

# M-Theory solutions with AdS factors

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## ABSTRACT

Solutions of  $D = 7$  maximal gauged supergravity are constructed with metrics that are a product of a  $n$ -dimensional anti-de Sitter ( $AdS$ ) space, with  $n = 2, 3, 4, 5$ , and certain Einstein manifolds. The gauge fields have the same form as in the recently constructed solutions describing the near-horizon limits of M5-branes wrapping supersymmetric cycles. The new solutions do not preserve any supersymmetry and can be uplifted to obtain new solutions of  $D = 11$  supergravity, which are warped and twisted products of the  $D = 7$  metric with a squashed four-sphere. Some aspects of the stability of the solutions are discussed.

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# 1 Introduction

A rich class of supersymmetric solutions of  $D = 10$  and  $D = 11$  supergravity have recently been found that describe the near-horizon limits of branes wrapping supersymmetric cycles [1]–[21]. These solutions are dual to the supersymmetric field theories arising on the branes. There are two key features of the construction of the supergravity solutions. First, they are initially constructed in a suitable gauged supergravity and then subsequently uplifted to  $D = 10$  or  $D = 11$ . Secondly, the ansatz for the gauge fields in the gauged supergravity are determined by the geometry of the normal bundle of the supersymmetric cycle. This latter feature is a manifestation of the fact that the field theories arising on the wrapped branes are coupled to external R-symmetry currents, or “twisted”, in order to preserve supersymmetry [22].

For almost all cases of M-fivebranes wrapping  $d$ -dimensional supersymmetric cycles,  $D = 11$  supergravity solutions were found that include an  $(7 - d)$ -dimensional anti-de Sitter factor [1, 3, 5, 16]. More precisely, the solutions are warped and twisted products of an anti-de Sitter factor, a cycle with an Einstein metric and a squashed four-sphere. The  $AdS$  factor indicates that, in the infrared, at length scales much larger than the size of the cycle, the corresponding  $(6 - d)$ -dimensional field theory on the wrapped M-fivebranes is super-conformal, a fact which *a priori* was not at all clear.

Here we would like to report on a new class of solutions of  $D = 11$  supergravity with  $AdS$  factors that preserve no supersymmetry. They are obtained in maximal  $D = 7$  gauged supergravity using exactly the same ansatz that was used to find the supersymmetric wrapped M5-brane solutions, and consequently they have a similar  $D = 11$  structure. In the supersymmetric  $AdS$  solutions the Einstein metric on the cycle typically has negative curvature. In the new non-supersymmetric solutions we find a more democratic mixture of both negatively and positively curved cycles.

Our main interest in these solutions is that they might provide M-theory duals of new conformal field theories with no supersymmetry. Since the  $AdS$  factors that appear range from  $n = 5$  to  $n = 2$ , these would be dual to conformal field theories in dimensions ranging from four to one. The most interesting case could be the dualities between the  $AdS_5$  solutions and four-dimensional field theories. In this context, to our knowledge, the only previously known solutions of this type are the compactifications on Kähler–Einstein six-dimensional manifolds given in [23]. A necessary condition for the correspondence is that the solutions be stable. We have checked that certain perturbations of the scalar fields satisfy the Breitenlohner-Freedman bound [24, 25,

26] in most cases, but we will leave a full investigation of this issue to future work. We note that we do not know of any stability analysis that has been performed for the solutions in [23] just mentioned. Additional instabilities might arise because of the presence of massless fields [27, 28]. Furthermore, one also needs to establish that there are non-perturbative instabilities that could be of the general form discussed in [29, 30]. We shall not attempt to address these issues here. Assuming that the correspondence is valid, we note that the central charges of the conformal field theories would be proportional to  $N^3$  as in the supersymmetric cases [1, 3, 5, 16].

The plan of the paper is as follows. In section 2 we describe the essential aspects of the ansatz that we employ to find the new solutions in  $D = 7$  gauged supergravity. As noted, the ansatz includes a  $d$ -dimensional cycle with an Einstein metric and this is used to consistently truncate the theory, via Kaluza-Klein reduction on the cycle, to an effective  $(7 - d)$ -dimensional theory of gravity coupled to two scalar fields. (The exception is the case of five-cycles, i.e.  $d = 5$ , where we find the solutions directly from the  $D = 7$  equations of motion.) Some general comments concerning the stationary points and their stability in this reduced theory then precedes a more detailed description of the different cases in the following sections. In section 3, 4, 5 and 6 we discuss the  $AdS_5$ ,  $AdS_4$ ,  $AdS_3$  and  $AdS_2$  solutions, respectively. The different cases are labelled by the kind of supersymmetric cycle that a fivebrane can wrap and for completeness we have included the supersymmetric  $AdS$  solutions already found in [1, 3, 5, 16]. The paper concludes in section 7, where we have included a table summarising both the supersymmetric and non-supersymmetric solutions. We also comment on the possible connection between our new non-supersymmetric solutions and the supersymmetric conformal field theories arising on M5-branes wrapping supersymmetric cycles.

## 2 Effective actions and obtaining AdS solutions

## 2.1 Maximal $D = 7$ gauged supergravity and ansatz

The Lagrangian for the bosonic fields of maximal gauged supergravity in  $D = 7$  is given by [31]

$$\begin{aligned} \mathcal{L} = & \sqrt{-g} \left[ R + \frac{1}{2} m^2 (T^2 - 2T_{ij}T^{ij}) - P_{\mu ij} P^{\mu ij} - \frac{1}{2} (\Pi_A^i \Pi_B^j F_{\mu\nu}^{AB})^2 \right. \\ & \left. - m^2 \left( \Pi^{-1}{}^A{}_i S_{\mu\nu\rho, A} \right)^2 \right] - 6m\delta^{AB} S_A \wedge F_B \\ & + \sqrt{3}\epsilon_{ABCDE}\delta^{AG} S_G \wedge F^{BC} \wedge F^{DE} + \frac{1}{8m} (2\Omega_5[B] - \Omega_3[B]) \end{aligned} \quad (2.1)$$

Here  $A, B = 1, \dots, 5$  denote indices of the  $SO(5)_g$  gauge-group, while  $i, j = 1, \dots, 5$  denote indices of the  $SO(5)_c$  local composite gauge-group, which are raised and lowered with  $\delta^{ij}$  and  $\delta_{ij}$ . The 14 scalar fields  $\Pi_A^i$  are given by the coset  $SL(5, \mathbb{R})/SO(5)_c$  and transform as a  $\mathbf{5}$  under both  $SO(5)_g$  (from the left) and  $SO(5)_c$  (from the right). The term that gives the scalar kinetic term,  $P_{\mu ij}$ , and the  $SO(5)_c$  composite gauge field,  $Q_{\mu ij}$ , are defined as the symmetric and antisymmetric parts of  $(\Pi^{-1})^A{}_i (\delta_A^B \partial_\mu + 2mB_{\mu A}{}^B) \Pi_B^k \delta_{kj}$ , respectively. Here  $B_A^B$  are the  $SO(5)_g$  gauge fields with field strength  $F^{AB} = \delta^{AC} F_C^B$ , and note that the gauge coupling constant is given by  $2m$ . The four-form field strength  $F_A$  for the three-form potential  $S_A$ , is given by the covariant derivative  $F_A = dS_A + 2mB_A^B S_B$ . The potential terms for the scalar fields are expressed in terms of  $T_{ij} = \Pi^{-1}{}^A{}_i \Pi^{-1}{}^B{}_j \delta_{AB}$  with  $T = \delta^{ij} T_{ij}$ . Finally,  $\Omega_3[B]$  and  $\Omega_5[B]$  are Chern-Simons forms for the gauge fields  $B$  that will not play a role in this paper. We use a ‘‘mostly plus’’ signature convention for the metric. The supersymmetry transformations of the fermions in these conventions were given in [5].

