

Implications of Superconformal Symmetry for Interacting (2,0) Tensor Multiplets

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Abstract

We study the structure of the four-point correlation function of the lowest-dimension 1/2 BPS operators (stress-tensor multiplets) in the (2,0) six-dimensional theory.

We first discuss the superconformal Ward identities and the group-theoretical restrictions on the corresponding OPE. We show that the general solution of the Ward identities is expressed in terms of a single function of the two conformal cross-ratios (“prepotential”).

Using the maximally extended gauged seven-dimensional supergravity, we then compute the four-point amplitude in the supergravity approximation and identify the corresponding prepotential. We analyze the leading terms in the OPE by performing a conformal partial wave expansion and show that they are in agreement with the non-renormalization theorems following from representation theory.

The investigation of the (2,0) theory is carried out in close parallel with the familiar four-dimensional $\mathcal{N} = 4$ super-Yang-Mills theory.

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1 Introduction

One theory of particular interest which has emerged on the scene of the AdS/CFT duality [1]-[3] is the superconformal six-dimensional theory of $(2, 0)$ self-dual tensor multiplets. It is conjectured to describe the world-volume fluctuations of the M-theory five-branes and this explains the special rôle the $(2, 0)$ theory might play in the possible formulations of M-theory [4].

Our actual knowledge of the $(2, 0)$ theory is very limited, primarily due to the lack of a field theory formulation. This theory does not allow a dimensionless coupling and hence a perturbative approach. This is quite different from the superconformal $\mathcal{N} = 4$ Yang-Mills theory in four dimensions, where the coupling is merely a free parameter. On the other hand, the conformal superalgebras in $d = 4$ and $d = 6$ and their unitary irreducible representations (UIR) have a very similar structure. Moreover, both $\mathcal{N} = 4$ SYM with a gauge group $SU(N)$ and the $(2, 0)$ theory possess well-defined supergravity duals: In the large N limit and for a large t'Hooft coupling the former is dual to type IIB ten-dimensional supergravity compactified on $AdS_5 \times S^5$, while the latter is dual to eleven-dimensional supergravity compactified on $AdS_7 \times S^4$, provided the number of tensor multiplets grows like N^3 . Thus, representation theory confronts us with the problem to understand what precisely makes these two theories so different and whether they have something in common. The aim of the present paper is to provide a partial answer to this question. At the same time, we put the AdS/CFT correspondence to another non-trivial test.

We exploit two different but complementary approaches. The first is to study the operator product expansion (OPE) of two stress-tensor multiplets. These are the simplest non-trivial examples of the so-called $1/2$ BPS operators. Their conformal dimension is protected from quantum corrections by conformal supersymmetry, but their OPE has a rich spectrum of both protected and unprotected multiplets. In Section 2.1 we recall the known facts about this OPE both in $d = 4$ and in $d = 6$ and make a detailed comparison of the OPE spectra of the two theories. In particular, we point out the different realization of what one may call Konishi-like multiplets. In $d = 4$ these are represented by operators bilinear in the fundamental fields which have canonical dimension and satisfy conservation conditions in the free theory, but develop anomalous dimension in the presence of interaction. This is related to the fact that the corresponding superconformal UIRs lie at the unitarity bound of the *continuous* series of representations. At the same time, other bilinear operators, also at the unitarity bound of the continuous series, remain protected. This is due to the kinematics of the OPE, i.e., to the properties of the three-point functions which these operators may form with the two $1/2$ BPS operators.

In $d = 6$ the picture is quite different. There the operators at the unitarity bound of the continuous series of UIRs are trilinear and cannot appear in the OPE. The closest analogs of the Konishi-like bilinear operators belong to a *discrete* series of UIRs with quantized dimension.

Thus, they are automatically protected by unitarity. The lowest-dimension unprotected multiplets in this OPE correspond to UIRs lying above the unitarity bound of the continuous series and are realized by quadrilinear operators.

The second approach is based on the superconformal Ward identities for the four-point function of stress-tensor multiplets. In $d = 4$ they are known to restrict the freedom in the amplitude to just two functions of conformally invariant variables, one depending on two such variables and the other on one variable. An additional, dynamical mechanism which generates the quantum corrections to the amplitude by insertion of the SYM action, fixes the function of one variable at its free theory value (“partial non-renormalization”). Once again, the situation changes in $d = 6$. In Sections 2.2 and 3 we show that the analogous Ward identities are solved in terms of a single function of two variables, which we call “prepotential”. In other words, in $d = 6$ the “partial non-renormalization” is purely a kinematical effect. We consider this new phenomenon as an indication that there exists no smooth interpolation between the two fixed conformal points, the free one and the dual of the supergravity theory.

