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Symmetry Breaking Boundary States and Defect Lines

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Abstract

We present a large and universal class of new boundary states which break part of the chiral symmetry in the underlying bulk theory. Our formulas are based on coset constructions and they can be regarded as a non-abelian generalization of the ideas that were used by Maldacena, Moore and Seiberg to build new boundary states for $\mathrm{SU}(N)$. We apply our expressions to construct defect lines joining two conformal field theories with possibly different central charge. Such defects can occur e.g. in the AdS/CFT correspondence when branes extend to the boundary of the AdS-space.

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1 Introduction

During the last years, the microscopic techniques of boundary conformal field theory have been developed into a powerful tool that allows to study D-branes in curved backgrounds with finite curvature. Most of these investigations, however, focus on boundary conditions that preserve the full chiral symmetry in the bulk theory. The latter is typically much larger than the Virasoro algebra that must be unbroken to guarantee conformal invariance. Constructing boundary theories with the minimal Virasoro symmetry tends to lead into non-rational models which are notoriously difficult to control.

Nevertheless, some progress has been made in this direction. Boundary conditions with the minimal Virasoro symmetry were systematically investigated for 1-dimensional flat targets [1, 2, 3, 4]. In-spite of this remarkable progress, such a complete control over conformal boundary conditions should be considered exceptional and it is probably very difficult to achieve for more complicated backgrounds. Less ambitious programs focus on intermediate symmetries which are carefully selected so as to render the boundary theory rational.

One possibility is to work with orbifold chiral algebras. This has been explored in great detail by several groups (see e.g. [5, 6] and also [7]) and it has lead to new boundary theories in group manifolds and other backgrounds. More recently, Maldacena, Moore and Seiberg [8, 9] have proposed further symmetry breaking boundary states for the $\mathrm{SU}(N)$ WZW-model. Their construction employs the chiral algebra of the $\mathrm{SU}(N)/\mathrm{U}(1)^{N-1}$ coset theory (see also [10] for a similar analysis in a non-compact background). Our aim here is to turn these ideas into a more general procedure that provides a large class of new boundary theories. The construction involves coset chiral algebras with non-abelian denominators and after some more technical refinements it has a good chance even to exhaust all rational boundary theories.

To make this paper self-contained, we shall start our exposition with some background material on 2D boundary conformal field theory and on coset chiral algebras. This then enters crucially into our construction of the new boundary theories in the third section. After presenting formulas for the boundary states and computing the associated open string spectra we discuss the relation with the D-branes constructed in [8, 9]. Finally, we shall sketch how our new states can be used to describe defect lines separating two different conformal field theories. Such systems have been studied by various authors (see e.g. [11, 12, 13, 14, 15, 16, 17]) and they are known to appear e.g. in the context of the AdS_3/CFT_2 correspondence where AdS_2-branes can end on the boundary of AdS_3 [18] (see also [19, 20, 21]). In fact, it was mainly the interest in the latter that has motivated the present work even though the results we present can have many other applications.

2 Background from Conformal Field Theory

In this section we collect some background material about 2D boundary conformal field theory (BCFT) and coset chiral algebras. One of the main aims is to set up the notations we are using throughout this work. Readers who are familiar with the relevant techniques from conformal field theory may skip this section and consult it only to look up our conventions.

2.1 Some boundary conformal field theory

Let us start by reviewing some basic elements of (boundary) conformal field theory. Our presentation will closely follow the reference [7]. The central ingredient in any CFT is its chiral algebra $\mathcal{A}$ which contains the Virasoro algebra. We shall restrict ourselves to the so-called rational algebras $\mathcal{A}$ possessing a finite set $\mathrm{Rep}(\mathcal{A})$ of 'physical' irreducible representations. Furthermore, we assume that the two chiral algebras $\mathcal{A}$ and $\bar{\mathcal{A}} \cong \mathcal{A}$ of the bulk theory are identical. This is not the most general situation as there exist also so-called heterotic CFTs with different left and right-moving chiral algebras (see [22, 23] for instance).

To fully specify the bulk CFT we still need to characterize its field content. The space of fields decomposes into irreducible representations for the product of the two chiral algebras,

$$\mathcal{H} \;=\; \bigoplus_{\mu,\bar{\mu}\in\mathrm{Rep}(\mathcal{A})} Z^{\mu\bar{\mu}}\; \mathcal{H}_\mu \otimes \bar{\mathcal{H}}_{\bar{\mu}}$$

with some numbers $Z^{\mu\bar{\mu}} \in \mathbb{N}_0$. We call the set of all pairs $(\mu, \bar{\mu})$ that contribute to $\mathcal{H}$ (including the multiplicities) the *spectrum* of the (bulk) theory and denote it by

$$\mathrm{Spec} \;=\; \left\{ \, (\mu, \bar{\mu}|\eta) \;\middle|\; \eta = 1, \dots, Z^{\mu\bar{\mu}} \, \right\} \quad .$$

Since time translation in the usual radial quantization is generated by the sum $L_0 + \bar{L}_0$ of the zero modes of the chiral Virasoro fields, the spectrum of the theory may be captured by the toroidal partition function

$$Z(q,\bar{q}) \;=\; \mathrm{tr}_{\mathcal{H}}\, q^{L_0 - c/24}\, \bar{q}^{\bar{L}_0 - c/24} \;=\; \sum_{\mu,\bar{\mu}\in \mathrm{Rep}(\mathcal{A})} Z^{\mu\bar{\mu}}\; \chi_\mu(q)\, \bar{\chi}_{\bar{\mu}}(\bar{q})$$

where the argument $q = \exp(2\pi i \tau)$ is determined by the modulus $\mathrm{Im}\ \tau > 0$ of the torus. The number c is the central charge of the Virasoro algebra and the characters of the chiral algebra are defined by

$$\chi_\mu(q) \;=\; \mathrm{tr}_{\mathcal{H}_\mu}\, q^{L_0 - c/24} \quad .$$

Consistency requires the partition function to be invariant under modular transformations $T : \tau \mapsto \tau + 1$ and $S : \tau \mapsto -1/\tau$ which may be represented unitarily on the characters as follows

$$\begin{aligned} T\chi_\mu(\tau) &= \chi_\mu(\tau+1) \;=\; e^{2\pi i (h_\mu - c/24)}\, \chi_\mu(\tau) \\ S\chi_\mu(\tau) &= \chi_\mu(-1/\tau) \;=\; \sum_{\nu\in\mathrm{Rep}(\mathcal{A})} S_{\mu\nu}\, \chi_\nu(\tau) \end{aligned}$$

where we introduced the conformal weight h_μ. For later use it is convenient to summarize some properties of the modular S-matrix,

$$S_{\mu\nu} \;=\; S_{\nu\mu} \qquad\qquad S_{\mu^+\nu} \;=\; \bar{S}_{\mu\nu} \qquad\qquad \sum_{\lambda\in\mathrm{Rep}(\mathcal{A})} \bar{S}_{\mu\lambda}\, S_{\nu\lambda} \;=\; \delta^\mu_\nu \quad . \tag{1}$$

Imposing invariance of the spectrum under modular transformations gives severe restrictions on the numbers $Z^{\mu\bar{\mu}}$. Nevertheless, there exist choices $Z^{\mu\bar{\mu}} = \delta^{\mu\bar{\mu}}$ and $Z^{\mu\bar{\mu}} = \delta^{\mu\bar{\mu}^+}$, the so-called diagonal and charge conjugation invariants, which are always allowed.

We are interested in boundary conditions which preserve the chiral algebra $\mathcal{A}$. When we specify boundary theories through the associated boundary states $|B\rangle$, the choice of the boundary condition is implemented by gluing conditions of the form

$$\big(\phi(z) - \Omega\bar{\phi}(\bar{z})\big)|B\rangle = 0 \qquad \text{for} \qquad z = \bar{z} \quad . \tag{2}$$

Here, the reflection of left into right movers is described by a gluing automorphism $\Omega \in \mathrm{Aut}(\mathcal{A})$ which must leave the energy momentum tensor invariant in order to preserve conformal symmetry. The automorphism Ω induces a permutation $\omega : \mathrm{Rep}(\mathcal{A}) \to \mathrm{Rep}(\mathcal{A})$ on the set of representations which leaves invariant the vacuum representation (see e.g. [24] for details). It is then easy to see that for each element

$$(\mu,\eta) \in \mathrm{Spec}^\omega \ = \ \big\{ \, (\mu,\eta) \, \big| \, (\mu,\bar\mu|\eta) \in \mathrm{Spec} \text{ and } \bar\mu = \omega(\mu^+) \big\}$$

in the ω-symmetric part of the spectrum one can construct a so-called Ishibashi (or *generalized coherent*) state $|\mu,\eta\rangle\rangle$. These states are normalized by

$$\langle\langle \mu,\eta | q^{\frac{1}{2}(L_0+\bar L_0 - c/12)} |\nu,\epsilon\rangle\rangle \ = \ \delta^\mu_\nu \, \delta^\eta_\epsilon \, \chi_\mu(q)$$

and they constitute a complete linear independent set of solutions to the linear equations (2). Although the Ishibashi states are often said to live in the bulk Hilbert space $\mathcal{H}$, one should bear in mind that they are not normalizable in the standard sense.

Naively one could think that all linear combinations

$$|b\rangle \ = \sum_{(\mu,\eta)\in\mathrm{Spec}^\omega} \frac{\psi_b{}^{(\mu,\eta)}}{\sqrt{S_{0\mu}}} \, |\mu,\eta\rangle\rangle \tag{3}$$

would lead to consistent boundary states. There exists, however, the important *Cardy constraint* which arises from world-sheet duality or from an exchange of open and closed string channel in a more string theoretic language. More precisely, one has

$$\begin{aligned} Z_{ab}(q) \ &= \ \langle a | \tilde q^{\frac{1}{2}(L_0+\bar L_0 - c/12)} | b\rangle \ = \sum_{(\mu,\eta)\in\mathrm{Spec}^\omega} \frac{\bar\psi_a{}^{(\mu,\eta)}\psi_b{}^{(\mu,\eta)}}{S_{0\mu}} \, \chi_\mu(\tilde q) \\ &= \sum_{\substack{(\mu,\eta)\in\mathrm{Spec}^\omega \\ \nu\in\mathrm{Rep}(\mathcal{A})}} \frac{\bar\psi_a{}^{(\mu,\eta)}\psi_b{}^{(\mu,\eta)} S_{\nu\mu}}{S_{0\mu}} \, \chi_\nu(q) \ \equiv \sum_{\nu\in\mathrm{Rep}(\mathcal{A})} \big(n_\nu\big)_b{}^a \, \chi_\nu(q) \end{aligned}$$

where $\tilde q$ is obtained from q by modular transformation, i.e. $\tilde q = e^{-2\pi i/\tau}$. All characters $\chi_\nu(q)$ in the second line must have non-negative integer coefficients $\big(n_\nu\big)_b{}^a$ since we want to interprete the whole expression as an open string partition function. Consistent boundary states which correspond to the gluing automorphism Ω may be generated from a set $\mathcal{B}^\omega$ of elementary boundary states. As a criterion for elementarity we shall use the requirement $\big(n_0\big)_b{}^a = \delta^a_b$. It states that the identity field should only live between identical boundary conditions and that it should appear with multiplicity one. Every consistent boundary state may be represented as a superposition of elementary boundary states with non-negative integer coefficients. Let us emphasize that the boundary states do not only depend on the gluing automorphism Ω but also on the bulk partition function under consideration.

One can show [7] that the matrices n_ν form a non-negative integer valued matrix representation (NIM-rep) of the fusion ring of the CFT, i.e.

$$n_\lambda \, n_\mu \;=\; \sum_{\nu\in\mathrm{Rep}(\mathcal{A})} N_{\lambda\mu}{}^{\nu}\, n_\nu \qquad \text{and} \qquad n_{\lambda^+} \;=\; (n_\lambda)^T \tag{4}$$

where the fusion rules of $\mathcal{A}$ are denoted by $N_{\lambda\mu}{}^{\nu}$. Let us also remark that the classification of NIM-reps for a given fusion ring is not sufficient to construct consistent BCFTs. In fact, many NIM-reps are known to possess no physical interpretation [25].

