AEI-2001-136 LPTENS 01/43
hep-th/0111189

D-Branes in Coset Models

Stefan Fredenhagen
Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut
Am Mühlenberg 1, D–14476 Golm, Germany *

Volker Schomerus
Laboratoire de Physique Théorique de l'École Normale Supérieure
24 rue Lhomond, F-75231 Paris Cedex 05, France &
Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut
Am Mühlenberg 1, D-14476 Golm, Germany †

November 22, 2001

Abstract

The analysis of D-branes in coset models G/H provides a natural extension of recent studies on branes in WZW-theory and it has various interesting applications to physically relevant models. In this work we develop a reduction procedure that allows to construct the non-commutative gauge theories which govern the dynamics of branes in G/H. We obtain a large class of solutions and interprete the associated condensation processes geometrically. The latter are used to propose conservation laws for the dynamics of branes in coset models at large level k. In super-symmetric theories, conserved charges are argued to take their values in the representation ring of the denominator theory. Finally, we apply the general results to study boundary fixed points in two examples, namely for parafermions and minimal models.

*email: stefan@aei.mpg.de
†email: vschomer@aei.mpg.de

1 Introduction

The study of D-branes on compact group manifolds has taught us a lot about the classification and the dynamics of branes in curved but highly symmetric backgrounds (see e.g. [1, 2, 3, 4, 5, 6, 7, 8, 9]). Part of this analysis applies directly to relevant string backgrounds, namely to string theory in near horizon geometries of NS5 or D3 branes. In both cases, a 3-sphere, i.e. the group manifold of SU(2), appears as part of the background.

But there is a more important aspect of these developments that is deeply rooted in CFT model building. In fact, the WZW models that are used to describe strings on group manifolds appear as the basic building blocks for all the coset and orbifold constructions of exactly solvable string backgrounds. It is therefore natural to analyze how much of the known properties of strings and branes on group manifolds descends down to less symmetric coset spaces G/H. This has been initiated in a number of recent papers [10, 11, 12, 13, 14] and it is our main subject below.

Brane configurations (of so-called "Cardy type") in a coset model G/H can be labeled by representations P of the Lie algebra $\mathfrak{g} \oplus \mathfrak{h}$. Let us remark that not all representations are to be admitted here and that there exist some identifications between representations that are associated with exactly the same brane configuration. These issues will be addressed in more detail in Section 3. Admissible irreducible representations P of $\mathfrak{g} \oplus \mathfrak{h}$ correspond to elementary branes while reducible representations enter when we want to describe superpositions of the elementary branes.

In Section 4 we shall construct the effective non-commutative gauge theory for such branes P in a certain limiting regime of the coset model. These theories are obtained by some kind of dimensional reduction from the fuzzy gauge theories that control the dynamics of Cardy type branes on group manifolds [7, 15]. They are (constrained) matrix models involving a Yang-Mills and a Chern-Simons like term.

A large number of solutions to these non-commutative gauge theories is constructed and interpreted as a formation of bound states in Section 5. We shall show that two brane configurations P and Q are related by condensation if the representations one obtains by restriction to the diagonally embedded $\mathfrak{h} \subset \mathfrak{g} \oplus \mathfrak{h}$ are equivalent,

$$P|_{\mathfrak{h}} \;\cong\; Q|_{\mathfrak{h}} \qquad \text{where} \qquad \mathfrak{h} = \mathfrak{h}_{\text{diag}} \subset \mathfrak{g} \oplus \mathfrak{h} \quad .$$

Here, the subscript 'diag' refers to the diagonal embedding $X \mapsto X \oplus X$ of $\mathfrak{h}$ into $\mathfrak{g} \oplus \mathfrak{h}$. For the criterion to make sense, we use the identification between brane configurations and representations of $\mathfrak{g} \oplus \mathfrak{h}$.

In non-supersymmetric backgrounds there will be many other processes that involve tachyonic (relevant) fields and these lead to further relations between brane configurations. But in super-symmetric models there is a fairly good chance that the criterion above exhausts all the possibilities. In this case, our findings suggest that the conserved charges of Cardy-type D-branes in a coset G/H take their values in the representation ring of the denominator H, i.e. there is one conserved charge for each irreducible representation of H. This also fits nicely with the structure of Ramond-Ramond charge lattices found in certain Kazama-Suzuki models [16]. For a trivial denominator $H = e$, there appears just one irreducible representation and hence we recover the known result that Cardy type branes on group manifolds carry only D0-brane charge. At present, our studies of the charge group in coset models are restricted to a limiting regime and one expects that they receive the same type of corrections that appear for group manifolds [17, 18, 19, 20, 21].

In the last section of this paper we shall apply our general results to branes in parafermion and minimal models. For minimal models our general results will provide a large number of new candidates for flows between boundary theories extending previous related work in conformal field theory [22, 23, 24, 25, 26] (see also [27]) and we shall propose a nice and very suggestive geometrical interpretation. The results on parafermions and N=2 super-conformal minimal models have been (partly) announced before in [11].

2 Review of branes in group manifolds

In this section we shall review some of the results on branes in a group manifold G. Strings moving on G are described by the WZW-model and so we will start by recalling some facts about the affine Lie algebras obtained from chiral currents in these theories. Following the work of Cardy [28], we shall then present the solution of the boundary WZW problem and its geometric interpretation [2]. Finally, we discuss the low energy effective action of branes on group manifolds [7].

WZW models for a compact simple simply connected group G are parametrized by one discrete parameter k which is known as the *level*. We can think of k as controlling the volume (or 'size') of the group manifolds. From the basic group valued field g of the model one can construct (anti-)holomorphic currents $J, \bar{J}$ taking values in the Lie algebra $\mathfrak{g}$. Throughout this work we are interested in boundary theories in which the currents are subjected to the following boundary condition at $z = \bar{z}$

$$J^\alpha(z) \ = \ \bar{J}^\alpha(\bar{z}) \quad \text{for} \quad \alpha = 1, \dots, D = \dim G \quad . \tag{2.1}$$

These boundary conditions were shown in [2] to describe branes localized along conjugacy classes of the group and they are equipped with a non-vanishing B-field. The stability of the associated super-symmetric theories was established in [8, 9], at least in the large volume regime. At finite level k, they can be shown to possess a tachyon free spectrum [5, 7].

The WZW-model with boundary condition (2.1) can be solved using ideas that go back to the work of Cardy [28] and Runkel [29] (see [5, 6] for applications to the WZW model). The solution uses data from the representation theory of affine Lie algebras which we shall recall briefly also to set up our notations. We shall label sectors of the theory by elements l taken from a finite set $\mathcal{J}_k^{\mathfrak{g}}$. The corresponding state spaces $\mathcal{H}_l^{\mathfrak{g}}$ are generated from irreducible representations $V^l \subset \mathcal{H}_l^{\mathfrak{g}}$ of the finite dimensional Lie algebra $\mathfrak{g} = \text{Lie}G$. This implies that the sectors of the theory at finite k form a subset of the set $\mathcal{J}^{\mathfrak{g}}$ of irreducible representations of $\mathfrak{g}$, i.e. $\mathcal{J}_k^{\mathfrak{g}} \subset \mathcal{J}^{\mathfrak{g}}$. We will often identify the elements $l \in \mathcal{J}_k^{\mathfrak{g}}$ with the corresponding element of $\mathcal{J}^{\mathfrak{g}}$.

On $\mathcal{H}_l^{\mathfrak{g}}$ there exists an action of the Virasoro algebra whose generators we denote by $L_n^{\mathfrak{g}}$ making explicit reference to the Lie algebra $\mathfrak{g}$. The space $V^l \subset \mathcal{H}_l^{\mathfrak{g}}$ consists of ground states with conformal dimension $h_l^{\mathfrak{g}} \sim \Delta_l/(k + c^\vee)$ where Δ_l is the value of the quadratic Casimir in the representation $l \in \mathcal{J}^{\mathfrak{g}}$ and $c^\vee$ denotes the dual Coxeter number. As we send k to infinity, the conformal dimension of these ground states vanishes.

Let us now consider the WZW-model associated with the diagonal modular invariant,

$$Z(q, \bar{q}) \ = \sum_{l \in \mathcal{J}^{\mathfrak{g}}} |\chi_l^{\mathfrak{g}}(q)|^2 \ \ ,$$

where $\chi_l^{\mathfrak{g}}(q)$ denotes the character of the sector l. The choice of Z implies that the bulk fields of the boundary theories we are about to discuss are obtained as descendants of primary fields $\Phi^{l,l}(z, \bar{z}) = \Phi^l(z, \bar{z})$, one for each sector of the affine Lie algebra. The space of bulk fields comes equipped with two commuting representations of the affine Lie algebra. Below, we shall frequently make use of the descendants

$$\Phi^l_{nm}(z, \bar{z}) \quad \text{for} \quad l \in \mathcal{J}_k \tag{2.2}$$

and n, m each label vectors from a basis of the representation space V^l. The fields in the list (2.2) are obtained from the primary fields by acting with zero modes of the two commuting affine Lie algebras. They correspond to ground states in the bulk theory and their conformal dimensions are $(h, \bar{h}) = (h_l^{\mathfrak{g}}, h_l^{\mathfrak{g}})$.

Following the analysis of Cardy, the boundary WZW model with condition (2.1) admits as many different solutions as there are sectors $l \in \mathcal{J}_k^{\mathfrak{g}}$. We will denote the boundary theories by capital letters $L, \dots$. These boundary theories can be characterized by the 1-point functions of bulk fields, i.e. by the the coupling of closed string modes to the brane. According to [28], these couplings are given by the modular matrix

$S^{\mathfrak{g}}$ as follows,

$$\langle\, \Phi^l_{nm}(z,\bar z)\,\rangle_L \;=\; \frac{S^{\mathfrak{g}}_{L\,l}}{\sqrt{S^{\mathfrak{g}}_{0\,l}}}\,\frac{\delta_{nm}}{|z-\bar z|^{2h_l}}\quad . \tag{2.3}$$

For the solution of the model, it would have been sufficient to present the couplings for the primary fields only, but we have decided to include all the fields from the list (2.2). On the right hand side this gives rise to the trivial factor δ_{nm}. Following a procedure suggested in [30, 6] (see also [12]), the localization region of the branes can be read off from the 1-point functions (2.3). The results of such an analysis confirm the findings of [2] that Cardy type branes are localized along conjugacy classes

$$C_L \;=\; \{\, g\in G \mid g = u g_L u^{-1} \text{ for } u\in G\,\} \quad . \tag{2.4}$$

Here g_L is some fixed group element which depends on the brane label L (see e.g. [6] for details).

Information on the spectrum of open string states on the branes (2.3) is encoded in the fusion rules. More precisely, the space of open strings stretching between two branes L_1 and L_2 is contained in

$$\mathcal{H}^{\mathfrak{g};L_2}_{L_1} \;=\; \bigoplus_l\; N^{\mathfrak{g};L_2}_{L_1\,l}\;\mathcal{H}^{\mathfrak{g}}_l \tag{2.5}$$

where $N^{\mathfrak{g}}$ are the fusion rules of $\widehat{\mathfrak{g}}_k$. There is a boundary field associated with each state in this state space and one can show that the operator product expansion of any two such fields is determined by the fusing matrix [29] (see also [5, 6, 31]).