In the absence of a perturbative formulation of the (2,0) theory one can only test the above general predictions via the AdS/CFT correspondence. In Sections 2.3 and 4 we use the maximally extended gauged seven-dimensional supergravity to derive the corresponding four-point amplitude and verify that it satisfies the constraints found in Section 3. We provide an explicit simple formula for the six-dimensional gravity-induced prepotential and show how it is related to its four-dimensional analogue. Finally, in Section 5 we analyze the leading terms in the conformal partial wave expansion of the supergravity four-point amplitude and show that they are in complete agreement with the OPE structure discussed in Section 2.1. Some technical details are gathered in the appendices.

2 Overview and summary of the results

2.1 OPE of stress-tensor multiplets

One can learn a lot both about the kinematics and the dynamics of a (super)conformal theory by studying the OPE of various operators. In the context of the AdS/CFT correspondence the so-called 1/2 BPS short operators¹ are of particular interest since, on the one hand, their conformal dimension is quantized (“protected”) in the CFT, and on the other hand, they can be identified with the Kaluza-Klein excitations in the AdS supergravity spectrum [3, 5]. They correspond to states which are annihilated by half of the supercharges. The simplest example of a 1/2 BPS operator is the stress-tensor multiplet \mathcal{O}^I whose lowest component is a scalar of dimension $\ell = d - 2$ belonging to the vector representation of the R symmetry group $\text{SO}(6) \sim$

¹In the AdS/CFT literature the BPS operators are often called Chiral Primary Operators (CPO). This name does not seem very adequate in the (2,0) theory, where all spinors are chiral.

SU(4) or SO(5) \sim USp(4) in the cases $d = 4$ or $d = 6$, respectively.

Before discussing the OPE of 1/2 BPS operators, it is useful to recall some known facts [6] about the UIRs of the $d = 4$ $\mathcal{N} = 4$ and the $d = 6$ (2,0) superconformal algebras PSU(2,2/4) and OSp(8*/4), correspondingly. They are labeled by the quantum numbers of the lowest-weight state $\mathcal{D}(\ell; J_1, J_2; a_1, a_2, a_3)$ (for PSU(2,2/4)) or $\mathcal{D}(\ell; J_1, J_2, J_3; a_1, a_2)$ (for OSp(8*/4)). Here ℓ is the conformal dimension, J_i label the Lorentz group SO(3,1) \sim SL(2, \mathbb{C}) or SO(5,1) \sim SU*(4) irrep, and $[a_i]$ are the Dynkin labels of the SU(4) or USp(4) irrep, correspondingly. We will be interested in superconformal UIRs which can appear in the OPE of two 1/2 BPS operators; since the latter are Lorentz scalars, the former must be vector-like with “spin” s , i.e., with $J_1 = J_2 = s/2$ for $d = 4$ and $J_1 = J_3 = 0, J_2 = s$ for $d = 6$. Below we list the relevant UIRs: ²

$$\begin{aligned} & \text{PSU}(2,2/4), \quad J_1 = J_2 = s/2 : \\ & \text{A)} \quad \ell \geq 2 + s + a_1 + a_2 + a_3, \\ & \text{C)} \quad \ell = a_1 + a_2 + a_3, \quad s = 0; \end{aligned} \tag{1}$$

$$\begin{aligned} & \text{OSp}(8^*/4), \quad J_1 = J_3 = 0, \quad J_2 = s : \\ & \text{A)} \quad \ell \geq 6 + s + 2(a_1 + a_2), \\ & \text{B)} \quad \ell = 4 + s + 2(a_1 + a_2), \\ & \text{C)} \quad \ell = 2 + 2(a_1 + a_2), \quad s = 0, \\ & \text{D)} \quad \ell = 2(a_1 + a_2), \quad s = 0. \end{aligned} \tag{2}$$

In both cases series A is continuous whereas B, C and D are isolated and contain operators with “quantized” conformal dimension. Series C in $d = 4$ and D in $d = 6$ correspond to the BPS states. In particular, if only $a_2 \neq 0$ we obtain 1/2 BPS states. The $d = 4$ and $d = 6$ stress-tensors belong to the 1/2 BPS multiplets $\mathcal{D}(2; 00; 020)$ and $\mathcal{D}(4; 000; 02)$, respectively.