There is a class of boundary conditions which was constructed by Cardy more than ten years ago [26]. In the original setup, these boundary conditions require that Ω is the identity and that we are working with the charge conjugated modular invariant, i.e. $Z^{\mu\bar{\mu}} = \delta^{\mu\mu^+}$. Hence, we can identify $\mathrm{Spec}^{\mathbf{id}}$ with $\mathrm{Rep}(\mathcal{A})$. It is then easy to solve the Cardy condition by the boundary states

$$|\nu\rangle \;=\; \sum_{\lambda\in\mathrm{Rep}(\mathcal{A})} \frac{S_{\nu\lambda}}{\sqrt{S_{0\lambda}}}\, |\lambda\rangle\!\rangle \tag{5}$$

where $\nu \in \mathcal{B}^{\mathbf{id}} \cong \mathrm{Rep}(\mathcal{A})$. Indeed, the Verlinde formula for fusion coefficients [27]

$$N_{\mu\nu}{}^{\lambda} \;=\; \sum_{\sigma\in\mathrm{Rep}(\mathcal{A})} \frac{\bar{S}_{\lambda\sigma} S_{\mu\sigma} S_{\nu\sigma}}{S_{0\sigma}} \tag{6}$$

immediately implies

$$Z_{\mu\nu}(q) \;=\; \sum_{\lambda\in\mathrm{Rep}(\mathcal{A})} N_{\mu^+\nu}{}^{\lambda}\, \chi_\lambda(q) \quad .$$

For later convenience let us summarize some important properties of fusion rules which may easily be proved by means of the Verlinde formula (6) using the properties (1) for the modular S-matrix,

$$N_{0\mu}{}^{\sigma} = \delta_{\mu}^{\sigma} \ , \qquad N_{\mu\nu}{}^{\sigma} = N_{\nu\mu}{}^{\sigma} = N_{\mu\sigma^+}{}^{\nu^+} = N_{\mu^+\nu^+}{}^{\sigma^+} \tag{7}$$

$$\sum_{\sigma\in\mathrm{Rep}(\mathcal{A})} N_{\lambda\nu}{}^{\sigma} N_{\mu\sigma}{}^{\rho} = \sum_{\sigma\in\mathrm{Rep}(\mathcal{A})} N_{\mu\lambda}{}^{\sigma} N_{\sigma\nu}{}^{\rho} \ . \tag{8}$$

The first equation guarantees that the identity field propagates only between identical boundary conditions. In addition, these relations imply that the matrices $(N_\lambda)_\mu{}^\sigma = N_{\lambda\mu}{}^\sigma$ form a representation (4) – the adjoint representation – of the fusion algebra.

2.2 The coset construction

One of the basic tools to build new conformal field theories is the so-called coset or GKO construction [28]. Although the main ideas in this section and in the rest of the paper apply to a rather general class of coset theories, we shall specialize most of our presentation to affine Kac-Moody algebras and their cosets. The formulation we have chosen, however, suggests the appropriate generalization.

Let G be a semi-simple simply connected compact group and $\hat{\mathfrak{g}}_k$ the associated affine Kac-Moody algebra. The latter generates a chiral algebra that we denote by $\mathcal{A}(\mathrm{G}) = \mathcal{A}(\hat{\mathfrak{g}}_k)$. Now we want to choose a semi-simple subgroup P of G. Up to isomorphism, the embedding $\mathfrak{p} \hookrightarrow \mathfrak{g}$ of the corresponding Lie algebras can be defined by giving a projection $\mathcal{P} : L_w^{(\mathfrak{g})} \to L_w^{(\mathfrak{p})}$ from the weight lattice of $\mathfrak{g}$ to the weight lattice of $\mathfrak{p}$. This projection is just dual to the injection of Cartan subalgebras. The embedding of Lie algebras may be lifted to an embedding $\hat{\mathfrak{p}}_{k'} \hookrightarrow \hat{\mathfrak{g}}_k$ of untwisted affine Kac-Moody algebras where the levels are related by $k' = x_e k$ with embedding index x_e (see [29] for instance). By the GKO construction [28] one may then define the coset chiral algebra $\mathcal{A}(\mathrm{G/P})$ such that the energy momentum tensors satisfy $T^{\mathrm{G}} = T^{\mathrm{G/P}} + T^{\mathrm{P}}$ and all the chiral fields generating $\mathcal{A}(\mathrm{G/P}) \subset \mathcal{A}(\mathrm{G})$ commute with those in $\mathcal{A}(\mathrm{P})$. It was shown in [30, 31, 32, 33] that the coset chiral algebra describes the symmetry of the G/P gauged WZW model. There are (at least) two equivalent ways of analysing the coset chiral algebra. First, there is a more geometric one which is discussed for example in [29]. In our presentation we will follow the simple current approach [34, 35, 36, 37] as this allows a straightforward generalization to cosets which do not arise from WZW theories.

We start with a discussion of simple currents $J \in \text{Rep(G)} = \text{Rep}\big(\mathcal{A}(\text{G})\big)$ which are characterized by the property that the fusion $(J) \star (\mu)$ of J with any other sector $\mu \in \text{Rep(G)}$ contains exactly one representation. We shall denote the latter by $J\mu \in \text{Rep(G)}$. Since the vacuum representation is a simple current and the fusion product is commutative, the set of all simple currents forms an abelian group $\mathcal{Z}(\text{G})$. In almost all cases,[1] this group is isomorphic to the center of the Lie group G. This also means that $\mathcal{Z}(\text{G})$ is in one-to-one correspondence with symmetries of the Dynkin diagram of the affine Lie algebra $\hat{\mathfrak{g}}$ modulo those of the Dynkin diagram of $\mathfrak{g}$.

Let us now summarize some well-known properties of simple currents. It turns out that simple current transformations satisfy

$$S^{\text{G}}_{J\mu\ \nu} \;=\; e^{2\pi i Q_J(\nu)}\, S^{\text{G}}_{\mu\nu} \quad . \tag{9}$$

The number $Q_J(\nu)$ is defined modulo integers and it is called the *monodromy charge* of ν with respect to J. It is possible to show that monodromy charges are related to conformal weights by the formula

$$Q_J(\nu) \;=\; h_J + h_\nu - h_{J\nu} \quad \text{mod } 1 \quad . \tag{10}$$

The relation (9) has some wide reaching consequences. In particular, iterated application implies

$$Q_{J^n}(\nu) \;=\; nQ_J(\nu) \quad , \quad Q_J(J'\nu) + Q_{J'}(\mu) \;=\; Q_{J'}(J\mu) + Q_J(\nu) \quad .$$

For a simple current J of order N, i.e. an element $J \in \mathcal{Z}(G)$ satisfying $J^N = (0)$ (the vacuum representation), the first relation means that the monodromy charge $\exp\big(2\pi i Q_J(\nu)\big)$ is an N^{th} root of unity. Simple currents provide symmetries of the fusion rules. Indeed, the S-matrix symmetry (9) in combination with the Verlinde formula (6) implies

$$N_{\mu J\nu}{}^{J\sigma} \;=\; N_{\mu\nu}{}^{\sigma} \quad . \tag{11}$$

With this preparation on simple currents we can now address our main aim to describe properties of coset chiral algebras.

It is convenient to distinguish the sectors of the G and P theories by using different types of labels,

$$\hat{\mathfrak{g}}_k : \ \mu, \nu, \rho, \ldots \in \text{Rep(G)} \qquad\qquad \hat{\mathfrak{p}}_{k'} : \ a, b, c, \ldots \in \text{Rep(P)} \quad .$$

[1] The only exception is E_8 at level 2.

The generic coset sectors are labeled by tupels (μ, a) satisfying some constraints which are known as branching selection rules and depend on the specific algebras and embeddings. To be concrete, any allowed pair (μ, a) has to satisfy

$$\mathcal{P}\mu - a \ \in \ \mathcal{P}\mathcal{Q} \tag{12}$$

where $\mathcal{Q}$ denotes the root lattice of $\mathfrak{g}$. If this relation would not be satisfied, there would be no chance to find a weight in the weight system of μ that is projected onto a. We denote the set of allowed coset labels by

$$\mathrm{All}(\mathrm{G}/\mathrm{P}) \ = \ \left\{ (\mu, a) \,\middle|\, \mathcal{P}\mu - a \in \mathcal{P}\mathcal{Q} \right\} \ \subset \ \mathrm{Rep}(\mathrm{G}) \times \mathrm{Rep}(\mathrm{P}) \ \ .$$

In addition, certain pairs (μ, a) need to be identified because they give rise to one and the same sector [38, 39]. Generically, this field identification exactly corresponds to elements in the common center $\mathcal{Z}(\mathrm{G}) \cap \mathcal{Z}(\mathrm{P})$ of the groups G and P.[2]

Before we continue, let us make the last statement precise. To this end, we pick two elements $J \in \mathcal{Z}(\mathrm{G})$ and $J' \in \mathcal{Z}(\mathrm{P})$. We say that the pair (J, J') lies in the common center if the relation

$$Q_J(\mu) \ = \ Q_{J'}(\mathcal{P}\mu) \tag{13}$$

holds for all weights $\mu \in \mathrm{Rep}(\mathrm{G})$. The abelian group of all pairs (J, J') satisfying this condition shall be denoted by $\mathcal{G}_{\mathrm{id}}$. By construction, $\mathcal{G}_{\mathrm{id}}$ is a subgroup of the product $\mathcal{Z}(\mathrm{G}) \times \mathcal{Z}(\mathrm{P})$. Sometimes this group also is called identification group. Note that it depends explicitly on the embedding.

The first application of this identification group $\mathcal{G}_{\mathrm{id}}$ is that it allows to reformulate the branching selection rule (12) in a completely algebraic way. It turns out that the allowed weights may be described by

$$\mathrm{All}(\mathrm{G}/\mathrm{P}) \ = \ \left\{ (\mu, a) \,\middle|\, Q_J(\mu) = Q_{J'}(a) \text{ for all } (J, J') \in \mathcal{G}_{\mathrm{id}} \right\} \ \ . \tag{14}$$

Moreover, we can now also address the issue of field identification. Let us note that for generic sectors $(\mu, a), (\nu, b) \in \mathrm{All}(\mathrm{G}/\mathrm{P})$, the modular S-matrix of the coset theory is given by

$$S^{\mathrm{G}/\mathrm{P}}_{(\mu,a)(\nu,b)} \ = \ |\mathcal{G}_{\mathrm{id}}| \, S^{\mathrm{G}}_{\mu\nu} \ \bar{S}^{\mathrm{P}}_{ab} \ \ . \tag{15}$$

[2] In so-called Maverick cosets (see e.g. [40]) and cosets arising from conformal embeddings this statement is not true. Conformal embeddings, however, are restricted to $k = 1$ and all known Maverick cosets also are a low level phenomenon.

If we act on the first weight by an element $(J, J') \in \mathcal{G}_{\rm id}$ we obtain

$$S^{\rm G/P}_{(J\mu,J'a)(\nu,b)} \;=\; e^{2\pi i(Q_J(\nu)-Q_{J'}(b))}\, S^{\rm G/P}_{(\mu,a)(\nu,b)} \;=\; S^{\rm G/P}_{(\mu,a)(\nu,b)}$$

from equation (9). The phase factor vanishes because of the branching selection rule expressed in the relation (14). This hints towards an identification of the sectors $(J\mu, J'a)$ and (μ, a). In fact, under certain simplifying assumptions one can show that inequivalent irreducible representations of the coset theory are labeled by elements

$$[\mu, a] \in {\rm Rep(G/P)} \;=\; {\rm All(G/P)}/\mathcal{G}_{\rm id} \quad . \tag{16}$$

Complications arise when there exist fixed points, i.e. sectors with the property $(J\mu, J'a) = (\mu, a)$ for at least one pair $(J, J') \in \mathcal{G}_{\rm id}$. In this case, the representation spaces carry (reducible) representations of the relevant stabilizer subgroup of $\mathcal{G}_{\rm id}$. Determining the irreducible constituents and the associated modular data is known as fixed point resolution [41, 36, 42, 43]. We will circumvent these technical difficulties and assume in the following that our field identification has no fixed points. Under these circumstances all orbits $\mathcal{G}_{\rm id}(\mu, a)$ have the same length $|\mathcal{G}_{\rm id}|$.

From the expression (15) and the Verlinde formula we can easily deduce the following expression for the fusion coefficients of the coset model,

$$N_{[\mu,a],[\nu,b]}^{[\sigma,c]} \;=\; \sum_{(J,J')\in\mathcal{G}_{\rm id}} N_{\mu\nu}^{J\sigma}\; N_{ab}^{J'c} \quad . \tag{17}$$

Below we shall also need a projector which implements the branching selection rule (14). This is rather easy to introduce by the explicit formula

$$P(\mu, a) \;=\; \frac{1}{|\mathcal{G}_{\rm id}|} \sum_{(J,J')\in\mathcal{G}_{\rm id}} e^{2\pi i(Q_J(\mu)-Q_{J'}(a))} \quad . \tag{18}$$

The definition of All(G/P) directly implies that $P(\mu, a) = 1$ for all (μ, a) in the set All(G/P) and that it vanishes otherwise.

