

# Superstring on PP-Wave Orbifold from Large-N Quiver Gauge Theory \*

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## abstract

We extend the proposal of Berenstein, Maldacena and Nastase to Type IIB superstring propagating on pp-wave over  $\mathbf{R}^4/\mathbf{Z}_k$  orbifold. We show that first-quantized free string theory is described correctly by the large  $N$ , fixed gauge coupling limit of  $\mathcal{N} = 2$   $[U(N)]^k$  quiver gauge theory. We propose a precise map between gauge theory operators and string states for both untwisted and twisted sectors. We also compute leading-order perturbative correction to the anomalous dimensions of these operators. The result is in agreement with the value deduced from string energy spectrum, thus substantiating our proposed operator-state map.

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# 1 Introduction

Berenstein, Maldacena and Nastase (BMN) [1] have recently put forward a remarkable proposal, extending the regime of gauge theory description from supergravity to the full-fledged closed string theory. The idea is to utilize conserved global charges in the gauge theory and reorganize the perturbative expansion. For example, for  $\mathcal{N} = 4$  super Yang-Mills theory in four-dimensions, correlation functions involving operators of scaling dimension  $\Delta$  and global charge  $J$  are reorganizable in a calculable manner in the limit:

$$\lambda^2 = g_{\text{YM}}^2 N \rightarrow \infty, \quad g_{\text{eff}}^2 = \frac{\lambda^2}{J^2} \rightarrow \text{finite}, \quad (\Delta - J) \rightarrow \text{finite}. \quad (1)$$

In this limit, effective expansion parameter is set by  $g_{\text{eff}}^2$ , in sharp contrast to the 't Hooft large-N limit, where the expansion parameter is set by the 't Hooft coupling  $\lambda^2$ .

In the  $\mathcal{N} = 4$  super Yang-Mills theory, operators with  $(\Delta - J) > 0$  are long supermultiplets, hence, correspond to massive string oscillator modes of Type IIB string theory. In the new limit Eq.(1),  $J \sim \sqrt{N}$  is large of order  $\mathcal{O}(1/\sqrt{g_s})$  at weak string coupling limit. The limit Eq.(1) turns out to correspond in the Type IIB string theory to the so-called Penrose limit of the  $AdS_5 \times S^5$  background, yielding pp-wave spacetime with transverse  $\mathbf{R}^4 \times \mathbf{R}^4$  geometry and homogeneous RR 5-form field strength [2]–[4]. It amounts to boosting the background around a great circle in  $S^5$  and rescaling so that a neighborhood around null geodesics is blown up. As such, the BMN limit of  $\mathcal{N} = 4$  super Yang-Mills theory is interpretable as gauge theory description for discrete light-cone quantization of Type IIB superstring. Interestingly, in the Penrose limit of  $AdS_5 \times S^5$ , the total number of isometries as well as spacetime supersymmetries remain the same. Rather, the limit yields a contraction of the  $SU(2, 2|4)$  superconformal algebra (see [5] for an explicit demonstration).

A central technical feature that facilitates this correspondence is the phenomenon that anomalous dimensions of a certain class of long multiplet operators are parametrically suppressed. Relevance of this sort of operators to the string theory has been first emphasized by Polyakov [6]. In the proposal of BMN, these operators play the prominent role in that they describe the creation and annihilation operators of string oscillation modes.

An immediate question is whether the proposal is applicable to a more nontrivial background of the pp-wave front. In this paper, we extend the BMN proposal to the simplest yet nontrivial situation: pp-wave orbifold — the homogeneous pp-wave background (part of) whose transverse space is orbifolded. Specifically, we will consider orbifolding one of the two  $\mathbf{R}^4$  subspaces transverse to the propagation null vector. Our motivation comes from various corners. First, the plane-wave background considered by BMN preserves all 32 spacetime supersymmetries. It is clearly of interest to investigate if the BMN proposal is extendible to plane-wave backgrounds with fewer number of spacetime supersymmetries. The simplest way to reduce

the supersymmetry is to orbifold part of the transverse space. Second, as shown in [1, 7, 8], the plane-wave background acts as a harmonic potential to the string, and hence dynamical distinction between untwisted and twisted states is less clear. It is thus of intrinsic interest to see if one can find a precise map between Type IIB string oscillation modes and quiver gauge theory operators, both for untwisted and twisted sectors.