The new solutions we find here are obtained from the same ansatz used in [5] and [16]. The geometry is taken to be a product of a  $(7 - d)$ -dimensional space with a  $d$ -dimensional cycle  $\Sigma_d$ , so that

$$ds^2 = e^{-2d\phi} ds_{7-d}^2 + e^{2(5-d)\phi} d\bar{s}^2(\Sigma_d) \quad (2.2)$$

where  $ds_{7-d}^2$  is the metric on the uncompact space, and  $d\bar{s}_d^2$  is the metric on the  $d$ -cycle,  $\Sigma_d$ . The scalar field  $\phi$  is assumed to depend on the coordinates on  $ds_{7-d}^2$  and the exponents are chosen so that the reduced  $(7 - d)$ -dimensional effective action has a conventional Einstein term. (Note that in section 5.6 where we discuss the geometry arising from fivebranes wrapped on a product of cycles this metric ansatz is generalised slightly, to allow for a different conformal factor for each cycle. However, it is still precisely the same ansatz used in [16].) Ultimately, we will be interested in

solutions where  $\phi$  is a constant and the  $(7-d)$ -dimensional space is  $AdS$ . For each case we will present the  $AdS$  radius for the line element  $ds_{7-d}^2$  without the conformal factor, as this is proportional to the central charge of the putative dual conformal field theory. Our definition of the radius  $R$  of  $AdS_n$  is given by

$$R_{\mu\nu} = -\frac{n-1}{R^2}g_{\mu\nu} \quad (2.3)$$

Again following the ansatz for supersymmetric solutions, the metric on the  $\Sigma_d$  is assumed to be at least Einstein, satisfying

$$\bar{R}_{ab} = lm^2\bar{g}_{ab} \quad (2.4)$$

where we can always rescale so that  $l = 0, \pm 1$ . (In fact, we will only find  $AdS$  solutions for the cases  $l = \pm 1$ .) The Einstein condition implies that the Riemann tensor can be written

$$\bar{R}_{abcd} = \bar{C}_{abcd} + \frac{2lm^2}{d-1}\bar{g}_{a[c}\bar{g}_{d]b} \quad (2.5)$$

where  $\bar{C}$  is the Weyl tensor and is only present for  $d \geq 4$ . The additional conditions that are placed on  $\bar{C}$  will be discussed case by case later. Note that, for comparison with conventions used in [5, 16], we have  $e^{2(5-d)\phi} = m^2e^{2g}$ .

The ansatz for the  $SO(5)$  gauge fields incorporates the twisting required for supersymmetric solutions and is specified by the spin connection with respect to the metric on  $\Sigma_d$ . In general, we will decompose the  $SO(5)$  symmetry into  $SO(p) \times SO(q)$  with  $p+q=5$ , and excite the gauge fields in the  $SO(p)$  subgroup. The precise form was given in [5, 16] and will be summarized case by case below.

In order to respect the  $SO(p) \times SO(q)$  decomposition, the ansatz for the scalar fields, is restricted to a single scalar mode:

$$\Pi_A^i = \text{diag}(e^{q\lambda}, \dots, e^{q\lambda}, e^{-p\lambda}, \dots, e^{-p\lambda}) \quad (2.6)$$

where we have  $p$  followed by  $q$  entries. This implies that the composite gauge field  $Q$  is then determined by the gauge fields via  $Q^{ij} = 2mB^{ij}$ .

The form of the three-form potentials  $S_A$  and four-form field strengths  $F_A$ , is, in general, determined by the gauge fields and scalar fields, via the  $S_A$  equation of motion. As discussed case by case in [5, 16], there are two distinct classes of ansatz. For  $d \leq 3$  and two cases with  $d = 4$ , one can always set  $S_A = F_A = 0$ . For the remaining cases with  $d = 4$  and for  $d = 5$ , it is consistent to take  $F_A = 0$ , but now  $S_A$  is non-zero. In particular, for the  $d = 4$  cases we always have  $p = 4$  and  $q = 1$ , so that

the  $SO(5)$  index  $A$  is naturally labelled via the split  $A = (m, 5)$  where  $m = 1, \dots, 4$ . We then have  $S_m = 0$  while

$$S_5 = -\frac{1}{2\sqrt{3}} c e^{-8\lambda-4\phi} e^0 \wedge e^1 \wedge e^2 \quad (2.7)$$

where  $(e^0, e^1, e^2)$  are an orthonormal frame for  $ds_{7-d}^2$ , and the constant  $c$  is given by

$$c = \frac{1}{8m^2} \epsilon^{abcd} \epsilon_{mnpq} F_{ab}^{mn} F_{cd}^{pq} \quad (2.8)$$

where  $a, b = 1, \dots, 4$  denote coordinates on  $\Sigma$ . (Note the normalisation of  $c$  is slightly different from that used in [5, 16].) The value of  $c$  depends, through  $F^{mn}$ , on the ansatz for the gauge fields, and so changes case by case. The  $d = 5$  case will be discussed separately in section 6.

Note that much of the structure of this ansatz [5, 16], following [1], arose from requiring the solutions to be supersymmetric. Given this is no longer a requirement, there are, of course, a number of generalisations one might consider. One very simple possibility when  $q > 1$ , is to break the  $SO(q)$  symmetry in the scalar field space so that we now have a set of scalars  $\lambda_1, \dots, \lambda_q$  with

$$\Pi_A^i = \text{diag}(e^{\sum \lambda_i}, \dots, e^{\sum \lambda_i}, e^{-p\lambda_1}, \dots, e^{-p\lambda_q}) \quad (2.9)$$

Actually, this possibility could *a priori* be consistent with supersymmetry as it does not destroy the twisting. We note that we did, in fact, consider generalisations of this type. We find that this leads to one new  $AdS$  solution beyond those found using the simpler ansatz (2.6). Note that this analysis revealed that a solution presented in [34] is not in fact a solution to the equations of motion.

The solutions of  $D = 7$  gauged supergravity containing  $AdS$  factors that we obtain via this ansatz can then be used to obtain solutions to  $D = 11$  supergravity using the uplifting formulae given in [31, 32, 33]. In particular the  $D = 11$  metric takes the form

$$ds_{11}^2 = \Delta^{-2/5} [e^{-2d\phi} ds_{7-d}^2 + e^{2(5-d)\phi} d\bar{s}^2(\Sigma_d)] + m^{-2} \Delta^{4/5} [e^{2q\lambda} DY^a DY^a + e^{-2p\lambda} dY^i dY^i] \quad (2.10)$$

where

$$DY^a = dY^a + 2mB^{ab}Y^b \\ \Delta^{-6/5} = e^{-2q\lambda} Y^a Y^a + e^{2p\lambda} Y^i Y^i \quad (2.11)$$

The indices run over  $a = 1, \dots, p$  and  $i = p + 1, \dots, 5$ , and  $(Y^a, Y^i)$  are constrained coordinates on  $S^4$  satisfying  $Y^a Y^a + Y^i Y^i = 1$ . The presence of non-zero  $\lambda$  and  $B^{ab}$

mean that the sphere is squashed and twisted. The four-form field strength of the eleven-dimensional supergravity is proportional to the volume form on the squashed  $S^4$  together with additional terms due to the gauging. Its precise form is given in [32, 33].

## 2.2 Truncated effective theory

In all but the  $d = 5$  case which will be dealt with separately in section 6, our procedure for finding solutions is as follows. Using the fact that in our ansatz the  $(7 - d)$ -metric and the two scalars  $\phi$  and  $\lambda$  do not depend on the co-ordinates of the cycle  $\Sigma_d$ , we first truncate the seven-dimensional gauged supergravity to obtain an effective  $(7 - d)$ -dimensional theory based on these fields. We then look for stationary points of the effective potential for the scalar fields, which in all cases turn out to be at points where  $V$  is negative. Thus these correspond to solutions where the  $(7 - d)$ -dimensional space is  $AdS$ .