If we let k tend to infinity while keeping the brane labels L_1, L_2 fixed, the space of ground states stays finite and it is easy to identify it with $\mathrm{Hom}(V^{L_1}, V^{L_2})$, i.e. with the space of linear maps between the two finite dimensional representation spaces V^{L_1} and V^{L_2} of the Lie algebra $\mathfrak{g}$. For $L = L_1 = L_2$, the space $\mathrm{Hom}(V^L, V^L)$ comes equipped with a natural product ("matrix multiplication") and it is exactly this product that one obtains from the OPE of open string vertex operators in the limit $k\to\infty$ [5]. Its non-commutativity can be nicely explained by the presence of a non-vanishing B-field on the branes.

After these remarks on the (non-commutative) geometry of branes in group manifolds we are prepared to review their low energy effective gauge theory. Consider some configuration $P = \sum P_L(L)$ of Cardy-type branes on the group manifold which contains P_L branes of type (L) on top of each other. In the following we will not distinguish in notation between such a brane configuration P and the associated representation P of $\mathfrak{g}$. In particular, we shall denote by V^P the reducible representation space $V^P = \sum P_L V^L$. It was shown in [7] that the effective action for the brane configuration P is given by a linear combination of a Yang-Mills and a Chern-Simons term for a set of fields $\mathrm{A}_\alpha \in \mathrm{End}(V^P)$,

$$\mathsf{S}_P \;=\; \mathsf{S}_{\mathrm{YM}} + \mathsf{S}_{\mathrm{CS}} \;=\; \frac{1}{4}\,\mathrm{tr}\,\left(\mathrm{F}_{\alpha\beta}\,\mathrm{F}^{\alpha\beta}\right) - \frac{i}{2}\,\mathrm{tr}\,\left(\mathrm{f}^{\alpha\beta\gamma}\,\mathrm{CS}_{\alpha\beta\gamma}\right) \tag{2.6}$$

where we defined the 'curvature form' $\mathrm{F}_{\alpha\beta}$ and some non-commutative analogue $\mathrm{CS}_{\alpha\beta\gamma}$ of the Chern-Simons form by the expressions

$$\mathrm{F}_{\alpha\beta}(\mathrm{A}) \;=\; i\,\mathrm{L}_\alpha \mathrm{A}_\beta - i\,\mathrm{L}_\beta \mathrm{A}_\alpha + i\,[\mathrm{A}_\alpha\,,\,\mathrm{A}_\beta] + \mathrm{f}_{\alpha\beta\gamma}\mathrm{A}^\gamma \tag{2.7}$$

$$\mathrm{CS}_{\alpha\beta\gamma}(\mathrm{A}) \;=\; \mathrm{L}_\alpha \mathrm{A}_\beta\,\mathrm{A}_\gamma + \frac{1}{3}\,\mathrm{A}_\alpha\,[\mathrm{A}_\beta\,,\,\mathrm{A}_\gamma] - \frac{i}{2}\,\mathrm{f}_{\alpha\beta\delta}\,\mathrm{A}^\delta\,\mathrm{A}_\gamma \quad . \tag{2.8}$$

We have introduced the symbol L_α to denote the 'infinitesimal translation' $\mathrm{L}_\alpha \mathrm{A} = [P(t_\alpha), \mathrm{A}]$ where t_α denote the generators of the Lie algebra $\mathfrak{g}$. Gauge invariance of (2.6) under the gauge transformations

$$\mathrm{A}_\alpha \;\rightarrow\; \mathrm{L}_\alpha \Lambda \;+\; i\,[\,\mathrm{A}_\alpha\,,\,\Lambda\,] \quad \text{for} \quad \Lambda \in \mathrm{End}(V^P)$$

follows by straightforward computation. Similar gauge theories on matrix ("fuzzy") geometries [32, 33] have been studied before they were shown to appear in string theory (see e.g. [34, 35, 36, 37, 38, 39]).

From eq. (2.6) we obtain the following equations of motion for the elements $Q_\alpha := P(t_\alpha) + \mathrm{A}_\alpha \in \mathrm{End}(V^P)$,

$$\Big[Q^\alpha \;, [\, Q_\alpha \,,\, Q_\beta \,] \,-\, i\, \mathrm{f}_{\alpha\beta\gamma}\, Q^\gamma \Big] \;=\; 0 \quad . \tag{2.9}$$

Solutions of these equations (2.9) describe possible condensates of our brane configuration P. There exists one type of solutions that is particularly interesting. Obviously, we can satisfy the equations (2.9) by choosing Q_α to be any $\dim(V^P)$-dimensional representation of the Lie algebra $\mathfrak{g}$. The associated solution is then given by $\mathrm{A}_\alpha = Q_\alpha - P(t_\alpha)$. As it was argued in [7], this solution describes a process of the form

$$(P) \;\stackrel{\mathrm{A}=Q-P}{\longrightarrow}\; (Q) \quad .$$

Support for this statement comes from both the open string sector and the coupling to closed strings (see [7]). On the one hand, we can compare the tension of D-branes in the final configuration Q with the value of the action $\mathsf{S}_P(\mathrm{A})$ at the classical solution A. On the other hand, we can look at fluctuations around the chosen solution and compare their dynamics with the low energy effective theory S_Q of the brane configuration Q. In formulas, this means that

$$\mathsf{S}_P(\mathrm{A}+\delta\mathrm{A}) \;\stackrel{!}{=}\; \mathsf{S}_P(\mathrm{A}) + \mathsf{S}_Q(\delta\mathrm{A}) \quad \text{with} \quad \mathsf{S}_P(\mathrm{A}) \;\stackrel{!}{=}\; \ln \frac{g_Q}{g_P} \quad . \tag{2.10}$$

The second requirement expresses the comparison of tensions in terms of the g-factors [40] of the involved conformal field theories (see e.g. [7] for more details). All equalities must hold to the order in $(1/k)$ that we used when we constructed the effective actions.

These results imply that condensation processes of Cardy type brane configurations P on group manifolds possess only one invariant: the dimension $\dim(V^P)$ of the representation P. We can easily identify this invariant with the D0 brane charge. In fact, a particular initial configuration is given by the representation $P = P_0[0]$, i.e. by choosing the trivial representation of $\mathfrak{g}$ with multiplicity P_0. It corresponds to a configuration in which P_0 point-like branes are placed on top of each other at the group unit $e \in G$. Now we are advised to pick any $\dim(V^P) = P_0$-dimensional representation Q of $\mathfrak{g}$. The latter decomposes into a sum of irreducible representations $Q = \sum Q_L[L]$. If Q is irreducible, the final state contains a single extended brane of charge P_0. A well known example of this phenomenon is the formation of spherical branes on $S^3 \cong \mathrm{SU}(2)$ which was discussed extensively in the past (see [7] and also [41],[42]). Similar effects have been described for branes in RR-background fields [43]. The advantage of our scenario with NSNS-background fields is that it can be treated in perturbative string theory so that string effects may be taken into account (see [20]).

3 Boundary coset models

From now on let $H \subset G$ denote some simple simply connected subgroup of G. We want to study the associated G/H coset model. A more precise formulation of this theory requires a bit of preparation (more details can be found e.g. in [44]). We shall label the sectors $\mathcal{H}^{\mathfrak{h}}_{l'}$ of the affine Lie algebra with $\widehat{\mathfrak{h}}_{k'}$ labels $l' \in \mathcal{J}^{\mathfrak{h}}_{k'}$. Note that the sectors of the numerator theory carry an action of the denominator algebra $\widehat{\mathfrak{h}}_{k'} \subset \widehat{\mathfrak{g}}_k$ and under this action each sector $\mathcal{H}^{\mathfrak{g}}_l$ decomposes according to

$$\mathcal{H}^{\mathfrak{g}}_l \; = \; \bigoplus_{l'} \mathcal{H}_{(l,l')} \otimes \mathcal{H}^{\mathfrak{h}}_{l'} \quad .$$

Here we have introduced the infinite dimensional spaces $\mathcal{H}_{(l,l')}$ which we want to interprete as sectors of the coset chiral algebra. The latter is usually hard to describe explicitly, but at least it is known to contain a Virasoro field with modes

$$L_n \; = \; L^{\mathfrak{g}}_n - L^{\mathfrak{h}}_n \quad . \tag{3.1}$$

One may easily check that they obey the usual exchange relations of the Virasoro algebra with central charge given by $c = c^{\mathfrak{g}} - c^{\mathfrak{h}}$.

Note that some of the spaces $\mathcal{H}_{(l,l')}$ may vanish simply because a given sector $\mathcal{H}^{\mathfrak{h}}_{l'}$ of the denominator theory may not appear as a subsector in a given $\mathcal{H}^{\mathfrak{g}}_l$. This motivates to introduce the set

$$\mathcal{E} \;=\; \{\,(l,l') \in \mathcal{J}^{\mathfrak{g}}_k \times \mathcal{J}^{\mathfrak{h}}_{k'} \,|\, \mathcal{H}_{(l,l')} \neq 0\,\} \;\; .$$

Elements of $\mathcal{E}$ do not yet label sectors of the coset models. In fact, different elements of $\mathcal{E}$ may correspond to the same sector, i.e. there is an equivalence relation

$$(l,l') \sim (m,m') \;\Leftrightarrow\; \mathcal{H}_{(l,l')} \cong \mathcal{H}_{(m,m')} \;\; .$$

At this point we want to make one assumption, namely that all the equivalence classes we find in $\mathcal{E}$ contain the same number N_0 of elements. This holds true for many important examples and it guarantees that the sectors of the coset theory are simply labeled by the equivalence classes, i.e. $\mathcal{J} = \mathcal{E}/\sim$. [1] It is then also easy to spell out explicit formulas for the fusion rules and the S-matrix of the coset model. These are given by

$$N^{(k,k')}_{(l,l')\,(m,m')} \;=\; \sum_{(n,n')\sim(k,k')} N^{\mathfrak{g};n}_{l\,m}\, N^{\mathfrak{h};n'}_{l'\,m'} \;\; , \tag{3.2}$$

$$S_{(l,l')(m,m')} \;=\; N_0\, S^{\mathfrak{g}}_{l\,m}\, \bar{S}^{\mathfrak{h}}_{l'\,m'} \tag{3.3}$$

where the bar over the second S-matrix denotes complex conjugation.

