

Black hole entropy and thermodynamics from symmetries

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ABSTRACT

Given a boundary of spacetime preserved by a $Diff(S^1)$ sub-algebra, we propose a systematic method to compute the zero mode and the central extension of the associated Virasoro algebra of charges. Using these values in the Cardy formula, we may derive an associated statistical entropy to be compared with the Bekenstein-Hawking result.

To illustrate our method, we study in detail the BTZ and the rotating Kerr-adS₄ black holes (at spatial infinity and on the horizon). In both cases, we are able to reproduce the area law with the correct factor of 1/4 for the entropy. We also recover within our framework the first law of black hole thermodynamics.

We compare our results with the analogous derivations proposed by Carlip and others. Although similar, our method differs in the computation of the zero mode. In particular, the normalization of the ground state is automatically fixed by our construction.

1 Introduction

There have been a lot of interest in understanding black hole entropy from a microscopical description, using either string theory [1] or loop quantum gravity [2]. A third symmetry-based approach originated by Strominger [3] and Birmingham, Sachs and Sen [4] is attracting more and more interest [5]-[20]. This method assumes that the symmetries of a black hole horizon are good enough, first to survive a quantification, and second to count the density of states at a given energy. The main argument in favor of these new ideas is probably that they seem to work, although we still do not clearly understand why.

Let us start by summarizing the key points of this calculation: Given a boundary of spacetime (spatial infinity for Strominger [3] or the black hole horizon in Carlip [6, 8] and Solodukhin [7] works), we need to identify the diffeomorphisms which preserve the boundary conditions. Let us assume that their associated algebra contains at least one sub-algebra isomorphic to $Diff(S^1)$. This sub-algebra can be parametrized by a (infinite) discrete set of vector fields $\xi_n^\mu(x)$, $n \in \mathbf{Z}$ that satisfy

$$i \{ \xi_m, \xi_n \} = (m - n) \xi_{m+n}, \quad (1)$$

where $\{ , \}$ denotes the Lie bracket between vector fields (the spacetime index μ has been omitted).

The next step consists in constructing, either using an Hamiltonian [21, 22] or a covariant-Lagrangian formalism [23]-[38], the charges associated with each of these vector fields:

$$\xi_n(x) \leftrightarrow Q_n. \quad (2)$$

This is usually a difficult and “boundary condition dependent” task. Indeed, only in the cases where the asymptotic conditions which generate the algebra (1) are fully understood, Q_n can be derived methodically.

But let us assume that we found these charges. A bracket is then defined by $[Q_m, Q_n] := \delta_{\xi_m} Q_n$ where δ_{ξ_m} is the Lie derivative along ξ_m^μ which acts only on the fields (but not on the vectors ξ_n^μ). In this situation, a notable theorem by Brown and Henneaux [22] shows an isomorphism between the two brackets $\{.,.\}$ and $[.,.]$ up to central charges (a Lagrangian version of this theorem is derived in [31, 38]). In particular, from the algebra (1), we get,

$$i [Q_m, Q_n] = (m - n) Q_{m+n} + \frac{c}{12} m(m^2 - 1) \delta_{m+n}, \quad (3)$$

for some central extension c to be determined. We emphasize here that the freedom of shifting Q_0 by a constant has been fixed by the contribution linear in m to the central charge (and then Q_{-1} , Q_0 and Q_1 form a proper $sl(2, \mathbb{R})$ sub-algebra).

Once Q_0 and c are found, the next idea is to insert them in a Cardy-like formula [39] to find the entropy of the black hole

$$S = 2\pi \sqrt{\frac{c}{6} Q_0}. \quad (4)$$

The last step is simply to check whether this calculation reproduces or not the well-known semi-classical result.

It is still under discussion why the formula (4) would or should work¹ (see the nice references [5, 8, 14]). In fact, the Cardy's result comes from a quantum calculation in a two dimensional conformal field theory which has not been identified in our gravitational theory. Moreover, it is not obvious why the *classical* central charge of equation (3) would then be protected by quantum corrections. Therefore, only a full understanding of the quantum theory which governs the black hole horizon would be able to answer these questions.

However, following Strominger [3], we can still investigate whether, when and how this Cardy equation works. This is the purpose of this manuscript. To proceed, we first propose a straightforward method (based on one single equation) to compute Q_0 and c on the horizon of any D -dimensional black hole ($D \geq 3$). We then show in specific examples that these derived Q_0 and c indeed reproduce the area law formula (with the correct normalization) when used in equation (4). Our construction is strongly inspired from Carlip's results [6, 8], although it differs in the construction of Q_0 . A detailed comparison with these works is carried out.

In section 2, we explain and motivate our new construction of Q_0 and c , and compare it with previous results found in the literature (see also appendix A). In section 3, we study the examples of BTZ black hole at spatial infinity and on the horizon (the Carlip boundary conditions are discussed in appendix B and the Eddington-Finkelstein change of coordinates in appendix C). In both cases, we find that equation (4) correctly reproduces the Bekenstein-Hawking entropy. We then show how the first law of black hole thermodynamics is embedded in our framework. Finally, we extend our investigation to a general four-dimensional Kerr-adS black hole (which covers

¹Using either the derived Q_0 and c , or some "effective" ones.

Schwarzschild, Kerr and adS-Schwarzschild black holes). We again find a perfect agreement for the entropy computed on the horizon using the Cardy formula.

