emission beam axis fixed in the star and inclined with respect to $\boldsymbol{n_d}$, modulation at frequency $\dot{\psi}$ is observed.

For insignificant pinning ($J_{sf} \ll \Delta I_d\Omega$), the body-frame precession frequency reduces to the classic elastic-body result: $\dot{\psi} \simeq -(\Delta I_d/I_{c,0})\Omega$. However, for significant pinning ($J_{sf} \gg \Delta I_d\Omega$), the precession is much faster, $\dot{\psi} \simeq -J_{sf}/I_{c,0}$, as originally shown by Shaham (1977). Taking $J_{sf} = I_p\Omega_s$, where I_p is the moment of inertia of the pinned superfluid and Ω_s is the magnitude of its angular velocity, $\dot{\psi} = -(I_p/I_{c,0})\Omega_s \simeq -(I_p/I_{c,0})\Omega$. If the entire crust superfluid is pinned and $I_{c,0}$ is the moment of inertia of the solid only, $I_p/I_{c,0}$ is $\simeq 2$ for most equations of state, giving extremely fast precession with $\dot{\psi} \simeq -2\Omega$. If pinning is imperfect, but vortices move with respect to the lattice against a strong drag force, the precessional dynamics resembles that for perfect pinning (Sedrakian, Wasserman & Cordes 1999).

The angle θ' between $\boldsymbol{\Omega}$ and $\boldsymbol{J}$, is also a constant of the motion (see Fig. 1). For θ and θ' both small, these two angles are related by

$$\theta' \simeq \frac{(\Delta I_d\Omega + J_{sf})}{J - J_{sf} - \Delta I_d\Omega}\theta. \quad (4)$$

For insignificant pinning ($J_{sf} \ll \Delta I_d\Omega$), θ' is $\simeq (\Delta I_d/I_{c,0})\theta \ll \theta$. For significant pinning ($J_{sf} \gg \Delta I_d\Omega$), $\theta' \simeq (J/J_{sf} - 1)^{-1}\theta$. If the core is decoupled from the solid, J_{sf} can be $\simeq J$, and θ' can exceed θ. If the entire inner crust superfluid is pinned, $J_{sf}/J \simeq I_s/(I_s + I_i)$, where I_s is the moment of inertia of the crust superfluid and I_i is the moment of inertia of the lattice. For most equations of state, J_{sf}/J is about 0.7 in this case, giving $\theta' \simeq 2\theta$. On the other hand, if the crust and core are tightly coupled, $\theta' \simeq (J_{sf}/J)\theta \lesssim 10^{-2}\theta$.

3 FORCES ON THE VORTEX LATTICE IN A PRECESSING STAR

We have seen that significant vortex pinning produces a precession frequency that is orders of magnitude faster than the periodicities observed in PSR B1828-11 and other candidate precessing pulsars. We now study the stability of the pinned state in a precessing star. When making estimates, we take f_p (the maximum pinning force/length that the crustal nuclei can exert on the pinned superfluid vortices) to be constant through the crust.

We begin by estimating f_p from the angular momentum requirements of giant glitches in pulsars. The standard explanation for giant glitches – that they represent transfer of angular momentum from the more rapidly rotating inner crust superfluid to the crust via catastrophic unpinning (Anderson & Itoh 1975) – yields a lower limit for f_p. As the stellar crust slows under electromagnetic torque, vortex pinning fixes the angular velocity of the crust superfluid. As the velocity difference grows, a Magnus force develops on the pinned vortices. If the crust and superfluid are rotating about the same axis (and therefore not precessing) the force per unit length of vortex is $f_m = \rho_s \kappa r_\perp \omega$, where $\rho_s \simeq 10^{14}$ g cm^{-3} is the superfluid density, $\kappa \equiv h/2m_n$ is the quantum of circulation (m_n is the neutron mass), $r_\perp$ is the distance of the vortex line from the rotation axis and ω is the angular velocity lag between the crust and the pinned superfluid (see eq. 7 below). For a critical value $f_{m,c} = f_p$, vortices cannot remain pinned. The corresponding lag velocity is $\omega_c = f_p/\rho\kappa r_\perp$. Treating the crust as a thin shell of constant density, the excess angular momentum stored in the pinned superfluid (moment of inertia I_p) is $\Delta J = (3\pi/8)(f_p/\kappa)(I_p/R)$. Suppose that in a glitch some or all of the excess angular momentum is delivered to the crust; the crust suffers a spin-up of $\Delta\Omega \leq \Delta J/I_c$, where I_c is the moment of inertia of the crust plus any part of the star tightly coupled to it over timescales much shorter than the timespan of glitch observations. The glitch magnitude is as much as