The OPE of two 1/2 BPS operators is restricted by the kinematics in the sense that it can contain only the operators whose quantum numbers $\mathcal{D}(\ell; J_m; a_n)$ allow them to form a non-vanishing three-point function with the two 1/2 BPS operators [7]. The case of interest for us is the OPE of two stress-tensor multiplets. Then the allowed R symmetry irreps are obtained by decomposing the tensor products

$$\text{SU}(4) : \quad [020] \times [020] = [040]_0 + [121]_0 + [202]_0 + [020]_1 + [101]_1 + [000]_2; \tag{3}$$

$$\text{USp}(4) : \quad [02] \times [02] = [04]_0 + [40]_0 + [22]_0 + [02]_1 + [20]_1 + [00]_2, \tag{4}$$

where the subscript indicates the value of the integer $k = 2 - \frac{1}{2} \sum a_i$. In ref. [7–11] it was shown that the existence of a non-vanishing three-point function for $k = 0, 1$ implies certain selection rules. In particular, the dimension of the operators appearing in the OPE becomes quantized.

²Series B of PSU(2,2/4) contains chiral superfields with $J_1 J_2 = 0$, so it is not relevant here.

No such selection rules are found for $k = 2$. More specifically, in terms of the classifications (1) and (2) the picture is as follows.

$\mathbf{k} = 0$:

The three irreps with $k = 0$ in (3) and (4) belong to the series C ($d = 4$) and D ($d = 6$), and therefore they are 1/2 or 1/4 BPS states. The corresponding operators are “protected”, i.e., their conformal dimension $\ell = 2(d - 2)$ cannot be modified by the interaction.

$\mathbf{k} = 1$:

In this case all the operators are protected as well. However, this time, besides 1/2 or 1/4 BPS short operators of dimension $\ell = d - 2$, we encounter a new species of protected operators, the so-called “semishort” operators (see, e.g., [12]). They have different interpretations in four and six dimensions.

In $d = 4$ the semishort operators with spin $s \geq 0$ lie at the unitarity bound $\ell = s + 4$ of the continuous series A. They satisfy conservation-like conditions in superspace (which imply the existence of conserved component tensors in the multiplet). For this reason they may also be called “current-like”. We stress that in $d = 4$ the semishort operators are *a priori* not protected, since their dimension can be continuously varied above the unitarity bound. However, the careful analysis of the corresponding three-point function [7–11] shows that the kinematics of the particular OPE under consideration protects the dimension of the semishort operators with $k = 1$, so that they remain at the unitarity bound even in the presence of interaction. A well-known example of a *protected semishort* operator is the so-called \mathcal{O}_{20}^4 corresponding to the UIR $\mathcal{D}(4; 00; 020)$, first discovered in refs. [13, 14].

In $d = 6$ the semishort operators with $k = 1$ correspond to UIRs from the isolated series B with quantized dimension $\ell = s + 8$ [9, 15]. Since their conformal dimension cannot be continuously modified, they are automatically protected by unitarity. In this respect they resemble the BPS short operators which belong to an isolated series of UIRs as well. Note that the existence of an isolated series of semishort operators is specific to the six-dimensional superconformal algebras $\text{OSp}(8^*/2\mathcal{N})$.

$\mathbf{k} = 2$:

This is the most interesting case since only it involves *unprotected* operators. As can be seen from (3) and (4), these are R symmetry singlets. Here the analysis of the three-point functions produces no further selection rules. Still, a particular type of operators can be singled out. Again, the situation is different in four and in six dimensions.

In $d = 4$ the operators with $k = 2$ lying at the unitarity bound $\ell = s + 2$ have twist $\ell - s = 2$. So, they correspond to bilinears made out of the free $\mathcal{N} = 4$ SYM field-strength superfields. Still in the free case, these bilinears satisfy conservation conditions which make them semishort. However, this conservation does not reflect any symmetry of the interacting theory, therefore such operators develop anomalous dimension and drift away from the unitarity bound. So, the

semishort operators with $k = 2$ are *unprotected*. The best-known example of this type is the Konishi multiplet (a singlet scalar of dimension 2), but there exists an infinite series of similar operators with spin which we call Konishi-like. Their anomalous dimension at one loop has been calculated in ref. [16, 14, 17–19].