2 A direct derivation of Q_0 and c

The charges associated with the diffeomorphism invariance of general relativity depend on the boundary considered. In concrete, there is *no* general formula like $Q = \int_{bd} \mathcal{Q}[g]$, for some density $\mathcal{Q}[g]$, which would be defined independently of the boundary “ bd ”. This is due to the fact that an unambiguous construction of $\mathcal{Q}[g]$ depends necessarily on the boundary conditions² [21, 22].

On the other hand, a general boundary-independent formula for the *variation* of the charge $\delta Q[g]$ can be derived from first principles [21, 24, 25, 26, 29]. This formula depends *only on the gauge symmetry and on the equations of motion* of the theory [29]. For gravity in any spacetime dimension $D \geq 3$ with or without cosmological constant, the variation of the charge Q_n associated with the diffeomorphism $\xi_n^\mu(x)$ is given by [34, 35]:

$$\delta Q_n = \int_{bd} \frac{1}{16\pi G} \left(2 \nabla_\sigma \xi_n^\tau \delta \left(\sqrt{|g|} g^{\sigma[\mu} \delta_\tau^{\nu]} \right) + 6 \delta \left(\Gamma_{\rho\sigma}^\tau \right) \sqrt{|g|} g^{\sigma[\mu} \delta_\tau^{\nu]} \xi_n^\rho \right) dS_{\mu\nu} \quad (5)$$

with $dS_{\mu\nu}$ the bi-normal to the intersection between the spacetime boundary considered and a (partial) Cauchy hypersurface.

The equation (5) is valid on *any boundary* of spacetime whose boundary conditions are compatible with the variational principle³. In particular, it has been used at spatial infinity [34] to recover a covariant formulation of the ADM mass [23] and also on a Brane-World to properly define its associated conserved charges [35]. The purpose of this paper is to study this equation (5) on the horizon of a black hole. We then assume that there exists a properly defined set of boundary conditions on the horizon compatible with the variational principle (although we will not need to find it explicitly). The main point is that equation (5) will then allow us first, to compute

²A general formula for the charges was recently given in [38] following the methods of Anderson and Torres [27]. This formalism requires that the metric approaches a given background metric on the boundary. However in some cases as Brane-World scenarios, no background metric is needed in order to define conserved charges [35].

³A given set of boundary conditions are compatible with the variational principle if there exists a Lagrangian \mathcal{L} in the class $\mathcal{L} \sim \mathcal{L} + \partial_\mu \Sigma^\mu$, such that on-shell $\delta \int \mathcal{L} = 0$ (with no boundary terms).

directly the zero mode Q_0 and the central charge c and second, to derive the first law of black hole thermodynamics.

To proceed, let us use the general formula (5) with $\delta = \delta_{\xi_m}$ that is a Lie derivative along $\xi_m^\mu(x)$ given by

$$\delta_\xi g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)}, \quad \delta_\xi \Gamma_{\rho\sigma}^\tau = \frac{1}{2}g^{\tau\nu} (\nabla_\rho\delta_\xi g_{\sigma\nu} + \nabla_\sigma\delta_\xi g_{\rho\nu} - \nabla_\nu\delta_\xi g_{\rho\sigma}). \quad (6)$$

Dropping the following total derivative

$$\frac{6}{16\pi G}\partial_\rho \left(\sqrt{|g|}\xi_n^{[\mu}\nabla^\nu\xi_m^{\rho]} \right), \quad (7)$$

the equation (5) then reduces to:

$$\begin{aligned} [Q_m, Q_n] &= \delta_{\xi_m} Q_n \\ &= \int_{bd} \frac{\sqrt{|g|}}{16\pi G} [\nabla^\mu\xi_n^\nu\nabla_\rho\xi_m^\rho - \nabla^\mu\xi_m^\nu\nabla_\rho\xi_n^\rho + 2\nabla_\rho\xi_n^\mu\nabla^\rho\xi_m^\nu \\ &\quad + (\mathcal{R}^{\mu\nu}{}_{\rho\sigma} - 2\delta_\rho^\mu\mathcal{R}^\nu{}_\sigma)\xi_n^\rho\xi_m^\sigma - (\mu \leftrightarrow \nu)] dS_{\mu\nu}, \end{aligned} \quad (8)$$

with $\mathcal{R}^{\mu\nu}{}_{\rho\sigma}$ the Riemann tensor of the metric.

Results similar to equation (8) can be found in the literature [18, 38], and are described in the appendix A. In addition, an alternative definition for the bracket between two charges was given in the reference [8] (see equation (A.1)). As shown in section 3, this other bracket fails to reproduce the known charges at spatial infinity. We will also discuss its applicability on the horizon.

The main point is that equation (8) can be used for any metric which satisfies Einstein equations, and not necessarily for a fixed background metric. Then, given a black hole solution and a subset of diffeomorphisms $\xi_n(x)$, we can compute the left-hand side of equation (3) for any n and m using the single formula (8). In particular, we can consider $m = -n = 1$ and $m = -n = 2$:

$$i [Q_1, Q_{-1}] = 2Q_0. \quad (9)$$

$$i [Q_2, Q_{-2}] = 4Q_0 + \frac{c}{2}. \quad (10)$$

The left-hand side of these equations being known, it is then easy to single out Q_0 and c .

We would like to insist on the following very important point: the normalization of Q_0 is fixed in our proposal by requiring that the charges Q_{-1} ,

Q_0 and Q_1 form a $sl(2, \mathbb{R})$ sub-algebra. This means that the right-hand side of equations (3), (9) and (10) are completely determined and we cannot shift Q_0 by some constant anymore. The *claim* is then that the Q_0 and the c computed in that way on the horizon are the effective quantities to be used in the Cardy formula (4). We show how this works for specific examples in the next section. Of course, it would be more gratifying to have a general proof of this statement; a proof for instance based on a quantum conformal theory living on the horizon. This is however out of the scope of this work.