This paper is organized as follows. In section 2, we study the discrete light-cone quantization of the Type IIB superstring on pp-wave orbifold, and obtain the energy spectrum. In section 3, we analyze gauge-invariant operators in the dual,  $\mathcal{N} = 2$  quiver gauge theory, and find precise correspondence with the spectrum obtained in section 2. In section 4, we compute perturbatively the anomalous dimension of  $(\Delta - J) = 1$  operators at leading order and find agreement with the light-cone energy spectrum of section 2.

Shortly after [1], several preprints with various generalizations have appeared [9, 10, 11]. In particular, [12], which has substantial overlap with part of our work, was posted on the archive while we were in the process of writing up our results.

## 2 Type IIB Superstring on pp-wave Orbifold

Begin with dynamics of a Type IIB superstring on pp-wave background. The background, supported by homogeneous RR 5-form and dilaton fields, is given by

$$ds^2 = -4dx^+ dx^- - \mu^2(\vec{x}^2 + \vec{y}^2)(dx^+)^2 + d\vec{x}^2 + d\vec{y}^2, \quad (2)$$

$$F_{+1234} = F_{+5678} = \mu, \quad (3)$$

$$e^\phi \equiv g_s = \text{constant}, \quad (4)$$

where  $(\vec{x}, \vec{y}) \in \mathbf{R}^4 \times \mathbf{R}^4$ , and is known to be maximally supersymmetric, preserving all 32 spacetime supersymmetries. It was argued that, in the background Eqns.(2)–(4), the Type IIB superstring is exactly solvable [7, 8], owing mainly to the fact that the light-cone worldsheet dynamics is described by *free* fields, albeit being massive.

Recently, it was found that the pp-wave background Eqns.(2)–(4) is related to the other known maximally supersymmetric background –  $AdS_5 \times S^5$  with RR 5-form flux threaded on the five-sphere – via the Penrose limit along a large circle of the  $S^5$  [3]. Note that the isometry group of the eight-dimensional space transverse to the null propagation direction is  $SO(4) \times SO(4)$ : while the spacetime geometry is invariant under  $SO(8)$ , the 5-form field strength breaks it to  $SO(4) \times SO(4)$ . In the Green-Schwarz action of the Type IIB string in the plane-wave background, the reduction of the isometry is due to the coupling of spinor fields to the background RR 5-form field strength.

One is interested in reducing the number of supersymmetries preserved by the background. As alluded to in the introduction, one can reduce the 32 supersymmetries to 16 supersymmetries

by taking a  $\mathbf{Z}_k$  orbifold of the  $\mathbf{R}^4$  subspace parametrized by  $\vec{y}$ . The orbifold action is defined by

$$g : (z^1, z^2) \longrightarrow (\omega z^1, \omega z^2) \quad \text{where} \quad \omega = e^{\frac{2\pi i}{k}}. \quad (5)$$

Here,  $z^1 \equiv \frac{1}{\sqrt{2}}(y^6 + iy^7)$ ,  $z^2 \equiv \frac{1}{\sqrt{2}}(y^8 - iy^9)$ . The orbifold action  $g$  acts on spacetime fields as  $g = \exp\left(\frac{2\pi i}{k}(J_{67} - J_{89})\right)$ ,  $J_{67}$  and  $J_{89}$  being the rotation generators in the 67 and 89 planes, respectively. Defined so, the orbifold of the pp-wave background is actually derivable from the Penrose limit of  $AdS_5 \times S^5/\mathbf{Z}_k$  taken along the great circle of the  $S^5$  that is fixed by the orbifold.