It should be pointed out that the Konishi-like operators are not present in the OPE derived from gauged $\mathcal{N} = 8$ supergravity. This was demonstrated in ref. [13] by analyzing the supergravity four-point function of 1/2 BPS operators found in ref. [20]. A common lore to explain their absence in the strongly coupled $\mathcal{N} = 4$ theory is to say that they develop large anomalous dimension $\ell \sim (g_{YM}^2 N)^{1/4}$ as the t'Hooft coupling $g_{YM}^2 N$ tends to infinity, and thus they drop out of the spectrum. Note that the peculiar asymptotic behavior $(g_{YM}^2 N)^{1/4}$ has not yet been obtained by field theory means and it remains a prediction of string theory.

The six-dimensional case is rather different. Here the multiplets with $k = 2$ lying at the unitarity bound $\ell = 6 + s$ of the continuous series A have twist 6, so in the free theory they could be realized only by trilinear operators. Such operators cannot appear in the OPE of two stress-tensor multiplets (bilinears). Therefore we should look for analogs of the Konishi-like multiplets among the bilinear composites with $k = 2$. In $d = 6$ they should have twist 4 and we see that they can only appear in the isolated series B. Thus, we may say that in $d = 6$ the Konishi-like semishort multiplets are protected by unitarity.

Being protected operators in $d = 6$, the Konishi-like multiplets are not expected to appear in the supergravity-induced OPE. This can be anticipated on the general grounds of the AdS/CFT correspondence, because there is no field in the spectrum of the corresponding supergravity theory dual to any of these currents. In Section 4, by using gauged seven-dimensional $\mathcal{N} = 4$ supergravity [21], we compute the four-point amplitude of the lowest dimension 1/2 BPS operators in the $(2, 0)$ theory. Subsequently, in Section 5 we indeed demonstrate the absence of the Konishi-type currents in the supergravity-induced OPE. On the other hand, the Konishi-like multiplets are present in the free OPE of two 1/2 BPS operators. In our opinion, the fact that they drop out of the spectrum of the interacting theory clearly demonstrates the absence of a superconformal theory that could smoothly interpolate between the free CFT and the CFT dual to the eleven-dimensional supergravity on the $AdS_7 \times S^4$ background.

Note that the $d = 6$ OPE under consideration does not contain operators from series C. Indeed, since in the $k = 2$ channel $a_1 = a_2 = 0$, they should have the dimension of the fundamental field $\ell = 2$.

2.2 Four-point function of stress-tensor multiplets

The complete, i.e., both kinematical and dynamical information about the OPE of two stress-tensor multiplets is encoded in their four-point correlation function. We have already seen that the kinematics (or, in other terms, superconformal representation theory) strongly restricts the

content of the OPE. We should expect to see the implications of these restrictions on the four-point amplitude. The easiest and most economic way to do this is to use the superconformal Ward identities. Below we summarize the already known results about this four-point amplitude in $d = 4$ [22–26, 19] and compare them to our new results in the six-dimensional case.

We would like to stress that the four-point amplitude of 1/2 BPS short multiplets that we consider is rather special in the sense that superconformal symmetry is powerful enough to restore the complete superspace dependence solely from the knowledge of the lowest ($\theta = 0$) component of the amplitude. Indeed, the 1/2 BPS short superfields depend on half of the Grassmann variables. Thus, a four-point function of this type depends on $4 \times (1/2) = 2$ full sets of odd variables. At the same time, the superconformal algebra has two sets of odd shift-like generators (Q and S supersymmetry). This leaves no room for nilpotent superconformal invariants made out of the odd variables and thus the θ expansion is completely fixed.

The lowest component of this amplitude corresponds to the correlator of four scalar operators \mathcal{O}^I of dimension $\ell = d - 2$ in the vector representation of the R symmetry group SO(6) (for $d = 4$) or SO(5) (for $d = 6$):