Let us note that (l,l') is an element of $\mathcal{E}$ if (but not only if) the representation l' of the finite dimensional Lie algebra $\mathfrak{h}$ appears as a sub-representation of the representation l for $\mathfrak{g}$. The equivalence classes of such special pairs form a subset $\mathcal{J}^r \subset \mathcal{J}$. Sectors in the subset $\mathcal{J}^r$ are also distinguished because the conformal dimension of their ground state satisfies the equality

$$h_{(l,l')} \;=\; h^{\mathfrak{g}}_l - h^{\mathfrak{h}}_{l'} + n$$

with $n = 0$. For other pairs $(l,l') \in \mathcal{J}$, n is a (non-vanishing) positive integer. This means in particular, that fields associated with the sectors in $\mathcal{J} \setminus \mathcal{J}^r$ necessarily have

[1] For more general cases, there are further sectors that cannot be constructed within the sectors of the numerator theory.

conformal dimension $h, \bar{h} > 1$ and hence they are irrelevant. We shall see in Appendix A that the sectors labeled by elements of $\mathcal{J}^r$ play a special role when we analyse the brane geometry.

The boundary theories we are going to look at are associated with the diagonal modular invariant bulk partition function,

$$Z(q,\bar{q}) \;=\; \sum_{(l,l')} \chi_{(l,l')}(q)\chi_{(l,l')}(\bar{q}) \quad .$$

We want to impose trivial gluing conditions along the boundary in which each left moving chiral field of the coset theory is glued to its right moving partner. Under this condition, the associated boundary theories can be constructed using Cardy's solution [28]. It asserts that the model has as many boundary conditions as there are sectors of the coset algebra. We will label them with $(L, L') \in \mathcal{J}$. For a given theory (L, L') the couplings of closed string modes to the boundary are essentially given by the S-matrix, i.e.

$$\langle\, \phi_{(l,l')}(z,\bar{z})\, \rangle_L \;=\; \frac{S^{\mathfrak{g}}_{l\,L}\bar{S}^{\mathfrak{h}}_{l'\,L'}}{\sqrt{S^{\mathfrak{g}}_{0\,l}\bar{S}^{\mathfrak{h}}_{0l'}}}\;\frac{1}{|z-\bar{z}|^{2h_{(l,l')}}} \quad . \tag{3.4}$$

Here we have used the explicit formula for the S-matrix of the coset theory that was given in eq. (3.3). Let us also write down the spectrum of open string modes stretching in between two branes (L_1, L_1') and (L_2, L_2'),

$$Z^{(L_2,L_2')}_{(L_1,L_1')}(q) \;=\; \sum_{(l,l')} N^{\mathfrak{g};L_2}_{L_1\,l}\; N^{\mathfrak{h};L_2'}_{L_1'\,l'}\; \chi_{(l,l')}(q) \quad . \tag{3.5}$$

This formula involves the fusion rules of the coset model that were spelled out in eq. (3.2). Let us point out that we can think of these elementary Cardy branes as being labeled by a set of irreducible representations $[L, \bar{L}']$ of $\mathfrak{g} \oplus \mathfrak{h}$. Here $\bar{L}'$ denotes the representation conjugate to L'. This way of associating a representation $[L, \bar{L}']$ to the brane configuration (L, L') turns out to be rather convenient. More complicated brane configurations P involve reducible representations of the same Lie algebra. As in the case of branes on group manifolds we will often identify brane configurations P with the associated representation of $\mathfrak{g} \oplus \mathfrak{h}$.

The geometry of the Cardy type branes in coset models was recently uncovered in [13] (see also [14]). To describe the answer we need some more notation. Recall first that geometrically the quotient G/H is formed with respect to the adjoint action of H on G, i.e. two points on G are identified if they are related by conjugation with an element of $H \subset G$. We denote the projection from G to the space G/H of H orbits by $\pi^G_{G/H}$. Furthermore, we use C^G_L to refer to the conjugacy class of G along which the brane with L is localized and similarly for $C^H_{L'}$. The latter is a conjugacy class in H. Through the embedding of H into G, we can regard it as a subset of G. Now we construct the set $C_{(L,L')}$ of all elements in G which are of the form uv^{-1} where $u \in C^G_L$ and $v \in C^H_{L'}$. This set is left invariant by conjugation with elements of H and hence it can be projected down to G/H. The claim of [13] is that the brane (L, L') is localized along the resulting subset $C^{G/H}_{(L,L')}$ of G/H,

$$C^{G/H}_{(L,L')} \; = \; \pi^G_{G/H} \left(C^G_L \; (C^H_{L'})^{-1} \right) \; \subset \; G/H \quad . \tag{3.6}$$

We shall extract this result from an analysis of the 1-point function in Appendix A.

4 Coset branes and fuzzy gauge theories

In this section we will discuss the effective non-commutative gauge theory that describes the dynamics of branes in coset theories. Given some configuration of coset branes, i.e. a representation of $\mathfrak{g} \oplus \mathfrak{h}$, we construct some parent action which is essentially identical to the field theory (2.6). The effective field theory of coset branes is then obtained by a suitable reduction. Our construction can be derived from conformal field theory. While we explain most of this in Appendix B, we provide the derivation for special brane configurations of the form $P = \sum P_L(L, 0)$ in the second subsection below.

4.1 Construction of the effective field theory

We have reviewed the effective action for branes on group manifolds in Section 2. The result has been discussed for a WZW model involving a single affine Lie algebra $\widehat{\mathfrak{g}}_k$. For our purposes below, we need the action for cases where the underlying affine Lie algebra is a direct sum of algebras with different levels k_r. From the resulting action we will then obtain the effective field theory of coset branes by reduction.

A coset model involves two chiral algebras $\widehat{\mathfrak{g}}$ and $\widehat{\mathfrak{h}}$ in the numerator and the denominator, respectively. In general, these possess decompositions of the form $\widehat{\mathfrak{g}} = \widehat{\mathfrak{g}}_1 \oplus \cdots \oplus \widehat{\mathfrak{g}}_R$ and $\widehat{\mathfrak{h}} = \widehat{\mathfrak{h}}_1 \oplus \cdots \oplus \widehat{\mathfrak{h}}_{R'}$ with possibly different levels $k_1, \ldots, k_R; k'_1, \ldots, k'_{R'}$ appearing in each summand. We will study a regime of the model in which some of the levels are sent to infinity while others stay finite. Let us assume that the decompositions above have been arranged such that $k_1, \ldots, k_S$ and $k'_1, \ldots k'_{S'}$ become large.

In this limiting regime we intend to study Cardy type brane configurations $P = \sum P_{L\,L'}(L, L')$ where L, L' are multi-labels of the form $L = (L_1, \ldots, L_S, 0, \ldots, 0)$ and $L' = (L'_1, \ldots, L'_{S'}, 0, \ldots, 0)$ in which the representation labels for the small directions are chosen to be trivial[2]. As we explained before, such a brane configuration gives rise to a representation $P = \sum P_{L\,L'}[L, \bar{L}']$ of the Lie algebra $\mathfrak{g} \oplus \mathfrak{h}$.

The field theory we are going to spell out now involves a number of gauge fields A_α where α label a basis in $\mathfrak{g} \oplus \mathfrak{h}$, i.e. it runs through the values $1, \ldots, \dim \mathfrak{g} + \dim \mathfrak{h}$. The gauge fields A_α are elements of the space $\mathrm{End}(V^P)$ which depends on the choice of our initial brane configuration P. Let us also introduce the derivatives L_α as follows

$$\mathrm{L}_\alpha \mathrm{A} \;=\; \begin{cases} [\, P(t_\alpha)\,,\, \mathrm{A}\,] & \text{for} \quad \alpha \le \dim \mathfrak{g} \\ i\,[\, P(t_\alpha)\,,\, \mathrm{A}\,] & \text{for} \quad \alpha > \dim \mathfrak{g} \end{cases} \quad . \tag{4.1}$$

Note that we have absorbed an extra factor $\sqrt{-1}$ into the definition of $\mathrm{L}_\alpha, \alpha > \dim \mathfrak{g}$. This will turn out to be rather convenient in the following. In these notations, we are

[2] In the limit of large k the theory is essentially independent of the labels $L_{S+1}, \ldots, L_R$, $L_{S'+1}, \ldots, L_{R'}$

now able to introduce the following action,

$$\mathsf{S}_P^{\rm WZW}({\rm A}) \;=\; \mathsf{S}_{\rm YM}({\rm A}) + \mathsf{S}_{\rm CS}({\rm A}) \;=\; \frac{1}{4}{\rm tr}\;({\rm F}_{\alpha\beta}{\rm F}^{\alpha\beta}) - \frac{i}{2}{\rm tr}\;(\tilde{\rm f}^{\alpha\beta\gamma}{\rm CS}_{\alpha\beta\gamma}) \tag{4.2}$$

where $\tilde{\rm f}^{\alpha\beta\gamma} \;=\; {\rm f}^{\alpha\beta\gamma}$ if $\alpha,\beta,\gamma \leq \dim\mathfrak{g}$ and $\tilde{\rm f}^{\alpha\beta\gamma} \;=\; i{\rm f}^{\alpha\beta\gamma}$ otherwise. Let us also note that the indices α are raised and lowered using the open string metric

$$G^{\alpha\,\alpha'} \;=\; \frac{2}{k(\alpha)}\,\delta^{\alpha\,\alpha'}\;. \tag{4.3}$$

Here, the function $k(\alpha)$ has been introduced such that it takes the value k_r (or $k'_{r'}$) if α refers to a basis element in the Lie-algebra $\mathfrak{g}_r$ (or $\mathfrak{h}_{r'}$).

We use effective action S^{WZW} as a master theory from which we descend to the effective description of branes in coset conformal field theory. For the reduction it is convenient to switch to a new basis of $\mathfrak{g}\oplus\mathfrak{h}$ which makes reference to the embedding $\mathfrak{h}\subset\mathfrak{g}$. We shall employ $a = 1,\dots,\dim\mathfrak{g}-\dim\mathfrak{h}$ when we label directions perpendicular to $\mathfrak{h}\subset\mathfrak{g}$ while labels $i = 1,\dots,\dim\mathfrak{h}$ and $\tilde{\imath} = 1,\dots,\dim\mathfrak{h}$ stand for directions along $\mathfrak{h}\subset\mathfrak{g}$ and $\mathfrak{h}$, respectively.

The idea is now to perform a reduction of the theory (4.2) by imposing the following constraints

$$\begin{aligned} {\rm A}_i \;=\; {\rm A}_{\tilde{\imath}} \;=\; 0 \\ (i{\rm L}_i + {\rm L}_{\tilde{\imath}}){\rm A}_a + {\rm f}_{ia}^{b}{\rm A}_b \;=\; 0\;. \end{aligned} \tag{4.4}$$

These conditions allow to rewrite the effective action in the form

$$\mathsf{S}_P({\rm A}) \;=\; \frac{1}{4}{\rm tr}\;({\rm F}_{ab}{\rm F}^{ab}) - \frac{i}{2}{\rm tr}\;({\rm f}^{\alpha\beta\gamma}{\rm CS}_{\alpha\beta\gamma}) \tag{4.5}$$

which, together with the constraints (4.4), determine the brane dynamics in coset models. The field strength F and the Chern-Simons form CS are defined as before in (2.7),(2.8), but with ${\rm A}_i, {\rm A}_{\tilde{\imath}}$ set to zero. Formulas (4.5,4.4) constitute the central result of this section.