3 The new boundary states

Our aim now is to construct new boundary states for a theory whose partition function is given by the charge conjugated modular invariant of the chiral algebra $\mathcal{A}$(G). We shall analyse this theory with respect to some intermediate chiral algebra $\mathcal{A}(\mathrm{P})$. This will lead us to a set of boundary conditions extending the usual Cardy type conditions. Explicit expressions for the boundary states and the associated open string spectra are provided for different gluing conditions.

3.1 Decomposition of the bulk modular invariant

Before going into the discussion of the boundary states, let us present the general idea of our construction. As usual, our starting point is some bulk theory with a state space that is assumed to be charge conjugated with respect to some chiral algebra $\mathcal{A}(\mathrm{G})$,

$$\mathcal{H}^{\mathrm{G}} \;\cong \bigoplus_{\mu\in\mathrm{Rep}(\mathrm{G})} \mathcal{H}^{\mathrm{G}}_{\mu}\otimes\bar{\mathcal{H}}^{\mathrm{G}}_{\mu^+} \;\; .$$

But now we want to construct boundary states which break at least some part of the chiral symmetry. To be precise, we only want to preserve the subalgebra

$$\mathcal{A}(\mathrm{G/P})\oplus\mathcal{A}(\mathrm{P}) \;\hookrightarrow\; \mathcal{A}(\mathrm{G})$$

at the boundary. It is then natural to decompose the full state space according to the action of the smaller chiral algebra. Under the restriction to $\mathcal{A}(\mathrm{G/P})\oplus\mathcal{A}(\mathrm{P})$, the irreducible representations of $\mathcal{A}(\mathrm{G})$ can be reduced to

$$\mathcal{H}^{\mathrm{G}}_{\mu} \;\cong \bigoplus_{(\mu,a)\in\mathrm{All(G/P)}} \mathcal{H}^{\mathrm{G/P}}_{(\mu,a)}\otimes\mathcal{H}^{\mathrm{P}}_{a} \;\; .$$

Note that the sum is restricted to those values of a for which the branching selection rule (14) is satisfied. The last relation also illustrates why we wanted to preserve the chiral algebra $\mathcal{A}(\mathrm{G/P})\oplus\mathcal{A}(\mathrm{P})$, not only the subalgebra $\mathcal{A}(\mathrm{P})$: as representations of $\mathcal{A}(\mathrm{G/P})$ are generically infinite-dimensional the resulting theory would become non-rational otherwise. The decomposition of the full space reads

$$\mathcal{H}^{\mathrm{G}} \;\cong \bigoplus_{(\mu,a),(\mu,\bar{a})\in\mathrm{All(G/P)}} \mathcal{H}^{\mathrm{G/P}}_{(\mu,a)}\otimes\mathcal{H}^{\mathrm{P}}_{a}\otimes\bar{\mathcal{H}}^{\mathrm{G/P}}_{(\mu,\bar{a})^+}\otimes\bar{\mathcal{H}}^{\mathrm{P}}_{\bar{a}^+} \;\; . \tag{19}$$

In terms of partition functions, the decomposition can be expressed as follows,

$$Z \;=\; \sum_{\mu\in\mathrm{Rep}(\mathcal{A})}\Big|\,\chi^{\mathrm{G}}_{\mu}\Big|^2 \;=\; \sum_{\mu\in\mathrm{Rep}(\mathcal{A})}\Big|\sum_{(\mu,a)\in\mathrm{All(G/P)}}\chi^{\mathrm{G/P}}_{(\mu,a)}\,\chi^{\mathrm{P}}_{a}\Big|^2 \;\; . \tag{20}$$

To simplify notations we have used that the characters are invariant under the substitution $\mu \mapsto \mu^+$. Hence, on the level of partition functions, we do not distinguish between the diagonal and the charge conjugated modular invariant. Let us stress that the theory is not charge conjugated with respect to the smaller chiral algebra. In particular, the boundary states preserving the smaller chiral algebra can not be constructed by Cardy's solution.

In our setting we are free to choose two different gluing automorphisms $\Omega^{\mathrm{G/P}}$ and Ω^{P} for chiral fields in the two individual parts of the reduced chiral algebra and to require

$$\begin{aligned} \big(\phi(z) - \Omega^{\mathrm{G/P}}\bar{\phi}(\bar{z})\big)|B\rangle &= 0 \\ \big(\psi(z) - \Omega^{\mathrm{P}}\bar{\psi}(\bar{z})\big)|B\rangle &= 0 \end{aligned}$$

for arbitrary fields $\phi \in \mathcal{A}(\mathrm{G/P})$ and $\psi \in \mathcal{A}(\mathrm{P})$. Note that these conditions ensure the Virasoro field $T^{\mathrm{G}} = T^{\mathrm{G/P}} + T^{\mathrm{P}}$ of the theory to be preserved along the boundary. Naively one might think that boundary states satisfying these gluing conditions can be factorized into boundary states of the two chiral algebras $\mathcal{A}(\mathrm{G/P})$ and $\mathcal{A}(\mathrm{P})$. However, this is not true because the partition function does not factorize.

We will certainly not be able to solve the boundary theories for an arbitrary choice of $\Omega^{\mathrm{G/P}}$ and Ω^{P}. In the next subsection we shall discuss the special case in which both these gluing automorphisms are trivial. After that, we address a more general possibility in which $\Omega^{\mathrm{G/P}}$ is still trivial while any choice of Ω^{P} is allowed.

3.2 Trivial gluing automorphisms

We start with boundary conditions for which the left and right movers are glued trivially, $\Omega = \Omega^{\mathrm{G/P}} \otimes \Omega^{\mathrm{P}} = \mathbf{id} \otimes \mathbf{id}$. This induces the identity map $\omega = \mathbf{id} \times \mathbf{id}$ on the set $\mathrm{Rep(G/P)} \times \mathrm{Rep(P)}$ of sectors. The constituents of the Hilbert space $\mathcal{H}^{\mathrm{G}}$ which are left-right-symmetric with respect to the automorphism ω are given by

$$\mathcal{H}^{\mathrm{G/P}}_{(\mu,a)} \otimes \mathcal{H}^{\mathrm{P}}_{a} \otimes \bar{\mathcal{H}}^{\mathrm{G/P}}_{(\mu,a)^+} \otimes \bar{\mathcal{H}}^{\mathrm{P}}_{a^+} \quad .$$

Hence, Ishibashi states are labeled unambigously by pairs $(\mu, a) \in \mathrm{All(G/P)}$, i.e. μ, a run over all representations such that the branching selection rule (14) is satisfied. Let us point out that in these labels, no field identification is made. We choose the standard normalization [7] of Ishibashi states such that

$$\langle\!\langle(\mu,a)|\,\tilde{q}^{\frac{1}{2}(L_0+\bar{L}_0-c/12)}\,|(\nu,b)\rangle\!\rangle \;=\; \delta^{\mu}_{\nu}\,\delta^{a}_{b}\;\chi^{\rm G/P}_{(\mu,a)}(\tilde{q})\,\chi^{\rm P}_{a}(\tilde{q}) \quad .$$

As we shall see, the elementary boundary states are labeled by elements (ρ,r) from the set $\mathcal{B}^{\mathbf{id}\times\mathbf{id}} = \big(\mathrm{Rep(G)}\times\mathrm{Rep(P)}\big)/\mathcal{G}_{\rm id}$. Their expansion in terms of Ishibashi states reads

$$|(\rho,r)\rangle \;=\; \sum_{(\mu,a)\in \mathrm{All(G/P)}} B^{(\mu,a)}_{(\rho,r)}\,|(\mu,a)\rangle\!\rangle \tag{21}$$

with coefficients $B^{(\mu,a)}_{(\rho,r)}$ being determined by the modular S-matrix of the G and the P theory through the simple formula

$$B^{(\mu,a)}_{(\rho,r)} \;=\; \frac{S^{\rm G}_{\rho\mu}}{\sqrt{S^{\rm G}_{0\mu}}}\,\frac{\bar{S}^{\rm P}_{ra}}{\bar{S}^{\rm P}_{0a}} \quad . \tag{22}$$

The proof of this claim proceeds in several steps. Let us first note that (ρ,r) and $(J\rho,J'r)$ lead to the same boundary state. This is a simple consequence of eq. (9) and the definition (14) of All(G/P). We will show now that the proposed boundary states possess a consistent open string spectrum. Finally, it remains to demonstrate that the identity field propagates in between two boundary conditions if and only if these two boundary conditions are identical.

Let us begin by computing the open string spectrum in between two boundary conditions (ρ_1,r_1) and (ρ_2,r_2),[3]

$$\begin{aligned} Z \;&=\; Z_{(\rho_1,r_1),(\rho_2,r_2)}(q) \;=\; \langle(\rho_1,r_1)|\,\tilde{q}^{\frac{1}{2}(L_0+\bar{L}_0-c/12)}\,|(\rho_2,r_2)\rangle \\ &=\; \sum_{(\mu,a),[\nu,b],c}\Big[\,\bar{B}^{(\mu,a)}_{(\rho_1,r_1)}B^{(\mu,a)}_{(\rho_2,r_2)}S^{\rm G/P}_{(\mu,a),(\nu,b)}S^{\rm P}_{ac}\Big]\,\chi^{\rm G/P}_{(\nu,b)}(q)\,\chi^{\rm P}_{c}(q) \\ &=\; |\mathcal{G}_{\rm id}|\sum_{(\mu,a),[\nu,b],c}\left[\frac{\bar{S}^{\rm G}_{\rho_1\mu}S^{\rm G}_{\rho_2\mu}S^{\rm G}_{\nu\mu}}{S^{\rm G}_{0\mu}}\;\frac{S^{\rm P}_{r_1a}\bar{S}^{\rm P}_{r_2a}\bar{S}^{\rm P}_{ba}S^{\rm P}_{ca}}{S^{\rm P}_{0a}\bar{S}^{\rm P}_{0a}}\right]\chi^{\rm G/P}_{(\nu,b)}(q)\,\chi^{\rm P}_{c}(q) \quad . \end{aligned}$$

In the second step we inserted our expression for the coefficients of the boundary states and formula (15) for the S-matrix of the coset model. Note that

[3] To save space we omit the ranges of the summation indices. The summation rules are: $(\mu,a)\in$ All(G/P), $[\mu,a]\in$ Rep(G/P) and all other (single) indices run over Rep(G) or Rep(P), respectively.

the coefficients of the individiual characters on the right hand side are not expected to be integers since we still sum over labels which are related by the action of the identification group. Now we use that the quantum dimensions S_{ra}/S_{0a} form a representation of the fusion algebra,

$$\frac{S^{\rm P}_{r_1a}}{S^{\rm P}_{0a}}\,\frac{\bar S^{\rm P}_{r_2a}}{\bar S^{\rm P}_{0a}} \;=\; \sum_{d\in{\rm Rep(P)}} N_{r_1r_2^+}{}^d\,\frac{S^{\rm P}_{da}}{S^{\rm P}_{0a}} \quad ,$$

and obtain

$$Z \;=\; |\mathcal{G}_{\rm id}| \sum_{(\mu,a),[\nu,b],c,d} N_{r_1r_2^+}{}^d \left[\frac{\bar S^{\rm G}_{\rho_1\mu} S^{\rm G}_{\rho_2\mu} S^{\rm G}_{\nu\mu}}{S^{\rm G}_{0\mu}}\,\frac{S^{\rm P}_{da}\bar S^{\rm P}_{ba} S^{\rm P}_{ca}}{S^{\rm P}_{0a}}\right] \chi^{\rm G/P}_{(\nu,b)}(q)\,\chi^{\rm P}_c(q) \quad .$$

If the sum over the pairs (μ,a) was not restricted by the branching selection rule (14), the quotients of S-matrices could be evaluated by means of the Verlinde formula (6). But as it stands, this step can not be performed so easily. However, we can implement the constraint my means of the projector $P(\mu,a)$ which has been defined in (18). This yields

$$\begin{aligned} Z \;&=\; |\mathcal{G}_{\rm id}| \sum_{\mu,a,[\nu,b],c,d} P(\mu,a)\, N_{r_1r_2^+}{}^d \left[\frac{\bar S^{\rm G}_{\rho_1\mu} S^{\rm G}_{\rho_2\mu} S^{\rm G}_{\nu\mu}}{S^{\rm G}_{0\mu}}\,\frac{S^{\rm P}_{da}\bar S^{\rm P}_{ba} S^{\rm P}_{ca}}{S^{\rm P}_{0a}}\right] \chi^{\rm G/P}_{(\nu,b)}(q)\,\chi^{\rm P}_c(q) \\ &=\; \sum_{\substack{\mu,a,[\nu,b],c,d\\ (J,J')\in\mathcal{G}_{\rm id}}} \frac{e^{2\pi i Q_J(\mu)}}{e^{2\pi i Q_{J'}(a)}}\, N_{r_1r_2^+}{}^d \left[\frac{\bar S^{\rm G}_{\rho_1\mu} S^{\rm G}_{\rho_2\mu} S^{\rm G}_{\nu\mu}}{S^{\rm G}_{0\mu}}\,\frac{S^{\rm P}_{da}\bar S^{\rm P}_{ba} S^{\rm P}_{ca}}{S^{\rm P}_{0a}}\right] \chi^{\rm G/P}_{(\nu,b)}(q)\,\chi^{\rm P}_c(q) \quad . \end{aligned}$$