$$\begin{aligned} \langle \mathcal{O}^{I_1}(x_1) \dots \mathcal{O}^{I_4}(x_4) \rangle = & \\ & a_1(s, t) \frac{\delta^{I_1 I_2} \delta^{I_3 I_4}}{(x_{12}^2 x_{34}^2)^{d-2}} + a_2(s, t) \frac{\delta^{I_1 I_3} \delta^{I_2 I_4}}{(x_{13}^2 x_{24}^2)^{d-2}} + a_3(s, t) \frac{\delta^{I_1 I_4} \delta^{I_2 I_3}}{(x_{14}^2 x_{23}^2)^{d-2}} \\ & + b_1(s, t) \frac{C^{I_1 I_3 I_2 I_4}}{(x_{13}^2 x_{14}^2 x_{23}^2 x_{24}^2)^{\frac{d-2}{2}}} + b_2(s, t) \frac{C^{I_1 I_2 I_3 I_4}}{(x_{12}^2 x_{14}^2 x_{23}^2 x_{34}^2)^{\frac{d-2}{2}}} + b_3(s, t) \frac{C^{I_1 I_2 I_4 I_3}}{(x_{12}^2 x_{13}^2 x_{24}^2 x_{34}^2)^{\frac{d-2}{2}}}. \end{aligned} \quad (5)$$

Here s, t are the conformal cross-ratios

$$s = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad t = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}.$$

The six tensor structures $\delta^{I_1 I_2} \delta^{I_3 I_4}$, $C^{I_1 I_2 I_3 I_4}$ (and permutations) are invariant tensors of SO(6) (or SO(5)) and are related to the six channels in the OPE (3) (or (4)). In general, we define the invariant tensors $C^{I_1 \dots I_n}$ as $\text{tr}(C^{I_1} \dots C^{I_n})$, where the matrices C_{ij}^I , which are symmetric and traceless in their lower indices, realize a basis of the corresponding vector representation of the R symmetry group.

Among the six coefficient functions in (5) only two are independent, for example, $a_1(s, t)$ and $b_2(s, t)$. The others are obtained from the crossing symmetry relations

$$\begin{aligned} a_1(s, t) &= a_3(t, s) = a_1(s/t, 1/t) \\ a_2(s, t) &= a_2(t, s) = a_3(s/t, 1/t) \\ b_1(s, t) &= b_3(t, s) = b_1(s/t, 1/t) \\ b_2(s, t) &= b_2(t, s) = b_3(s/t, 1/t) \end{aligned} \quad (6)$$

The amplitude (5) must obey superconformal Ward identities which follow from the 1/2 BPS

nature of the supermultiplets. In the four-dimensional case they take the form of two first-order PDEs for the independent coefficient functions [24]:

$d = 4$ Ward identities:

$$\begin{aligned}\partial_t b_2 &= \frac{s}{t} \partial_s a_3 - \partial_s a_1 - \frac{s+t-1}{s} \partial_t a_1 \\ \partial_s b_2 &= \frac{t}{s} \partial_t a_1 - \partial_t a_3 - \frac{s+t-1}{t} \partial_s a_3\end{aligned}\tag{7}$$

In six dimensions the corresponding equations look very similar (see Section 3 for the derivation):

$d = 6$ Ward identities:

$$\begin{aligned}\partial_t b_2 &= \frac{s^2}{t^2} \partial_s a_3 - \frac{t}{s} \partial_s a_1 - \frac{t(s+t-1)}{s^2} \partial_t a_1 \\ \partial_s b_2 &= \frac{t^2}{s^2} \partial_t a_1 - \frac{s}{t} \partial_t a_3 - \frac{s(s+t-1)}{t^2} \partial_s a_3\end{aligned}\tag{8}$$

However, the small change in the coefficients from eqs. (7) to eqs. (8) results in an important difference when it comes to their general solution.

In ref. [25] it was found that the general solution ³ of the $d = 4$ Ward identities (7) is parametrized by *two independent functions*, one of two variables and the other of a single variable. ^{4 5} It was further shown in ref. [25] that the function of one variable can be set to its free-theory value by evoking a dynamical mechanism. It consists in employing Intriligator's insertion procedure [27] which gives the quantum (interacting) part of the amplitude as the result of the insertion of the SYM action into it. Thus, combining kinematics with dynamics, the full solution of the $d = 4$ superconformal Ward identities is reduced to

$$\begin{aligned}a_1(s, t) &= A + sF(s, t), \\ b_2(s, t) &= B + (1 - s - t)F(s, t),\end{aligned}$$

where A and B are constants determined from the free $\mathcal{N} = 4$ theory.

The non-trivial part of the amplitude is therefore encoded in the *single function of two variables* $F(s, t)$ satisfying the crossing-symmetry conditions

$$F(s, t) = F(t, s) = 1/t F(s/t, 1/t).\tag{9}$$

³We recall some details in Appendix 1.