To be explicit, after truncation, the effective Lagrangian in  $7 - d$  dimensions is given by

$$\mathcal{L} = \sqrt{-g} \left[ R - 5d(5 - d) (\partial\phi)^2 - 5pq (\partial\lambda)^2 - V(\phi, \lambda) \right] \quad (2.12)$$

where  $g_{\mu\nu}$  is the metric for the  $(7 - d)$ -dimensional line element  $ds_{7-d}^2$  appearing in (2.2). The first two terms arise from the reduction of the seven-dimensional curvature  $R$  and scalar kinetic  $P^2$  terms in (2.1). The effective potential  $V$  comes from the seven-dimensional curvature  $R$  and the remaining terms and is given by

$$\begin{aligned} m^{-2} V(\phi, \lambda) = & -\frac{1}{2} e^{-2d\phi} \left[ p(p - 2) e^{-4q\lambda} + 2pq e^{2(p-q)\lambda} + q(q - 2) e^{4p\lambda} \right] \\ & - dl e^{-10\phi} + k e^{4q\lambda + 2(d-10)\phi} + \frac{1}{2} c^2 e^{-8\lambda - 16\phi} \end{aligned} \quad (2.13)$$

where

$$k = \frac{1}{2m^2} \bar{g}^{ac} \bar{g}^{bd} F_{ab}^{mn} F_{cd}^{mn} \quad (2.14)$$

and so depends on the ansatz for the gauge fields. The effective potential depends on the integers  $d$ ,  $p$  and  $q = 5 - p$ , the sign  $l$  of the curvature of the Einstein metric on the cycle, as well as the constants  $c$  and  $k$ , all of which vary case by case. It is useful to distinguish between those cases with  $c = 0$  and those with  $c \neq 0$ . Calculating  $k$  in each case, one can show that, in general,

$$\begin{aligned} \text{if } d = 4 \text{ and } p = 4: & \quad c \neq 0, \quad k = c \\ \text{otherwise:} & \quad c = 0, \quad k = \frac{d(2d + 2p - dp)}{8p} \end{aligned} \quad (2.15)$$

Note that, as usual, one must derive this action via substitution of the ansatz into the equations of motion. Substitution directly into the action can lead to the wrong equations of motion. In particular, here one would obtain the wrong sign for the final term in (2.13). More generally we have in fact shown that this reduction is in fact consistent. In other words, that *any* solution to the equations of motion of the reduced theory give rise to a solution of  $D = 7$  gauged supergravity.

Before turning to the general problem of finding the stationary points of  $V$ , note that, since we know that the ansatz admits supersymmetric solutions, we might expect that the effective action has a simple supersymmetric generalisation. In particular, it may well be possible to extend the ansatz we are considering to obtain a consistently truncated supersymmetric theory. We shall leave such an investigation to future work, but let us note that we can recast the effective potential in terms of a putative superpotential  $W$  as follows:

$$\frac{1}{2}V = K^{ab} (\partial_a W \partial_b W) - \beta^2 W^2 \quad (2.16)$$

where  $a$  and  $b$  label scalar fields  $A^a = (\lambda, \phi)$ ,  $K^{ab}$  is the inverse of the sigma-model metric for the scalar kinetic terms  $2K_{ab}\partial A^a\partial A^b = 5d(5-d)(\partial\phi)^2 + 5pq(\partial\lambda)^2$  and the constant  $\beta^2$  depends on the dimension of the effective theory and is given by

$$\beta^2 = \frac{2(6-d)}{5-d} \quad (2.17)$$

Explicitly, we have

$$W = \frac{1}{4}me^{-d\phi} (pe^{-2q\lambda} + qe^{2p\lambda}) + \frac{1}{8}mdle^{2q\lambda+(d-10)\phi} - \frac{1}{4}mce^{-4\lambda-8\phi} \quad (2.18)$$

provided that  $k$  and  $c$  are given as in (2.15). For the four-dimensional case, where  $d = 3$ , in the supersymmetric extension we must have two additional bosonic fields to partner  $\phi$  and  $\lambda$  to form two chiral multiplets. The form (2.16), can be derived from  $N = 1$  supergravity coupled to two chiral superfields truncated to the  $\phi$  and  $\lambda$  sector. In other dimensions, the form naturally generalises that obtained in, for example, [35] for the case a single scalar field.

### 2.3 Stationary points and stability

For solutions with constant  $ds_{7-d}^2$  curvature, and in particular  $AdS$  space, the scalars  $\phi$  and  $\lambda$  are constant. Thus we need to find the stationary points of the potential  $V$ . Let us introduce the variables

$$\begin{aligned} x &= e^{10\lambda} \\ y &= le^{2(d-5)\phi+2(q-p)\lambda} \end{aligned} \quad (2.19)$$



The conditions for a stationary point of  $V$  imply, in all cases, that

$$x = \frac{2y + p}{(4k/d)y^2 + (p - 3)} \quad (2.20)$$

together with, when  $d = p = 4$  (so  $c \neq 0$ ),

$$(y + 1) (cy^2 + 1) (cy^3 + 3cy^2 - 2y - 3) = 0 \quad (2.21)$$

while otherwise, i.e. when  $d \neq 4$  or  $p \neq 4$  (so  $c = 0$ ),

$$(y + 1) (k(dp - 2p - 4d + 4)y^3 - k(dp - 10p + 24)y^2 - d(p - 3)(2y + 3)) = 0 \quad (2.22)$$

Note that in every case (except when  $d = 2p$ ) there is a solution

$$y = -1$$

$$x = \begin{cases} \frac{2p}{2p-d} & \text{if } d \neq 4 \text{ or } p \neq 4 \\ \frac{2}{c+1} & \text{if } d = p = 4 \end{cases} \quad (2.23)$$

As we will see, these solutions correspond to the supersymmetric fixed points as found in [5, 16]. Note that, since by definition,  $y/l > 0$  and  $x > 0$ , these cases always have  $l = -1$  so the cycle has negative curvature. In general, since  $c > 0$ , the remaining non-supersymmetric solutions come from roots of the cubic factors in (2.21) and (2.22). It turns out that in all cases, the value of the potential at the stationary point  $V_0$  is negative. Thus all our solutions correspond to  $AdS$  fixed points. In general, the radius  $R$  of  $AdS_n$  (here  $n = 7 - d$ ) is given by

$$R^2 = -\frac{(n-1)(n-2)}{V_0} \quad (2.24)$$

One important question is whether these  $AdS$  solutions are stable to fluctuations. In the following, we do not make a full stability analysis for fluctuations in all possible modes in the seven-dimensional supergravity theory (2.1). Instead, we concentrate on the scalar modes  $\lambda$  and  $\phi$  which already have non-trivial values in the solutions. In particular, we calculate the mass eigenvalues of the  $(\lambda, \phi)$  fluctuations about the fixed points. It is important to recall that in  $AdS$  space, instability is characterised not by simply a negative mass-squared  $M^2$ , but rather a negative mass-squared violating the Breitenlohner–Freedman (BF) bound [24, 25, 26]. For  $AdS_n$  we have

$$M^2 R^2 \geq -\frac{1}{4}(n-1)^2 \quad (2.25)$$

In fact, we actually considered more general fluctuations. In particular, allowing a set of supergravity scalars  $\lambda_1, \dots, \lambda_q$  as in (2.9), breaking the  $SO(q)$  symmetry in the scalar field space. We showed that this is again a consistent truncation and that it does in fact lead to instabilities in three cases.

### 3 $AdS_5 \times \Sigma_2$ solutions

We start by considering the  $AdS$  solutions with  $d = 2$ , when  $\Sigma_d$  is a two-cycle. The  $D = 7$  gauged supergravity metric (2.2) in the solutions is then given by

$$ds^2 = \frac{e^{-4\phi} R^2}{r^2} [ds^2(\mathbb{R}^{1,3}) + dr^2] + e^{6\phi} d\bar{s}^2(\Sigma_2) \quad (3.1)$$

where  $e^{6\phi}$  and  $R^2$  are constants. Since the two-cycle has an Einstein metric it is either an  $S^2$ ,  $\mathbb{R}^2$ ,  $H^2$  or a quotient of these spaces by a discrete group of isometries. Here and throughout we will not find any  $AdS$  solutions based on flat-cycles, so we will not mention them further. There are two cases to be considered: the first uses the ansatz used to obtain the supersymmetric solutions corresponding to the near horizon limits of M5-branes wrapping Kähler two-cycles in Calabi–Yau two-folds [1], while the second uses the ansatz corresponding to M5-branes wrapping Kähler two-cycles in Calabi–Yau four-folds [1]. Let us discuss each case in turn. For the stability analysis, note that in this case the BF bound is  $M^2 R^2 \geq -4$ , where  $M^2$  is the mass-squared of the fluctuation.