4.2 Derivation of the action for $P = \sum P_L(L,0)$

The effective theory we proposed in the previous subsection can be derived from boundary conformal field theory. We want to explain this here at least for a restricted set of brane configurations $P = \sum P_L(L,0)$ in which the denominator labels are all set to zero, i.e. $L'_r = 0$. This implies that the state space of the configuration contains only sectors of the form $\mathcal{H}_{(l,0)}$ and it simplifies the discussion considerably. For the general case the reader is referred to Appendix B where we sketch the main ideas of the derivation.

Since the contribution of the denominator theory is trivial, we can restrict our attention to the numerator theory with chiral algebra $\widehat{\mathfrak{g}}$. This theory has a state space of the form

$$\mathcal{H}^P \;=\; \bigoplus \; P_L \; N^{\mathfrak{g};L}_{Ll} \; \mathcal{H}^{\mathfrak{g}}_l$$

where $\mathcal{H}_l$ denotes the state space of the sector l of the chiral algebra $\widehat{\mathfrak{g}}$. To get rid of excitations in the direction of $\mathfrak{h} \subset \mathfrak{g}$, we have to impose the conditions

$$J^i_n \, \psi \;=\; 0 \quad \text{for} \quad n \geq 0 \quad \text{and} \quad i = 1, \dots, \dim \mathfrak{h} \tag{4.6}$$

on $\psi \in \mathcal{H}^{\mathfrak{g}}_l$. The subspace of states that solves these constraints is given by

$$\bigoplus \; P_L \; N^{\mathfrak{g};L}_{Ll} \; \mathcal{H}_{(l,0)} \otimes |0\rangle^{\mathfrak{h}} \;\simeq\; \bigoplus P_L \; N^{\mathfrak{g};L}_{Ll} \; \mathcal{H}_{(l,0)} \;\subset\; \mathcal{H}^P \quad . \tag{4.7}$$

Omitting the vector $|0\rangle^{\mathfrak{h}}$ is justified here because it has vanishing conformal dimension and the operator product expansions of the associated identity field are all trivial. Hence, the isomorphism indicated by $\simeq$ is canonical, i.e. it preserves all the structure that we need to compute the effective theory. Obviously, our assumption $L'_r = 0$ is crucial at this point.

These observations give a a good handle to compute the effective action for the coset model from the known effective action for the G WZW-model. All we have to do is to implement the restrictions (4.6) described above directly on the fields A_α of the effective theory. This gives

$$\mathrm{A}^i \;=\; 2i\alpha'\mathrm{f}^{i\gamma\alpha}\mathrm{L}_\gamma\mathrm{A}_\alpha \qquad \text{for all} \quad i \tag{4.8}$$

$$i\mathrm{L}_i\mathrm{A}_\beta + \mathrm{f}_{i\beta}{}^{\alpha}\mathrm{A}_\alpha \;=\; 0 \qquad \text{for all} \quad i,\ \beta \tag{4.9}$$

where the first constraint follows from eq. (4.6) for $n > 0$ and the second constraint is obtained with $n = 0$. By using (4.8) and (4.9) for $\beta = j$, we can express A_i through A_a. Thus we eliminate A_i from the action and are left with an action only containing A_a subjected to the condition

$$i\mathrm{L}_i\mathrm{A}_b + \mathrm{f}_{ib}{}^{a}\mathrm{A}_a \;=\; 0 \quad \text{for all} \quad i, b\ . \tag{4.10}$$

Furthermore it turns out that the terms in the action coming from the A_i are strongly suppressed against other terms. This is suggested already by the appearance of α' in equation (4.8) and it allows us to neglect these terms in the action to leading order so that $A_i = 0$. The resulting theory agrees with the prescription given in Subsection 4.1.

5 Solutions and Condensation Processes

Having found the effective theory (4.5, 4.4) of coset branes we shall now proceed to discuss a large class of solutions and their interpretation as condensation processes. In [11] we reported on the results from the reduction procedure in the parafermion case, SU(2)/U(1). This section will be a generalization to arbitrary fixed point free coset models.

5.1 Solutions

To obtain the equations of motion we vary the action (4.5) under the constraint (4.4). It is easy to see that the variation vanishes away from the configurations fulfilling the constraints so the resulting equations are the same as in the unconstrained problem,

$$\mathrm{L}^\alpha\mathrm{F}_{\alpha b} + [\mathrm{A}^a, \mathrm{F}_{ab}] \;=\; 0\ . \tag{5.1}$$

Together with the constraints (4.4), eqs. (5.1) determine the dynamics.

Consider a configuration $P = \sum_{L,L'} P_{L\,L'}(L, L')$. The fields A_a are matrices on which we can act with derivatives L_α. These can be expressed through commutators with the matrices $P_\alpha = P(t_\alpha)$ given by the corresponding representation P. Suppose now that we found a decomposition $P_\alpha = Q_\alpha - Q'_\alpha$ [3] such that Q is a representation of $\mathfrak{g} \oplus \mathfrak{h}$,

$$
\begin{aligned}
[\, Q_\alpha \,,\, Q_\beta \,] \;&=\; i\, \mathrm{f}_{\alpha\beta}{}^{\gamma}\, Q_\gamma \;, \\
\text{and} \qquad Q'_i \,+\, Q'_{\tilde{\imath}} \;&=\; 0 \;.
\end{aligned}
$$

Then

$$
\mathrm{A}_a \;=\; Q_a - P_a \;=\; Q'_a \tag{5.2}
$$

is a solution fulfilling the equations of motion (5.1) and the constraints (4.4). This can be verified in a straightforward computation. Note that this construction generalizes the one that we sketched for the case of branes on group manifolds. As we will show in the next subsection also the interpretation of the solution is analogous: The solution describes a brane configuration given by the representation Q. The main difference is that for a non-trivial denominator, we are not free to choose any representation Q of $\mathfrak{g} \oplus \mathfrak{h}$ but have to satisfy the extra condition that the solution vanishes in the diagonal combination $Q'_i + Q'_{\tilde{\imath}}$. The latter becomes trivial for group manifolds since the set of directions i along the denominator is empty in this case.

Let us comment on the meaning of this extra condition which is equivalent to

$$
P_i \,+\, P_{\tilde{\imath}} \,=\, Q_i \,+\, Q_{\tilde{\imath}} \;,
$$

saying that P and Q are isomorphic as representations of the diagonally embedded $\mathfrak{h}_{\mathrm{diag}} \subset \mathfrak{g} \oplus \mathfrak{h}$. Using the identification of the solution with a condensation process we see that

[3] where Q' and Q are zero in 'small' directions, i.e. in directions corresponding to a small level

1. all processes we found leave the diagonal part $P|_{\mathfrak{h}_{\rm diag}}$ invariant
2. any two brane configurations P, Q satisfying $P|_{\mathfrak{h}_{\rm diag}} \sim Q|_{\mathfrak{h}_{\rm diag}}$ are connected through a condensation process.

It is finally worth noticing that there is a distinguished solution which exists for any configuration P, namely we can choose

$$Q'_a \;=\; -P_a\;,\; Q'_i = \;-P_i\;,\; Q'_{\tilde{\imath}} \;=\; P_i$$

The solution relates P to the following configuration Q

$$Q_a \;=\; 0\;,\; Q_i \;=\; 0\;,\; Q_{\tilde{\imath}} \;=\; P_i + P_{\tilde{\imath}}\;.$$

Since Q describes a superposition of $(0, L')$ branes, we have just shown that any brane configuration P can be related by a condensation process to a superposition of $(0, L')$-branes. This is in complete agreement with an investigation of closed string couplings in [16] for a family of Kazama-Suzuki coset models. There it was observed that the $(0, L')$ branes provide a basis for the lattice of Ramond-Ramond charges.

5.2 Interpretation of the solutions

The described solutions can be identified as a condensation process that leads either to or away from the initial brane configuration P. Let us reformulate the proposal for the configuration that is associated with the solution Q'. The set of matrices Q_α form a representation of $\mathfrak{g} \oplus \mathfrak{h}$. This representation can be decomposed into irreducible sub-representations $\bigoplus Q_{L\,L'} V_L \otimes V_{\bar{L}'}$. We now claim that this decomposition describes the brane configuration $\sum Q_{L\,L'}(L, L')$ we are looking for. Whether the process is a flow to or from this configuration depends on $\mathsf{S}(Q')$ being negative or positive, respectively. As evidence for this interpretation we will give here an analysis of D-brane tensions and of fluctuation spectra. What we will show can be summarized in the formula

$$\mathsf{S}_P(Q' + \mathrm{A}) \;=\; \mathsf{S}_P(Q') + \mathsf{S}_Q(\mathrm{A}) \quad \text{with} \quad \mathsf{S}_P(Q') \;=\; \ln \frac{g_Q}{g_P} \tag{5.3}$$

analogous to (2.10).

For the calculations it is useful to relate our reduced action to the unreduced WZW action (4.2). It can be shown that

$$\mathsf{S}_P(\mathrm{A}_a) = \mathsf{S}_P^{\mathrm{WZW}}(\mathrm{A}_a, \mathrm{A}_i = Q'_i, \mathrm{A}_{\tilde{\imath}} = iQ'_{\tilde{\imath}}) \tag{5.4}$$

for any $Q'_i = -Q'_{\tilde{\imath}}$ belonging to a solution (5.2) and for all A_a fulfilling the constraint (4.4). Note the appearance of a factor of i because of our conventions for the $\mathfrak{h}$-part.