$$\frac{\Delta\Omega}{\Omega} \leq \frac{3\pi}{8} \frac{f_p}{\rho_s \kappa R \Omega} \left(\frac{I_p}{I_c}\right), \tag{5}$$

yielding the following lower limit for the force on pinned vortices just prior to a giant glitch:

$$f_p \geq 10^{15} \left(\frac{\Delta\Omega/\Omega}{10^{-6}}\right) \left(\frac{\Omega}{80 \text{ rad s}^{-1}}\right) \left(\frac{I_p/I_c}{10^{-2}}\right)^{-1}. \tag{6}$$

Here $\Omega \simeq 80$ rad s^{-1} for the Vela pulsar and $\Delta\Omega/\Omega \simeq 10^{-6}$ is typically observed. Analyses of glitches in Vela and other pulsars show that $I_p/I_c \gtrsim 10^{-2}$ (Link, Epstein & Lattimer 1999); we have taken $I_p/I_c = 10^{-2}$ as a fiducial value. If glitches relax f_m to nearly zero, the above lower limit is an estimate for f_p. By comparison, the maximum amount by which the Magnus force can increase between glitches is $\Delta f_m = \rho_s \kappa r_\perp |\dot{\Omega}| t_g$, where $|\dot{\Omega}|$ is the crust's spindown rate and t_g is the average time interval between glitches. For Vela, $\Delta f_m \simeq 10^{15}$ dyne cm^{-1}.

We now compare $f_p \sim 10^{15}$ dyne cm^{-1}, which is sufficient to explain giant glitches, to the Magnus forces on vortices in a precessing neutron star. Define a Cartesian coordinate system (x, y, z) fixed in the crust and centered on the star, and let $(\boldsymbol{n_x}, \boldsymbol{n_y}, \boldsymbol{n_z})$ be the corresponding basis vectors. The superfluid flow $\boldsymbol{\Omega_s}$ past a pinned vortex segment creates a Magnus force per unit length of vortex at location $\boldsymbol{r} = (x, y, z)$ of (see, *e.g.*, Shaham 1977)

$$\boldsymbol{f}_m = \rho_s \kappa \boldsymbol{n_z} \times ([\boldsymbol{\Omega_s} - \boldsymbol{\Omega}] \times \boldsymbol{r}). \tag{7}$$

For simplicity, we assume vortex pinning along $\boldsymbol{n_z}$ and take the superfluid angular velocity to be $\boldsymbol{\Omega_s} = \Omega \boldsymbol{n_z}$. Consider an instant at which $\boldsymbol{\Omega}$ and $\boldsymbol{n_z}$ lie in the $y - z$ plane. The angular velocity of the crust is then $\boldsymbol{\Omega} \simeq \Omega(\alpha \boldsymbol{n_y} + \boldsymbol{n_z})$, where $\alpha \equiv \theta + \theta'$. The instantaneous Magnus force per unit length of vortex as a function of position in the star is