We then use the fact that the exponentials may be pulled into the S-matrices with the help of eq. (9). The result is

$$Z \;=\; \sum_{\substack{\mu,a,[\nu,b],c,d\\ (J,J')\in\mathcal{G}_{\rm id}}} N_{r_1r_2^+}{}^d \left[\frac{\bar S^{\rm G}_{\rho_1\mu} S^{\rm G}_{\rho_2\mu} S^{\rm G}_{J\nu\ \mu}}{S^{\rm G}_{0\mu}}\,\frac{S^{\rm P}_{da}\bar S^{\rm P}_{J'b\ a} S^{\rm P}_{ca}}{S^{\rm P}_{0a}}\right] \chi^{\rm G/P}_{(\nu,b)}(q)\,\chi^{\rm P}_c(q) \quad .$$

As the field identification demands $\chi^{\rm G/P}_{(\nu,b)} = \chi^{\rm G/P}_{(J\nu,J'b)}$, we may collect the summations over $(J,J')\in\mathcal{G}_{\rm id}$ and $[\nu,b]\in{\rm Rep(G/P)}$ to give a sum over $(\nu,b)\in{\rm All(G/P)}$. Then, applying in addition the Verlinde formula (6), we finally arrive at

$$Z_{(\rho_1,r_1),(\rho_2,r_2)} \;=\; \sum_{\substack{(\nu,b)\in{\rm All(G/P)}\\ c,d\in{\rm Rep(P)}}} N_{\rho_1^+\rho_2}{}^\nu\, N_{r_1^+r_2}{}^d\, N_{dc}{}^b\;\chi^{\rm G/P}_{(\nu,b)}(q)\,\chi^{\rm P}_c(q) \quad . \qquad (23)$$

In the last step we also used some symmetries of the fusion rule coefficients (7) and charge conjugation invariance of the characters. Thereby we have shown that Z can be expanded into characters of the chiral algebra $\mathcal{A}(\mathrm{G/P}) \oplus \mathcal{A}(\mathrm{P})$ with manifestly non-negative integer coefficients.

Finally, we now wish to convince ourselves that the vacuum representation appears exactly once in the boundary partition function (23) with two identical elementary boundary conditions and that it does not contribute whenever the two boundary conditions are different. This is somewhat obscured by the possible field identification. By a calculation similar to the previous one it is possible to rewrite the partition function in the form

$$Z_{(\rho_1,r_1),(\rho_2,r_2)} \;=\; \sum_{(J,J')\in\mathcal{G}_{\mathrm{id}}} \delta^{\rho_1}_{J\rho_2}\delta^{r_1}_{J'r_2}\,\chi^{\mathrm{G/P}}_{(J0,J'0)}(q)\,\chi^{\mathrm{P}}_0(q) \;+\; \ldots$$

where $\ldots$ stand for other contributions that do not contain the vacuum character of $\mathcal{A}(\mathrm{G/P}) \oplus \mathcal{A}(\mathrm{P})$. Hence, the identity field only appears if (ρ_1, r_1) and (ρ_2, r_2) are identical up to the action of $\mathcal{G}_{\mathrm{id}}$, in agreement with our claim.

Let us conclude with some comments. First note that the partition function (23) can not be obtained by decomposing the usual Cardy boundary partition function. In fact, if we decompose the latter into characters of $\mathcal{A}(\mathrm{G/P}) \oplus \mathcal{A}(\mathrm{P})$, we obtain

$$Z_{\rho_1\rho_2} \;=\; \sum_{\nu\in\mathrm{Rep(G)}} N_{\rho_1^+\rho_2}{}^{\nu}\,\chi^{\mathrm{G}}_\nu(q) \;=\; \sum_{(\nu,b)\in\mathrm{All(G/P)}} N_{\rho_1^+\rho_2}{}^{\nu}\,\chi^{\mathrm{G/P}}_{(\nu,b)}(q)\,\chi^{\mathrm{P}}_b(q)$$

where the labels b for the $\mathcal{A}(\mathrm{P})$-sector coincide with the second label of the coset theory. But this is not the case for most of the partition functions of our boundary states. In other words, our new boundary theories manifestly break some of the chiral symmetry $\mathcal{A}(\mathrm{G})$ in the bulk theory. Note, however, that the right hand side of the previous equation coincides with the partition function for the pair $(\rho_1, 0), (\rho_2, 0)$. In other words, for the states of the special form $|(\rho, 0)\rangle$, the maximal chiral symmetry is restored and these states can be identified with Cardy's boundary states.

3.3 The case of partially twisted boundary conditions

Our construction possesses a natural extension to cases in which we choose a non-trivial gluing automorphism for one of the factors $\mathcal{A}(\mathrm{G/P})$ or $\mathcal{A}(\mathrm{P})$. To be specific, we shall assume that the gluing automorphism $\Omega^{\mathrm{G/P}}$ remains trivial. A solution is then possible for any $\Omega^{\rm P}$, provided we can solve the corresponding P-theory with charge conjugated modular invariant partition function. For technical reasons we shall also assume that the identification group $\mathcal{G}_{\rm id}$ is trivial.

To begin with, let us briefly comment on the solution of the auxiliary P-theory with charge conjugate modular invariant partition function. As usual, boundary states for the gluing automorphism $\Omega^{\rm P}$ are built up from Ishibashi states $|a\rangle\rangle^{\rm P}$ where $a \in {\rm Rep(P)}$ and $a = \omega(a) := \omega^{\rm P}(a)$,

$$|\alpha\rangle^{\rm P} \;=\; \sum_{\omega(a)=a} \frac{\psi_\alpha{}^a}{\sqrt{S^{\rm P}_{0a}}}\,|a\rangle\rangle^{\rm P} \quad .$$

Here, α is chosen from the set $\alpha \in \mathcal{B}^\omega({\rm P})$. We will assume that the structure constants $\psi_\alpha{}^a$ are known explicitly. According to the general theory, they determine a NIM-rep through

$$\big(n_b\big)_\beta{}^\alpha \;=\; \sum_{\omega(a)=a} \frac{\bar{\psi}_\alpha{}^a\;\psi_\beta{}^a\;S^{\rm P}_{ba}}{S^{\rm P}_{0a}} \quad .$$

For detailed expressions we refer the reader to [5, 6, 7, 44, 45, 46] (see also [47, 48] for the limit $k \to \infty$).

With this solution of the auxiliary problem in mind, we can return to our main goal of finding new symmetry breaking boundary states for the G-theory. Once more, we have to determine which sectors in the decomposition (19) can contribute Ishibashi states. The condition is

$$\big((\mu,\bar{a}),\bar{a}\big)^+ \;\sim\; \big((\mu,a),\omega(a)\big)^+ \quad .$$

We did not write "=" because the labels must be related only up to a field identification in the coset part. Since the Rep(P) part is not subject to any field identification, the previous relation immediately implies $\bar{a} = \omega(a)$. Hence we are left to decide whether for given $(\mu, a) \in {\rm All(G/P)}$ we are able to find an element $(J, J') \in \mathcal{G}_{\rm id}$ of the field identification group such that

$$(J\mu, J'a) \;=\; \big(\mu,\omega(a)\big) \quad .$$

Up to now, we do not know how to determine all solutions to these equations in a systematic way. Obviously, such a classification depends strongly on the detailed structure of the field identification group and of its compatibility with the automorphism ω. In addition, it seems likely that one runs into troubles with field identification fixed points in the general case, a technical difficulty which we want to avoid.

We thus restrict ourselves to classes of embeddings for which the field identification group is trivial, i.e. $\mathcal{G}_{\rm id} = \{\mathbf{id}\}$. This assumption in turn implies that there exist no branching selection rules. In particular, the coset representations are given by the set Rep(G/P) = Rep(G) × Rep(P). Let us emphasize that there is a large set of coset models for which our assumption holds, including all theories with an E_8 subgroup (which has trivial center) and the maximal embeddings $A_{n-1} \hookrightarrow A_n$ at embedding index $x_e = 1$.

With our assumption $\mathcal{G}_{\rm id} = \{\mathbf{id}\}$ being made, the decomposition of the Hilbert space takes the particularly simple form

$$\mathcal{H}^{\rm G} \;\cong\; \bigoplus_{\substack{\mu\in{\rm Rep(G)}\\ a,\bar a\in{\rm Rep(P)}}} \mathcal{H}^{\rm G/P}_{(\mu,a)} \otimes \mathcal{H}^{\rm P}_{a} \otimes \;\bar{\mathcal{H}}^{\rm G/P}_{(\mu,\bar a)^+} \otimes \bar{\mathcal{H}}^{\rm P}_{\bar a^+} \quad . \tag{24}$$

Ishibashi states in this case are given by $|(\mu,a),a\rangle\rangle$ where $\mu \in$ Rep(G) and $a \in$ Rep(P) with $\omega(a) = a$. Using the coefficients ${\psi_\alpha}^a$ from the solution of the auxiliary P-theory, we define boundary states by

$$|(\rho,\gamma)\rangle\rangle \;=\; \sum_{\substack{\mu\in{\rm Rep(G)}\\ \omega(a)=a}} \frac{S^{\rm G}_{\rho\mu}}{\sqrt{S^{\rm G}_{0\mu}}}\, \frac{{\bar\psi_\gamma}^{\ a}}{\bar S^{\rm P}_{0a}}\, |(\mu,a),a\rangle\rangle \quad . \tag{25}$$

Note that the expression imitates the construction of the last subsection. Along the line of our previous computations, one can also work out the boundary partition function. It is given by the formula

$$Z^{\mathbf{id}\times\omega}_{(\rho_1,\gamma_1),(\rho_2,\gamma_2)}(q) \;=\; \sum_{\substack{\nu\in{\rm Rep(G)}\\ d\in{\rm Rep(P)}}} {N_{\rho_1^+\rho_2}}^{\nu}\, {N_{b^+c}}^{d}\, {\left(n_d\right)_{\gamma_1}}^{\gamma_2}\, \chi^{\rm G/P}_{(\nu,b)}(q)\, \chi^{\rm P}_{c}(q)$$

which contains the NIM-rep that comes with our solution of the P-theory. It is easy to check that this expression satisfies all consistency requirements.

Let us briefly comment on the generalization of this result to the case with non-trivial field identification group $\mathcal{G}_{\rm id} \neq \{\mathbf{id}\}$. The construction of the previous section suggests that one needs properties of the coefficients ${\psi_\alpha}^a$ which are similar to those for the S-matrix given in eq. (9). Such relations, however, have been worked out in [49] for a number of examples.

4 Orbifold construction and brane geometry

In [8, 9] Maldacena, Moore and Seiberg developed a construction of symmetry breaking branes on group manifolds that is based on an orbifolding. We shall now compare the results of their proposal with our algebraic analysis of symmetry breaking boundary states. After presenting the general ideas of the orbifold construction in the first subsection we show that it is capable of reproducing only a subset of our boundary states, namely those that are obtained by choosing an abelian denominator P, i.e. $\mathrm{Rep}(\mathrm{P}) = \mathcal{Z}(\mathrm{P})$. Under this condition, our new boundary states possess a simple geometrical interpretation which emerges as a by-product of our discussion.

4.1 The orbifold construction - a review

Our aim here is to study branes in a simple current orbifold of $\mathrm{G/P}\times\mathrm{P}$. Before we address this rather complicated background, let us make some introductory remarks on brane geometries in group manifolds, cosets and orbifolds. As we have mentioned before, the description of branes in simple simply-connected compact group manifolds G involves the WZW theory based on an affine Kac-Moody algebra $\hat{\mathfrak{g}}_k$ with *charge conjugation* bulk partition function. The value of the level k controls the size of the group manifold. It is well-known that maximally symmetric D-branes on group manifolds are localized along quantized (twisted) conjugacy classes

$$\mathcal{C}^\Omega_\mu \;=\; \left\{\, g h_\mu \Omega(g)^{-1} \,\middle|\, g \;\in\; \mathrm{G} \,\right\}$$

where Ω can be any automorphism of the group G [50, 51]. Even though some of these D-branes wrap trivial cycles, they are all stable due to the presence of a non-vanishing three form flux [52, 53, 54].