The problem of the global normalization of Q_0 , and how to fix it, is obviously a key feature if we want to compute the entropy *à la Cardy*. The usual Regge and Teitelboim [21] or the covariant symplectic methods [24, 25, 26, 29] give Q_0 only up to a constant shift. At spatial infinity, this constant is fixed by requiring that the charge vanishes on a “natural background”, as for instance Minkowski spacetime. On the horizon, the problem is more involved because it is hard to define a background on which Q_0 would be set to vanish. For instance, it is not clear to the author what would be a natural background for a Kerr black hole, near to the horizon (Minkowski space does not have an internal boundary). Finally, in the reference [19] the arbitrary shift in Q_0 was fixed *a posteriori* so that the Cardy formula gave the expected Bekenstein-Hawking result. However, this approach is quite unsatisfactory in the sense that the non-trivial check of the entropy calculation is lost.

In summary, given a black hole solution and a set of vector fields $\xi_n^\mu(x)$ satisfying equation (1), we can directly compute Q_0 and c (and then the entropy S) by making use of equations (8), (9) and (10).

3 Examples

In the following examples we use the method given in the previous section in order to compute Q_0 and c . We start with the rotating BTZ black hole in three dimensions, at spatial infinity and on the horizon. We then study the four dimensional adS-Kerr solution. We simultaneously compare our results with recent works on the subject [6, 8, 17]. Finally, we will also use equation (5) to derive the (local) first law of black hole thermodynamics.

3.1 The BTZ black hole at spatial infinity

The metric of the rotating BTZ black hole in Schwarzschild-type coordinates is given by [40]:

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 (N_\phi dt + d\phi)^2, \quad (11)$$

with

$$N^2(r) = -8MG + \frac{r^2}{l^2} + \frac{16J^2 G^2}{r^2} \quad (12)$$

$$N_\phi(r) = -\frac{4JG}{r^2}. \quad (13)$$

The horizon of the black hole is located at r_+ , defined by:

$$r_\pm^2 = 4MGl^2 \left(1 \pm \sqrt{1 - J^2/(Ml)^2} \right). \quad (14)$$

Moreover, the area, angular momentum and surface gravity on the horizon are respectively:

$$\mathcal{A} = 2\pi r_+, \quad (15)$$

$$\Omega = \frac{4JG}{r_+^2}, \quad (16)$$

$$\kappa = \frac{r_+^2 - r_-^2}{l^2 r_+}. \quad (17)$$

At spatial infinity ($r \rightarrow \infty$), the BTZ metric has the two sets of asymptotic Killing fields found by Brown and Henneaux [41]. A simple basis for these vector fields is:

$$\xi_n^+ = \frac{1}{2} e^{in(t/l+\phi)} (l, -irn, 1) + \left(O\left(\frac{1}{r^2}\right), O\left(\frac{1}{r}\right), O\left(\frac{1}{r^2}\right) \right) \quad (18)$$

$$\xi_n^- = \frac{1}{2} e^{in(t/l-\phi)} (l, -irn, -1) + \left(O\left(\frac{1}{r^2}\right), O\left(\frac{1}{r}\right), O\left(\frac{1}{r^2}\right) \right). \quad (19)$$

The overall normalization is chosen such that the Lie bracket between two vectors gives properly normalized $Diff(S^1)$ algebras,

$$i \{ \xi_m^+, \xi_n^+ \} = (m - n) \xi_{m+n}^+ \quad (20)$$

$$i \{ \xi_m^-, \xi_n^- \} = (m - n) \xi_{m+n}^- \quad (21)$$

$$i \{ \xi_m^+, \xi_n^- \} = 0. \quad (22)$$

We then use the metric (11) and the vector fields (18) and (19) in equation (8). After performing the integration over ϕ from 0 to 2π and taking the limit $r \rightarrow \infty$, we get:

$$[Q_m^+, Q_n^+] = -i\delta_{m+n} \left(m(lM - J) + m^3 \frac{l}{8G} \right) \quad (23)$$

$$[Q_m^-, Q_n^-] = -i\delta_{m+n} \left(m(lM + J) + m^3 \frac{l}{8G} \right) \quad (24)$$

$$[Q_m^+, Q_n^-] = 0. \quad (25)$$

The left hand sides of equations (9) and (10) can then be directly read from equations (23) and (24) for $m = -n = 1$ and $m = -n = 2$. A simple rearrangement then gives:

$$Q_0^+ = \frac{1}{2} (lM - J) + \frac{l}{16G} \quad (26)$$

$$Q_0^- = \frac{1}{2} (lM + J) + \frac{l}{16G} \quad (27)$$

$$c^+ = c^- = \frac{3l}{2G} \quad (28)$$

in perfect agreement with [40, 41].