Now let us expand our coset action for a solution Q'_a using the result for WZW models (2.10).

$$\begin{aligned}
\mathsf{S}_P(Q'_a + \mathrm{A}_a) &= \mathsf{S}_P^{\mathrm{WZW}}(Q'_a + \mathrm{A}_a\,,\, Q'_i\,,\, iQ'_{\tilde{\imath}}) \\
&= \mathsf{S}_P^{\mathrm{WZW}}(Q'_a + \mathrm{A}_a\,,\, Q'_i + \mathrm{A}_i\,,\, iQ'_{\tilde{\imath}} + \mathrm{A}_{\tilde{\imath}})|_{\mathrm{A}_i = \mathrm{A}_{\tilde{\imath}} = 0} \\
&= \mathsf{S}_P^{\mathrm{WZW}}(Q'_a\,,\, Q'_i\,,\, iQ'_{\tilde{\imath}}) \;+\; \mathsf{S}_Q^{\mathrm{WZW}}(\mathrm{A}_a\,,\, \mathrm{A}_i\,,\, \mathrm{A}_{\tilde{\imath}})|_{\mathrm{A}_i = \mathrm{A}_{\tilde{\imath}} = 0} \\
&= \mathsf{S}_P^{\mathrm{WZW}}(Q'_\alpha) \;+\; \mathsf{S}_Q(\mathrm{A}_a) \;\;.
\end{aligned}$$

This confirms our result that the fluctuations around the solution Q'_a are governed by the action corresponding to the brane configuration Q. To complete this argument we note that the constraint (4.4) for the A_a in the P-configuration is the same as in the Q configuration as

$$i\mathrm{L}_i + \mathrm{L}_{\tilde{\imath}} \;=\; i[P_i + P_{\tilde{\imath}},\; \cdot\;] \;=\; i[Q_i + Q_{\tilde{\imath}},\; \cdot\;]\;.$$

In the remaining part of this section we will show that the D-brane tensions are reproduced correctly by our solution, i.e.

$$\ln\frac{\sum Q_{L\,L'}g_{L,L'}}{\sum P_{L\,L'}g_{L,L'}} \;=\; \mathsf{S}(Q'_a)\;. \tag{5.5}$$

in some order of $1/k$. The g-factors are defined by

$$g_{L,L'} \;=\; \frac{S_{(L,L')(0,0)}}{\sqrt{S_{(0,0)(0,0)}}} \tag{5.6}$$

with the help of the coset S-matrices. The coset S-matrices are up to some constant factor just products of S-matrices of the involved affine Lie algebras in the numerator and the denominator, therefore

$$\frac{\sum Q_{L\,L'} g_{L,L'}}{\sum P_{L\,L'} g_{L,L'}} \;=\; \frac{\sum Q_{L\,L'} \prod S^{r}_{L_r 0} \prod S^{r'}_{L'_{r'} 0}}{\sum P_{L\,L'} \prod S^{r}_{L_r 0} \prod S^{r'}_{L'_{r'} 0}} \quad . \tag{5.7}$$

As we are performing a perturbative analysis in $1/k$ we need asymptotic expressions for S-matrices. Using expressions from [44] we find

$$S_{l0} \;=\; N(k)\, \dim(l) \Big(1 - \frac{\pi^2}{6(k+g^\vee)^2} g^\vee C_l + \mathcal{O}\big(\frac{1}{k^4}\big)\Big) \tag{5.8}$$

where $N(k)$ is some factor independent of l, $C_l = (l, l+2\rho)$ is the quadratic Casimir, and $g^\vee$ is the dual Coxeter number. We insert this expression into (5.7) and obtain

$$\begin{aligned}
\frac{\sum Q_{L\,L'} g_{L,L'}}{\sum P_{L\,L'} g_{L,L'}} \;=\; & \frac{\sum Q_{L\,L'} \dim{}_{L,L'}}{\sum P_{L\,L'} \dim{}_{L,L'}} \\
& - \frac{\pi^2}{6} \frac{1}{\sum Q_{L\,L'} \dim{}_{L,L'}} \sum Q_{L\,L'} \dim{}_{L,L'} \Big[\sum_r \frac{g^\vee_r C^r_{L_r}}{(k_r + g^\vee_r)^2} + \sum_{r'} \frac{g^\vee_{r'} C^{r'}_{L'_{r'}}}{(k_{r'} + g^\vee_{r'})^2} \Big] \\
& + \frac{\pi^2}{6} \frac{1}{\sum P_{L\,L'} \dim{}_{L,L'}} \sum P_{L\,L'} \dim{}_{L,L'} \Big[\sum_r \frac{g^\vee_r C^r_{L_r}}{(k_r + g^\vee_r)^2} + \sum_{r'} \frac{g^\vee_{r'} C^{r'}_{L'_{r'}}}{(k_{r'} + g^\vee_{r'})^2} \Big] + \mathcal{O}\big(\frac{1}{k^4}\big) \ .
\end{aligned}$$

We now want to check the condition (5.5) for our proposed interpretation. The value of the action is

$$\begin{aligned}
\mathsf{S}_P(Q'_a) \;&=\; \mathsf{S}^{\rm WZW}_P(Q'_\alpha) \\
&=\; \frac{1}{12} \mathrm{f}_{\alpha\beta}{}^{\gamma} \mathrm{f}^{\alpha\beta\delta} \mathrm{tr}\, (P_\gamma P_\delta - Q_\gamma Q_\delta) \quad .
\end{aligned}$$

Remembering that indices are raised and lowered with the help of the k-dependent open string metric (4.3), we can see that this result is of order $1/k^2$. Since in our case we have

$$\sum P_{L\,L'} \dim\,[L, L'] \;=\; \sum Q_{L\,L'} \dim\,[L, L'] \ ,$$

the left hand side of (5.5) is also of order $1/k^2$. It is then straightforward to show that indeed (5.5) is fulfilled to this order.

Let us briefly mention that it may happen that the action vanishes in the order $1/k^2$. This is the case if all 'large' directions [4] that are used in the construction of the solution are divided out. We will encounter such a case in the example of the minimal models. However, it can be shown that in this case the relation (5.5) is fulfilled also in the order $1/k^3$.

6 Examples: Parafermions and minimal models

In this final section we want to illustrate our very general results in three simple examples. It will become clear that the solutions we have constructed above are capable of describing brane processes with very different geometrical manifestations. In the case of minimal models we will recover the processes found in [24] among our solutions and we shall now provide a very nice geometrical picture for them.

6.1 Parafermions

Let us start by reviewing the construction of parafermion theories as $\widehat{\mathrm{su}}(2)_k/\widehat{\mathrm{u}}(1)_k$ cosets. The free bosonic U(1) theory is embedded such that its current gets identified with the component J^3 of the SU(2) current.

The numerator theory has sectors $\mathcal{H}_l^{\mathrm{su}(2)}$ where $l = 0, 1, \ldots, k$, the sectors $\mathcal{H}_m^{\mathrm{u}}$ of the denominator algebra $\widehat{\mathrm{u}}(1)_k$ carry a label $m = -k+1, \ldots, k$. We can label the sectors $\mathcal{H}_{(l,m)}$ of the coset model by pairs (l, m) of numerator and denominator levels. The possible pairs (l, m) are restricted by a selection rule forcing the sum $l + m$ to be even. Furthermore some pairs label the same sector so that we have to identify the pairs $(l, m) \sim (k - l, m + k)$. Here we take the label m to be $2k$-periodic. Note that this field identification has no fixed points.

[4]by large directions we mean those which belong to a large level k_r

Now we want to apply our general formalism to formulate the effective action for the parafermion branes. Let us illustrate here only the case where the branes have trivial label in the denominator part. We start with the effective action for the $\widehat{\mathrm{su}}(2)$-WZW model involving three fields $\mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3$. Our brane configuration $P = \sum P_{L\,0}[L, 0]$ determines the derivatives $\mathrm{L}_\alpha = [P(t_\alpha),\ \cdot\ ]$. The constraint (4.4) reads in the parafermion case

$$i\mathrm{L}_3 \mathrm{A}_a + \mathrm{f}_{3a}{}^b \mathrm{A}_b \ = \ 0 \quad a, b = 1, 2 \quad , \tag{6.1}$$

A_3 is set to zero. Eventually we arrive at the effective action for the coset theory. The result is

$$\mathsf{S}(\mathrm{A}_1, \mathrm{A}_2) \ = \ \frac{1}{4} \mathrm{tr}\ (\hat{\mathrm{F}}_{ab} \hat{\mathrm{F}}^{ab}) \tag{6.2}$$

where $a = 1, 2$ and $\hat{\mathrm{F}}_{ab} = i\mathrm{L}_a \mathrm{A}_b - i\mathrm{L}_b \mathrm{A}_a + i[\mathrm{A}_a, \mathrm{A}_b]$. Obviously, there is no Chern-Simons like term in this case simply for dimensional reasons.

Let us now analyze the effective theory on a single $(L, 0)$ brane. For $L > 0$ we find a solution of the form described in 5.1 given by the following non-constant field

$$\mathrm{A}_a \ = \ -P_a \ = \ -P(t_a) \tag{6.3}$$

which is rather easy to check here for the parafermions.

If we insert this solution into the action (6.2) we find a positive value, indicating that the brane is the decay product of some configuration with a higher mass. This configuration is a chain of adjacent branes

$$(0, -L) + (0, -L + 2) + \cdots + (0, L) \tag{6.4}$$

as can be deduced by the rules of Section 5.2. In the language of Section 5.1 our solution has $Q_\alpha = 0, Q_{\tilde{3}} = P_3$. The decomposition of this representation of $\mathrm{su}(2) \oplus \mathrm{u}(1)$ gives precisely the stated result (6.4).

In the parafermion theory we have an additional $\mathbb{Z}_k$-symmetry, the branes $(L, 0)$ and (L, M) behave in the same way. Thus we can generalize the identified processes to

$$(0, M-L) + (0, M-L+2) + \cdots + (0, M+L) \longrightarrow (L, M) \quad . \tag{6.5}$$

We observe that all branes can be constructed out of a fundamental set of $(0, M)$-branes.

6.2 N=2 Minimal models

Our results can easily be extended to the $N = 2$ supersymmetric minimal models. The latter are obtained as $\widehat{\mathrm{su}}(2)_k \oplus \widehat{\mathrm{u}}(1)_2 / \widehat{\mathrm{u}}(1)_{k+2}$ coset theories. Now we need three integers (l, m, s) to label sectors, where $l = 0, \ldots, k$, $m = -k-1, \ldots, k+2$ and $s = -1, 0, 1, 2$ are subjected to the selection rule $l + m + s =$ even. Maximally symmetric branes are labeled by triples (L, M, S) from the same set. We shall restrict our attention to the cases with $S = 0$.

The U(1) factor in the numerator contributes an additional field X which enters the effective action (6.2) minimally coupled to the gauge fields $\mathrm{A}_a, a = 1, 2$. The solution (6.3) carries over to the new theory if we set X$= 0$ and its interpretation is the same as in the parafermion case since the perturbation does not act in the $\widehat{\mathrm{u}}(1)_2$ part. It means once more that a chain of P adjacent (L=0)-branes decays into a single (L=P−1)-brane. This process admits for a very suggestive pictorial presentation. Using the geometric setting described in Section 3, we find the target space of the $N = 2$ minimal models as a disc with $k + 2$ equidistant punctures at the boundary. This was first described in [12]. Let us label the punctures by a $k + 2$-periodic integer $q = 0, \ldots, k+1$. A brane (L, M) is then represented through a straight line stretching between the points $q_1 = M - L - 1$ and $q_2 = M + L + 1$. In the described process, a chain of branes, each of minimal length, decays to a brane forming a straight line between the ends of the chain (see Fig. 1). In [45] similar pictures occur in a geometric description using the realization of $N = 2$ minimal models as Landau-Ginzburg models.

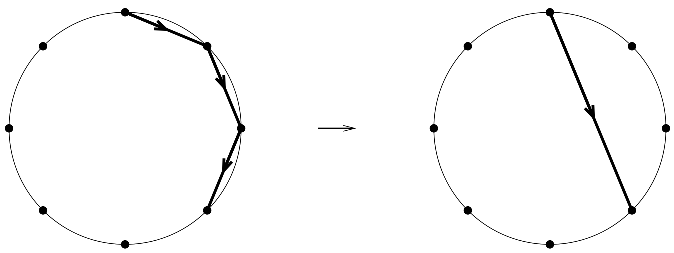

Figure 1: The chain (6.4) of branes can decay into a single brane (L, M).