#### 3.1 Kähler two-cycle in $CY_2$

For this case we take  $p = 2, q = 3$ , so the scalar field ansatz (2.6) reads

$$\Pi = \text{diag}(e^{3\lambda}, e^{3\lambda}, e^{-2\lambda}, e^{-2\lambda}, e^{-2\lambda}) \quad (3.2)$$

The  $SO(5)$  gauge fields are decomposed into  $SO(2) \times SO(3)$  and the  $SO(3)$  gauge fields are set to zero. The  $SO(2)$  gauge fields, denoted by  $B^{12}$ , are determined by the  $SO(2)$  spin-connection of the two-cycle so that

$$B^{12} = -\frac{1}{4m} \bar{\omega}_{ab} J^{ab} \quad (3.3)$$

where  $J$  is the Kähler structure of the two-cycle. This implies  $k = 1/2$ . As noted above  $c = 0$  for this example.

We find that the effective potential has two minima. The first occurs when  $l = -1$  and gives rise to the  $AdS_5 \times H^2$  supersymmetric solution found in [1]

$$\begin{aligned} e^{10\lambda} &= 2 \\ e^{6\phi} &= e^{2\lambda} \approx 1.1487 \\ m^2 R^2 &= 2^{4/3} \approx 2.5198 \end{aligned} \quad (3.4)$$

The second solution on the other hand is new and has  $l = +1$ . Thus it is an  $AdS_5 \times S^2$  solution and has

$$\begin{aligned} e^{10\lambda} &\approx 6.6056 \\ e^{6\phi} &\approx 1.1197 \\ m^2 R^2 &\approx 1.4623 \end{aligned} \tag{3.5}$$

This solution breaks all supersymmetry.

It is straightforward to determine the masses of the two scalar fields about these solutions. After diagonalising the mass matrix we find that  $\phi, \lambda$  give rise to fluctuations with mass-squared  $M^2$  satisfying:

$$M^2 R^2 = -4, 12 \tag{3.6}$$

for the supersymmetric solution, and

$$M^2 R^2 \approx -5.58, 22.1 \tag{3.7}$$

for the non-supersymmetric solution. Note that the BF bound is not violated for the supersymmetric solution, as expected, but is violated for the non-supersymmetric solution (3.5). This instability implies that this solution cannot be the dual of a new non-supersymmetric CFT.

For this case, one can generalise the ansatz to include two additional scalar fields as in (2.9). However, we have checked that this leads to no further  $AdS_5$  solutions.

### 3.2 Kähler two-cycle in $CY_3$

We now have  $p = 4, q = 1$ , so the scalar field ansatz is given by

$$\Pi = \text{diag} (e^\lambda, e^\lambda, e^\lambda, e^\lambda, e^{-4\lambda}) \tag{3.8}$$

For the gauge fields we first decompose  $SO(4) \rightarrow U(2) \sim U(1) \times SU(2)$  and then let the  $U(1)$  factor be determined by the spin connection of the two-cycle. In other words we can choose a basis such that the only non-zero components are given by

$$B^{12} = B^{34} = -\frac{1}{8m} \bar{\omega}_{ab} J^{ab} \tag{3.9}$$

This gives  $k = 1/4$  and we have  $c = 0$ .

We again find that there are two solutions. The first is  $AdS_5 \times H^2$ , that is  $l = -1$ , with

$$\begin{aligned} e^{10\lambda} &= \frac{4}{3} \\ e^{6\phi} &= \frac{3}{4}e^{4\lambda} \approx 0.84147 \\ m^2 R^2 &= \frac{9}{4} = 2.25 \end{aligned} \tag{3.10}$$

We thus recover the supersymmetric  $AdS_5 \times H^2$  fixed point first presented in [1]. The second solution on the other hand is again new and has  $l = -1$  so the space is  $AdS_5 \times H^2$ , with

$$\begin{aligned} e^{10\lambda} &\approx 1.5536 \\ e^{6\phi} &\approx 0.84824 \\ m^2 R^2 &\approx 2.2496 \end{aligned} \tag{3.11}$$

This solution does not preserve any supersymmetry.

The masses of the  $(\phi, \lambda)$  fluctuations about these solutions are given by

$$M^2 R^2 \approx -1.29, 9.29 \tag{3.12}$$

for the supersymmetric solution, and

$$M^2 R^2 \approx 1.40, 9.38 \tag{3.13}$$

and for the non-supersymmetric solution, so that in all cases the BF bound is satisfied.

## 4 $AdS_4 \times \Sigma_3$ solutions

These solutions are obtained when  $d = 3$  so  $\Sigma_d$  is a three-cycle. The  $D = 7$  gauged supergravity metric for  $AdS$  solutions is given by

$$ds^2 = \frac{e^{-6\phi} R^2}{r^2} [ds^2(\mathbb{R}^{1,2}) + dr^2] + e^{4\phi} d\bar{s}^2(\Sigma_3) \tag{4.1}$$

where  $e^{4\phi}$  and  $R^2$  are constants. Being Einstein, the three-cycle has constant curvature and for  $l = 1$  is  $S^3$  and for  $l = -1$  is  $H^3$ , or a quotient of these spaces by a discrete group of isometries. There are two cases to be considered: the first uses the ansatz used to obtain the supersymmetric solutions corresponding to M5-branes wrapping SLAG three-cycles in Calabi–Yau three-folds [5] (see also [34]), while the second uses the ansatz corresponding to M5-branes wrapping associative three-cycles in manifolds with  $G_2$  holonomy [5]. We discuss each case in turn and note that the BF bound now reads  $M^2 R^2 \geq -9/4$ .

## 4.1 SLAG three-cycle in $CY_3$

In this example we have  $p = 3, q = 2$  so the scalar field ansatz (2.6) reads

$$\Pi = \text{diag}(e^{2\lambda}, e^{2\lambda}, e^{2\lambda}, e^{-3\lambda}, e^{-3\lambda}) \quad (4.2)$$

The  $SO(5)$  gauge fields are decomposed into  $SO(3) \times SO(2)$ . The  $SO(2)$  gauge fields are set to zero while the  $SO(3)$  gauge fields are determined by the  $SO(3)$  spin-connection of the three-cycle. Thus if we denote the non-zero fields by  $B^{ab}$  with  $a, b = 1, 2, 3$  we have

$$B^{ab} = \frac{1}{2m} \bar{\omega}^{ab} \quad (4.3)$$

This implies  $k = 3/8$  and again  $c = 0$  for this example.

We find that the effective potential has two minima, both with  $l = -1$ . The first is the supersymmetric  $AdS_4 \times H^3$  solution of [5, 34]:

$$\begin{aligned} e^{10\lambda} &= 2 \\ e^{4\phi} &= \frac{1}{2} e^{8\lambda} \approx 0.87055 \\ m^2 R^2 &= \sqrt{2} \approx 1.4142 \end{aligned} \quad (4.4)$$

The second  $AdS_4 \times H^3$  solution is non-supersymmetric and was in fact first found in [34]:

$$\begin{aligned} e^{10\lambda} &= 10 \\ e^{4\phi} &\approx 1.0516 \\ m^2 R^2 &\approx 1.3608 \end{aligned} \quad (4.5)$$

The masses of the  $(\phi, \lambda)$  fluctuations about the supersymmetric solution are given by

$$M^2 R^2 \approx -1.12, 7.12 \quad (4.6)$$

while for the non-supersymmetric solution they are

$$M^2 R^2 \approx 0.555, 8.64 \quad (4.7)$$

These all satisfy the BF bound.

For this example we can consider adding additional scalar fields via (2.9). In particular one can take

$$\Pi = \text{diag}(e^{2\lambda}, e^{2\lambda}, e^{2\lambda}, e^{-3\lambda+\alpha}, e^{-3\lambda-\alpha}) \quad (4.8)$$

We have checked that the claim of [34] that this leads to an additional  $AdS$  fixed point is *not* correct. In particular  $\alpha$  is not real. It is interesting to note that the extra scalar field  $\alpha$  does not violate the BF bound. In particular we find

$$M^2 R^2 = 4, 68 \tag{4.9}$$

for the supersymmetric and non-supersymmetric fixed points, respectively.