The Strominger's derivation [3] of the black hole entropy using the Cardy formula is almost straightforward. There is just a subtle point which concerns the effective values of Q_0^\pm to be used in equation (4). As discussed in [40], the adS_3 metric is recovered from the BTZ solution (11) not by taking $M = 0$ but instead $M = -\frac{1}{8G}$ (and $J = 0$). Indeed, the generators (26) and (27) came naturally normalized such that $Q_0^\pm|_{adS_3} = 0$. However, there is no physical solution (a spacetime without naked singularities) between $M = -\frac{1}{8G}$ and $M = 0$. Therefore, the continuous spectrum of solutions starts with $M \geq 0$ (and $J^2 \leq M^2 l^2$). Following Strominger, the states responsible for the entropy are then fluctuations above the vacuum $M = J = 0$. The properly normalized zero modes which annihilate this vacuum are simply $\bar{Q}_0^\pm = Q_0^\pm - \frac{c^\pm}{24}$. Using these in the Cardy formula (4), we then recover the famous result:

$$S = 2\pi \sqrt{\frac{c^+}{6} \bar{Q}_0^+} + 2\pi \sqrt{\frac{c^-}{6} \bar{Q}_0^-} = \frac{\mathcal{A}}{4G}. \quad (29)$$

Note finally that if we work with the bracket (A.1) instead of the formula (8) to compute the left-hand side of equations (9) and (10), we cannot

reproduce the known results (26)-(29). In fact, a direct calculation gives $\tilde{Q}_0^\pm = \frac{r^2}{4lG} + \frac{Ml}{2} + \frac{l}{16G}$ and $\tilde{c}^\pm = \frac{3l}{2G}$. Although the central charges are unchanged, the charges \tilde{Q}_0^\pm are equal to each other (the dependence in J is lost) and diverge at $r \rightarrow \infty$. Now, even when we use the renormalized $\tilde{Q}_0^\pm = \frac{Ml}{2}$ in the Cardy formula, we cannot recover the semi-classical value for the entropy.

3.2 The BTZ black hole at the horizon

The idea is to repeat the above exercise on the horizon. To proceed, we first need to identify a family of $Diff(S^1)$ vector fields which preserve the structure of the horizon. Following the work of Carlip [8] summarized in the appendix B, we consider a set of diffeomorphisms given by (see equation (B.12)):

$$\xi_n^\mu = T_n \chi^\mu + R_n \rho^\mu \quad (30)$$

where $\chi^\mu = (1, 0, \Omega)$ is the Killing field whose norm vanishes on the horizon. The vector ρ^μ is defined by formula (B.3).

Let us then consider the general ansatz (obviously motivated by the search of a $Diff(S^1)$ algebra) for the functions T_n ,

$$T_n = \frac{1}{\alpha + \Omega} e^{in(\phi + \alpha t + f(r))}, \quad (31)$$

and therefore (to leading order, see equation (B.13)),

$$R_n = -in \frac{\alpha + \Omega}{\kappa} T_n. \quad (32)$$

The requirement that the vector fields ξ_n satisfy the $Diff(S^1)$ algebra (1) first, fixes the overall normalization and second, implies that the arbitrary function $f(r)$ has to be well-behaved on the horizon when Schwarzschild coordinates are used. That is $f(r)$ should be *finite* as $r \rightarrow r_+$. This condition is indeed a consequence of equation (B.14). In fact, in Schwarzschild coordinates, $\rho^\mu \nabla_\mu T_n \sim (r - r_+) \partial_r f(r)$ vanishes on the horizon only if $f(r)$ is well defined on \mathcal{H} . The constant parameter α will be discussed later.

So, using the vector fields (30) in equation (8), integrating over ϕ and taking the limit $r \rightarrow r_+$, we get:

$$[Q_m, Q_n] = -i \delta_{m+n} m^3 \frac{\mathcal{A}}{8\pi G} \frac{\alpha + \Omega}{\kappa}. \quad (33)$$

We see that the dependence in the arbitrary function $f(r)$, completely disappears from the final expression (33). Note however that the regularity of $f(r)$ on the horizon was required in order to obtain this result. Next, we use this formula (33) together with equations (9) and (10) to single out the zero-mode and the central charge:

$$Q_0 = \frac{\mathcal{A}}{16\pi G} \frac{\alpha + \Omega}{\kappa} \quad (34)$$

$$c = \frac{3\mathcal{A}}{2\pi G} \frac{\alpha + \Omega}{\kappa}. \quad (35)$$

Using finally these results in the Cardy formula (4), we obtain:

$$S = \frac{\mathcal{A}}{4G} \frac{\alpha + \Omega}{\kappa}. \quad (36)$$

This coincides with the usual semiclassical formula only for

$$\alpha = \kappa - \Omega, \quad (37)$$

which was imposed by hand in [8].

One can however intuitively argue in favor of this assignment (37): the natural variables on the horizon are $v_- = \phi - \Omega t$ (since $\chi^\mu \nabla_\mu v_- = 0$) and t , with periods 2π and $\frac{2\pi}{\kappa}$ respectively. However, this period $\frac{2\pi}{\kappa}$ is based on the semi-classical computation of Gibbons and Hawking [42], which we would like to avoid in our symmetry-based calculation of thermodynamical quantities. An independent way for understanding equation (37) is expected to exist, but unknown to the author.

In the rest of this subsection we compare the results (34), (35) and (36) with previous works [6, 8, 17]. First, our equation (33) is in essence equivalent to equation (3.8) of [6] (with $\alpha + \Omega = \frac{2\pi}{T}$). However, the construction of our zero mode (34) differs from the one given in the reference [6] (which was later criticized in [9]).

Let us now use the general ansatz (31) in the modified bracket (A.1). After integrating over ϕ and taking the limit $r \rightarrow r_+$, we get:

$$[[Q_m, Q_n]] = -i\delta_{m+n} \left(m \frac{\mathcal{A}}{4\pi G} \frac{\kappa}{\alpha + \Omega} + m^3 \frac{\mathcal{A}}{8\pi G} \frac{\alpha + \Omega}{\kappa} \right). \quad (38)$$

Then using the same argument above equations (9) and (10), we find

$$\tilde{Q}_0 = \frac{\mathcal{A}}{8\pi G} \frac{\kappa}{\alpha + \Omega} + \frac{\mathcal{A}}{16\pi G} \frac{\alpha + \Omega}{\kappa} \quad (39)$$

$$\tilde{c} = \frac{3\mathcal{A}}{2\pi G} \frac{\alpha + \Omega}{\kappa}. \quad (40)$$

First, a naive use of \tilde{Q}_0 and \tilde{c} in the Cardy formula (4) does not work. Now, even if we assume that equation (37) holds, the equation (4) gives $S = \sqrt{3} \frac{A}{4G}$.