In Figure 1 we have tacitly assumed that the processes we identified in the large k regime persist to finite values of k. For branes on SU(2), analogous results were described in [17]. Similar systematic investigations in case of other CFT backgrounds can be performed [46]. In any case, the results of [25, 26] and the comparison with exact studies (see e.g. [22]) in particular models display a remarkable stability of the RG flows as we move away from the decoupling limit.

6.3 Minimal models

The minimal models are constructed as a $\widehat{\text{su}}(2)_k \oplus \widehat{\text{su}}(2)_1 / \widehat{\text{su}}(2)_{k+1}$ coset. The embedding of the denominator theory is diagonal. The sectors of the numerator theory are labeled by two integers (l, s) where $l = 0 \ldots k, s = 0, 1$. Together with a label $l' = 0 \ldots k+1$ from the denominator we label the sectors of the coset model by triples (l, s, l'). From the coset construction we find the selection rule that $l + s + l'$ has to be even and the field identification $(l, s, l') \sim (k - l, 1 - s, k + 1 - l')$. Because of the selection rule, s is determined by fixing l and l' so that we can label the sectors by pairs (l, l').

Now we want to formulate the effective action using our general formalism. Let us again start with a configuration of branes (L, L') that have trivial label in the denominator su(2), $L' = 0$. On such a configuration we have six fields $\mathrm{A}_a, \mathrm{B}_a$ corresponding to directions in the first and the second $\widehat{\text{su}}(2)$-part of the numerator respectively. The action governing the dynamics of these fields is constructed as in Section 4.1. The constraints (4.4) translate into

$$\mathrm{B}_a \;=\; -\mathrm{A}_a - i\mathrm{f}_{ab}{}^c\mathrm{L}_c\mathrm{A}^b \quad \text{for all} \quad a \tag{6.6}$$

and

$$i\mathrm{L}_a\mathrm{A}_b + \mathrm{f}_{ab}{}^c\mathrm{A}_c \;=\; 0 \quad \text{for all} \quad a, c\ . \tag{6.7}$$

By the first of these relations we can eliminate B_a from the action. The action is expanded according to powers of the level k. As leading terms we find

$$\mathsf{S}(\mathrm{A}) \;=\; -\frac{1}{2k}\mathrm{tr}\ (\mathrm{L}_a\mathrm{A}_b\mathrm{L}^a\mathrm{A}^b) - \frac{2i}{3k}\mathrm{f}^{abc}\mathrm{tr}\ (\mathrm{A}_a\mathrm{A}_b\mathrm{A}_c)\ . \tag{6.8}$$

Taking the k-dependent metric into account, we note that the action is of order $1/k^3$. With the help of (6.7) the derivatives can be eliminated and we get

$$\mathsf{S}(\mathrm{A}) \;=\; \frac{2}{k^2}\mathrm{tr}\ (\mathrm{A}_a\mathrm{A}^a) - \frac{2i}{3k}\mathrm{f}^{abc}\mathrm{tr}\ (\mathrm{A}_a\mathrm{A}_b\mathrm{A}_c)\ . \tag{6.9}$$

To find solutions we have to find an extremum of this action where the fields have to fulfill (6.7). Applying our general results to this example we see that we have to look for solutions $\mathrm{A}_a = Q'_a$ where the Q'_a commute with the Q_a and where $-Q' = S$ is a representation of su(2).

Let us go into an example by considering a single $(L, 0)$ brane, $L > 0$. In this case the L+1-dimensional representation $S_a = P_a$ is the only possibility for S. We can easily calculate the value of the action for this solution (after proper normalization, more details about normalization can be found in [7]) and obtain

$$\mathsf{S}(-P) \;=\; \frac{\pi^2}{3k^3}L(L+2) > 0\ . \tag{6.10}$$

The solution describes the flow from a different brane configuration with higher mass to the $(L, 0)$-brane. From our general rules we can identify this configuration as a single $(0, L)$-brane. Thus, we observe here the decay process $(0, L) \longrightarrow (L, 0)$ which coincides precisely with the results of [24].

This process gives us the possibility to determine the effective action of the $(0, L)$-brane by considering the fluctuation spectrum,

$$\mathsf{S}_{(0,L)}(\mathrm{A}) = \mathsf{S}_{(L,0)}(-P + \mathrm{A}) - \mathsf{S}_{(L,0)}(-P) \tag{6.11}$$

$$= +\frac{1}{2k}\mathrm{tr}\ (\mathrm{L}_a \mathrm{A}_b \mathrm{L}^a \mathrm{A}^b) - \frac{2i}{3k}\mathrm{f}^{abc}\mathrm{tr}\ (\mathrm{A}_a \mathrm{A}_b \mathrm{A}_c)\ , \tag{6.12}$$

which looks the same as the action for the $(L, 0)$-brane except the change of sign in front of the kinetic term. The constraint on the fields (6.7) does not change. This is the expected result since the kinetic term comes now from the $\mathfrak{h}$-part and therefore comes with a different sign.

Our next example will be a configuration of one $(L, 0)$-brane I and one $(L + 2, 0)$ brane II. The fields A_a are then described by quadratic matrices of size $2L + 4$ which we can understand as consisting of four blocks I-I, I-II, II-I, II-II where the block I-I describes modes of strings with both ends on brane I and so on.

$$\mathrm{A} = \left(\begin{array}{|c|c|} \hline \text{I-I} & \text{I-II} \\ \hline \text{II-I} & \text{II-II} \\ \hline \end{array}\right) \begin{array}{l} \} \, L+1 \\ \} \, L+3 \end{array} \tag{6.13}$$

The matrices P which implement the derivatives can be decomposed in $P = P^I + P^{II}$ where P^I has entries only in the I-I block and P^{II} only in the II-II block.

Besides the solutions $-P^I$ and $-P^{II}$ we find two more coming from a 2-dimensional and an $L + 2$-dimensional representation, $-S^2$ and $-S^{L+2}$. This is easily understood because these are just the representations appearing as tensor factors in the sum of representations,

$$[L + 1] \oplus [L + 3] \simeq [2] \otimes [L + 2]\ . \tag{6.14}$$

The value of the action is positive, so we deal here with a decay process to the $(L, 0), (L+2, 0)$ system. By using the general interpretation of Section 5.2 we can immediately deduce the starting configuration: For the solution by the 2-dimensional representation it is the brane $(L+1, 1)$, for the other solution it is the brane $(1, L+1)$.

The described analysis of brane processes carries over to more general brane configurations. We find that any (L, L')-brane finally decays into a configuration with trivial denominator labels,

$$(L, L') \longrightarrow (\,|L-L'|, 0) + (\,|L-L'|+2, 0) + \cdots + (L+L', 0)\ . \tag{6.15}$$

All branes with nontrivial label from the denominator part are unstable and decay into configurations of branes with trivial denominator part. Which branes appear in the decay product is determined by the rules of how a tensor product of representations is decomposed into irreducible representations. These are exactly the processes described in [24]. But our analysis shows more, namely that any two configurations $\sum P_{L\,L'}(L, L')$ and $\sum Q_{L\,L'}(L, L')$ are connected by a process if

$$\sum P_{L\,L'}\, L \otimes L' \sim \sum Q_{L\,L'}\, L \otimes L'\ .$$

For example, any brane (L, L') can be constructed as condensate from L=0-branes,

$$(0,\, |L-L'|\,) + (0,\, |L-L'|+2) + \cdots + (0, L+L') \longrightarrow (L, L')\ .$$

Recently there has been a study of RG flows in minimal models [27] extending the work of [24]. All fixed points discovered there by a thorough CFT-investigation can also be found from our general coset analysis.

We can use our insights on the geometry from Section 3 to visualize the processes. The target space of minimal models is a cylinder where the ends are squeezed to a line (see Fig. 2). The simple $(L, 0)$-branes are point-like branes sitting at the top or at the bottom depending on L being odd or even. The value of L varying between 0 and k determines the position along the cylinder (see Fig. 2). The generic branes are

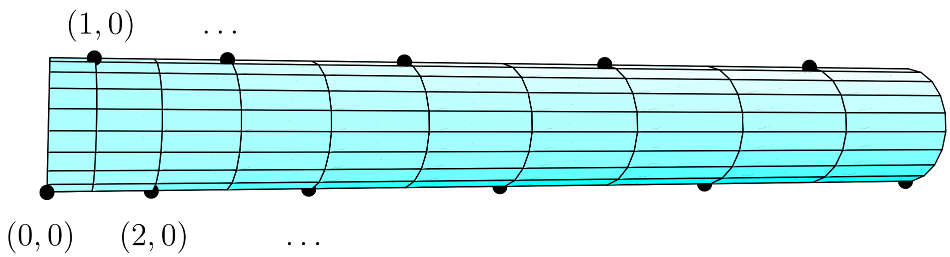

Figure 2: Geometric interpretation: The picture shows the underlying geometry of the minimal models together with the possible point-like branes of the form $[L, 0]$ sitting at the top and at the bottom of a cylinder with squeezed ends. The right end of the cylinder is cut.

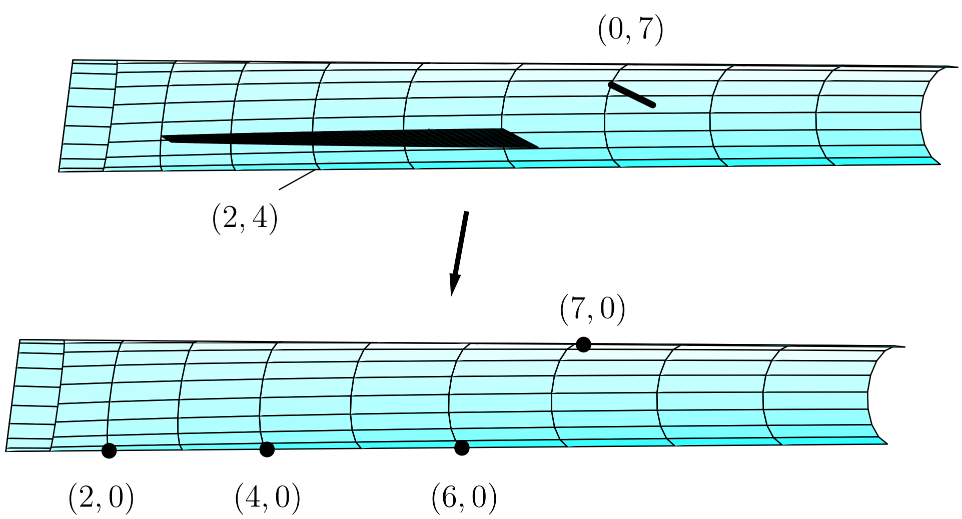

Figure 3: Processes in the minimal model geometry of Fig. 2 with removed front wall. Two processes are shown: (a) A one-dimensional string-like brane $(0, 7)$ decays into one point-like brane at the top. (b) A two-dimensional brane $(2, 4)$ decays into a configuration of point-like branes at the bottom.

two-dimensional planar branes (L, L'). The extension along the cylinder is between $|L - L'|$ and $L + L'$, the vertical position is given by L'. If L is zero these branes degenerate to string-like branes of type $(0, L')$. Only the point-like branes are stable, the other types of branes decay into configurations of point-like branes as is illustrated in Fig. 3. This is reminiscent of the phenomena that were observed in the study of tachyon condensation (see e.g. [47, 48, 49, 50, 51]).