⁴It should be mentioned that a similar picture has been observed many years ago by Fradkin, Palchik and Zaikin [28]. They have studied the conformal correlator of one conserved current with three scalar operators. By imposing the Ward identity for the current they have found a differential equation whose solution has exactly the same functional freedom. However, the difference between their study and ours is in the fact that the functions determining their correlator are obtained as derivatives of our coefficients a_i and b_i (when reconstructing the corresponding component of our superamplitude starting from the lowest one). Thus, the solution of our constraints lies one level deeper than that of ref. [28].

⁵For a recent discussion in which crossing symmetry is not imposed see ref. [19].

It includes all (non-)perturbative corrections to the free field amplitude. This prediction of the superconformal Ward identities and of the dynamical insertion procedure about the form of the amplitude, called “partial non-renormalization” in ref. [25], has been confirmed by all the available perturbative [29], instanton [30] and strong coupling [20] results.

In six dimensions the solution of the Ward identities (8) is directly given in terms of *one unconstrained function of two variables* (see Section 3):

$$\begin{aligned} a_1(s, t) &= A + s^4 \Delta \left(\frac{1}{\lambda^3} t \mathcal{F}(s, t) \right), \\ b_2(s, t) &= B + s^2 t^2 \Delta \left(\frac{1}{\lambda^3} (1 - s - t) \mathcal{F}(s, t) \right), \end{aligned}$$

where A and B are additive integration constants, Δ is a second-order differential operator,

$$\Delta = s \partial_{ss} + t \partial_{tt} + (s + t - 1) \partial_{st} + 3 \partial_s + 3 \partial_t, \quad (10)$$

and $\lambda = \sqrt{(s + t - 1)^2 - 4st}$ is its discriminant. Here the function $\mathcal{F}(s, t)$ satisfies the same crossing-symmetry relations as its four-dimensional analogue (recall (9)):

$$\mathcal{F}(s, t) = \mathcal{F}(t, s) = 1/t \mathcal{F}(s/t, 1/t). \quad (11)$$

Again, it encodes the dynamics of the theory and, in particular, it comprises all M-theory corrections to the leading supergravity result. However, in $d = 4$ this function itself is a coefficient function of the amplitude, whereas in $d = 6$ it plays the rôle of a *prepotential* in the sense that all the coefficients can be obtained from it by applying derivatives. It would be very interesting to find out whether this prepotential has a deeper origin.

We would like to underline once more the important difference between the four- and six-dimensional cases. In $d = 4$ one can reduce the freedom in the amplitude to just *one unconstrained function* by combining kinematics (the superconformal Ward identities) with dynamics (the insertion formula). The latter relies on the existence of a certain nilpotent superconformal five-point covariant with rather special properties [31, 25]. Our attempts to find a similar construction in $d = 6$ were unsuccessful. This again points at the absence of a Lagrangian formulation of the six-dimensional theory. However, now we see that in $d = 6$ the kinematics (superconformal symmetry) alone leaves *exactly the same freedom*, the single function $\mathcal{F}(s, t)$.

Our final remark concerns an alternative explanation of the rôle of the function of one variable in the four-dimensional amplitude, recently discussed by Dolan and Osborn [19]. They relate this function to the possible exchanges only of protected operators in the OPE (the first five channels in the decomposition (3)). Indeed, it is easy to show that Intriligator’s insertion procedure forbids such exchanges [11], and so it is natural to expect that this fixes the function at its free-theory value. However, in six dimensions similar protected channels exist, but the insertion procedure cannot be applied. Still, we do not find a function of one variable here. It

would be interesting to understand this phenomenon from the OPE point of view advocated in ref. [19]. One might speculate about the different behavior of the protected semishort operators in $d = 4$ which lie at the boundary of the continuous series A, and of those in $d = 6$ which belong to the isolated series B.

2.3 Obtaining the prepotential \mathcal{F} from gauged supergravity

Since no field-theory formulation of the interacting $(2, 0)$ six-dimensional theory is available, the way to check the general predictions we have found here is to compute the amplitude via the AdS/CFT correspondence and to try to identify the prepotential \mathcal{F} . We perform this program in Section 4 by using the gauged seven-dimensional $\mathcal{N} = 4$ supergravity and find a perfect agreement. In particular, we show that the supergravity four-point amplitude of the $1/2$ BPS operators is generated by the following very simple prepotential

$$\mathcal{F}(s, t) = \frac{240}{N^3} \frac{\lambda^3}{st^2} \bar{D}_{7333}(s, t), \quad (12)$$

together with the integration constants

$$A = 1, \quad B = \frac{1}{N^3}. \quad (13)$$

The conformally covariant functions $\bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}(s, t)$ are defined in Appendix 2.