$$\boldsymbol{f}_m = \boldsymbol{n_y} \rho_s \kappa \Omega \alpha z. \tag{8}$$

For $\Omega = 16$ rad s^{-1} (PSR B1828-11), the inferred α of 3° and a density $\rho_s = 10^{14}$ g cm^{-3}, $|\boldsymbol{f}_m|$ exceeds 10^{17} dyne cm^{-1} at $z = R$, a factor of ~ 100 larger than the minimum force on vortices before a giant glitch in Vela. A vortex segment must unpin if $f_m > f_p$, or,

$$\frac{|z|}{R} > \frac{f_p}{\rho_s \kappa R \Omega \alpha} \equiv \frac{h}{R}, \tag{9}$$

where R is the stellar radius. For $\alpha = 3^\circ$ and $\Omega = 16$ rad s^{-1} (PSR B1828-11), $f_p = 10^{15}$ dyne cm^{-1} and $\rho_s = 10^{14}$ g cm^{-1}, h is $\simeq 0.007R$; only vortex segments in a region of height $h \ll R$ are not unpinned directly by the Magnus force (see Fig. 2). We next consider whether the vortex array can be in static equilibrium with respect to the crust when only a portion of the array is pinned very near the equatorial plane. We find that the Magnus force on the unpinned segments exerts a torque about $z = 0$ which leads to further unpinning.

As vortex segments unpin at $|z| \geq h$, the angle between $\boldsymbol{n_z}$ and $\boldsymbol{n_\Omega}$ will assume a new value $\alpha' < \alpha$. The value of α' will depend on how much pinning there is initially, but it will not become less than θ (corresponding to J_{sf} becoming effectively zero; recall discussion following eq. 4). The precession frequency will also be less than before, as there is less pinned vorticity to exert torque on the crust. If the vortex segments are to remain anchored at $|z| < h$, the vortex array must bend under the Magnus force exerted on unpinned segments. The extent to which vortices can bend is determined by their self-energy or *tension*, which arises primarily from the kinetic energy of the flow about them. For a single vortex, the tension is $T \simeq (\rho_s \kappa^2/4\pi) \ln(r_v k)^{-1}$, where r_v is the vortex core dimension and k is the characteristic wavenumber of the bend in the vortex and $kr_v \ll 1$ (Sonin 1987). This tension, which is $\sim 10^8$ dyne, has a negligible effect on the dynamics if the vortex is bending over macroscopic dimensions. However, if a bundle of vortex lines bends, the effective tension per vortex is enormous. A bundle of N vortices has N times the circulation of a single vortex, and hence N^2 times the tension: $T_N \simeq N^2 \rho_s \kappa^2/4\pi) \ln(r_v k)^{-1}$. The effective tension per vortex is $T_{\rm eff} = T_N/N$. We estimate the angle β through which the vortex array can bend by taking the pinning at $|z| < h$ to be infinitely strong, and seek a new static configuration of a vortex line with tension $T_{\rm eff}$ under the Magnus force. We will find that $\beta \ll \alpha'$, so we approximate the Magnus force in the new equilibrium as unchanged by the small displacement of the line away from its original pinning axis. Let $\boldsymbol{u}(z)$ denote the displacement of a section of a vortex from its original pinning position along $\boldsymbol{n_z}$. In static equilibrium, the shape of the vortex in the region $|z| \geq h$ is given by

$$T_{\rm eff} \boldsymbol{u}''(z) = \boldsymbol{n_y} \rho_s \kappa \alpha' \Omega z. \tag{10}$$