A large variety of backgrounds arise from WZW models as cosets G/P and orbifolds of the form G/Γ. Maximally symmetric branes in coset theories with chiral algebra $\mathcal{A}(\mathrm{G/P})$ and charge conjugation bulk partition function are localized along the image of $\mathcal{C}^\Omega_\mu\big(\mathcal{C}^\Omega_a\big)^{-1}$ under the projection from G to G/P [8, 55, 56, 57]. D-branes on orbifolds G/Γ can be represented by summing over all their pre-images in G, at least as long as they do not contain fixed points for the action of Γ on G.

After this preparation, we now want to look at orbifolds obtained from product geometries of the form $\mathrm{G/P} \times \mathrm{P}$. We request the orbifold group Γ

to be generated by simple currents, i.e. $\Gamma \subset \mathcal{Z}(\mathrm{G/P}) \times \mathcal{Z}(\mathrm{P})$. The sectors $\big([\mu,a],b\big)$ of $\mathcal{A}(\mathrm{G/P}) \oplus \mathcal{A}(\mathrm{P})$ fall into orbits $\big[[\mu,a],b\big]$ with respect to the action of Γ. With each of these orbits we associate two numbers, namely the monodromy charges $Q_J\big(\big[[\mu,a],b\big]\big)$, $J \in \Gamma$, and the order $|\mathcal{S}_{[[\mu,a],b]}|$ of the stabilizer subgroup. The orbifold bulk partition function is then given by (see e.g. [36])

$$Z^{\mathrm{orb}}(q) \;=\; \sum_{Q_\Gamma([[\mu,a],b])=0} \Big|\mathcal{S}_{[[\mu,a],b]}\Big| \left| \sum_{([\nu,c],d)\in[[\mu,a],b]} \chi^{\mathrm{G/P}}_{(\nu,c)}(q)\chi^{\mathrm{P}}_d(q) \right|^2 \quad . \tag{26}$$

Boundary states of the orbifold theory can be obtained from the Cardy states $|[\mu,a],b\rangle = |[\mu,a]\rangle^{\mathrm{G/P}} \otimes |b\rangle^{\mathrm{P}}$ of the charge conjugated covering theory by averaging over the action of the orbifold group Γ. This leads to boundary states of the form (see e.g. [58])

$$\big|\big[[\mu,a],b\big]\big\rangle \;=\; \frac{1}{\sqrt{|\Gamma|}} \sum_{(J,J')\in\Gamma} |J[\mu,a],J'b\rangle \tag{27}$$

where the labels $\big[[\mu,a],b\big]$ of boundary states now take values in the set $\big(\mathrm{Rep}(\mathrm{G/P}) \times \mathrm{Rep}(\mathrm{P})\big)/\Gamma$. The geometric interpretation of these boundary states is obvious from our remarks above. It is also easy to calculate the boundary partition function

$$Z^{\mathrm{orb}}_{[[\mu,a],b],[[\nu,c],d]} \;=\; \sum_{(J,J')\in\Gamma} \; \sum_{[\sigma,e]\in\mathrm{Rep}(\mathrm{G/P}),f\in\mathrm{Rep}(\mathrm{P})} N^{J[\sigma,e]}_{[\mu,a]^+[\nu,c]}\, N^{J'f}_{b^+d}\, \chi^{\mathrm{G/P}}_{[\sigma,e]}\, \chi^{\mathrm{P}}_f \quad .$$

When the orbifold action has fixed points some of these states may be resolved further, but we will not discuss this issue. The main point here was to outline how one can obtain branes in the background (26). They are labeled by elements of $\big(\mathrm{Rep}(\mathrm{G/P}) \times \mathrm{Rep}(\mathrm{P})\big)/\Gamma$ and come with an obvious geometric interpretation. Moreover, in [59] one can find explicit formulas for the boundary operator product expansions in such boundary theories (see also [60] for a generalization to orbifolds with fixed points).

Let us note that open string theory on conformal field theory orbifolds was pioneered by Sagnotti and collaborators starting from [61] and systematized in [62, 63] (see also e.g. [64, 65, 66]). Important contributions were made later by Behrend et al. [67, 7] and by Fuchs et al. [5, 68, 6] (see also [69, 70]).

4.2 Comparison with the new boundary states

Our task now is to find a choice for the orbifold group $\Gamma = \Gamma_0 \subset \mathcal{Z}(\mathrm{G/P}) \times \mathcal{Z}(\mathrm{P})$ such that the partition function (26) coincides with the charge conjugated modular invariant of the G-theory, i.e. with the expression (20). In more geometric terms, the condition on the orbifold group is $\mathrm{G} = (\mathrm{G/P} \times \mathrm{P})/\Gamma_0$. It will turn out that the existence of an appropriate group Γ_0 imposes strong constraints on the choice of P. Once these constraints have been formulated, we shall compare the boundary states (27) with our new boundary states (21, 22).

To formulate necessary conditions for the equivalence of the partition function (20) of the G-theory with one of the orbifold partition functions (26), we shall concentrate on terms that contain a factor $\chi_{[0,0]}\chi_0$ from the holomorphic sector,

$$Z \;=\; \sum\nolimits_a \chi^{\mathrm{G/P}}_{[0,0]}\; \chi^{\mathrm{P}}_0\; \bar{\chi}^{\mathrm{G/P}}_{[0,a]}\; \bar{\chi}^{\mathrm{P}}_a \;+\; \dots \quad ,$$
$$Z^{\mathrm{orb}} \;=\; \sum_{(J,J')\in\Gamma} \chi^{\mathrm{G/P}}_{[0,0]}\; \chi^{\mathrm{P}}_0\; \bar{\chi}^{\mathrm{G/P}}_{J[0,0]}\; \bar{\chi}^{\mathrm{P}}_{J'0} \;+\; \dots \quad .$$

The summation over a in the first expression is restricted such that $(0,a) \in \mathrm{All(G/P)}$. We can now read off one important condition for the equivalence: all the labels $a \in \mathrm{Rep(P)}$ that appear in the summation must be simple currents of $\mathcal{A}(\mathrm{P})$, i.e. elements of $\mathcal{Z}(\mathrm{P})$. Under this condition we can set

$$\Gamma \;=\; \Gamma_0 \;=\; \big\{\big([0,a],a\big)\;\big|\;(0,a)\in \mathrm{All(G/P)}\big\} \;\subset \mathcal{Z}(\mathrm{G/P})\times\mathcal{Z}(\mathrm{P}) \quad .$$

By projection on the first or second factor, Γ_0 can be identified with a subgroup of both $\mathcal{Z}(\mathrm{P})$ and $\mathcal{Z}(\mathrm{G/P})$. If the identification group $\mathcal{G}_{\mathrm{id}}$ is trivial, it follows that all sectors of $\mathcal{A}(\mathrm{P})$ must be simple currents, i.e. P must be abelian. In cases with non-trivial field identification, the orbifold construction with Γ_0 can reproduce the partition function of the G-theory even if some of the sectors of P are not simple currents. We shall provide one example at the end of this section.

A more detailed comparison of the bulk partition functions reveals a second necessary condition for the desired equivalence. Namely, one can see that the orbifold and the G-theory can only agree if $\Gamma = \Gamma_0$ acts transitively on the sets $\mathrm{All}_\mu(\mathrm{G/P}) := \big\{(\mu,b)\in \mathrm{All(G/P)}\big\}$. In particular, this implies that $\big|\mathrm{All(G/P)}/\Gamma_0\big| = |\mathrm{Rep(G)}|$.

We are now prepared to compare the brane spectra of the orbifold construction with the spectra obtained in Section 3. In the following analysis we assume that P is abelian which is the case for all the examples considered in [8, 9]. The orbifold construction of the background works for a slightly larger class of cases, but in such cases the brane spectra can be different, at least before resolving possible fixed points of Γ_0 (see below). Assuming that $\mathrm{Rep}(\mathrm{P}) = \mathcal{Z}(\mathrm{P})$, we want to verify first that both constructions provide the same number of boundary states. This amounts to saying that

$$\big(\mathrm{Rep}(\mathrm{G}/\mathrm{P}) \times \mathrm{Rep}(\mathrm{P})\big)/\Gamma_0 \;\cong\; \big(\mathrm{Rep}(\mathrm{G}) \times \mathrm{Rep}(\mathrm{P})\big)/\mathcal{G}_{\mathrm{id}} \quad . \tag{28}$$

By our assumption on P, the action of Γ_0 has no fixed points. The same holds true automatically for the action of $\mathcal{G}_{\mathrm{id}}$. Therefore, our results of Section 3 apply and it is easy to compute the order of the two sets in relation (28). For the set on the left hand side we find that

$$\left|\frac{\mathrm{Rep}(\mathrm{G}/\mathrm{P}) \times \mathrm{Rep}(\mathrm{P})}{\Gamma_0}\right| \;=\; \frac{|\mathrm{All}(\mathrm{G}/\mathrm{P})| \cdot |\mathrm{Rep}(\mathrm{P})|}{|\Gamma_0| \cdot |\mathcal{G}_{\mathrm{id}}|} \;=\; \frac{|\mathrm{Rep}(\mathrm{G})| \cdot |\mathrm{Rep}(\mathrm{P})|}{|\mathcal{G}_{\mathrm{id}}|} \quad .$$

This agrees with the number of new boundary states on the right hand side of eq. (28). If we drop the assumption $\mathrm{Rep}(\mathrm{P}) = \mathcal{Z}(\mathrm{P})$ the action of Γ_0 can have fixed points so that the number of unresolved branes is smaller than the number of branes we obtained from our construction.

To compare the open string spectra of the two sets of branes we have to go a step further and choose an explicit isomorphism between the labels. Let us propose

$$\vartheta : \big[[\mu, b], c\big] \;\mapsto\; (\mu, b - c) \quad .$$

Note that $b - c \in \mathrm{Rep}(\mathrm{P})$ makes sense for two elements $b, c \in \mathrm{Rep}(\mathrm{P})$ since we assume $\mathrm{Rep}(\mathrm{P}) = \mathcal{Z}(\mathrm{P})$ to be an abelian group. Furthermore, ϑ is well-defined because the action of Γ_0 on the labels $\big([\mu, b], c\big) \in \mathrm{Rep}(\mathrm{G}/\mathrm{P}) \times \mathrm{Rep}(\mathrm{P})$ adds the same a to b and c so that their difference $b - c$ is left invariant. In writing down the pair $(\mu, b - c)$ we have to pick a representative (μ, b) of the sector $[\mu, b]$. This is unique up to the action of the identification group $\mathcal{G}_{\mathrm{id}}$. But different representatives are mapped to the same $\mathcal{G}_{\mathrm{id}}$-orbit in $\mathrm{Rep}(\mathrm{G}) \times \mathrm{Rep}(\mathrm{P})$. Obviously, ϑ is surjective and hence, by our counting above, it is a bijection between the two sets of labels.

It is now straightforward to compare the boundary partition function coming from our construction with those arising from the orbifold analysis. In the following we shall identify the elements $\big([0,a],a\big)\in\Gamma_0$ with $a\in\mathcal{Z}(\mathrm{P})$. We first calculate the boundary partition function from the orbifold point of view. Using the formula (17) we obtain

$$Z^{\mathrm{orb}}_{[[\mu,b_1],c_1],[[\nu,b_2],c_2]} \;=\; \sum_{\substack{a\in\Gamma_0,(\sigma,d)\in\mathrm{All(G/P)}\\ e\in\mathrm{Rep(P)}}} N_{\mu\nu}{}^{\sigma}\; N_{b_1^+\,b_2}{}^{a+d}\; N_{c_1^+\,c_2}{}^{a+e}\; \chi^{\mathrm{G/P}}_{(\sigma,d)}\,\chi^{(\mathrm{P})}_{e}\quad .$$

In this particular example, the fusion coefficients for the P-part are well-known and parts of the sum may be carried out. A careful calculation leads to

$$Z^{\mathrm{orb}}_{[[\mu,b_1],c_1],[[\nu,b_2],c_2]} = \sum_{(\sigma,d)\in\mathrm{All(G/P)}} N_{\mu\nu}{}^{\sigma}\,\chi^{\mathrm{G/P}}_{(\sigma,d)}\,\chi^{(\mathrm{P})}_{d+c_2-c_1+b_1-b_2}\quad .$$

Let us now consider the boundary partition function for the corresponding weights (μ,b_1-c_1) and (ν,b_2-c_2) in our approach. Again, a careful analysis yields

$$\begin{aligned} Z_{(\mu,b_1-c_1),(\nu,b_2-c_2)} \;&=\; \sum_{\substack{(\sigma,d)\in\mathrm{All(G/P)}\\ e,f\in\mathrm{Rep(P)}}} N_{\mu\nu}{}^{\sigma}\; N_{(b_1-c_1)^+\,(b_2-c_2)}{}^{f}\; N_{fe}{}^{d}\; \chi^{\mathrm{G/P}}_{(\sigma,d)}\,\chi^{(\mathrm{P})}_{e}\\ &=\; \sum_{(\sigma,d)\in\mathrm{All(G/P)}} N_{\mu\nu}{}^{\sigma}\,\chi^{\mathrm{G/P}}_{(\sigma,d)}\,\chi^{(\mathrm{P})}_{d+c_2-c_1+b_1-b_2}\quad . \end{aligned}$$

This agrees with the result of the orbifold construction and thus proves the equivalence of the two approaches.