It is interesting to compare this supergravity-induced solution with the theory of η free $(2, 0)$ tensor multiplets. For this theory the prepotential \mathcal{F} vanishes, while the constants A, B equal

$$A = 1, \quad B = \frac{4}{\eta}. \quad (14)$$

If one would try to view the supergravity solution \mathcal{F} as being obtained from the free one $\mathcal{F} = 0$ by some smooth deformation, then one should obviously set $\eta = 4N^3$. The factor $4N^3$ was found in ref. [32] by studying the absorption rate of longitudinally polarized gravitons by M5 branes. The same factor appears as the universal coefficient between the free and the AdS two- and three-point correlators of the stress tensor [33], as well as between the free and the AdS type B conformal anomaly [34]. Since all the non-trivial dynamics is encoded in the prepotential, we see that $4N^3$ is just what is needed to match the integration constants of the free and of the supergravity-induced four-point amplitudes. In Section 4 we discuss however that the existence of a smooth superconformal deformation from the free to the supergravity theory appears to be in conflict with unitarity.⁶

⁶One way to smoothly connect the free and the supergravity amplitudes is to multiply the supergravity prepotential by a function $f(g)$ of some “coupling constant” g , such that $f(0) = 0$ and $f(1) = 1$. This is a trivial possibility that would explain the decoupling of the Konishi-like multiplets from the supergravity OPE by the vanishing of the corresponding OPE coefficients. In what follows we discard such deformations.

It is instructive to compare the six-dimensional prepotential (12) with the “potential” which generates the strongly-coupled $d = 4$ $\mathcal{N} = 4$ amplitude found via the AdS/CFT correspondence [20, 25]: ⁷

$$F(s, t) = -\frac{24}{N^2} \frac{1}{t} \bar{D}_{4222}(s, t), \quad A = 1, \quad B = \frac{4}{N^2}. \quad (15)$$

Since differentiating a \bar{D} -function with respect to s or t amounts to raising by unity the values of two of its indices, we see that the six-dimensional prepotential is obtained from the four-dimensional potential by *dressing* it with a certain third-order differential operator (see Section 4). It would be interesting to find out if this operator has some intrinsic meaning.

Finally, we observe that we might construct infinite towers of superconformal four-point amplitudes both in the $d = 4$ and $d = 6$ theories as follows:

$$F(s, t) \sim \frac{1}{t} \bar{D}_{3\Delta-2, \Delta, \Delta, \Delta}(s, t), \quad \mathcal{F}(s, t) \sim \frac{\lambda^3}{st^2} \bar{D}_{3\Delta-2, \Delta, \Delta, \Delta}(s, t), \quad (16)$$

where $\Delta = 1, 2, \dots$. These functions are symmetric and satisfy (11) as a consequence of the corresponding symmetry properties (68) and (69) of the \bar{D} -functions. One could ask the question whether any of the amplitudes (16) (or their linear combinations), other than (12) and (15), together with some appropriate integration constants A and B , has an OPE free from Konishi-like multiplets. If not, this might explain the distinguished rôle of the amplitudes (12) and (15).

3 General structure of the four-point amplitude

We begin this section by recalling, just in a few words, the procedure which leads to the $d = 4$ Ward identities (7). The origin of these constraints can be traced back to the fact that the $\mathcal{N} = 4$ SYM stress-tensor multiplet is 1/2 BPS short. The natural framework for describing BPS shortness is harmonic superspace [35]. The constraints (7) can be derived directly in $\mathcal{N} = 4$ harmonic superspace [36], but it is easier to do this using its $\mathcal{N} = 2$ version (both methods are explained in detail in ref. [24]; for a recent rederivation of the same constraints without using harmonic superspace see ref. [19]). The main point is that when reconstructing the full harmonic superspace dependence of the four-point amplitude starting from its lowest component (5), one encounters harmonic singularities already at the level $(\theta)^2$. Their absence, i.e., the requirement of harmonic analyticity, is equivalent to imposing irreducibility under the R symmetry group. It is precisely this requirement which leads to the constraints (7). Once these constraints have been imposed, it can be shown that no new harmonic singularities appear at the higher levels of the θ expansion of the amplitude.