7 Conclusion

In this work a rather general picture of condensation processes in a certain limiting regime of coset models has been developed. We have managed to show that two brane configurations P and Q on a brane are related by some flow if the restrictions of the corresponding representations to the diagonally embedded $\mathfrak{h} \subset \mathfrak{g} \oplus \mathfrak{h}$ are equivalent. This shows that the conserved charges must take values in the representation ring of the denominator or in some quotient thereof in case there are further processes. Our present work can be regarded as a generalization of previous work in conformal field theory to more general brane configurations and a large class of coset theories. The use of non-commutative gauge theories made it possible to keep track of the large number of boundary couplings.

It is of obvious interest to go beyond the limit in which some of the levels are sent to infinity and to study the pattern of flows for finite values of the level, i.e. deep in the stringy regime. In case of string theory on group manifolds such an extension can be performed with the help of the 'absorption of the boundary spin'- principle that was formulated by Affleck and Ludwig [52, 53]. We will propose an appropriate generalization of this idea in a forthcoming publication [46]. It is interesting to remark that coset models typically possess brane processes at finite k which cannot be seen in the limiting regime (see [24] for an example in unitary minimal models), i.e. these condensation processes are not deformations of a process one can study in the 'geometric regime'.

An obvious extension of our analysis is to go beyond the Cardy case and to incorporate e.g. boundary theories that are obtained from branes localized along the twined conjugacy classes on the group manifold G [6]. The latter arise when we glue left- and right moving currents of the WZW-model for the group G with some automorphism Ω of $\mathfrak{g}$,

$$J^\alpha(z) \;=\; \Omega(\bar{J}^\alpha)(\bar{z}) \quad \text{for} \quad \alpha = 1, \dots, D = \dim G \;\; . \tag{7.1}$$

The associated branes have been shown to be localized along the following twined conjugacy classes

$$C_g^{G;\omega} \;=\; \{\, g' \in G \mid g' = ug\omega(u^{-1}) \text{ for } u \in G \,\} \;\; .$$

Here, ω denotes the automorphism of the group G that comes with Ω. These twined branes on group manifolds descend to the coset G/H provided that ω can be restricted to the subgroup $H \subset G$. In the latter case it induces an automorphism on H and we can construct the corresponding twined conjugacy classes $C_h^{H;\omega}$. The induced branes of the coset model are localized along

$$C_{(g,h)}^{G/H;\omega} \;=\; \pi^G_{G/H}\left(C_g^{G;\omega}\; (C_h^{H;\omega})^{-1} \right) \;\subset\; G/H \;\; .$$

To show that this prescription is consistent one has to show that the adjoint action of H on G leaves the space $C_g^{G;\omega}\; (C_h^{H;\omega})^{-1}$ invariant. The dynamics of such twined branes in coset models can be studied once more by a reduction from the theory of twined branes on group manifolds. The latter was constructed recently in [54].

Acknowledgments: We would like to thank I. Brunner, M. Gaberdiel, K. Gawedzki, K. Graham, T. Quella, A. Recknagel and N. Reis for stimulating and very useful discussions.

A On brane geometry in coset models

In this appendix we plan to derive the geometric interpretation (3.6) of D-branes in coset models that was found in [13] from the 1-point functions (3.4). The main idea is borrowed from an analogous discussion of brane geometry on group manifolds [6] and it generalizes the constructions of [12].

It is useful to recall very briefly how one decodes the brane geometry on group manifolds (2.4) for the 1-point functions (2.3). To begin with one needs to set up a correspondence between functions on the group and the bulk fields whose conformal dimension vanishes when we send k to infinity, i.e. the fields listed in (2.2). This correspondence is obvious since the space $Fun(G)$ of functions on the group G is spanned by the matrix elements $D^l_{nm}(g)$ of irreducible representations where l runs through the set $\mathcal{J}^{\mathfrak{g}}$. With such a relation between bulk fields and functions in mind, the formula (2.3) suggests to introduce a set of functions $T^{\mathfrak{g}}_L : G \to \mathbb{C}$,

$$T^{\mathfrak{g}}_L(g) \ := \ \sum_{l\in\mathcal{J}_k} \frac{S^{\mathfrak{g}}_{L\,l}}{\sqrt{S^{\mathfrak{g}}_{0\,l}}}\, \delta_{nm}\, D^l_{nm}(g)$$

in which the basis element $D^l_{nm} \in Fun(G)$ is weighted with the strength of the coupling of the associated closed string mode to the brane L. The function $T^{\mathfrak{g}}_L$ can be shown to possess a peak along a conjugacy class C^G_L of G [6]. This confirms the geometrical interpretation of the gluing condition (2.1) uncovered in [2].

After this preparation we want to turn to the case of branes in a coset G/H. Now we need to find a correspondence between bulk fields from the set $\mathcal{J}^r$ and a set of functions on the coset space G/H where the denominator H acts on G by conjugation. To construct such functions, we rewrite G/H as a coset of the form $G \times H/H \times H$ in which the two factors H in the denominator act by left and right multiplication on $G \times H$, respectively,

$$u_l(g,h) \ = \ (ug,uh) \quad , \quad v_r(g,h) \ = \ (gv^{-1},hv^{-1})$$

for all $u, v \in H$ and $(g, h) \in G \times H$. The equivalence of the two coset constructions is based on the equality $G \times H/H = G$ and it uses the fact that after dividing out one copy of H from $G \times H$, the second factor H acts by conjugation on G rather than by left- or right translation.

Our aim now is to argue that there exists a correspondence between the coset fields labeled by $\mathcal{J}^r$ and functions on $G \times H/H \times H$, i.e. $H \times H$-invariant functions on $G \times H$. To this end, let us note that such invariant functions are obtained by averaging elements of $Fun(G \times H)$ over the group $H \times H$ of translations, i.e.

$$\int_{H \times H} d\mu_{H\times H}(u,v)\ F(ugv^{-1}, uhv^{-1}) \ \ .$$

If we apply this averaging prescription to the basis $D^l_{nm}(g)\overline{D}^{l'}_{rs}(h)$ of $Fun(G \times H)$ we obtain a non-vanishing invariant function whenever the representation l' of the finite dimensional Lie algebra $\mathfrak{h}$ is contained in l. This means that for each bulk field labeled by elements from the set $\mathcal{J}^r$ there exists a function on $G \times H/H \times H$. We can now apply the same procedure as in the WZW case to read off the geometry of the branes in the coset theory, i.e. we define a function

$$\begin{aligned} T_{(L,L')} &= \sum_{(l,l')} \frac{S^{\mathfrak{g}}_{l\,L}\bar{S}^{\mathfrak{h}}_{l'\,L'}}{\sqrt{S^{\mathfrak{g}}_{0\,l}\bar{S}^{\mathfrak{h}}_{0l'}}} \int d\mu_{H\times H}(u,v) D^l_{mm}(ugv^{-1})\overline{D}^{l'}_{ss}(uhv^{-1}) \\ &= \int d\mu_{H\times H}(u,v) T^{\mathfrak{g}}_L(ugv^{-1})\overline{T}^{\mathfrak{h}}_{L'}(uhv^{-1}) \ \ . \end{aligned} \tag{A.1}$$

Since $T^{\mathfrak{g}}_L(g)\overline{T}^{\mathfrak{h}}_{L'}(h)$ is localized along the product $C^G_L \times C^H_{L'} \subset G \times H$, we have just shown that the coset brane is localized along the image of this product in the coset space $G \times H/H \times H$. Rephrased in terms of the more conventional coset G/H this means that the coset brane (L, L') is localized along the image of the space $C^G_L(C^H_{L'})^{-1}$, in agreement with eq. (3.6).

B Effective action for general branes

It this section we sketch the derivation of the effective action for a general coset brane configuration from conformal field theory. We begin by looking at the following product $\mathcal{H}^{\mathfrak{g}\ M}_{L} \otimes \mathcal{H}^{\mathfrak{h}\ \bar{M}'}_{\bar{L}'}$ of state spaces for boundary theories of the G and H WZW model. Within such a space we want to find the state space $\mathcal{H}^{(M,M')}_{(L,L')}$ of the coset theory. In a first step let us impose the constraints

$$J^i_n \, \psi \;=\; J^{\tilde{\imath}}_n \, \psi \;=\; 0 \qquad \text{for all} \;\; n > 0 \;, \tag{B.1}$$

and $i, \tilde{\imath}$ run through the usual range. This restricts us to the ground states for the actions of $\widehat{\mathfrak{h}} \subset \widehat{\mathfrak{g}}$ on the first factor and of $\widehat{\mathfrak{h}}$ on the second. With the help of eq. (3.5) we can conclude that the resulting subspace of states satisfying eqs. (B.1) has the form

$$\bigoplus_{l,m,n} N^{\mathfrak{g};M}_{L\,l}\, \mathcal{H}_{(l,m)} \otimes V^{\mathfrak{h}}_m \otimes N^{\mathfrak{h};\bar{M}'}_{\bar{L}'\,n} V^{\mathfrak{h}}_n \tag{B.2}$$

where $V^{\mathfrak{h}}_m$ denotes the space of ground states in $\mathcal{H}^{\mathfrak{h}}_m$ and we sum over all m such that (l, m) is a sector of the coset model. If we now require the additional invariance condition

$$(J^i_0 + \tilde{J}^i_0)\; \psi \;=\; 0 \tag{B.3}$$

then the only contribution in the sum will come from $m = \bar{n}$ and the invariant part of $V^{\mathfrak{h}}_m \otimes V^{\mathfrak{h}}_n$ is one-dimensional. This means that after imposing the two constraints (B.1,B.3), we are left with the space

$$\bigoplus_{l,m} N^{\mathfrak{g};M}_{L\,l}\, N^{\mathfrak{h};M'}_{L'\,m}\; \mathcal{H}_{(l,m)} \tag{B.4}$$

which is isomorphic to the state space $\mathcal{H}^{(M,M')}_{(L,L')}$ of the boundary coset model. In this way we have prepared states of the coset theory from states of the product of boundary WZW models.