In the references [8, 17], the zero-mode was computed using the Komar integral (A.3). The result is indeed consistent with equation (39), up to a constant shift:

$$J_0 = \tilde{Q}_0 - \frac{\tilde{c}}{24} = \frac{A}{8\pi G} \frac{\kappa}{\alpha + \Omega}. \quad (41)$$

Then, if we use this effective J_0 in the Cardy formula, we find $S = \sqrt{2} \frac{A}{4G}$, independently of the value of α . This $\sqrt{2}$ -anomaly was first pointed out in [17]. Indeed, up to a change of variable described in appendix C, the “extended symmetries” proposed in [17] are nothing but a special case of equation (31), namely for $\alpha = \Omega$.

Finally, in Carlip second paper [8], the relation $\alpha = \kappa - \Omega$ is assumed from the beginning and therefore equation (41) gives $\frac{A}{8\pi G}$. In order to avoid the $\sqrt{2}$ anomaly discussed before, another $-\frac{c}{24}$ was then removed from J_0 . From our point of view, where the global normalization of \tilde{Q}_0 is fixed by the form of the algebra (3), this last step looks a little artificial.

Therefore, only in the case where $\frac{c}{24}$ is removed twice from the “natural” generator (39) and equation (37) is assumed to hold, the bracket (A.1) succeeded in reproducing the correct black hole entropy. In that sense, our derivation based on equation (8) seems more straightforward since it only depends on the equality (37).

3.3 The first law of thermodynamics

The first law of black hole thermodynamics is also encoded in equation (5). This can indeed be proven in a general way using covariant symplectic methods [43, 37]. Now for a given black hole, we can check this first law in a simple way. The basic idea is to consider the metric $g_{\mu\nu}$ as a functional of r_+ and J (instead of M and J). The BTZ metric is therefore given by equations (11) and (13), together with:

$$N^2 = \frac{1}{l^2 r^2} \left(r^2 - r_+^2 \right) \left(r^2 - \left(\frac{4GlJ}{r_+} \right)^2 \right). \quad (42)$$

In this case, a variation of $g_{\mu\nu}$ is given by

$$\delta g_{\mu\nu} = \frac{\partial g_{\mu\nu}}{\partial r_+} \delta r_+ + \frac{\partial g_{\mu\nu}}{\partial J} \delta J. \quad (43)$$

Then, using the timelike Killing vector $\xi^t = (1, 0, 0)$ and the explicit form (43) of $\delta g_{\mu\nu}$ in equation (5), we get

$$\delta Q_t = \left(-\frac{4GJ^2}{r_+^3} + \frac{r_+}{4Gl^2} \right) \delta r_+ + \frac{4JG}{r_+^2} \delta J. \quad (44)$$

Using then that⁴ $\delta Q_t = \delta M$ together with the definitions (15), (16) and (17), the equation (44) can be rewritten as:

$$\delta M = \frac{1}{8\pi G} \kappa \delta A + \Omega \delta J. \quad (45)$$

Note that this result is independent of the boundary considered in the equation (5). In fact, the integral does not depend on r (when computed with ξ^t and with the BTZ metric).

3.4 Four dimensional black holes

Let us consider the Kerr-adS₄ black hole [44] (see also [45]-[48]):

$$\begin{aligned} ds^2 = & -\frac{\Delta_r}{\Sigma} \left(dt - \frac{a \sin^2(\theta)}{\Xi} d\phi \right)^2 + \frac{\Sigma}{\Delta_r} dr^2 + \frac{\Sigma}{\Delta_\theta} d\theta^2 \\ & + \frac{\Delta_\theta \sin^2(\theta)}{\Sigma} \left(a dt - \frac{r^2 + a^2}{\Xi} d\phi \right)^2 \end{aligned} \quad (46)$$

with,

$$\begin{aligned} \Delta_r = & (r^2 + a^2) \left(1 + \frac{r^2}{l^2} \right) - 2MGr \\ = & \Delta'_{r_+} (r - r_+) + \left(6\frac{r_+^2}{l^2} + 1 + \frac{a^2}{l^2} \right) (r - r_+)^2 \\ & + 4\frac{r_+}{l^2} (r - r_+)^3 + \frac{1}{l^2} (r - r_+)^4 \end{aligned} \quad (47)$$

$$\Delta'_{r_+} = \left. \frac{\partial \Delta_r}{\partial r} \right|_{r=r_+} = r_+ \left(3\frac{r_+^2}{l^2} + \frac{a^2}{l^2} + 1 - \frac{a^2}{r_+^2} \right) \quad (48)$$

⁴It is easy to check that ξ^t is associated with the mass of the black hole. In fact, using that $\xi^t = \frac{1}{l}(\xi_0^+ + \xi_0^-)$ (see equations (18) and (19)), we get $Q_t = \frac{1}{l}(Q_0^+ + Q_0^-) = M + \frac{1}{8G}$ (see equations (26) and (27)).

$$\Delta_\theta = 1 - \frac{a^2}{l^2} \cos^2(\theta) \quad (49)$$

$$\Sigma = r^2 + a^2 \cos^2(\theta) \quad (50)$$

$$\Xi = 1 - \frac{a^2}{l^2}, \quad (51)$$

where r_+ is the highest root of Δ_r .