In the light-cone gauge, Type IIB superstring on the pp-wave background Eqns.(2)–(4) is described by eight worldsheet scalars  $x^I$  and eight worldsheet Majorana fermions  $(\theta_1, \theta_2)$ , all of which are free but massive. The masses of the scalars and the fermions are equal by worldsheet supersymmetry (which descends from the light-cone gauge fixing of the Green-Schwarz action) and equal the RR 5-form field strength,  $\mu$ . Both  $\theta_1, \theta_2$  are positive chirality Majorana-Weyl spinors of  $SO(9, 1)$ , obeying the light-cone gauge condition  $\Gamma^+ \theta_i = 0$ . Decomposing the worldsheet fields into  $SO(4)_1 \times SO(4)_2$  subgroups,

$$x^I = (\vec{x}, \vec{y}) \rightarrow (\vec{x}, z^1, z^2), \quad g : \vec{x} \longrightarrow \vec{x}, \quad z^m \longrightarrow \omega z^m, \quad (6)$$

$$\theta \equiv \frac{1}{\sqrt{2}}(\theta_1 + i\theta_2) \rightarrow (\chi^\alpha, \xi^{\dot{\alpha}}), \quad g : \chi^\alpha \longrightarrow \chi^\alpha, \quad \xi^{\dot{\alpha}} \longrightarrow \Omega^{\dot{\alpha}}_{\beta} \xi^{\dot{\beta}}. \quad (7)$$

Here,  $\alpha$  and  $\dot{\alpha}$  are spinor indices of  $SO(4)_2$ , ranging over 1,2. We have suppressed the spinor indices of  $SO(4)_1$  under which  $\chi^\alpha$  carry positive chirality, while  $\xi^{\dot{\alpha}}$  carry negative one.  $\Omega = \text{diag}(\omega, \omega^{-1})$ , viz.  $\xi^1$  and  $\xi^2$  transform oppositely under the  $\mathbf{Z}_k$  orbifold action. It is convenient to combine  $\xi^1, \bar{\xi}^2$  into a Dirac spinor  $\xi$ , and  $\bar{\xi}^1$  and  $\xi^2$  into its conjugate  $\bar{\xi}$  and analogously for  $\chi$  and  $\bar{\chi}$ . As the worldsheet theory is free, it is straightforward to quantize the Type IIB superstring in each twisted sector, the only difference among various sectors being the monodromy of the worldsheet fields sensitive to the orbifolding, viz.  $z^1, z^2$  and  $\xi$ . The other worldsheet fields remain periodic as usual. The monodromy conditions in the  $q$ -th twisted sector,  $q = 0, \dots, k-1$ , are given by

$$z^m(\sigma + 2\pi\alpha' p^+, \tau) = \omega^q z^m(\sigma, \tau), \quad \xi(\sigma + 2\pi\alpha' p^+, \tau) = \omega^q \xi(\sigma, \tau), \quad (8)$$

and result in fractional moding,  $n(q) = n + \frac{q}{k}$  ( $n \in \mathbf{Z}$ ) of the corresponding oscillator modes.

Physical states are obtainable by applying the bosonic and fermionic creation operators to the light-cone vacuum  $|0, p^+\rangle_q$  of each  $q$ -th twisted sector. They ought to satisfy additional constraints ensuring the level-matching condition:

$$\sum_{n \in \mathbf{Z}} n N_n = 0, \quad \sum_{n \in \mathbf{Z}} n(q) (N_{n(q)} - \bar{N}_{-n(q)}) = \sum_{n \in \mathbf{Z}} (n(q) N_{n(q)} + n(-q) \bar{N}_{n(-q)}) = 0, \quad (9)$$

and  $\mathbf{Z}_k$  invariance. The bosonic creation operators consist of

$$\vec{a}_n^\dagger, \quad \text{and} \quad \alpha_{n(q)}^{\dagger m}, \quad \bar{\alpha}_{n(-q)}^{\dagger m} \quad (n \in \mathbf{Z}). \quad (10)$$

Here,  $\vec{a}_n$  are the  $\vec{x}$  oscillators, whereas  $\alpha_{n(q)}^m$  and  $\bar{\alpha}_{n(-q)}^m$  are  $z^m$  and  $\bar{z}^m$  oscillators, respectively. The fermionic creation operators consist, in obvious notation, of

$$\chi_n^\dagger, \quad \bar{\chi}_n^\dagger \quad \text{and} \quad \xi_{n(q)}^\dagger, \quad \bar{\xi}_{n(-q)}^\dagger. \quad (11)$$