Let a vortex follow $\boldsymbol{n_z}$ in the region $|z| < h$, so that $\boldsymbol{u}'(\pm h) = 0$. Integrating eq. (10) once gives the angle between a section of vortex and $\boldsymbol{n_z}$ of $\beta \simeq \rho_s \kappa \alpha' \Omega (z^2 - h^2)/2T_{\rm eff}$. The number N of vortices that must bend is $\simeq n_v 2\pi R \Delta R$, where $n_v = 2\Omega_s/\kappa$ is the vortex areal density and ΔR is the thickness of the pinning region (approximately the thickness of the inner crust). The longest vortices that pass through the region $x - y$ plane in the inner crust extend to a height $z_0 \simeq \sqrt{2R\Delta R}$. Evaluating β at $z = z_0/2$ gives $\beta \simeq \alpha'/4 \ln(r_v k)^{-1}$. Taking a bending wavenumber $k = 1/z_0$ gives $\beta \simeq 0.006\alpha'$. Hence, the vortex array is far too stiff to bend over an angle $\simeq \alpha'$. This means that the unpinned segments are prevented by tension from assuming a new static configuration in which the Magnus force is small *unless* further unpinning occurs at $|z| < h$. We now show that further unpinning is likely.

We treat individual vortices as infinitely stiff *when the entire vortex array is bending.* The Magnus force on a vortex exerts a torque about $z = 0$. A segment can remain pinned and in static equilibrium with respect to the crust only if pinning forces in the region $|z| < h$ can exert a compensating torque. Suppose a given vortex line has a length $H > h$. The Magnus force on this line exerts a torque about $z = 0$ of

$$\boldsymbol{N}_m = \int_{-H}^{H} dz\, \mathbf{r} \times \mathbf{f_m} = \frac{2}{3} \boldsymbol{n_x} \rho_s \kappa \Omega \alpha' H^3. \qquad (11)$$

If the section of the line at $-h \le z \le h$ is pinned, the lattice can exert a compensating pinning torque of *at most*

$$\boldsymbol{N}_p = 2\boldsymbol{n_x} \int_0^h dz\, z f_p = \boldsymbol{n_x} f_p h^2, \qquad (12)$$

assuming the crust does not crack. Taking $f_p = \rho_s \kappa \Omega \alpha h$ (eq. 9), the torque on the unpinned segment exceeds that by the lattice if the length of the segment satisfies

$$H > \left(\frac{3\alpha}{2\alpha'}\right)^{1/3} h. \qquad (13)$$

These segments "unzip" from their pinning bonds, and are forced through the lattice by the Magnus force. Segments shorter than H exist only in the outermost region of the inner crust, in a region of extent $\delta R \simeq H^2/2R$ in the $x - y$ plane (see Fig. 2). Combining eqs. (9) and (13), we estimate the extent of the pinning region to be

$$\frac{\delta R}{R} \simeq \frac{1}{2} \left(\frac{3}{2\alpha'\alpha^2}\right)^{2/3} \left(\frac{f_p}{\rho_s \kappa R \Omega}\right)^2. \qquad (14)$$

The pinning region is largest if the core is decoupled, and if unpinning changes θ' from $\simeq 2\theta$ to $\ll \theta$; in this case $\alpha' \simeq \alpha/3$. For $\alpha = 3^\circ$ and $\Omega = 16$ rad s^{-1}, we obtain $\delta R < 1$ m. Pinning in this outermost region of the crust probably cannot occur at all, but if it does, the moment of inertia of the pinned superfluid in this small region would be too small to significantly affect the spin dynamics of the star. If the core is tightly-coupled to the crust, δR is smaller by a factor of $\simeq 2$. Our estimate for f_p applies in the regions of the crust where pinning is expected to be strongest; hence vortex unpinning is likely to be more effective almost everywhere else in the crust. Vortex pinning is even more difficult to sustain for precessing stars with higher spin rates.

The pinning strength f_p can be considerably larger than 10^{15} dyne cm^{-1} without affecting our conclusions. For example, a pinning strength as large as $f_p = 4 \times 10^{16}$ dyne cm^{-1} gives $\delta R = 0.1R$ (assuming core decoupling), again probably preventing significant pinning. We conclude that f_p in the range $10^{15} \lesssim f_p \lesssim 4 \times 10^{16}$ dyne cm^{-1} is sufficient to explain giant glitches in young pulsars such as Vela, but insufficient to sustain vortex pinning in PSR B1828-11.