The area, angular velocity and the surface gravity (equation (B.6)) of the horizon are [49]:

$$\mathcal{A} = \int_{r=r_+} \sqrt{g_{\theta\theta}g_{\phi\phi}} d\theta d\phi = 4\pi \frac{r_+^2 + a^2}{\Xi} \quad (52)$$

$$\Omega = - \left. \frac{g_{t\phi}}{g_{\phi\phi}} \right|_{r=r_+} = \frac{a \Xi}{r_+^2 + a^2} \quad (53)$$

$$\kappa = \frac{\Delta'_{r_+}}{2(r_+^2 + a^2)} \quad (54)$$

The mass and angular momentum are given by⁵:

$$\mathcal{M} = \frac{M}{\Xi} \quad (55)$$

$$\mathcal{J} = \frac{aM}{\Xi^2}. \quad (56)$$

Note that the Kerr solution, together with its charges and its thermodynamical quantities can be obtained from the metric (46) by taking the limit $l \rightarrow \infty$. The Schwarzschild solution then follows after setting a to zero. Moreover, all our following results will remain valid in these limits.

Following section 3.2, the asymptotic diffeomorphisms on the horizon are given by equation (30). The null Killing vector field is now $\chi^\mu = (1, 0, 0, \Omega)$. We choose a natural four-dimensional extension of the ansatz (31) for the functions T_n

$$T_n = \frac{1}{\alpha + \Omega} e^{in(\phi + \alpha t + f(r, \theta))}. \quad (57)$$

The normalization of T_n has been fixed by the algebra (1). Moreover, we can verify that the constraint (B.14) again requires that the arbitrary function $f(r, \theta)$ has to be well-defined (finite) in the limit $r \rightarrow r_+$.

⁵We used the superpotential of Katz, Bičák and Lynden-Bell [23] (see also [34]) to compute \mathcal{M} and \mathcal{J} (respectively associated with the Killing vectors ∂_t and $-\partial_\phi$). The background metric to be used is the adS_4 spacetime seen by a rotating observer which is obtain from (46) by setting $M = 0$ [47].

We next use the metric (46) and the vector fields ξ_n^μ (derived from equations (30) and (57)) in the formula (8). After a tedious calculation, which involves integrating over ϕ (from 0 to 2π) and over θ (from 0 to π) and taking the limit $r \rightarrow r_+$, we finally get:

$$[Q_m, Q_n] = -i\delta_{m+n}m^3 \frac{\mathcal{A}}{8\pi G} \frac{\alpha + \Omega}{\kappa}, \quad (58)$$

which is identical to (33). We can then follow the same argument to conclude that the semiclassical entropy is again recovered if equation (37) is satisfied.

Note that this derivation of the Kerr-adS₄ black hole entropy remains equally valid for the Kerr and the Schwarzschild solutions since the limits $l \rightarrow \infty$ and $a \rightarrow 0$ are always well defined.

We finally check that equation (5) indeed reproduces the first law of thermodynamics. To proceed, we use the same trick outlined by equation (43), that is, we re-express the metric (46) as a function of r_+ and a in order to compute $\delta g_{\mu\nu}$. Using this varied metric and the timelike Killing vector $\xi^t := (1, 0, 0, 0)$ in equation (5), we get:

$$\begin{aligned} \delta Q_t &= \frac{\Delta'_{r_+}}{2r_+\Xi G} \delta r_+ + \frac{a \left(1 + \frac{r_+^2}{l^2}\right) \left(3\frac{r_+^2}{l^2} + \frac{a^2}{l^2} + 2\right)}{2r_+\Xi^2 G} \delta a \\ &= \frac{1}{8\pi G} \kappa \delta A + \Omega \delta J. \end{aligned} \quad (59)$$

We used equations (52), (53), (54) and (56) in the second line. Note moreover that the computed integral (5) is independent of r , and therefore remains valid on any boundary (at spatial infinity or on the horizon).

It is important to compare the result (59) with the first law derived explicitly in the reference [47]. We first need to rewrite δQ_t in term of the mass (55). The simplest way to proceed is to re-consider the metric as a functional of M and a . Using now $\delta g = \frac{\partial g}{\partial M} \delta M + \frac{\partial g}{\partial a} \delta a$ in equation (5), we find:

$$\delta Q_t = \delta \mathcal{M} + \frac{a\mathcal{M}}{l^2\Xi} \delta a \quad (60)$$

Combining equations (59) and (60), we find after some simple algebra:

$$\delta \left(\frac{\mathcal{M}}{\Xi} \right) = \frac{1}{8\pi G} \kappa \delta A + \tilde{\Omega} \delta J, \quad (61)$$

where $\tilde{\Omega} = \Omega + \frac{a}{l^2}$ is the angular velocity at infinity [46]. We then recovered the result given in [47].

4 Conclusion

We have presented a new method to derive the zero-mode Q_0 and the central charge c of a given black hole solution. The entropy computed using the Cardy formula then coincides with the Bekenstein-Hawking formula. A simple derivation of the first law of black hole thermodynamics using our framework was also given.

We found a one-parameter family of $Diff(S^1)$ algebras which preserve the Carlip boundary conditions on the horizon. However, only one of them gives the correct entropy, namely for $\alpha = \kappa - \Omega$. We expect that there exists an additional natural constraint to be imposed on the horizon which would single out this particular $Diff(S^1)$ algebra. Finally, if new $Diff(S^1)$ algebras are found on the horizon, our formula (8) could then be used to check their ability to derive the black hole entropy.