Acting the fermionic zero mode oscillators to the light-cone vacua and projecting onto  $\mathbf{Z}_k$  invariant states, one fills out  $\mathcal{N} = 2$  gravity and tensor supermultiplets of the plane wave background. The action of the bosonic oscillators on these gives rise to a whole tower of multiplets, much as in the  $AdS_5 \times S^5$  case. As an example, we have four invariant states with a single bosonic oscillator

$$\vec{a}_0^\dagger |0, p^+\rangle_q, \quad (12)$$

and states with two bosonic oscillators are

$$a_n^{\dagger \mu} a_{-n}^{\dagger \nu} |0, p^+\rangle_q, \quad \alpha_{n(q)}^{\dagger l} \bar{\alpha}_{-n(q)}^{\dagger m} |0, p^+\rangle_q. \quad (13)$$

In the  $\mathbf{Z}_2$  case there are additional invariant states built from two  $z^m$  or two  $\bar{z}^m$  oscillators. However, they do not satisfy the level matching condition, Eq.(9).

One straightforwardly obtains the light-cone Hamiltonian in the  $q$ -th twisted sector as

$$H_{\text{LC}}(q) = \sum_{n \in \mathbf{Z}} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}} + \sum_{n \in \mathbf{Z}} (N_{n(q)} + \bar{N}_{-n(q)}) \sqrt{\mu^2 + \frac{n(q)^2}{(\alpha' p^+)^2}}. \quad (14)$$

The first sum is over those oscillators which are not sensitive to the orbifold. Positive modes label ‘left’ movers, negative ones ‘right’ movers,  $N_n$  ( $N_{n(q)}$  and  $\bar{N}_{-n(q)}$ ) is the total occupation number of bosons and fermions. The ground state energy is cancelled between bosons and fermions. This corresponds to a choice of fermionic zero mode vacuum that explicitly breaks the  $SO(8)$  symmetry, which is respected by the metric but not the field strength background, to  $SO(4)_1 \times SO(4)_2$  [8].

### 3 Operator Analysis in $\mathcal{N} = 2$ Quiver Gauge Theory

It is known [13] that Type IIB supergravity on  $AdS_5 \times (S^5/\mathbf{Z}_k)$  is dual to  $\mathcal{N} = 2$   $[U(N)]^k$  quiver gauge theory, the worldvolume theory of  $kN$  D3-branes sitting at the orbifold singularity. In light of discussions in the previous section, one anticipates that Type IIB superstring on pp-wave orbifold is dual to a new perturbative expansion of the quiver gauge theory at large  $N$

and *fixed* gauge coupling  $g_{\text{YM}}^2 = 4\pi g_s k$ . The factor of  $k$  in the relation between the string and the gauge coupling is standard, and is easily deducible from moving the D3-branes off the tip of the orbifold into the Higgs branch. See also [14]. In the new expansion, one focuses primarily on states with conformal weight  $\Delta$  and  $U(1)_R$  charge  $J$  which scale as  $\Delta, J \sim \sqrt{N}$ , whose difference  $(\Delta - J)$  remains finite in the large  $N$  limit.  $U(1)_R$  is the subgroup of the original  $SU(4)_R$  symmetry of  $\mathcal{N} = 4$  super Yang-Mills theory, which on the gravity side corresponds to the  $S^1$  fixed under the orbifolding; this  $U(1)_R$  together with the  $SU(2)_1$  subgroup of the remaining  $SO(4) \simeq SU(2)_1 \times SU(2)_2$  that commutes with  $\mathbf{Z}_k \subset SU(2)_2$  forms the  $R$ -symmetry group of  $\mathcal{N} = 2$  supersymmetric gauge theory.