4.3 An instructive example

For our general comparison of brane spectra in the previous subsection we assumed that P is abelian, i.e. that all sectors of $\mathcal{A}(\mathrm{P})$ are simple currents. This assumption was sufficient for the equivalence of the bulk partition functions but not necessary when the identification group $\mathcal{G}_{\mathrm{id}}$ is non-trivial. In this subsection we shall present one such example.

To begin with, let us set $\mathcal{A}(\mathrm{G})=\mathcal{A}\big(\widehat{\mathfrak{su}}(2)_{k_1}\oplus\widehat{\mathfrak{su}}(2)_{k_2}\big)$. This chiral algebra has several subalgebras $\mathcal{A}(\mathrm{P})$ that we could choose for our construction of boundary states. There are various abelian subalgebras that we could use such as $\mathcal{A}(\mathrm{P})=\mathcal{A}\big(\widehat{\mathfrak{u}}(1)_{k_1}\big)$ or $\mathcal{A}(\mathrm{P})=\mathcal{A}\big(\widehat{\mathfrak{u}}(1)_{k_1}\oplus\widehat{\mathfrak{u}}(1)_{k_2}\big)$ etc. To make things a bit more interesting we shall pick a non-abelian subalgebra, namely the chiral algebra that is generated by the diagonally embedded subalgebra $\widehat{\mathfrak{su}}(2)_{k_1+k_2}$. The corresponding projection of weights is given by $\mathcal{P}(\mu,\alpha)=\mu+\alpha$. Sectors of the coset theory are labeled by triples (μ,α,a) with $\mu\leq k_1, \alpha\leq k_2$, $a\leq k_1+k_2$ and the branching selection rule $\mu+\alpha-a=0 \mod 2$. One can show that there is only one non-trivial field identification current $(k_1,\, k_2,\, k_1+k_2)$. It gives rise to the field identification

$$(\mu,\alpha,a) \;\sim\; (k_1-\mu,\, k_2-\alpha,\, k_1+k_2-a) \quad .$$

Since we want to avoid fixed points of the field identification we have to consider the situation where at least one of the levels is odd.

We now specialise to the case $k_1=k_2=1$ for which the coset algebra is the chiral algebra of the Ising model. The relevant lists of sectors are,

$$\begin{aligned}
\mathrm{Rep}(\mathrm{G}) &= \mathrm{Rep}\big(\widehat{\mathfrak{su}}(2)_1\oplus\widehat{\mathfrak{su}}(2)_1\big) \;=\; \big\{(0,0),(0,1),(1,0),(1,1)\big\}\\
\mathrm{Rep}(\mathrm{P}) &= \mathrm{Rep}\big(\widehat{\mathfrak{su}}(2)_2\big) \;=\; \big\{0,1,2\big\}\\
\mathrm{Rep}(\mathrm{G/P}) &= \big\{(0,0,0)\sim(1,1,2),\,(0,0,2)\sim(1,1,0),\,(0,1,1)\sim(1,0,1)\big\} \quad .
\end{aligned}$$

Next, we have to decompose the charge conjugated modular invariant partition function for $\mathcal{A}\big(\widehat{\mathfrak{su}}(2)_1\oplus\widehat{\mathfrak{su}}(2)_1\big)$ into characters of the reduced chiral algebra. In our case this reads,

$$\begin{aligned}
Z &= \Big|\chi^{\mathrm{G}}_{(0,0)}\Big|^2+\Big|\chi^{\mathrm{G}}_{(0,1)}\Big|^2+\Big|\chi^{\mathrm{G}}_{(1,0)}\Big|^2+\Big|\chi^{\mathrm{G}}_{(1,1)}\Big|^2 \qquad (29)\\
&= \Big|\chi^{\mathrm{G/P}}_{(0,0,0)}\chi^{\mathrm{P}}_0+\chi^{\mathrm{G/P}}_{(0,0,2)}\chi^{\mathrm{P}}_2\Big|^2+2\Big|\chi^{\mathrm{G/P}}_{(0,1,1)}\chi^{\mathrm{P}}_1\Big|^2+\Big|\chi^{\mathrm{G/P}}_{(0,0,2)}\chi^{\mathrm{P}}_0+\chi^{\mathrm{G/P}}_{(0,0,0)}\chi^{\mathrm{P}}_2\Big|^2
\end{aligned}$$

where we already took the field identification into account. Following the results of Section 3.2 for trivial gluing conditions on the reduced chiral algebra, we find six boundary states with labels from the set

$$\begin{aligned}
\mathcal{B}=\big\{&(0,0,0)\sim(1,1,2),\,(1,0,0)\sim(0,1,2),\,(0,1,0)\sim(1,0,2),\\
&(1,1,0)\sim(0,0,2),\,(0,0,1)\sim(1,1,1),\,(1,0,1)\sim(0,1,1)\big\} \quad .
\end{aligned}$$

The four boundary states which are in the $\mathcal{G}_{\mathrm{id}}$-orbit of the labels $(\mu,\alpha,0)$ with trivial last entry can be identified with the four Cardy states of the model. All of them preserve the full chiral algebra $\mathcal{A}(\mathrm{G})$. For the remaining two boundary states we find

$$
\begin{aligned}
Z_{(0,0,1),(0,0,1)} &= \chi^{G/P}_{(0,0,0)}\chi^{P}_{0} + \chi^{G/P}_{(0,0,2)}\chi^{P}_{2} + \chi^{G/P}_{(0,0,0)}\chi^{P}_{2} + \chi^{G/P}_{(0,0,2)}\chi^{P}_{0} &&= \chi^{G}_{(0,0)} + \chi^{G}_{(1,1)}\\
Z_{(0,0,1),(1,0,1)} &= 2\chi^{G/P}_{(0,1,1)}\chi^{P}_{1} &&= \chi^{G}_{(1,0)} + \chi^{G}_{(0,1)}\\
Z_{(1,0,1),(1,0,1)} &= \chi^{G/P}_{(0,0,0)}\chi^{P}_{0} + \chi^{G/P}_{(0,0,2)}\chi^{P}_{2} + \chi^{G/P}_{(0,0,0)}\chi^{P}_{2} + \chi^{G/P}_{(0,0,2)}\chi^{P}_{0} &&= \chi^{G}_{(0,0)} + \chi^{G}_{(1,1)}\ .
\end{aligned}
$$

In particular, these boundary conditions preserve the full chiral symmetry! This is rather accidental and it is related to the fact that $\widehat{\mathfrak{su}}(2)_1 \oplus \widehat{\mathfrak{su}}(2)_1$ possesses an outer automorphism which acts by exchanging the two summands. With our construction we just recovered the two boundary states which belong to the associated twisted gluing condition. Note, however, that the spectra of open strings which stretch in between the four Cardy and the two non-Cardy type branes do only preserve the reduced chiral symmetry.

Before we conclude this section let us observe that the partition function (29) actually is an orbifold partition function obtained with the orbifold group

$$\Gamma_0 \;=\; \big\{\big([0,0,0],0\big),\big([0,0,2],2\big)\big\} \;\cong\; \mathbb{Z}_2\ .$$

In fact, the partition function of our model is recovered from the general expression (26) with the help of $Q_{([0,0,2],2)}\big(\big[[\mu,\alpha,a],b\big]\big) = (b-a)/2$ and using that the weight $\big[[0,1,1],1\big]$ is invariant under Γ_0. On the other hand, $\mathrm{P} = \widehat{\mathfrak{su}}(2)_2$ is not abelian since $(1) \in \mathrm{Rep}\big(\widehat{\mathfrak{su}}(2)_2\big)$ is not a simple current.

The orbifold group Γ_0 acts on the set $\mathrm{Rep}(\mathrm{G/P}) \times \mathrm{Rep}(\mathrm{P})$. Under this action, the nine elements of the latter are grouped into four orbits of length 2 and one fixed point. Hence, before resolution of the fixed point one obtains five boundary states of the form (27). But the one brane $|([0,1,1],1)\rangle$ which is associated with the fixed point of Γ_0 can be resolved into a sum of two elementary branes. In this way we recover all the six branes with symmetry $\mathcal{A}(\mathrm{G/P}) \oplus \mathcal{A}(\mathrm{P})$ from the orbifold construction. Note that in our approach the issue of fixed point resolution did not arise.

5 Product geometries and defect lines

Our final goal is to apply our general formalism to tensor products of two or more conformal field theories. Such product theories appear in particular whenever the background geometry splits into several factors. Moreover, tensor products also arise in the description of defect lines in 2D systems due to the so-called ‘folding trick’. We shall explain the general ideas behind these two types of applications in the first subsection and then illustrate the constructive power of our formulas by considering products of WZW models.

5.1 Boundary states in tensor products

As we have just noted, there exist at least two important motivations for the analysis of branes in product theories. First of all, many string backgrounds are obtained as products from several factors, interesting examples for our purposes being $AdS_3 \times S^3 \times T^4$ or $AdS_3 \times S^3 \times S^3 \times \mathbb{R}$. Some boundary states for such theories can be factored accordingly so that they are simply products of boundary states for each of the individual factors. But this does not exhaust all possibilities, as one can understand most easily by considering the simple product $S^1 \times S^1$. Since there are only point-like and space-filling branes on a circle, products of the corresponding boundary states can only give point-like branes, 1-dimensional branes running parallel to one of the two chosen circles and space filling branes with vanishing magnetic field. 1-dimensional branes which run diagonally through the 2-dimensional space and, closely related, space filling branes with a B-field are not factorisable. In this example, the factorisable branes suffice at least to generate all the possible RR-charges. However, this is not true for many other product geometries. In the case of $S^3 \times S^3$, for example, stable factorisable branes carry only 0-brane charge. But K-theory predicts the existence of additional branes with non-vanishing 3-brane charge which can not be built up from branes on the factors S^3. Hence, there is a strong demand for additional boundary states. We shall show below that our ideas can be fruitfully applied in this context.

There exists another – superficially very different – setup which leads to exactly the same type of problems. It arises by considering a one-dimensional quantum system with a defect (see e.g. [12, 13, 14, 15, 16] and [11, 17] for higher dimensional analogues), or, more generally, two different systems on the half-lines $x < 0$ and $x > 0$ which are in contact at the origin. The defect or contact at $x = 0$ could be totally reflecting, or more interestingly it could be partially (or fully) transmitting. To fit such system into our general discussion, we apply the usual folding trick (see Figure 1). After such a folding, the defect or contact is located at the boundary of a new system on the half-line. In the bulk, the new theory is simply a product of the two models that were initially placed to both sides of the contact at $x = 0$. Factorizing boundary states for

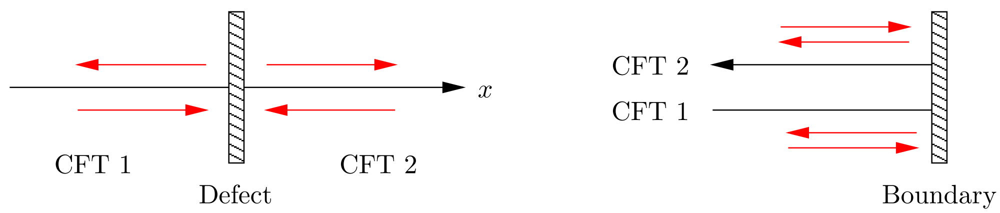

Figure 1: The folding trick relates a system on the real line with a defect to a tensor product theory on the half line.

the new product theory on the half line correspond to totally reflecting defects or contacts. With our new boundary states we can go further and couple the two systems in a non-trivial way. Since we always start with conformal field theories $\mathrm{H}_1 = \mathrm{H}_<$ and $\mathrm{H}_2 = \mathrm{H}_>$ on either side of $x = 0$, it is natural to look for contacts that preserve conformal invariance. This requires to preserve the sum of the two Virasoro algebras of the individual theories. After folding the system, the preserved Virasoro algebra is diagonally embedded into the product theory $\mathrm{G} = \mathrm{H}_1 \times \mathrm{H}_2$. Of course, one can often embed a larger chiral algebra P and then look for defects that preserve this extended symmetry. This is exactly the setup to which our general ideas apply.