We have then shown that the Cardy formula is able to handle a microscopical black hole entropy calculation with success. We hope to better understand in the future the quantum conformal theory on the horizon which is responsible for this remarkable result.

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Appendix A: Some comments on equation (8)

The purpose of this first appendix is to compare the equation (8) with similar results given in the literature.

Let us first assume that we have a natural background metric denoted by $\bar{g}_{\mu\nu}$ on a given boundary (for instance flat metric at spatial infinity). We can then normalize the charges Q_n such that $Q_n[\bar{g}] = 0$. As a consequence of the formula (3), the equation (8) evaluated on this background $\bar{g}_{\mu\nu}$ gives then the central charge. We can then compare this result with other formulas derived in the literature.

First, it is straightforward to check that on-shell ($\mathcal{R}_{\mu\nu} = \frac{2}{D-2}\Lambda g_{\mu\nu}$), equation (8) evaluated on $\bar{g}_{\mu\nu}$ perfectly agrees with the central charge formula found by Koga [18] using the covariant symplectic methods. This is not really a surprise since the “covariant Regge-Teitelboim” method [29] used to derive equation (5) is equivalent to the symplectic techniques developed

by Ashtekar [24], Wald [25] and collaborators. This was shown in [38], and will be detailed also in [50].

A similar formula for the central charge was derived by Barnich and Brandt [38]. There is again a good agreement with Koga's and our results, up to the term $(\bar{\nabla}^\rho \xi_m^\nu + \bar{\nabla}^\nu \xi_m^\rho)(\bar{\nabla}^\mu \xi_{\rho n} + \bar{\nabla}_\rho \xi_n^\mu) dS_{\mu\nu}$. Although this extra contribution vanishes for all known examples⁶, it would be of interest to understand its meaning.

An alternative to equation (8) for the bracket between two charges was derived by Carlip in [8] (see equation (3.3) (and (3.6)) of that paper). This bracket, also used by Jing and Yan [13] and by Dreyer, Ghosh and Wiśniewski [17], is denoted here by $[[\cdot, \cdot]]$:

$$[[Q_m, Q_n]] = \int_{bd} \frac{\sqrt{|g|}}{16\pi G} [\xi_m^\mu \nabla_\rho (\nabla^\rho \xi_n^\nu - \nabla^\nu \xi_n^\rho) - \xi_n^\mu \nabla_\rho (\nabla^\rho \xi_m^\nu - \nabla^\nu \xi_m^\rho) + \xi_m^\mu \xi_n^\nu \mathcal{L} - (\mu \leftrightarrow \nu)] dS_{\mu\nu} \quad (\text{A.1})$$

On-shell, the Lagrangian is related to the Ricci tensor by $\mathcal{R}_{\mu\nu} = \frac{1}{2}g_{\mu\nu}\mathcal{L}$. Then, it can be shown that the difference between equation (8) and equation (A.1) is (up to total derivatives analogous to (7))

$$\int_{bd} \frac{\sqrt{|g|}}{16\pi G} [\nabla^\mu (\xi_m^\rho \nabla_\rho \xi_n^\nu - \xi_n^\rho \nabla_\rho \xi_m^\nu) - (\mu \leftrightarrow \nu)] dS_{\mu\nu}. \quad (\text{A.2})$$

This is nothing but the (1/2) Komar superpotential [49] (also called Noether charge $\mathbf{Q}[g, \xi]$ in [25])

$$J[\xi] = \int_{bd} \frac{\sqrt{|g|}}{16\pi G} [\nabla^\mu \xi^\nu - (\mu \leftrightarrow \nu)] dS_{\mu\nu}, \quad (\text{A.3})$$

evaluated for the Lie bracket of the two vector fields ξ_m^μ and ξ_n^ν . As pointed out in a footnote by Koga [18], the mismatch comes from the equation (3.2) of the reference [8]. Indeed, if we assume that δ_ξ acts on the metric but *not* on the parameters, this equation (3.2) has to be modified to $\delta_{\xi_m} J[\xi_n] = (\xi_m \cdot \mathbf{d} + \mathbf{d}\xi_m \cdot) J[\xi_n] - J[\{\xi_m, \xi_n\}]$ (see for instance equation (4.8) of [29] for the analogous example of Yang-Mills theory). The last term $J[\{\xi_m, \xi_n\}]$ gives the additional contribution (A.2) on the horizon. In the examples of section 3, the extra term (A.2) does modify the zero-mode Q_0 but not the central charge.

⁶G. Barnich and F. Brandt, *private communication*.

Appendix B: Local Killing horizons : the Carlip's approach

The goal of this appendix is to summarize part of the Carlip's results [8] on the asymptotic symmetries of (local) Killing horizons. Although we present this work in a slightly rephrased way, the important results remain unchanged. We refer to the original paper for a detailed discussion.

Let us assume that a locally isolated Killing horizon can be characterized by some vector field χ^μ , whose norm

$$\chi^2 = 0. \tag{B.1}$$

vanishes on the horizon. We work on the stretched horizon, and we will take the limit $\chi^2 = 0$ at the end of the calculations.