The reason for the above scaling behaviour is that  $(\Delta - J)$  is identified with the light-cone Hamiltonian on the string theory side, whereas  $^1 \frac{J}{\sqrt{kN}} \sim p^+$ ,  $p^+$  being the longitudinal momentum carried by the string. When  $(\Delta - J) \ll J$ , the light-cone Hamiltonian Eq.(14) implies that on the gauge theory side there are operators obeying the following relation between the dimension  $\Delta$  and the  $U(1)_R$  charge  $J$  (we set  $\mu \equiv 1$ )

$$(\Delta - J)_n = \sqrt{1 + g_{\text{eff}}^2 n^2} \quad \text{and} \quad (\Delta - J)_{n(q)} = \sqrt{1 + g_{\text{eff}}^2 (n(q))^2}. \quad (15)$$

In the gauge theory, before orbifolding we have  $N \times N$  matrix valued fields, i.e. the gauge field  $A_\mu$ , complex scalars  $Z = \frac{1}{\sqrt{2}}(X^4 + iX^5)$  and  $\phi^m = (\phi^1, \phi^2) \equiv \frac{1}{\sqrt{2}}(X^6 + iX^7, X^8 - iX^9)$ , and fermions  $\chi$  and  $\xi$ . The fields  $\chi$  and  $\xi$  are spinors of  $SO(5, 1)$ , transforming as  $\mathbf{4}$  and  $\mathbf{4}'$ , respectively. For defining the  $\mathbf{Z}_k$  orbifolding in the gauge theory, we promote these fields to  $kN \times kN$  matrices  $\mathcal{A}_\mu, \mathcal{Z}, \Phi^m, \mathcal{X}$  and  $\Xi$  and project onto the  $\mathbf{Z}_k$  invariant components. The projection is ensured by the conditions

$$S\mathcal{A}_\mu S^{-1} = \mathcal{A}_\mu, \quad S\mathcal{Z}S^{-1} = \mathcal{Z}, \quad S\mathcal{X}S^{-1} = \mathcal{X} \quad (16)$$

and

$$S\Phi^m S^{-1} = \omega \Phi^m, \quad S\Xi S^{-1} = \omega \Xi. \quad (17)$$

where  $S = \text{diag}(1, \omega^{-1}, \omega^{-2}, \dots, \omega^{-k+1})$ , each block being proportional to the  $N \times N$  unit matrix.

The resulting spectrum is that of a four-dimensional  $\mathcal{N} = 2$  quiver gauge theory [15] with  $[U(N)]^k$  gauge group, containing hypermultiplets in the bi-fundamental representations of  $U(N)_i \times U(N)_{i+1}$ ,  $i \in \mathbf{Z} \bmod(k)$ . More precisely,  $\mathcal{A}_\mu, \mathcal{Z}$  and  $\mathcal{X}$  fill out  $k$   $\mathcal{N} = 2$  vector multiplets with the fermions transforming as doublets under  $SU(2)_R$  (as its Cartan generator

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<sup>1</sup> Since  $\int_{S^5/\mathbf{Z}_k} F_5 = N$ , the radius of  $AdS_5$  is proportional to  $(kN)^{1/4}$ .

is proportional to  $(J_{67} + J_{89})$ . The  $\mathcal{Z}$  field has the block-diagonal form

$$\mathcal{Z} = \begin{pmatrix} Z_1 & & & & & \\ & Z_2 & & & & \\ & & Z_3 & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & Z_k \end{pmatrix} \quad (18)$$

with zeros on the off-diagonal and the diagonal blocks being  $N \times N$  matrices of  $U(N)_i$ 's. The  $\mathcal{A}_\mu$  and  $\mathcal{X}$  fields take an analogous form. Likewise, the  $\Phi^m$  and  $\Xi$  fields fill out  $k$  hypermultiplets, in which the scalars are doublets under  $SU(2)_R$ , whereas the fermions are neutral. The  $\Phi^m$  fields take the form

$$\Phi^m = \begin{pmatrix} 0 & \phi_{12}^m & & & & \\ & 0 & \phi_{23}^m & & & \\ & & 0 & \cdot & & \\ & & & \cdot & \cdot & \\ & & & & \cdot & \cdot \\ \phi_{k1}^m & & & & & \cdot \end{pmatrix} \quad (19)$$

and analogously for  $\Xi$ .