5.2 Defect lines with jumping central charge

Our goal is now to construct examples of defect lines that join two conformal field theories with different central charge. Such situations are known to appear on the boundary of an AdS-space whenever there is a brane in the bulk which extends all the way to the boundary [19, 20, 21, 18]. To be specific, we will choose two WZW models based on the same semi-simple Lie group H but at different levels k_i and hence with different central charges $c_i = k_i \dim \mathrm{H}/(k_i + h^\vee)$. The boundary states we shall discuss may also be interpreted as D-branes in the product geometry $\mathrm{H}_1 \times \mathrm{H}_2$ in which the two factors may have different volume.

In this setup, the 'G-theory' is provided by the charge conjugated modular invariant partition function for $\mathcal{A}(\mathrm{G}) = \mathcal{A}(\mathrm{H}_1) \oplus \mathcal{A}(\mathrm{H}_2) = \mathcal{A}(\hat{\mathfrak{h}}_{k_1} \oplus \hat{\mathfrak{h}}_{k_2})$. Now we are instructed to choose some chiral subalgebra $\mathcal{A}(\mathrm{P})$. There are many different choices, but we shall use the affine algebra $\hat{\mathfrak{h}}_{k_1+k_2}$ which is embedded diagonally into $\hat{\mathfrak{h}}_{k_1} \oplus \hat{\mathfrak{h}}_{k_2}$. In other words, $\mathrm{P} \cong \mathrm{H}_D$ and $\mathcal{A}(\mathrm{H}_D) = \mathcal{A}(\hat{\mathfrak{h}}_{k_1+k_2})$.

We start by introducing some pieces of notation. As we have to deal with three *different* affine algebras $\hat{\mathfrak{h}}$, it is convenient to use different labels for the sectors of each of these algebras,

$$\hat{\mathfrak{h}}_{k_1} : \mu, \nu, \ldots \in \text{Rep}(\text{H}_1) \quad , \qquad \hat{\mathfrak{h}}_{k_2} : \alpha, \beta, \ldots \in \text{Rep}(\text{H}_2) \quad ,$$
$$\hat{\mathfrak{h}}_{k_1+k_2} : a, b, \ldots \in \text{Rep}(\text{H}_D) \quad .$$

In the case under consideration, the projection is given by $\mathcal{P}(\mu, \alpha) = \mu + \alpha$ and hence the branching selection rule (14) reduces to $\mu + \alpha - a \in \mathcal{Q}$ where $\mathcal{Q}$ denotes the root lattice of $\mathfrak{h}$. Consequently, the coset fields are labeled by triples

$$\big((\mu, \alpha), a\big) \in \text{Rep}(\text{H}_1 \times \text{H}_2 \times \text{H}_D) \quad \text{with} \quad \mu + \alpha - a \in \mathcal{Q}$$

which give rise to a set $\text{All}(\text{H}_1 \times \text{H}_2/\text{H}_D)$. Next we have to describe the field identifications. Let $\mathcal{Z}(\text{H}_i)$ be the centers of the H_i-theories which – under our assumptions – are all ismorphic. The common center is given by the diagonal subset

$$\big((J, J), J\big) \in \mathcal{Z}(\text{H}_1 \times \text{H}_2) \times \mathcal{Z}(\text{H}_D)$$

and leads us to the identification rules

$$\big((J\mu, J\alpha), Ja\big) \sim \big((\mu, \alpha), a\big) \quad .$$

Note in particular that no additional field identifications occur even in the case where the levels coincide, $k_1 = k_2 = k$, and the two types of fields μ, α take values in the same set.

After this preparation we can address the issue of field identification fixed points and spell out conditions for their absence. For the moment, let us focus on one of the factors and denote it by H. Every outer automorphism $J \in \mathcal{Z}(\text{H})$ is associated with a unique permutation π_J of affine fundamental weights. Denoting affine weights by square brackets, this action may be written as

$$J\,[\,\lambda_0\,,\,\ldots\,,\,\lambda_r\,] \;=\; [\,\lambda_{\pi_J(0)}\,,\,\ldots\,,\,\lambda_{\pi_J(r)}\,] \quad .$$

Thus the existence of a field identification fixed point is equivalent to finding an affine weight such that $\lambda_i = \lambda_{\pi_J(i)}$ for all $i = 0, \ldots, r$ and at least one non-trivial $J \in \mathcal{Z}(\text{H})$. The condition for the existence of such weights have been studied and the results for all simple Lie algebras are summarized in

Table 1. Note that the exceptional groups E_8, F_4 and G_2 have trivial centers and thus no field identification or selection rules.

To illustrate the rules summarized in Table 1, let us derive them for the special case of A_2. The group $\mathcal{Z}\big(A_2^{(1)}\big) \cong \mathbb{Z}_3$ is generated by the shift

$$J\,[\,\lambda_0, \lambda_1, \lambda_2\,] \;=\; [\,\lambda_2, \lambda_0, \lambda_1\,] \;\;.$$

In terms of non-affine weights, this action reads $J\,(\,\lambda_1, \lambda_2\,) = (\,k - \lambda_1 - \lambda_2, \lambda_1\,)$. Hence, a fixed point would have to satisfy $\lambda_1 = \lambda_2$ and $\lambda_1 = k - \lambda_1 - \lambda_2$, i.e. it should be given by $(k/3, k/3)$. Obviously this is not an element of the weight lattice unless the level k is a multiple of three.

Except from the B-series, we can always find levels for which the action of the center $\mathcal{Z}(\mathrm{H})$ on the weights has no fixed points. In the context of our construction, we have three different sets of labels on which this groups acts at the same time and it is sufficient if at least one of the values k_1, k_2 or $k_1 + k_2$ avoids the values specified in Table 1. In the following we shall assume that this condition is satisfied. Otherwise one would have to resolve the fixed points according to [43] which leads to technical difficulties but no conceptually new insights.

The rest is now straightforward. Note that the modular S matrix of the 'numerator theory' factorizes according to

$$S^{\mathrm{H}_1 \times \mathrm{H}_2}_{(\mu,\alpha)(\nu,\beta)} \;=\; S^{\mathrm{H}_1}_{\mu\nu}\; S^{\mathrm{H}_2}_{\alpha\beta} \;\;.$$

In this situation, the Verlinde formula (6) implies that the same holds for the fusion coefficients

$$N_{(\mu,\alpha)(\nu,\beta)}{}^{(\,\rho,\gamma)} \;=\; N_{\mu\nu}{}^{\rho}\; N_{\alpha\beta}{}^{\gamma} \;\;.$$

Our boundary states are now labeled by $\mathcal{G}_{\mathrm{id}}$-orbits of triples $\big((\rho,\gamma), r\big)$. When we finally insert these expressions into the formula (23), we can read off their boundary partition function,

$$Z(q) \;=\; \sum_{((\nu,\beta),b),c,d} \Big[N_{\rho_1^+\rho_2}{}^{\nu} N_{\gamma_1^+\gamma_2}{}^{\beta} N_{r_1^+ r_2}{}^{d} N_{dc}{}^{b} \Big]\, \chi^{\mathrm{H}_1\times\mathrm{H}_2/\mathrm{H}_D}_{((\nu,\beta),b)}(q)\; \chi^{\mathrm{H}_D}_{c}(q) \;\;.$$

When reinterpreted in terms of defects, these formulas provide us with a large set of possible junctions between two conformal field theories. Note that these have different central charge if $k_1 \neq k_2$.

Algebra	$A_n^{(1)}$	$B_n^{(1)}$	$C_n^{(1)}$	$D_n^{(1)}$	$E_6^{(1)}$	$E_7^{(1)}$
FPs for k in	$\bigcup_{1\neq s\mid(n+1)} s\mathbb{N}_0$	$\mathbb{N}_0$	$2\mathbb{N}_0$	$2\mathbb{N}_0$	$3\mathbb{N}_0$	$2\mathbb{N}_0$

Table 1: Existence of fixed points under simple current actions.

6 Conclusions and Outlook

In this work we proposed a new algebraic construction of symmetry breaking boundary states of some given bulk conformal field theory – the G-theory – with chiral algebra $\mathcal{A}(\mathrm{G})$. According to our prescription, one starts by choosing some rational subalgebra $\mathcal{A}(\mathrm{P})$ of the full chiral algebra $\mathcal{A}(\mathrm{G})$ together with a gluing automorphism Ω^{P}. Using the solution of the boundary problem for an auxiliary P-theory with the gluing automorphism Ω^{P}, we were able to build new boundary states for the G-theory (see formula (25)). For the simplest choice $\Omega^{\mathrm{P}} = \mathbf{id}$, the auxiliary P-theory is solved by Cardy's solution. In this way one obtains at least $|\mathrm{Rep}(\mathrm{G})| \cdot |\mathrm{Rep}(\mathrm{P})|/|\mathcal{G}_{\mathrm{id}}|$ boundary states of the G-theory for each admissible P (see eqs. (21, 22)). If all sectors of $\mathcal{A}(\mathrm{P})$ are simple currents, i.e. if $\mathcal{A}(\mathrm{P})$ is abelian, then our 'algebraic' boundary states coincide with the boundary states (27) which can be obtained from an orbifold construction of the type suggested in [8, 9]. But our formulas do not require P to be abelian and hence they provide a true generalization of the orbifold ideas.

We have also discussed how the new boundary states can be applied to obtain non-factorizing ('diagonal') branes in product geometries or, equivalently, to the construction of non-trivial defect lines in 2D conformal field theory. We presented examples in which two CFTs with different central charge are joined along the defect line. Such phenomena are known to appear in the AdS/CFT correspondence whenever branes extend to the boundary of the AdS-space [19, 20, 21, 18]. The jump of the central charge along the defect depends on the charges of the brane.

Following [21, 18], it would be interesting to compute the Casimir energy between two defects. In a stationary system the defects arrange the excitation modes such that the energy density between the defects is lower than in the outside region resulting in an attractive force between the defects. This is well-known for the electro-magnetic field between two conducting plates. In [18], the Casimir energy was calculated for a free boson system with defects which join regions with different compactification radii.

Let us stress that the number of new boundary states can be enlarged by a simple iteration of our construction. In addition to the chiral symmetry $\mathcal{A}(\mathrm{P})$ we have decided to preserve the coset algebra $\mathcal{A}(\mathrm{G/P})$ so as to render the boundary problem rational. But it would be possible to reduce the symmetry even further by choosing a rational chiral subalgebra $\mathcal{A}(\mathrm{P}') \hookrightarrow \mathcal{A}(\mathrm{G/P})$ which should then be preserved together with the chiral algebra $\mathcal{A}(\mathrm{G/P/P}')$ of the 'double coset'.

It is well known that in many backgrounds, e.g. $\mathrm{SU}(N \geq 4)$ or $S^3 \times S^3$, the standard constructions of boundary conformal field theory do not suffice to generate the whole lattice of RR-charges. Having obtained a very large class of new symmetry breaking boundary states, the situation is likely to improve drastically. It would be interesting to study more examples and to understand which charges are carried by our new branes.

Finally, let us also stress once more that we have only provided a list of new boundary states. These contain information about how closed strings couple to the associated branes and about the spectrum of open string modes. The operator product expansions of open string vertex operators (or boundary fields) contain additional data. They can be obtained as solutions of the sewing constraints which have been worked out by several authors [71, 72, 73, 7, 74]. These constraints were solved for a series of orbifold models in [75] and then more systematically for simple current orbifolds in [60]. Using our discussion from Section 4, the formalism of [60] can be applied to our present context whenever P is abelian and it provides the desired boundary operator product expansions. For more general choice of P, however, the problem remains to be solved.

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Note added in proof: After this paper was completed we noticed that our constructions provide branes which are localized along the sets

$$\mathcal{C}_\rho^{\mathrm{G},\Omega}\big(\mathcal{C}_r^{\mathrm{P},\Omega^{\mathrm{P}}}\big)^{-1} \subset \mathrm{G} \quad ,$$

where $\mathcal{C}_\rho^{\mathrm{G},\Omega}$ and $\mathcal{C}_r^{\mathrm{P},\Omega^{\mathrm{P}}}$ are twisted conjugacy classes. Details will appear elsewhere [76].

References

[1] M. R. Gaberdiel, A. Recknagel and G. M. T. Watts, *The conformal boundary states for SU(2) at level 1*, *Nucl. Phys.* **B626** (2002) 344–362 [hep-th/0108102].