## 4.2 Associative three-cycle in a manifold of $G_2$ holonomy

For this case we have  $p = 4, q = 1$  and the scalar field ansatz (2.6) is given by

$$\Pi = \text{diag}(e^\lambda, e^\lambda, e^\lambda, e^\lambda, e^{-4\lambda}) \tag{4.10}$$

The non-zero  $SO(5)$  gauge fields are taken to lie in an  $SU(2)_L \subset SU(2)_L \times SU(2)_R \subset SO(5)$  subgroup. If we denote the non-vanishing gauge fields by  $B^{mn}$ , with  $m, n = 1, 2, 3, 4$ , they satisfy  $B^+ = 0$ , and  $B^-$  is determined by the  $SO(3)$  spin-connection of the three-cycle:

$$\begin{aligned} B^{-23} &= -\frac{1}{4m}\bar{\omega}^{23} \\ B^{-31} &= -\frac{1}{4m}\bar{\omega}^{31} \\ B^{-12} &= -\frac{1}{4m}\bar{\omega}^{12} \end{aligned} \tag{4.11}$$

This implies  $k = 3/16$  and again  $c = 0$  for this example.

We find that the effective potential has two minima, both with  $l = -1$ . The first gives the supersymmetric  $AdS_4 \times H_3$  solution of [3] with

$$\begin{aligned} e^{10\lambda} &= \frac{8}{5} \\ e^{4\phi} &= \frac{5}{8}e^{4\lambda} \approx 0.75427 \\ m^2 R^2 &= 2^{-11/2}5^{5/2} \approx 1.2353 \end{aligned} \tag{4.12}$$

while the second gives a new non-supersymmetric  $AdS_4 \times H_3$  solution with

$$\begin{aligned} e^{10\lambda} &\approx 1.2839 \\ e^{4\phi} &\approx 0.75049 \\ m^2 R^2 &\approx 1.2362 \end{aligned} \tag{4.13}$$

The masses of the  $(\phi, \lambda)$  fluctuations satisfy

$$M^2 R^2 = 1.55, 6.45 \tag{4.14}$$

for the supersymmetric solution and

$$M^2 R^2 = -1.38, 6.40 \tag{4.15}$$

for the non-supersymmetric solution and all satisfy the BF bound.

## 5 $AdS_3 \times \Sigma_4$ solutions

These solutions are obtained when  $d = 4$  so that  $\Sigma_d$  is a four-cycle. The  $D = 7$  gauged supergravity metric for  $AdS$  solutions is given by

$$ds^2 = \frac{e^{-8\phi} R^2}{r^2} [ds^2(\mathbb{R}^{1,1}) + dr^2] + e^{2\phi} d\bar{s}^2(\Sigma_4) \tag{5.1}$$

where  $e^{2\phi}$  and  $R^2$  are constants. There are now a number of different possibilities for the Einstein metric on  $\Sigma_4$  depending on which case we are considering. In all but the case corresponding to a M5-branes wrapping a co-associative cycle in a  $G_2$ -holonomy manifold, we have  $k = c$ , again with a value depending on the particular case. Note that we do not give details of the case analogous to the M5-brane wrapping a Kähler four-cycle in a Calabi–Yau three-fold as we do not find any  $AdS_3$  solution, even after adding in extra scalar fields. Note that the BF bound now reads  $M^2 R^2 \geq -1$ .

### 5.1 Co-associative four-cycle in a manifold of $G_2$ holonomy

For this case we have  $p = 3$ ,  $q = 2$  so the scalar ansatz (2.6) is given by

$$\Pi = \text{diag}(e^{2\lambda}, e^{2\lambda}, e^{2\lambda}, e^{-3\lambda}, e^{-3\lambda}) \tag{5.2}$$

The non-zero gauge fields are taken to lie in  $SO(3) \subset SO(5)$ . We denote them by  $B^{mn}$ , with  $m, n = 1, 2, 3$  and they are determined by the anti-self-dual part of the spin connection of the four-cycle  $\omega^-$ . Explicitly we let

$$\begin{aligned} B^{23} &= -\frac{1}{m} \bar{\omega}^{-12} \\ B^{31} &= -\frac{1}{m} \bar{\omega}^{-13} \\ B^{12} &= -\frac{1}{m} \bar{\omega}^{-14} \end{aligned} \tag{5.3}$$

This implies  $k = 1/3$  and we have still have  $c = 0$ . The four-cycle is Einstein and also conformally half-flat, so that the Weyl tensor is self-dual. Note that for  $l = 1$  this means that it is either  $S^4$  or  $CP^2$  if it is compact. For  $l = -1$  we denote these spaces by  $C_-^4$ .

We find that the effective potential has only one minima with  $l = -1$ . This is the supersymmetric  $AdS_3 \times C_-^4$  fixed fixed point found in [5]:

$$\begin{aligned} e^{10\lambda} &= 3 \\ e^{2\phi} &= \frac{1}{3}e^{8\lambda} \approx 0.80274 \\ m^2 R^2 &= \frac{4}{9} = 0.44444 \end{aligned} \tag{5.4}$$

The masses of the  $(\phi, \lambda)$  fluctuations are given by

$$M^2 R^2 = 0, 40/9 \tag{5.5}$$

and so satisfy the BF bound.

We also can consider the more general ansatz for the scalar fields

$$\Pi = \text{diag}(e^{2\lambda}, e^{2\lambda}, e^{2\lambda}, e^{-3\lambda+\alpha}, e^{-3\lambda-\alpha}) \tag{5.6}$$

However, we find no new  $AdS$  fixed points. In addition we checked that the mass of the scalar field  $\alpha$  is given by

$$M^2 R^2 = 8 \tag{5.7}$$

and so again these modes do not destabilise the solution.

## 5.2 SLAG four-cycle in $CY_4$

We now have  $p = 4, q = 1$  so the scalar ansatz (2.6) takes the form

$$\Pi = \text{diag}(e^\lambda, e^\lambda, e^\lambda, e^\lambda, e^{-4\lambda}) \tag{5.8}$$

The non-zero gauge fields, denoted by  $B^{ab}$  with  $a, b = 1, 2, 3, 4$ , are taken to lie in an  $SO(4)$  subgroup of  $SO(5)$  and are determined by the  $SO(4)$  spin connection:

$$B^{ab} = \frac{1}{2m}\bar{\omega}^{ab} \tag{5.9}$$

From (2.14) and (2.15) we then have  $k = c = 1/3$ . The four-cycle is Einstein and now the Weyl tensor must vanish. In other words  $\Sigma_4$  is  $S^4$  for  $l = 1$  and  $H^4$  for  $l = -1$  or, as usual, a quotient of these spaces by a finite group of discrete isometries.

We now find three minima of the effective potential. For  $l = -1$ , we find the supersymmetric  $AdS_3 \times H^4$  fixed point of [5]:

$$\begin{aligned} e^{10\lambda} &= \frac{3}{2} \\ e^{2\phi} &= e^{-6\lambda} \approx 0.78405 \\ m^2 R^2 &= \frac{4}{9} \approx 0.44444 \end{aligned} \tag{5.10}$$



and also a non-supersymmetric  $AdS_3 \times H^4$  solution

$$\begin{aligned} e^{10\lambda} &\approx 1.2592 \\ e^{2\phi} &\approx 0.78367 \\ m^2 R^2 &\approx 0.44474 \end{aligned} \tag{5.11}$$

For  $l = 1$  we find the non-supersymmetric  $AdS_3 \times S^4$  solution

$$\begin{aligned} e^{10\lambda} &\approx 3.3694 \\ e^{2\phi} &\approx 0.23488 \\ m^2 R^2 &\approx 0.00080517 \end{aligned} \tag{5.12}$$

For supersymmetric solution the masses of the  $(\phi, \lambda)$  fluctuations are given by

$$M^2 R^2 \approx 0.697, 4.30 \tag{5.13}$$

while for the non-supersymmetric solutions, with  $l = -1$  we have

$$M^2 R^2 \approx -0.628, 4.31 \tag{5.14}$$

and for  $l = 1$  we get

$$M^2 R^2 \approx 5.24, 9.12 \tag{5.15}$$

In all cases the BF bound is satisfied.