The light-cone vacua of the type IIB superstring in the plane-wave orbifold ought to be described by  $p^- = 0$ . In the quiver gauge theory side, the vacuum then corresponds to  $(\Delta - J) = 0$  operators acting on the Fock space vacuum. What are the operators satisfying  $(\Delta - J) = 0$ ? Obviously, one can build  $k$  mutually orthogonal,  $\mathbf{Z}_k$  invariant single trace operators  $\text{Tr}[S^q \mathcal{Z}^J]$ . We propose that these operators are associated to the vacuum in the  $q$ -th twisted sector

$$\frac{1}{\sqrt{k} J N^{J/2}} \text{Tr}[S^q \mathcal{Z}^J] \quad \longleftrightarrow \quad |0, p^+\rangle_q, \quad (q = 0, \dots, k-1). \quad (20)$$

In what sense is this identification unique? After all, in the quiver gauge theory, it appears that the operators  $\text{Tr}[S^q \mathcal{Z}^J]$  for any  $q$  stand on equal footing. However, the orbifold action renders an additional ‘‘quantum’’  $\mathbf{Z}_k$  symmetry (see e.g. [16]) that acts on fields in the quiver gauge theory.<sup>2</sup> Specifically, one can take an element  $g$  in this quantum  $\mathbf{Z}_k$ ,  $g = e^{\frac{2\pi i}{k}}$ , to act on an arbitrary field  $T_{ij}$ ,  $i, j \in \mathbf{Z} \bmod(k)$ , as  $g : T_{ij} \longrightarrow T_{i+1, j+1}$ . In particular, one notes that  $g : \text{Tr}[S^q \mathcal{Z}^J] \longrightarrow \omega^q \text{Tr}[S^q \mathcal{Z}^J]$ . So one can indeed distinguish classes of operators on the quiver gauge theory side by their eigenvalues under the quantum  $\mathbf{Z}_k$  symmetry.

<sup>2</sup> This  $\mathbf{Z}_k$  should not to be confused with the spacetime  $\mathbf{Z}_k$  used for constructing the orbifold. By construction, under the orbifold action, all the fields are invariant.

Next, consider the eight twist invariant operators with  $(\Delta - J) = 1$ . They are

$$\frac{1}{kN^{(J+1)/2}} \text{Tr}[S^q \mathcal{Z}^J \mathcal{D}_\mu \mathcal{Z}] \longleftrightarrow a_0^{\dagger\mu} |0, p^+\rangle_q, \quad (21)$$

$$\frac{1}{kN^{(J+1)/2}} \text{Tr}[S^q \mathcal{Z}^J \mathcal{X}_{J=1/2}] \longleftrightarrow \chi_0^\dagger |0, p^+\rangle_q, \quad (22)$$

$$\frac{1}{kN^{(J+1)/2}} \text{Tr}[S^q \mathcal{Z}^J \overline{\mathcal{X}}_{J=1/2}] \longleftrightarrow \overline{\chi}_0^\dagger |0, p^+\rangle_q, \quad (23)$$

and hence identifiable with Type IIB supergravity modes (in each twisted sector) built out of a single zero-mode oscillator acting on the  $q$ -th vacuum. Here,  $\mathcal{D}_\mu \mathcal{Z} = \partial_\mu \mathcal{Z} + [\mathcal{A}_\mu, \mathcal{Z}]$ .

Operators corresponding to higher string states on the pp-wave orbifold arise as follows. Oscillators of non-zero level  $n$  corresponding to the fields not sensitive to the orbifold are identified with insertions of the operators  $\mathcal{D}_\mu \mathcal{Z}$ ,  $\mathcal{X}_{J=1/2}$  and  $\overline{\mathcal{X}}_{J=1/2}$  with a position dependent phase factor  $e^{\frac{2\pi i l}{J} n}$  in the trace  $\text{Tr}[S^q \mathcal{Z}^J]$ . For instance, for  $(\Delta - J) = 2$ ,

$$\frac{1}{\sqrt{k} J N^{J/2+1}} \sum_{l=1}^J \text{Tr}[S^q \mathcal{Z}^l \mathcal{D}_\mu \mathcal{Z} \mathcal{Z}^{J-l} \mathcal{D}_\nu \mathcal{Z}] e^{\frac{2\pi i l}{J} n} \longleftrightarrow a_n^{\mu\dagger} a_{-n}^{\nu\dagger} |0, p^+\rangle_q. \quad (24)$$

This is exactly the same as in the unorbifolded case – the insertion of the position-dependent phase factor ensures that the level-matching condition is satisfied and that the light-cone energy of the string states is reproduced correctly [1].