In the following, we just need to assume that the vector field χ^μ is a Killing vector on the horizon and a conformal Killing vector to first order⁷. Concretely, it is enough to require that:

$$\nabla_{(\mu}\chi_{\nu)} = O(\chi^2)g_{\mu\nu} + O(\chi^4). \tag{B.2}$$

Following Carlip's paper, the vector normal to the horizon is defined by

$$\rho_\mu := -\frac{1}{2\kappa}\nabla_\mu\chi^2. \tag{B.3}$$

Then, the norm of ρ_μ vanishes on the horizon (null hypersurface)

$$\rho^2 = O(\chi^2). \tag{B.4}$$

Now, the normalization factor κ in the definition of ρ_μ (B.3) is fixed by the requirement that close to the horizon,

$$\rho^2 = -\chi^2 + O(\chi^4). \tag{B.5}$$

The overall normalization of χ is therefore the only remaining free parameter. Moreover, from equations (B.2) and (B.3), κ can be identified with the surface gravity associated with χ ,

$$\kappa^2 := -\lim_{\chi^2 \rightarrow 0} \left(\frac{\nabla_\sigma\chi^2\nabla^\sigma\chi^2}{4\chi^2} \right) = -\lim_{\chi^2 \rightarrow 0} \left(\frac{a^\sigma a_\sigma}{\chi^2} \right), \tag{B.6}$$

⁷This is little weaker than the original assumption of Carlip that the vector χ^μ is a Killing vector in a neighborhood of the horizon.

with the acceleration given by $a^\sigma := \chi^\mu \nabla_\mu \chi^\sigma$.

We can use equations (B.2) and (B.3) to derive some useful identities,

$$\rho^\mu \chi_\mu = O(\chi^4), \quad (\text{B.7})$$

$$\mathcal{L}_\chi \rho_\mu = \nabla_\mu (\rho \cdot \chi) = O(\chi^2) \rho_\mu \quad (\text{B.8})$$

$$\mathcal{L}_\chi \rho^\mu = O(\chi^2) \rho^\mu. \quad (\text{B.9})$$

The next step is to identify a set of diffeomorphisms of the metric which preserve some of the above structure. In particular and following Carlip, we require that the position and the normal direction of the black hole (equations (B.1) and (B.3)) remain unchanged under $g_{\mu\nu} \rightarrow g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu}$, that is:

$$\delta_\xi \chi^2 = \chi^\mu \chi^\nu \mathcal{L}_\xi g_{\mu\nu} = O(\chi^4) \quad (\text{B.10})$$

$$\delta_\xi \rho_\mu = -\frac{1}{2\kappa} \nabla_\mu (\chi^\nu \chi^\sigma \mathcal{L}_\xi g_{\nu\sigma}) = \rho_\mu O(\chi^2) \quad (\text{B.11})$$

For the ansatz

$$\xi^\mu = R\rho^\mu + T\chi^\mu, \quad (\text{B.12})$$

the equations (B.10) and (B.11) are satisfied if

$$\mathcal{L}_\chi T + \kappa R = O(\chi^2). \quad (\text{B.13})$$

This equation then gives R in terms of T .

Finally, the closure of the Lie bracket between two asymptotic parameters (B.12) requires an additional constraint

$$\rho^\mu \nabla_\mu T = O(\chi^2) T. \quad (\text{B.14})$$

Note that the vector field ξ^μ does not need to be well defined on the horizon itself (this is indeed a coordinate dependent-statement). However, the associated charges should be finite.

Appendix C: The BTZ metric in Eddington-Finkelstein coordinates

In Eddington-Finkelstein coordinates, the BTZ black hole is [17]:

$$ds^2 = -N^2 dv^2 + 2dvdr + r^2(N_\varphi dt + d\varphi)^2 \quad (\text{C.1})$$

with N^2 and N_φ given as before by equations (12) and (13).

The change between the Eddington-Finkelstein coordinates (v, r, φ) and the usual Schwarzschild coordinates (t, r, ϕ) is simply:

$$v = t + A(r) \tag{C.2}$$

$$r = r \tag{C.3}$$

$$\varphi = \phi + B(r), \tag{C.4}$$

where the functions $A(r)$ and $B(r)$ satisfy :

$$\partial_r A = N^{-2} \tag{C.5}$$

$$\partial_r B = -N^{-2} N_\phi. \tag{C.6}$$

Integrating equations (C.5) and (C.6) explicitly, we find:

$$A = \frac{1}{2\kappa} \left(\log \left(\frac{r-r_+}{r+r_+} \right) - \frac{r_-}{r_+} \log \left(\frac{r-r_-}{r+r_-} \right) \right) \tag{C.7}$$

$$B = \frac{\Omega}{2\kappa} \left(\log \left(\frac{r-r_+}{r+r_+} \right) - \frac{r_+}{r_-} \log \left(\frac{r-r_-}{r+r_-} \right) \right). \tag{C.8}$$

We can then use the asymptotic behavior of A and B close to the horizon,

$$A \xrightarrow{r \rightarrow r_+} \frac{1}{2\kappa} \log(r-r_+) + O((r-r_+)^0) \tag{C.9}$$

$$B \xrightarrow{r \rightarrow r_+} \frac{\Omega}{2\kappa} \log(r-r_+) + O((r-r_+)^0) \tag{C.10}$$

to check that the ‘‘extended symmetries’’ proposed in [17]⁸,

$$T_n = \frac{1}{2\Omega} e^{in(\varphi + \Omega v - \frac{\Omega}{\kappa} \log(r-r_+))} \tag{C.11}$$

is in the coordinates t and ϕ , a special case of the general ansatz (31) (namely for $\alpha = \Omega$). We already saw in section 3.2 that the regular part of the functions $A(r)$ and $B(r)$ does contribute neither to the zero-mode nor to the central charge and therefore can be dropped out.

⁸Our conventions differs by a minus signs from those of [17], for the angular velocity and for the $Diff(S^1)$ algebra (1).

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