### 5.3 Kähler four-cycle in $CY_4$

We again have  $p = 4, q = 1$  with the scalar ansatz (2.6) of the form

$$\Pi = \text{diag}(e^\lambda, e^\lambda, e^\lambda, e^\lambda, e^{-4\lambda}) \tag{5.16}$$

The only non-vanishing  $SO(5)$  gauge fields are in the  $U(1)$  given by the decomposition  $U(1) \times SU(2) \approx U(2) \subset SO(5)$ . In this case the cycle must be Einstein and Kähler, denoted by  $K_+$  and  $K_-$  depending on whether  $l = 1$  or  $-1$ . Note that  $CP^2$  and  $CP^1 \times CP^1$  provide examples of compact  $K_+$ . The gauge fields are determined by the  $U(1)$  part of the  $SO(4)$  spin connection of the four-cycle via the decomposition  $U(1) \times SU(2) \approx U(2) \subset SO(4)$ . Explicitly, we take

$$B^{12} = B^{34} = -\frac{1}{8m} \bar{\omega}_{ab} J^{ab} \tag{5.17}$$

with all other gauge fields zero, where  $J^{ab}$  is the Kähler-form of the four-cycle. We then have  $k = c = 1/2$ .

We find that the effective potential has two minima. For  $l = -1$  we recover the  $AdS_3 \times K_-$  supersymmetric fixed point:

$$\begin{aligned} e^{10\lambda} &= \frac{4}{3} \\ e^{2\phi} &= e^{-6\lambda} \approx 0.84147 \\ m^2 R^2 &= \frac{9}{16} = 0.5625 \end{aligned} \tag{5.18}$$

and for  $l = 1$  we find the non-supersymmetric  $AdS_3 \times K_+$  solution

$$\begin{aligned} e^{10\lambda} &\approx 3.0972 \\ e^{2\phi} &\approx 0.30836 \\ m^2 R^2 &\approx 0.0027879 \end{aligned} \tag{5.19}$$

For the supersymmetric solution the masses of the  $(\phi, \lambda)$  fluctuations are given by

$$M^2 R^2 = 0, 40/9 \tag{5.20}$$

and for the non-supersymmetric solutions they are given by

$$M^2 R^2 \approx 5.16, 8.72 \tag{5.21}$$

All satisfy the BF bound.

## 5.4 Cayley four-cycle in a manifold of $Spin(7)$ holonomy

Again we have  $p = 4, q = 1$  with the scalar ansatz (2.6)

$$\Pi = \text{diag}(e^\lambda, e^\lambda, e^\lambda, e^\lambda, e^{-4\lambda}) \tag{5.22}$$

Now, however, we retain only the anti-selfdual part of the  $SO(4) \subset SO(5)$ , so that the non-zero gauged fields, denoted by  $B^{ab}$  with  $a, b = 1, \dots, 4$ , satisfy  $B^+ = 0$  together with

$$B^{-ab} = \frac{1}{2m} \bar{\omega}^{-ab} \tag{5.23}$$

Thus, they are determined by the anti-self dual part of the  $SO(4)$  spin connection,  $\omega^-$ , of the four-cycle. This gives  $k = c = 1/6$ . In addition to being Einstein, the four-cycle must now be conformally half-flat, which means that the Weyl tensor is self-dual. For  $l = 1$  this means that it is either  $S^4$  or  $CP^2$  if it is compact. For  $l = -1$  we denote these spaces by  $C_-^4$ .

We find three minima of the effective potential. For  $l = -1$  we find the supersymmetric  $AdS_3 \times C_-^4$  solution found in [5]

$$\begin{aligned} e^{10\lambda} &= \frac{12}{7} \\ e^{2\phi} &= e^{-6\lambda} \approx 0.72369 \\ m^2 R^2 &= \frac{49}{144} = 0.34028 \end{aligned} \tag{5.24}$$

as well as the non-supersymmetric  $AdS_3 \times C_-^4$  solution

$$\begin{aligned} e^{10\lambda} &\approx 1.1547 \\ e^{2\phi} &\approx 0.72346 \\ m^2 R^2 &\approx 0.34328 \end{aligned} \tag{5.25}$$

While with  $l = 1$  we find the non-supersymmetric  $AdS_3 \times C_+^4$  solution where  $C_+^4$  is either  $S^4$  or  $CP^2$ ,

$$\begin{aligned} e^{10\lambda} &= 4 \\ e^{2\phi} &= \frac{1}{3}e^{-6\lambda} \approx 0.14509 \\ m^2 R^2 &= 2^{-4}3^{-6} \approx 8.5734 \times 10^{-5} \end{aligned} \tag{5.26}$$

The masses of the  $(\phi, \lambda)$  fluctuations about the supersymmetric solution are given by

$$M^2 R^2 \approx 1.89, 4.15 \tag{5.27}$$

while for the non-supersymmetric solutions, with  $l = -1$  we find

$$M^2 R^2 \approx -1.50, 4.15 \tag{5.28}$$

and for  $l = 1$  we find

$$M^2 R^2 \approx 5.33, 9.78 \tag{5.29}$$

All satisfy the BF bound.

## 5.5 Complex-Lagrangian four-cycle in $HK_8$

Once more we have  $p = 4, q = 1$  and the scalar ansatz (2.6) takes the form

$$\Pi = \text{diag}(e^\lambda, e^\lambda, e^\lambda, e^\lambda, e^{-4\lambda}) \tag{5.30}$$

The non-zero  $SO(5)$  gauge fields lie in an  $U(2)$  subgroup and are given by the  $U(2)$  spin connection of the four-cycle which must be Kähler. Denoting the non-zero fields by  $B^{ab}$  with  $a, b = 1, 2, 3, 4$  we have

$$B^{ab} = \frac{1}{2m} \bar{\omega}^{ab} \quad (5.31)$$

This gives  $k = c = 1/6$ . In fact the Einstein Kähler cycle must actually have constant holomorphic sectional curvature. This means for  $l = 1$  we have  $CP^2$  and for  $l = -1$  we have the Bergmann metric  $B$  on a bounded domain in  $\mathbb{C}^2$ . As usual we can take quotients of these spaces and in particular for  $l = -1$  we can have compact spaces.

The effective potential has three minima. For  $l = -1$  we find the supersymmetric  $AdS_3 \times B$  solution found in [16]

$$\begin{aligned} e^{10\lambda} &= \frac{6}{5} \\ e^{2\phi} &= e^{-6\lambda} \approx 0.89638 \\ m^2 R^2 &= \frac{25}{36} \approx 0.69444 \end{aligned} \quad (5.32)$$

and a non-supersymmetric  $AdS_3 \times B$  solution:

$$\begin{aligned} e^{10\lambda} &\approx 1.3890 \\ e^{2\phi} &\approx 0.89582 \\ m^2 R^2 &\approx 0.69422 \end{aligned} \quad (5.33)$$

For  $l = 1$  we find a non-supersymmetric  $AdS_3 \times CP^2$  solution:

$$\begin{aligned} e^{10\lambda} &\approx 2.9378 \\ e^{2\phi} &\approx 0.37238 \\ m^2 R^2 &\approx 0.0065244 \end{aligned} \quad (5.34)$$

The masses of the  $(\phi, \lambda)$  fluctuations for the supersymmetric solution are given by

$$M^2 R^2 \approx -0.419, 4.58 \quad (5.35)$$

and for the non-supersymmetric solutions, for  $l = -1$

$$M^2 R^2 \approx 0.462, 4.55 \quad (5.36)$$

and for  $l = 1$

$$M^2 R^2 \approx 5.10, 8.44 \quad (5.37)$$

All satisfy the BF bound.

## 5.6 SLAG four-cycle in $CY_2 \times CY_2$

Again we take  $p = 4, q = 1$  and the scalars are given by

$$\Pi = \text{diag}(e^\lambda, e^\lambda, e^\lambda, e^\lambda, e^{-4\lambda}) \quad (5.38)$$

We first assume that the four-cycle is the product of two Einstein two-metrics each satisfying (2.4), since in this case the four-cycle is also Einstein. Being Einstein, each two-cycle has constant curvature and hence we are considering four-cycles of the form  $H^2 \times H^2$  for  $l = -1$  and  $S^2 \times S^2$  for  $l = +1$ , or a quotient thereof. The twisting is obtained by first decomposing  $SO(5) \rightarrow SO(4) \rightarrow SO(2) \times SO(2)$  (with  $\mathbf{4} \rightarrow (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$  in the last step) and identifying the  $SO(2)$  factors with the spin connections on each two cycle. Thus the only non-zero gauge fields are given by

$$B^{12} = \frac{1}{2m}\bar{\omega}^{12} \quad B^{34} = \frac{1}{2m}\bar{\omega}^{34} \quad (5.39)$$

This then gives  $k = c = 1$ .