As for the remaining string states involving oscillators with a fractional moding  $n(q)$  in the twisted sectors, we propose to identify them with insertions of the operators  $\Phi^m$  and  $\Xi_{J=1/2}$  together with the position-dependent phase factor  $e^{\frac{2\pi i l}{J} n(q)}$ . Similarly, insertions of  $\overline{\Phi}^m$  and  $\overline{\Xi}_{J=1/2}$  are accompanied with the phase factor  $e^{\frac{2\pi i l}{J} n(-q)}$ . Again, the prescription implements the level-matching condition and, as will be demonstrated in the next section, seems to yield the correct energy of the corresponding string states. For instance,

$$\frac{1}{\sqrt{k} J N^{J/2+1}} \sum_{l=1}^J \text{Tr}[S^q \mathcal{Z}^l \Phi^r \mathcal{Z}^{J-l} \overline{\Phi}^s] e^{\frac{2\pi i l}{J} n(q)} \longleftrightarrow \alpha_{n(q)}^{r\dagger} \overline{\alpha}_{-n(q)}^{s\dagger} |0, p^+\rangle_q. \quad (25)$$

Note that, for  $\mathbf{Z}_2$  orbifold, the state  $\sum_{k,l=1}^J \text{Tr}[S \mathcal{Z}^k \Phi^r \mathcal{Z}^l \overline{\Phi}^s \mathcal{Z}^{J-k-l}] e^{\frac{2\pi i}{J} (kn(1)+(k+l)m(1))}$  corresponding to  $\alpha_{n(q)}^{r\dagger} \alpha_{m(q)}^{s\dagger} |0, p^+\rangle_1$ , though being  $\mathbf{Z}_2$  invariant, vanishes for all  $m, n$  due to the cyclicity of the trace, as it should, cf. the remark below Eq.(13).

Finally, operators with insertions such as  $\mathcal{D}_\mu \mathcal{D}_\nu \mathcal{Z}$ ,  $\overline{\mathcal{Z}}$  or  $\mathcal{X}_{J=-1/2}$  in the trace are present at weak coupling, but should not be present at strong coupling, as there are no corresponding states in the string spectrum. As in [1], the reason for this might be related to the fact that these operators acquire a large anomalous dimension in this limit [1].



## 4 Anomalously Suppressed Anomalous Dimensions

In this section, in fixed  $g_{\text{YM}}^2$ , large- $N$  and large- $J$  perturbation theory, we shall be computing leading-order anomalous dimensions of  $(\Delta - J) = 1$  operators in  $\mathcal{N} = 2$  quiver gauge theory, and confirm that our proposal for the twisted sector operators reproduces the correct light-cone string energy spectrum. Amusingly, in the set-up we have outlined above, one can proceed the computations essentially parallel to those of [1].

The bosonic part of the (euclidean) quiver gauge theory action involving the transverse scalars is given by

$$S_{\text{YM}} = \frac{2}{g_{\text{YM}}^2} \int d^4x \text{Tr} \left[ |\mathcal{D}\mathcal{Z}|^2 + \sum_{m=1}^2 |\mathcal{D}\Phi^m|^2 - \frac{1}{2} \left( |[\mathcal{Z}, \bar{\mathcal{Z}}]|^2 + \sum_{m=1}^2 |[\Phi^m, \bar{\Phi}^m]|^2 + \sum_{m \neq n=1}^2 \left( |[\Phi^m, \bar{\Phi}^n]|^2 + |[\Phi^m, \Phi^n]|^2 \right) + 2 \sum_{m=1}^2 \left( |[\mathcal{Z}, \Phi^m]|^2 + |[\mathcal{Z}, \bar{\Phi}^m]|^2 \right) \right) \right]. \quad (26)$$

The trace ‘Tr’ runs over the  $kN \times kN$  matrices, the  $N \times N$  matrix blocks being invariant under the orbifold action.