Now we use the boundary operators of the WZW models to build boundary operators on the product space. These will then be shown to reduce to boundary fields of the coset theory when k is sent to infinity. The idea is to use fields of the form

$$J^a \, \Psi^{NL}_{(l,\nu)} \, \Psi^{\bar{N}'\bar{L}'}_{(l',\nu')} \, C^{ll'}_{a;\nu,\nu'} \ : \mathcal{H}^{\mathfrak{g}\,M}_{L} \otimes \mathcal{H}^{\mathfrak{h}\,\bar{M}'}_{\bar{L}'} \ \rightarrow \ \mathcal{H}^{\mathfrak{g}\,M}_{N} \otimes \mathcal{H}^{\mathfrak{h}\,\bar{M}'}_{\bar{N}'} \tag{B.5}$$

where ν, ν' label a basis in the representation spaces $V^l, V^{l'}$, respectively, and the coefficients C are chosen such that the operator is invariant under the obvious action of $\mathfrak{h}$. This choice of C guarantees that the operators respect the constraint (B.3). On the other hand, they are not compatible with our first set of constraints (B.1) simply because boundary primary fields usually map ground states into a linear combination which contains also excited states. But these excitations get suppressed for large values of the level so that the operators (B.5) do respect the conditions (B.1) in the limiting regime and hence they become the operators of the boundary theory at $k \to \infty$. This means that we have reduced the computation of 3^{rd} and 4^{th} order terms in our effective field theory to computations in the boundary WZW model for G and H. These calculations have been performed in [7] and they provide the corresponding terms in the action (4.2). But in the case of coset theories, we work only with a small subset of boundary fields from the WZW models which is specified by the constraints (B.1, B.3). They manifest themselves in the constraints (4.4) of the effective field theory.

It remains to discuss the quadratic terms in our effective action. These terms can be read off from the conformal dimensions. More precisely a mode (l, l') of the coset model contributes a quadratic term proportional to $h_{(l,l')}$. But in our construction of the theory from the two WZW models, (l, l') is accompanied by the field of weight $h_{l'}$ for the subalgebra $\widehat{\mathfrak{h}} \subset \widehat{\mathfrak{g}}$ and another field with the same weight being associated with the second WZW model. This would add up to $h_{(l,l')} + 2h_{l'} \neq h_{(l,l')}$. Our prescription to put an extra factor $\sqrt{-1}$ into the derivatives $\mathrm{L}_{\tilde{\imath}}$, accounts for the mismatch. This is due to the fact that the conformal weights are obtained from the quadratic Casimir which changes sign under the replacement $J \to iJ$. Hence, the extra factor i does produce the right quadratic terms $h_{(l,l')} + h_{l'} - h_{l'} = h_{(l,l')}$ in the effective action. It is easy to see that it does not change the higher order terms in the constrained model.

References

[1] C. Klimcik and P. Severa, *Open strings and D-branes in WZNW models*, Nucl. Phys. B **488** (1997) 653 [arXiv:hep-th/9609112].

[2] A. Y. Alekseev and V. Schomerus, *D-branes in the WZW model*, Phys. Rev. D **60** (1999) 061901 [arXiv:hep-th/9812193].

[3] K. Gawedzki, *Conformal field theory: A case study*, arXiv:hep-th/9904145.

[4] S. Stanciu, *D-branes in group manifolds*, JHEP **0001** (2000) 025 [arXiv:hep-th/9909163].

[5] A. Y. Alekseev, A. Recknagel and V. Schomerus, *Non-commutative world-volume geometries: Branes on SU(2) and fuzzy spheres*, JHEP **9909** (1999) 023 [arXiv:hep-th/9908040].

[6] G. Felder, J. Fröhlich, J. Fuchs and C. Schweigert, *The geometry of WZW branes*, J. Geom. Phys. **34** (2000) 162 [arXiv:hep-th/9909030].

[7] A. Y. Alekseev, A. Recknagel and V. Schomerus, *Brane dynamics in background fluxes and non-commutative geometry*, JHEP **0005** (2000) 010 [arXiv:hep-th/0003187].

[8] C. Bachas, M. R. Douglas and C. Schweigert, *Flux stabilization of D-branes*, JHEP **0005** (2000) 048 [arXiv:hep-th/0003037].

[9] J. Pawelczyk, *SU(2) WZW D-branes and their noncommutative geometry from DBI action*, JHEP **0008** (2000) 006 [arXiv:hep-th/0003057].

[10] S. Stanciu, *D-branes in Kazama-Suzuki models*, Nucl. Phys. B **526** (1998) 295 [arXiv:hep-th/9708166].

[11] S. Fredenhagen and V. Schomerus, *Brane dynamics in CFT backgrounds*, arXiv: hep-th/0104043.

[12] J. Maldacena, G. W. Moore and N. Seiberg, *Geometrical interpretation of D-branes in gauged WZW models*, JHEP **0107** (2001) 046 [arXiv:hep-th/0105038].

[13] K. Gawedzki, *Boundary WZW, G/H, G/G and CS theories*, arXiv:hep-th/0108044.

[14] S. Elitzur and G. Sarkissian, *D-branes on a gauged WZW model*, arXiv:hep-th/0108142.

[15] A. Y. Alekseev, A. Recknagel and V. Schomerus, *Open strings and noncommutative geometry of branes on group manifolds*, Mod. Phys. Lett. A **16** (2001) 325 [arXiv:hep-th/0104054].

[16] W. Lerche and J. Walcher, *Boundary rings and N = 2 coset models*, arXiv:hep-th/0011107.

[17] A. Alekseev and V. Schomerus, *RR charges of D2-branes in the WZW model*, arXiv:hep-th/0007096.

[18] S. Stanciu, *A note on D-branes in group manifolds: Flux quantization and D0-charge*, JHEP **0010** (2000) 015 [arXiv:hep-th/0006145].

[19] J. M. Figueroa-O'Farrill and S. Stanciu, *D-brane charge, flux quantization and relative (co)homology*, JHEP **0101** (2001) 006 [arXiv:hep-th/0008038].

[20] S. Fredenhagen and V. Schomerus, *Branes on group manifolds, gluon condensates, and twisted K-theory*, JHEP **0104** (2001) 007 [arXiv:hep-th/0012164].

[21] J. Maldacena, G. W. Moore and N. Seiberg, *D-brane instantons and K-theory charges*, arXiv:hep-th/0108100.

[22] F. Lesage, H. Saleur and P. Simonetti, *Boundary flows in minimal models*, Phys. Lett. B **427** (1998) 85 [arXiv:hep-th/9802061].

[23] P. Dorey, I. Runkel, R. Tateo and G. Watts, *g-function flow in perturbed boundary conformal field theories*, Nucl. Phys. B **578** (2000) 85 [arXiv:hep-th/9909216].

[24] A. Recknagel, D. Roggenkamp and V. Schomerus, *On relevant boundary perturbations of unitary minimal models*, Nucl. Phys. B **588** (2000) 552 [arXiv:hep-th/0003110].

[25] K. Graham, I. Runkel and G. M. Watts, *Minimal model boundary flows and c = 1 CFT*, Nucl. Phys. B **608** (2001) 527 [arXiv:hep-th/0101187].

[26] K. Graham, I. Runkel and G. M. Watts, *Boundary renormalisation group flows of minimal models*, published in Budapest 2000, Non-perturbative QFT methods and their applications 95-113.

[27] K. Graham, *On Perturbations of Unitary Minimal Models by Boundary Condition Changing Operators*, arXiv:hep-th/0111205.

[28] J. L. Cardy, *Boundary Conditions, Fusion Rules And The Verlinde Formula*, Nucl. Phys. B **324** (1989) 581.

[29] I. Runkel, *Boundary structure constants for the A-series Virasoro minimal models*, Nucl. Phys. B **549** (1999) 563 [arXiv:hep-th/9811178].

[30] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, *Classical p-branes from boundary state*, Nucl. Phys. B **507** (1997) 259 [arXiv:hep-th/9707068].

[31] G. Felder, J. Fröhlich, J. Fuchs and C. Schweigert, *Correlation functions and boundary conditions in RCFT and three-dimensional topology*, arXiv:hep-th/9912239.

[32] J. Hoppe, *Diffeomorphism Groups, Quantization And SU(Infinity)*, Int. J. Mod. Phys. A **4** (1989) 5235.

[33] J. Madore, *The Fuzzy sphere*, Class. Quant. Grav. **9** (1992) 69.

[34] H. Grosse, C. Klimcik and P. Presnajder, *Towards finite quantum field theory in noncommutative geometry*, Int. J. Theor. Phys. **35** (1996) 231 [arXiv:hep-th/9505175].

[35] H. Grosse, C. Klimcik and P. Presnajder, *Field theory on a supersymmetric lattice*, Commun. Math. Phys. **185** (1997) 155 [arXiv:hep-th/9507074].

[36] H. Grosse, C. Klimcik and P. Presnajder, *Simple field theoretical models on noncommutative manifolds*, arXiv:hep-th/9510177.

[37] U. Carow-Watamura and S. Watamura, *Noncommutative geometry and gauge theory on fuzzy sphere*, Commun. Math. Phys. **212** (2000) 395 [arXiv:hep-th/9801195].

[38] C. Klimcik, *A nonperturbative regularization of the supersymmetric Schwinger model*, Commun. Math. Phys. **206** (1999) 567 [arXiv:hep-th/9903112].

[39] J. Madore, *An Introduction to noncommutative differential geometry and its physical applications*, Cambridge University Press 1999

[40] I. Affleck and A. W. Ludwig, *Universal noninteger 'ground state degeneracy' in critical quantum systems*, Phys. Rev. Lett. **67** (1991) 161.

[41] K. Hashimoto and K. Krasnov, *D-brane solutions in non-commutative gauge theory on fuzzy sphere*, Phys. Rev. D **64** (2001) 046007 [arXiv:hep-th/0101145].

[42] Y. Hikida, M. Nozaki and Y. Sugawara, *Formation of spherical D2-brane from multiple D0-branes*, arXiv:hep-th/0101211.

[43] R. C. Myers, *Dielectric-branes*, JHEP **9912** (1999) 022 [arXiv:hep-th/9910053].

[44] P. Di Francesco, P. Mathieu and D. Senechal, *Conformal field theory*, *New York, USA: Springer (1997) 890 p.*

[45] K. Hori, A. Iqbal and C. Vafa, *D-branes and mirror symmetry*, arXiv:hep-th/0005247.

[46] S. Fredenhagen and V. Schomerus, *Boundary RG-flows in coset models*, in preparation

[47] A. Sen, *SO(32) spinors of type I and other solitons on brane-antibrane pair*, JHEP **9809** (1998) 023 [arXiv:hep-th/9808141].

[48] A. Sen, *Descent relations among bosonic D-branes*, Int. J. Mod. Phys. A **14** (1999) 4061 [arXiv:hep-th/9902105].

[49] A. Sen, *Non-BPS states and branes in string theory*, arXiv:hep-th/9904207.

[50] A. Recknagel and V. Schomerus, *Boundary deformation theory and moduli spaces of D-branes*, Nucl. Phys. B **545** (1999) 233 [arXiv:hep-th/9811237].

[51] J. A. Harvey, D. Kutasov and E. J. Martinec, *On the relevance of tachyons*, arXiv:hep-th/0003101.

[52] I. Affleck and A. W. Ludwig, *The Kondo effect, conformal field theory and fusion rules*, Nucl. Phys. B **352** (1991) 849.

[53] I. Affleck and A. W. Ludwig, *Critical theory of overscreened Kondo fixed points*, Nucl. Phys. B **360** (1991) 641.

[54] A.Yu. Alekseev, S. Fredenhagen, T. Quella and V. Schomerus, *Twisted D-branes*, in preparation