The effective potential has three minima. For  $l = -1$  we find the supersymmetric  $AdS_3 \times H^2 \times H^2$  solution found in [16]

$$\begin{aligned} e^{10\lambda} &= 1 \\ e^{2\phi} &= 1 \\ m^2 R^2 &= 1 \end{aligned} \quad (5.40)$$

as well as a non-supersymmetric  $AdS_3 \times H^2 \times H^2$  solution:

$$\begin{aligned} e^{10\lambda} &\approx 1.4678 \\ e^{2\phi} &\approx 0.99497 \\ m^2 R^2 &\approx 0.99531 \end{aligned} \quad (5.41)$$

For  $l = 1$  we find a non-supersymmetric  $AdS_3 \times S^2 \times S^2$  solution:

$$\begin{aligned} e^{10\lambda} &\approx 2.7523 \\ e^{2\phi} &\approx 0.48274 \\ m^2 R^2 &\approx 0.020723 \end{aligned} \quad (5.42)$$

The masses of the  $(\phi, \lambda)$  fluctuations are now given by

$$M^2 R^2 \approx -0.828, 4.83 \quad (5.43)$$

for supersymmetric solution, while for the non-supersymmetric solutions, for  $l = -1$

$$M^2 R^2 \approx 1.09, 4.70 \quad (5.44)$$

and for  $l = 1$

$$M^2 R^2 \approx 5.02, 8.07 \quad (5.45)$$

Again all satisfy the BF bound.

For this case we can consider a more general ansatz than we have considered so far. For the metric we take

$$ds^2 = \frac{e^{-8\phi} R^2}{r^2} [ds^2(\mathbb{R}^{1,1}) + dr^2] + e^{2\phi} [e^{2h} d\bar{s}^2(\Sigma_2^1) + e^{-2h} d\bar{s}^2(\Sigma_2^2)] \quad (5.46)$$

where we have introduced a new function  $e^{2h}$  which again only depends on the coordinates on the three-space. We also allow the signs of the curvature of the two two-cycles,  $l_1$  and  $l_2$  to be in general unequal. The scalar field ansatz is also generalised to include an extra scalar field

$$\Pi = \text{diag}(e^{\lambda+\alpha}, e^{\lambda+\alpha}, e^{\lambda-\alpha}, e^{\lambda-\alpha}, e^{-4\lambda}) \quad (5.47)$$

The gauge fields and the three-forms are determined as before. We have checked that this again leads to a consistently truncated three-dimensional theory of gravity coupled to four scalar fields. Let us record the effective three-dimensional Lagrangian:

$$\mathcal{L} = \sqrt{-g} \left[ R - 20 (\partial\phi)^2 - 20 (\partial\lambda)^2 - 4 (\partial h)^2 - 4 (\partial\alpha)^2 - V(\phi, \lambda) \right] \quad (5.48)$$

where the effective potential is now given by

$$\begin{aligned} m^{-2} V(\phi, \lambda) = & - 2e^{-10\phi} (l_1 e^{-2h} + l_2 e^{2h}) - \frac{1}{2} e^{-8\phi} (8e^{-4\lambda} + 8e^{6\lambda} \cosh(2\alpha) - e^{16\lambda}) \\ & + e^{4\lambda-12\phi} \cosh 4(\alpha - h) + \frac{1}{2} e^{-16\phi-8\lambda} \end{aligned} \quad (5.49)$$

In addition to the solutions found above we find one more, with  $l_1 = 1$ ,  $l_2 = -1$ , i.e.  $AdS_3 \times S^2 \times H^2$ , with

$$\begin{aligned} e^{10\lambda} &\approx 2.4453 \\ e^{2\alpha} &\approx 2.1367 \\ e^{2\phi} &\approx 0.73162 \\ e^{2h} &\approx 1.3430 \\ m^2 R^2 &\approx 0.17815 \end{aligned} \quad (5.50)$$

It is also interesting to determine the masses of the fluctuations of the scalar fields  $\alpha$  and  $h$  for the solutions (5.40), (5.41) and (5.42). For these cases we find a block-diagonal mass matrix when combined with  $\phi, \lambda$ . After diagonalising the new block, for the supersymmetric  $AdS_3 \times H^2 \times H^2$  solution (5.40) we find:

$$M^2 R^2 = -0.828, 4.83 \quad (5.51)$$

for the non-supersymmetric  $AdS_3 \times H^2 \times H^2$  solution (5.41) we find

$$M^2 R^2 = -1.18, 5.45 \quad (5.52)$$

which violates the BF bound, and for the non-supersymmetric  $AdS_3 \times S^2 \times S^2$  solution (5.42) we find

$$M^2 R^2 = -1.49, 8.33 \quad (5.53)$$

which again violates the BF bound. For the non-supersymmetric case (5.50), we find after diagonalising the non-block diagonal  $(\phi, \lambda, \alpha, h)$  mass matrix, the following masses:

$$M^2 R^2 = 9.56, 5.04, 4.33, -1.89 \quad (5.54)$$

which again violates the BF bound. Hence, of the product cycle cases, it is only the supersymmetric one that does not violate the BF bound, which is perhaps expected.

## 6 $AdS_2 \times \Sigma_5$ solutions

Unlike the previous cases, the new solutions with  $AdS_2$  factors were found directly from the  $D = 7$  equations of motion without deriving an effective two-dimensional theory. There are two cases to consider.

### 6.1 SLAG five-cycle in $CY_5$

The metric for this case is taken to be

$$ds^2 = \frac{R^2}{r^2} [-dt^2 + dr^2] + e^{2g} d\bar{s}^2(\Sigma_5) \quad (6.1)$$

The scalar fields are taken to be trivial

$$\Pi_A{}^i = \delta_A{}^i \quad (6.2)$$

in order to keep the full  $SO(5)$  symmetry, and the  $SO(5)$  gauge fields are given by the  $SO(5)$  spin-connection of the five-cycle:

$$B_{ab} = \frac{1}{2m} \bar{\omega}_{ab} \quad (6.3)$$

The five-cycle is taken to be not only Einstein, but to have constant curvature. In other words  $S^5$  for  $l = 1$  and  $H^5$  for  $l = -1$ . All five three-forms are now active and we find

$$S_a = -\frac{\sqrt{3}e^{-4g}}{32} e^0 \wedge e^r \wedge e^a \quad (6.4)$$

With this ansatz we only find two  $AdS_2$  solutions, both are supersymmetric and were found in [5]. The first has  $l = -1$ , i.e.,  $AdS_2 \times H^5$ , with

$$\begin{aligned} e^{2g} &= \frac{3}{4} \\ m^2 R^2 &= \frac{9}{16} = 0.5625 \end{aligned} \tag{6.5}$$

The second has  $l = +1$ , i.e.,  $AdS_2 \times S^5$ , with

$$\begin{aligned} e^{2g} &= \frac{1}{4} \\ m^2 R^2 &= \frac{1}{16} = 0.0625 \end{aligned} \tag{6.6}$$

## 6.2 SLAG five-cycle in $CY_3 \times CY_2$

For this case we consider an  $AdS$  metric ansatz of the form

$$ds^2 = \frac{R^2}{r^2} [-dt^2 + dr^2] + e^{2g_1} d\bar{s}^2(\Sigma_3) + e^{2g_2} d\bar{s}^2(\Sigma_2) \tag{6.7}$$

The metric on the three-cycle and the two-cycle are both Einstein with the signs of the curvature denoted by  $l$  and  $k$  respectively. The ansatz for the scalar fields preserves  $SO(3) \times SO(2)$  symmetry and is given by

$$\Pi = \text{diag}(e^{2\lambda}, e^{2\lambda}, e^{2\lambda}, e^{-3\lambda}, e^{-3\lambda}) \tag{6.8}$$

The  $SO(5)$  gauge fields are split via  $SO(5) \rightarrow SO(3) \times SO(2)$  and the  $SO(3)$  piece  $B^{ab}$ , with  $a, b = 1, 2, 3$ , is determined by the spin connection on the three-cycle, while the  $SO(2)$  piece  $B^{\alpha\beta}$ , with  $\alpha, \beta = 4, 5$ , is determined by the spin connection on the two-cycle. Explicitly:

$$B_{ab} = \frac{1}{2m} \bar{\omega}_{ab} \quad B_{\alpha\beta} = \frac{1}{2m} \bar{\omega}_{\alpha\beta} \tag{6.9}$$

The  $S$  equation of motion is solved by setting

$$S_a = -\frac{kl}{4\sqrt{3}} e^{4\lambda - 2g_1 - 2g_2} e^0 \wedge e^r \wedge e^a \tag{6.10}$$

with  $S_\alpha = 0$ .