Explicitly, the quartic interactions involving  $\mathcal{Z}$  with  $\Phi^m$  (the last two terms in Eq.(26)) are

$$\begin{aligned} & - \sum_{a=1}^k \sum_{m=1}^2 \frac{4}{g_{\text{YM}}^2} \int d^4x \text{tr} \left[ Z_a \phi_{a,a+1}^m \bar{Z}_{a+1} \bar{\phi}_{a+1,a}^m + Z_a \bar{\phi}_{a,a-1}^m \bar{Z}_{a-1} \phi_{a-1,a}^m \right] \\ & + \sum_{a=1}^k \sum_{m=1}^2 \frac{2}{g_{\text{YM}}^2} \int d^4x \text{tr} \left[ Z_a \phi_{a,a+1}^m \bar{\phi}_{a+1,a}^m \bar{Z}_a + Z_a \bar{\phi}_{a,a-1}^m \phi_{a-1,a}^m \bar{Z}_a \right. \\ & \quad \left. + \phi_{a,a+1}^m Z_{a+1} \bar{Z}_{a+1} \bar{\phi}_{a+1,a}^m + \bar{\phi}_{a,a-1}^m Z_{a-1} \bar{Z}_{a-1} \phi_{a-1,a}^m \right], \end{aligned} \quad (27)$$

the trace ‘tr’ now being over  $N \times N$  matrices of the  $a$ -th  $U(N)$  group. The first line contains ‘momentum-dependent’ interactions, while the second and third line ‘momentum-independent’ interactions, respectively.

The free field propagators are

$$\langle (Z_a)_i^j(x) (\bar{Z}_b)_k^l(0) \rangle = \delta_{ab} \delta_i^l \delta_k^j \frac{g_{\text{YM}}^2}{8\pi^2} \frac{1}{|x|^2}, \quad \langle (\phi_{a,a+1}^m)_i^j(x) (\bar{\phi}_{b+1,b}^n)_k^l(0) \rangle = \delta_{ab} \delta^{mn} \delta_i^l \delta_k^j \frac{g_{\text{YM}}^2}{8\pi^2} \frac{1}{|x|^2}. \quad (28)$$

If our proposed map between gauge theory operators and string modes is correct, according to Eq.(15), anomalous dimension of the operators  $O(x)$  in Eq.(25) is expected to receive perturbative corrections as

$$(\Delta - J)_{n(q)} = 1 + \frac{1}{2} g_{\text{eff}}^2 (n(q))^2 + \dots \quad (29)$$

We now demonstrate that this is precisely what one finds. At leading-order in perturbation theory, the logarithmically divergent contribution to the two-point function from the ‘momentum dependent’ interactions is obtained as

$$\langle O(x)O^\dagger(0) \rangle = \frac{\left(\frac{g_{\text{YM}}^2}{8\pi^2}\right)^\Delta}{|x|^{2\Delta}} \left[ 1 + \frac{g_{\text{YM}}^2 N}{2\pi^2} \cos \frac{2\pi n(q)}{J} \ln(|x|\Lambda) \right], \quad (30)$$

whereas, for the ‘momentum-independent’ interactions,  $\cos \frac{2\pi n(q)}{J}$  is replaced by  $-1$ . Other ‘momentum-independent’ interactions involving gauge bosons and scalar loops cancel, owing to the underlying  $\mathcal{N} = 2$  supersymmetry. Hence, at large  $J$  and  $N$ , the leading-order perturbative correction to the two-point correlation function is, up to overall normalization factor,

$$\langle O(x)O^\dagger(0) \rangle \sim |x|^{-2\Delta} \left[ 1 - \frac{g_{\text{YM}}^2 N}{J^2} (n(q))^2 \ln(|x|\Lambda) \right]. \quad (31)$$

This implies that

$$(\Delta - J)_{n(q)} = 1 + \frac{1}{2} \frac{g_{\text{YM}}^2 N}{J^2} (n(q))^2 + \dots, \quad (32)$$

reproducing precisely the anticipated perturbative correction Eq.(29), and hence the requisite light-cone energy spectrum Eq.(14) in the  $q$ -th twisted sector. Resummation of the leading logarithms, corresponding to multiple insertions of the above quartic interactions, is straightforward, and reproduces the full square-root form in Eq.(15). Again, this closely parallels the computation of [1].

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