4 THEORETICAL ESTIMATE OF THE VORTEX PINNING STRENGTH

The vortex-nucleus interaction arises from the density dependence of the superfluid gap. The details of this interaction are uncertain. In the densest regions of the inner crust, where most of the liquid moment of inertia resides, the interaction energy of a vortex segment with a nucleus is estimated to be $E_p \simeq 5$ MeV (Alpar 1977; Epstein & Baym 1988; Pizzochero, Viverit & Broglia 1997). The length scale of the interaction is comparable to the pairing coherence length $r_v \simeq 10$ fm, giving an interaction force of $F_p \simeq 5 \times 10^6$ dyne per nucleus. Above a density $\simeq 10^{14}$ g cm^{-3}, the vortex-nucleus interaction energy falls rapidly with density and the vortex core dimension r_v increases. The mass-averaged pinning force is thus smaller than 5×10^6 dyne; we will take $F_p = 10^6$ dyne as a fiducial value. Below a density of $\simeq 10^{13}$ g cm^{-3}, the vortex-nucleus interaction becomes repulsive. Here vortices could pin to the interstices of the lattice, but too weakly to play a significant role in the rotational dynamics of neutron stars.

The degree to which vortices pin to the lattice nuclei is a complex problem, but fortunately one where terrestrial analogs can provide guidance. The pinning of elastic "strings" to attractive potentials is a subject of current interest in condensed matter physics, arising, e.g., in the pinning of magnetic vortices to lattice defects in type II superconductors. We follow the general reasoning of Blatter et al. (1994) and D'Anna et al. (1997) to obtain a rough estimate of f_p for our problem.

Our estimate effectively treats the crust as an amorphous solid with random pinning sites. This description of the solid is appropriate according to recent calculations by Jones (1998b, 2001), though we believe our pinning estimate to be roughly correct for a regular lattice as long as the vortex does not closely follow one of the lattice basis vectors. In this context, it is important to realize that pinning arises only because the vortex can bend. If the tension T were infinite, the forces on the vortex by nearby nuclei would cancel on average (Jones 1991, 1998a). On the other hand, if the vortex tension T were small compared to F_p, the vortex would minimize its energy by adjusting its shape so as to intersect as many pinning nuclei as possible. In this case the spacing between pinned nuclei would equal the lattice spacing b ($\simeq 30$ fm), and the pinning force per unit length f_p would be approximately $F_p/b \simeq 3 \times 10^{17}$ dyne cm^{-1}. For superfluid vortices in the NS crust, typically $F_p/T \simeq 10^{-2}$, so vortices bend rather little, and f_p is considerably below F_p/b, as we now estimate.

The are essentially five physical parameters that together determine f_p: The vortex tension T, the pinning force per nucleus F_p, the vortex radius r_v, the nuclear radius r_n and the typical nuclear separation $b \equiv n_{nuc}^{-1/3}$. The tension of a single vortex is

$$T = \frac{\rho_s \kappa^2}{4\pi} \ln(k r_v)^{-1}, \qquad (15)$$

where k is the bending wavenumber. For $r_n << r_v$, we expect the value of r_n to be unimportant. While r_v and r_n are comparable in the denser regions of the crust (Pizzochero, Viverit & Broglia 1997), in the rough estimate we make below we treat the nuclei as points. We take as fiducial values: $F_p = 10^6$ dynes, $r_v = 10$ fm, and $b = 30$ fm.