[2] M. R. Gaberdiel and A. Recknagel, *Conformal boundary states for free bosons and fermions*, *JHEP* **11** (2001) 016 [hep-th/0108238].

[3] R. A. Janik, *Exceptional boundary states at c = 1*, *Nucl. Phys.* **B618** (2001) 675–688 [hep-th/0109021].

[4] A. Cappelli and G. D'Appollonio, *Boundary states of* $c = 1$ *and* $3/2$ *rational conformal field theories*, *JHEP* **02** (2002) 039 [hep-th/0201173].

[5] J. Fuchs and C. Schweigert, *Symmetry breaking boundaries. I: General theory*, *Nucl. Phys.* **B558** (1999) 419 [hep-th/9902132].

[6] L. Birke, J. Fuchs and C. Schweigert, *Symmetry breaking boundary conditions and WZW orbifolds*, *Adv. Theor. Math. Phys.* **3** (1999) 671–726 [hep-th/9905038].

[7] R. E. Behrend, P. A. Pearce, V. B. Petkova and J.-B. Zuber, *Boundary conditions in rational conformal field theories*, *Nucl. Phys.* **B570** (2000) 525–589 [hep-th/9908036].

[8] J. Maldacena, G. W. Moore and N. Seiberg, *Geometrical interpretation of D-branes in gauged WZW models*, *JHEP* **07** (2001) 046 [hep-th/0105038].

[9] J. Maldacena, G. W. Moore and N. Seiberg, *D-brane instantons and K-theory charges*, *JHEP* **11** (2001) 062 [hep-th/0108100].

[10] A. Rajaraman, *New AdS(3) branes and boundary states*, hep-th/0109200.

[11] D. M. McAvity and H. Osborn, *Conformal field theories near a boundary in general dimensions*, *Nucl. Phys.* **B455** (1995) 522–576 [cond-mat/9505127].

[12] M. Oshikawa and I. Affleck, *Boundary conformal field theory approach to the critical two-dimensional Ising model with a defect line*, *Nucl. Phys.* **B495** (1997) 533–582 [cond-mat/9612187].

[13] A. LeClair and A. W. W. Ludwig, *Minimal models with integrable local defects*, *Nucl. Phys.* **B549** (1999) 546–562 [hep-th/9708135].

[14] C. Nayak, M. Fisher, A. W. Ludwig and H. H. Lin, *Resonant multilead point-contact tunneling*, *Phys. Rev.* **B59** (1999) 15694.

[15] H. Saleur, *Lectures on non perturbative field theory and quantum impurity problems*, cond-mat/9812110.

[16] H. Saleur, *Lectures on non perturbative field theory and quantum impurity problems. II*, cond-mat/0007309.

[17] J. Erdmenger, Z. Guralnik and I. Kirsch, *Four-dimensional superconformal theories with interacting boundaries or defects*, *Phys. Rev.* **D66** (2002) 025020 [hep-th/0203020].

[18] C. Bachas, J. de Boer, R. Dijkgraaf and H. Ooguri, *Permeable conformal walls and holography*, *JHEP* **06** (2002) 027 [hep-th/0111210].

[19] A. Karch and L. Randall, *Locally localized gravity*, *JHEP* **05** (2001) 008 [hep-th/0011156].

[20] A. Karch and L. Randall, *Open and closed string interpretation of SUSY CFT's on branes with boundaries*, *JHEP* **06** (2001) 063 [hep-th/0105132].

[21] O. DeWolfe, D. Z. Freedman and H. Ooguri, *Holography and defect conformal field theories*, *Phys. Rev.* **D66** (2002) 025009 [hep-th/0111135].

[22] T. Gannon, *Partition functions for heterotic WZW conformal field theories*, *Nucl. Phys.* **B402** (1993) 729–753 [hep-th/9209042].

[23] T. Gannon and Q. Ho-Kim, *The rank four heterotic modular invariant partition functions*, *Nucl. Phys.* **B425** (1994) 319–342 [hep-th/9402027].

[24] A. Recknagel and V. Schomerus, *D-branes in Gepner models*, *Nucl. Phys.* **B531** (1998) 185–225 [hep-th/9712186].

[25] T. Gannon, *Boundary conformal field theory and fusion ring representations*, *Nucl. Phys.* **B627** (2002) 506–564 [hep-th/0106105].

[26] J. L. Cardy, *Boundary conditions, fusion rules and the Verlinde formula*, *Nucl. Phys.* **B324** (1989) 581.

[27] E. Verlinde, *Fusion rules and modular transformations in 2-D conformal field theory*, *Nucl. Phys.* **B300** (1988) 360.

[28] P. Goddard, A. Kent and D. I. Olive, *Virasoro algebras and coset space models*, *Phys. Lett.* **B152** (1985) 88.

[29] P. Di Francesco, P. Mathieu and D. Senechal, *Conformal Field Theory*. Graduate Texts in Contemporary Physics. Springer, New York, 1999.

[30] K. Gawedzki and A. Kupiainen, *G/H conformal field theory from gauged WZW model*, *Phys. Lett.* **B215** (1988) 119–123.

[31] K. Gawedzki and A. Kupiainen, *Coset construction from functional integrals*, *Nucl. Phys.* **B320** (1989) 625.

[32] W. Nahm, *Quantum field theory in one and two dimensions*, *Duke Math. J.* **54** (1987) 579.

[33] K. Bardakci, E. Rabinovici and B. Saering, *String models with $c < 1$ components*, *Nucl. Phys.* **B299** (1988) 151.

[34] A. N. Schellekens and S. Yankielowicz, *Extended chiral algebras and modular invariant partition functions*, *Nucl. Phys.* **B327** (1989) 673.

[35] A. N. Schellekens and S. Yankielowicz, *Modular invariants from simple currents: An explicit proof*, *Phys. Lett.* **B227** (1989) 387.

[36] A. N. Schellekens and S. Yankielowicz, *Simple currents, modular invariants and fixed points*, *Int. J. Mod. Phys.* **A5** (1990) 2903–2952.

[37] K. A. Intriligator, *Bonus symmetry in conformal field theory*, *Nucl. Phys.* **B332** (1990) 541.

[38] D. Gepner, *Field identification in coset conformal field theories*, *Phys. Lett.* **B222** (1989) 207.

[39] G. W. Moore and N. Seiberg, *Taming the conformal Zoo*, *Phys. Lett.* **B220** (1989) 422.

[40] D. C. Dunbar and K. G. Joshi, *Maverick examples of coset conformal field theories*, *Mod. Phys. Lett.* **A8** (1993) 2803–2814 [hep-th/9309093].

[41] A. N. Schellekens and S. Yankielowicz, *Field identification fixed points in the coset construction*, *Nucl. Phys.* **B334** (1990) 67.

[42] J. Fuchs, B. Schellekens and C. Schweigert, *From Dynkin diagram symmetries to fixed point structures*, *Commun. Math. Phys.* **180** (1996) 39–98 [hep-th/9506135].

[43] J. Fuchs, B. Schellekens and C. Schweigert, *The resolution of field identification fixed points in diagonal coset theories*, *Nucl. Phys.* **B461** (1996) 371–406 [hep-th/9509105].

[44] V. B. Petkova and J. B. Zuber, *Boundary conditions in charge conjugate sl(N) WZW theories*, hep-th/0201239.

[45] M. R. Gaberdiel and T. Gannon, *Boundary states for WZW models*, *Nucl. Phys.* **B639** (2002) 471–501 [hep-th/0202067].

[46] T. Quella, I. Runkel and C. Schweigert, *An algorithm for twisted fusion rules*, math.qa/0203133. To appear in Adv. Theor. Math. Phys.

[47] A. Yu. Alekseev, S. Fredenhagen, T. Quella and V. Schomerus, *Non-commutative gauge theory of twisted D-branes*, hep-th/0205123. To appear in Nucl. Phys. B.

[48] T. Quella, *Branching rules of semi-simple Lie algebras using affine extensions*, *J. Phys.* **A35** (2002) 3743–3754 [math-ph/0111020].

[49] H. Ishikawa, *Boundary states in coset conformal field theories*, *Nucl. Phys.* **B629** (2002) 209–232 [hep-th/0111230].

[50] A. Yu. Alekseev and V. Schomerus, *D-branes in the WZW model*, *Phys. Rev.* **D60** (1999) 061901 [hep-th/9812193].

[51] G. Felder, J. Fröhlich, J. Fuchs and C. Schweigert, *The geometry of WZW branes*, *J. Geom. Phys.* **34** (2000) 162–190 [hep-th/9909030].

[52] C. Bachas, M. R. Douglas and C. Schweigert, *Flux stabilization of D-branes*, *JHEP* **05** (2000) 048 [hep-th/0003037].

[53] J. Pawelczyk, *SU(2) WZW D-branes and their noncommutative geometry from DBI action*, *JHEP* **08** (2000) 006 [hep-th/0003057].

[54] P. Bordalo, S. Ribault and C. Schweigert, *Flux stabilization in compact groups*, *JHEP* **10** (2001) 036 [hep-th/0108201].

[55] K. Gawedzki, *Boundary WZW, G/H, G/G and CS theories*, hep-th/0108044.

[56] S. Elitzur and G. Sarkissian, *D-branes on a gauged WZW model*, *Nucl. Phys.* **B625** (2002) 166–178 [hep-th/0108142].

[57] S. Fredenhagen and V. Schomerus, *D-branes in coset models*, *JHEP* **02** (2002) 005 [hep-th/0111189].

[58] I. Brunner and V. Schomerus, *D-branes at singular curves of Calabi-Yau compactifications*, *JHEP* **04** (2000) 020 [hep-th/0001132].

[59] I. Brunner and V. Schomerus, *On superpotentials for D-branes in Gepner models*, *JHEP* **10** (2000) 016 [hep-th/0008194].

[60] K. Matsubara, V. Schomerus and M. Smedback, *Open strings in simple current orbifolds*, *Nucl. Phys.* **B626** (2002) 53–72 [hep-th/0108126].

[61] G. Pradisi and A. Sagnotti, *Open string orbifolds*, *Phys. Lett.* **B216** (1989) 59.

[62] M. Bianchi and A. Sagnotti, *On the systematics of open string theories*, *Phys. Lett.* **B247** (1990) 517–524.

[63] M. Bianchi and A. Sagnotti, *Twist symmetry and open string Wilson lines*, *Nucl. Phys.* **B361** (1991) 519–538.

[64] M. Bianchi, G. Pradisi and A. Sagnotti, *Planar duality in the discrete series*, *Phys. Lett.* **B273** (1991) 389–398.

[65] G. Pradisi, A. Sagnotti and Y. S. Stanev, *Planar duality in $SU(2)$ WZW models*, *Phys. Lett.* **B354** (1995) 279–286 [`hep-th/9503207`].

[66] G. Pradisi, A. Sagnotti and Y. S. Stanev, *The open descendants of nondiagonal $SU(2)$ WZW models*, *Phys. Lett.* **B356** (1995) 230–238 [`hep-th/9506014`].

[67] R. E. Behrend, P. A. Pearce, V. B. Petkova and J.-B. Zuber, *On the classification of bulk and boundary conformal field theories*, *Phys. Lett.* **B444** (1998) 163 [`hep-th/9809097`].

[68] J. Fuchs and C. Schweigert, *Symmetry breaking boundaries. II: More structures, examples*, *Nucl. Phys.* **B568** (2000) 543 [`hep-th/9908025`].

[69] L. R. Huiszoon, A. N. Schellekens and N. Sousa, *Open descendants of non-diagonal invariants*, *Nucl. Phys.* **B575** (2000) 401–415 [`hep-th/9911229`].

[70] A. N. Schellekens, *Open strings, simple currents and fixed points*, `hep-th/0001198`.

[71] D. C. Lewellen, *Sewing constraints for conformal field theories on surfaces with boundaries*, *Nucl. Phys.* **B372** (1992) 654–682.

[72] G. Pradisi, A. Sagnotti and Y. S. Stanev, *Completeness conditions for boundary operators in 2D conformal field theory*, *Phys. Lett.* **B381** (1996) 97–104 [`hep-th/9603097`].

[73] I. Runkel, *Boundary structure constants for the A-series Virasoro minimal models*, *Nucl. Phys.* **B549** (1999) 563 [`hep-th/9811178`].

[74] G. Felder, J. Fröhlich, J. Fuchs and C. Schweigert, *Correlation functions and boundary conditions in RCFT and three-dimensional topology*, `hep-th/9912239`.

[75] I. Runkel, *Structure constants for the D-series Virasoro minimal models*, *Nucl. Phys.* **B579** (2000) 561 [hep-th/9908046].

[76] T. Quella and V. Schomerus. To appear.