It is useful to define

$$\begin{aligned} 4y &= e^{-2g_1 - 2\lambda} \\ 4z &= e^{-2g_2 - 2\lambda} \\ x &= e^{10\lambda} \end{aligned} \tag{6.11}$$



We then find that the equations of motion for the above ansatz are solved providing

$$\begin{aligned}
40lyx &= 28x^2y^2 - 16z^2 - 3 - 12x + 64y^2z^2x^2 \\
40kzx &= -12x^2y^2 + 64z^2 - 3 - 12x + 384y^2z^2x^2 \\
0 &= x - 1 + 8z^2 - 4x^2y^2 - 32x^2y^2z^2
\end{aligned} \tag{6.12}$$

and we find the following three solutions. The first has  $k = l = -1$ , i.e.  $AdS_2 \times H^3 \times H^2$ , and is the supersymmetric solution found in [16]

$$\begin{aligned}
e^{10\lambda} &= 2 \\
e^{2g_1} = e^{2g_2} &= e^{-2\lambda} \approx 0.87055 \\
m^2R^2 &\approx 0.43528
\end{aligned} \tag{6.13}$$

On the other hand for  $k = -l = 1$ , i.e.  $AdS_2 \times H^3 \times S^2$ , we get

$$\begin{aligned}
e^{10\lambda} &= 2 \\
e^{2g_1} = 3e^{2g_2} &= e^{-2\lambda} \approx 0.87055 \\
m^2R^2 &\approx 0.14509
\end{aligned} \tag{6.14}$$

or

$$\begin{aligned}
e^{10\lambda} &\approx 0.22921 \\
e^{2g_1} &\approx 0.45751 \\
e^{2g_2} &\approx 0.95097 \\
m^2R^2 &\approx 0.30432
\end{aligned} \tag{6.15}$$

## 7 Summary

We have found a large class of new solutions to  $D = 7$  gauged supergravity that are products of  $AdS_{7-d}$  space with an Einstein space  $\Sigma_d$ . These can be uplifted to obtain new solutions in  $D = 11$  supergravity. We have summarised the solutions found here, as well as the supersymmetric solutions found previously in [1, 3, 5, 16] in table 1.

It would be very interesting to determine which of our new solutions are stable, as this is a necessary condition for them to be dual to non-supersymmetric conformal field theories. Our preliminary analysis involving a small number of perturbations in  $D = 7$  gauged supergravity only found that four of the sixteen new solutions were unstable in the sense that the masses of the perturbations violate the Breitenlohner-Freedman bound [24, 25, 26]. Of course further instabilities might be found when

spacetime	embedding	cycle	supersymmetry	$m^2 R^2$
$AdS_5$	Kähler 2-cycle in $CY_2$	$H^2$	yes	2.5198
		$S^2$	no	1.4623*
	Kähler 2-cycle in $CY_3$	$H^2$	yes	2.25
		$H^2$	no	2.2496
$AdS_4$	SLAG 3-cycle in $CY_3$	$H^3$	yes	1.4142
		$H^3$	no	1.3608
	Associative 3-cycle	$H^3$	yes	1.2353
$AdS_3$	Coassociative 4-cycle	$C_-^4$	yes	0.44444
		SLAG 4-cycle in $CY_4$	$H^4$	yes
	Kähler 4-cycle in $CY_4$	$H^4$	no	0.44474
		$S^4$	no	0.00080517
	Cayley 4-cycle	$K_-^4$	yes	0.5625
		$K_+^4$	no	0.0027879
	CLAG 4-cycle in $HK_8$	$C_-^4$	yes	0.34028
		$C_-^4$	no	0.34328
		$CP^2, S^4$	no	0.000085734
	SLAG 4-cycle in $CY_2 \times CY_2$	$B$	yes	0.69444
		$B$	no	0.69422
		$CP^2$	no	0.0065244
$H^2 \times H^2$		yes	1	
$AdS_2$	SLAG 5-cycle in $CY_5$	$H^2 \times H^2$	no	0.99531*
		$S^2 \times S^2$	no	0.020723*
		$S^2 \times H^2$	no	0.17815*
		$H^5$	yes	0.5625 <sup>†</sup>
SLAG 5-cycle in $CY_2 \times CY_3$	$S^5$	yes	0.0625 <sup>†</sup>	
	$H^3 \times H^2$	yes	0.45328 <sup>†</sup>	
	$S^2 \times H^3$	no	0.14509 <sup>†</sup>	
		$S^2 \times H^3$	no	0.30432 <sup>†</sup>

Table 1: Table of solutions: \* denotes a solution shown to be unstable,  $C_-$  and  $K_{\pm}$  are conformally half-flat and Kähler–Einstein metrics with the subscript denoting positive or negative scalar curvature and  $B$  is the Bergmann metric. Note that we can also take quotients of all cycles by discrete groups of isometries and this preserves supersymmetry. <sup>†</sup> denotes radius in the seven-dimensional metric.

including further perturbations of the gauged supergravity or more general perturbations of  $D = 11$  supergravity.

Assuming that at least some of the new solutions are stable, further insight into the putative dual conformal field theories could be found by connecting them to other supersymmetric conformal field theories by gravitational flows. Recall that in [1, 3, 5, 16] supergravity solutions were found that flowed from an  $AdS_7$ -type region to the  $AdS_{7-d} \times \Sigma_d$  fixed points. The  $AdS_7$  region, which in Poincare-type co-ordinates has  $\mathbb{R}^{6-d} \times \Sigma_d$  slices, corresponds to the UV limit describing the  $(2, 0)$  superconformal field theory on the wrapped fivebranes on  $\mathbb{R}^{6-d} \times \Sigma_d$ . The  $AdS_{7-d} \times \Sigma_d$  fixed point at the end of the flow describes the IR physics in  $6 - d$  dimensions, where one is considering length scales much larger than the size of the cycle.

It seems likely that there could also be supergravity solutions that flow from the same kind of  $AdS_7$  type region to the new  $AdS_{7-d} \times \Sigma_d$  fixed points. By considering the fall-off of the various fields, one would then be able to interpret the  $AdS_{7-d} \times \Sigma_d$  fixed points as the IR physics arising from the  $(2, 0)$  superconformal field theory on wrapped fivebranes on  $\mathbb{R}^{6-d} \times \Sigma_d$  with certain supersymmetry breaking operators switched on. Unlike the supersymmetric flows in [1, 3, 5, 16], which were found by analysing first-order BPS equations, these new flows would be non-supersymmetric and would have to be found by solving second-order equations.

It might also be possible to find gravity flows to or from the supersymmetric  $AdS \times \Sigma$  IR fixed points and the new non-supersymmetric  $AdS \times \Sigma$  solutions, each with the same  $\Sigma$ . By assuming a c-theorem, the direction of these flows are determined from the  $AdS$  radii as this is proportional to the central charge of the conformal field theory. For example, from table 1, one might look for a flow from the supersymmetric  $AdS_5 \times H^2$  fixed point to the non-supersymmetric  $AdS_5 \times H^2$  solution. On the other hand one might look for a flow from the non-supersymmetric  $AdS_4 \times H^3$  solution to the supersymmetric  $AdS_4 \times H^3$  fixed point.

Finally we note that it would also be straightforward to apply the techniques used in this paper to find new non-supersymmetric fixed points and flows in other supergravity duals. In particular, one expects a variety of  $AdS_2 \times \Sigma_2$  fixed point solutions for the M2-brane based on the supersymmetric ansatz in [11], similarly  $AdS_3 \times \Sigma_2$  and  $AdS_2 \times \Sigma_3$  for the D3-brane following [1, 4] (we understand that this is being investigated in [37]), and also new non-supersymmetric solutions for the NS5-brane based on the ansatzes in [2] and [13, 14, 17, 18]. Note that solutions based on different non-supersymmetric deformations of [2] have recently be considered in [36].

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