If T were infinite, so that the vortex could not bend toward nuclei, the typical distance between nuclei along the vortex from random overlaps would be $(\pi r_v^2 n_{nuc})^{-1} = b(b^2/\pi r_v^2) \sim 3b$. For finite T, the vortex can bend to intersect extra nuclei, but does so only on sufficiently long length scales, due to competition between the attractive nuclear potentials and the elastic energy of the deforming vortex. Call these extra nuclei (that the vortex intersects due to bending) the "pinning nuclei". The pinning nuclei can bend the vortex over a *pinning correlation length* L_p. Let u be the transverse distance by which the vortex deviates from straight over a length L_p. For u of order r_v or greater, the distance L_p between successive pinning nuclei is of order $\sim (n_{nuc}u^2)^{-1} = b(b/u)^2$. The gain in binding energy due to each pinning nucleus is of order $\sim F_p r_v$, while the energy cost of bending is $\sim T(u^2/L_p^2)L_p$. Bending to intersect an extra nucleus becomes energetically favorable when these two energies are comparable, giving a pinning correlation length of

$$L_p \sim b\left(\frac{T}{F_p}\right)^{1/2}\left(\frac{b}{r_v}\right)^{1/2}. \tag{16}$$

To calculate the vortex tension, we take the bending wavenumber to be $k = 1/L_p$. For a superfluid mass density of $\rho_s = 10^{14}$ g cm^{-3} and the fiducial values given above, we solve eqs. [15] and [16] simultaneously and find $L_p \simeq 20b$. The binding energy/length of the bent vortex is $e_b \sim F_p r_v/L_b$. The maximum Magnus force/length that the vortex can withstand before unpinning is thus $f_p \simeq e_b/r_v \simeq F_p/L_p$, which for our fiducial parameters is $f_p \sim 2\times10^{16}$ dyne cm^{-1}. The deviation of the bent vortex from straight is $u = (L_p F_p r_v/T)^{1/2}$; this distance is comparable to r_v for our parameters, as we assumed *a priori*.

Glitch observations and the precession period of PSR B1828-11 suggest a pinning strength in the range $10^{15} \lesssim f_p \lesssim 4\times10^{16}$ dyne cm^{-1}. The above pinning estimate, though crude, shows that such pinning strengths are theoretically sensible.

5 DISCUSSION

In the most convincing example of pulsar free precession, PSR B1828-11, we have shown that vortex pinning is unstable for a reasonable pinning strength and wobble angle: the Magnus force on pinned vortices is sufficient to unpin all of the vortices of the inner crust. In support of this conclusion, we obtained in Section 4 a theoretical estimate of the maximum pinning force/length f_p. The large vortex tension increases the distance between effective pinning sites relative to the case of small tension. We estimated f_p to be $\sim 10^{16}$ dyne cm^{-1}, smaller by a factor of ~ 10 than the value obtained assuming a pinning spacing equal to the lattice spacing. Our estimated f_p is nevertheless large enough to account for the giant glitches seen in radio pulsars, like Vela. Our work here leads to at least one falsifiable prediction: *PSR B1828-11, or any other pulsar, should not exhibit giant glitches while precessing.* (Small glitches could be explained by some mechanism other than vortex unpinning, *e.g.*, crustquakes).

In Section 3 we showed that partially-pinned vortex configurations cannot be static. We did not attempt to solve for the dynamics of the unpinned superfluid vortices or the effects on the precession of the crust; we leave that as an interesting problem for future work.

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Figure 1. The constant angles in free precession. The symmetry axis $\boldsymbol{n_d}$ and spin axis $\boldsymbol{\Omega}$ span a plan containing $\boldsymbol{J}$. In the inertial frame, $\boldsymbol{n_d}$ and $\boldsymbol{\Omega}$ rotate about $\boldsymbol{J}$ at approximately the spin frequency.

Figure 2. The Magnus force on a pinned vortex in a precessing star. At heights $|z| > h$, the Magnus force exceeds the pinning force per length of vortex, and the vortex segments are unpinned directly by the Magnus force. Unpinned segments then torque free most segments that extend through $z = 0$, except for a small annulus of lens-like cross section of height H and thickness δR (shown with gray shading). For clarity, fewer vortices are shown on the right side of the figure.