In $d = 6$ one could go through exactly the same steps in order to obtain the new constraints (8). However, there is a much faster way which consists in simply adapting the $d = 4$ constraints

⁷See also Appendix D of ref. [19], where a similar formula for $F(s, t)$ was extracted from the results of [20].

(7) to the case $d = 6$. The key observation is that the coefficient functions a_1, a_3, b_2 appear in (7) only through their first-order derivatives. The origin of these derivatives is in the completion of the conformal invariant s to a full superconformal invariant $\hat{s} = s + \theta$ -terms (and similarly for t). Expanding, e.g., $a_1(\hat{s}, \hat{t})$ up to the level $(\theta)^2$ gives rise to the terms $\partial_s a_1, \partial_t a_1$. It is not hard to show that \hat{s}, \hat{t} are exactly the same both in $d = 4$ and in $d = 6$. Next, the rational coefficients in (7) originate from the “propagator” factors $1/(x_{12}^2 x_{34}^2)^{d-2}$, etc. in (5). These differ in $d = 4$ and $d = 6$, as can be seen most clearly by pulling out one of them in front of the amplitude (only the relevant terms are shown):

$$\frac{1}{(x_{12}^2 x_{34}^2)^{d-2}} \left[a_1(s, t) \delta^{I_1 I_2} \delta^{I_3 I_4} + \left(\frac{s}{t}\right)^{d-2} a_3(s, t) \delta^{I_1 I_4} \delta^{I_2 I_3} + \left(\frac{s}{t}\right)^{\frac{d-2}{2}} b_2(s, t) C^{I_1 I_2 I_3 I_4} + \dots \right]$$

Now, the completion of these propagator factors to full superconformal covariants does not affect the derivative terms in (7). Therefore, in order to pass from $d = 4$ to $d = 6$ it is sufficient to just redefine the coefficient functions as follows:

$$a_1^{d=6} \rightarrow a_1^{d=4}, \quad a_3^{d=6} \rightarrow \frac{s^2}{t^2} a_3^{d=4}, \quad b_2^{d=6} \rightarrow \frac{s}{t} b_2^{d=4}. \quad (17)$$

This redefinition should be done in (7) so that the derivatives do not act on the factors in (17). The result is precisely the constraints (8).

We remark that due to the crossing symmetry relations (recall (6))

$$a_3(t, s) = a_1(s, t), \quad b_2(s, t) = b_2(t, s) \quad (18)$$

the second equation both in (7) and in (8) is automatically satisfied.

From here on we concentrate on the solution of the $d = 6$ constraints (8). The integrability condition for this system of PDEs is

$$t \Delta \left(\frac{a_1}{s^2} \right) = s \Delta \left(\frac{a_3}{t^2} \right) \quad (19)$$

where Δ was defined in (10). To solve this second-order PDE we make the substitution

$$\frac{a_1}{s^2} = \frac{s}{\lambda^3} G(s, t), \quad \frac{a_3}{t^2} = \frac{t}{\lambda^3} G(t, s) \quad (20)$$

and change the variables s, t to the “normal coordinates” x, y for the hyperbolic operator Δ :

$$x = \rho s, \quad y = \rho t, \quad (21)$$

where $\rho = 2(1 - s - t + \lambda)^{-1}$ and $\lambda = \sqrt{(1 - s - t)^2 - 4st}$. After this change eq. (19) becomes

$$y (\partial_y + x^2 \partial_x + (1 - xy) \partial_{xy}) G(x, y) = x (\partial_x + y^2 \partial_y + (1 - xy) \partial_{xy}) G(y, x).$$

Then we set

$$G(x, y) = \phi(x, y) + \gamma(x, y),$$

where

$$\phi(x, y) = \phi(y, x), \quad \gamma(x, y) = -\gamma(y, x) \quad (22)$$

and perform one more change of variables

$$\sigma = \ln(xy), \quad \tau = \ln \frac{x}{y}.$$

In these new variables the symmetry conditions (22) become

$$\phi(\sigma, \tau) = \phi(\sigma, -\tau), \quad \gamma(\sigma, \tau) = -\gamma(\sigma, -\tau) \quad (23)$$

and equation (19) takes the form

$$\phi_\tau = \gamma_{\sigma\sigma} - \gamma_{\tau\tau} - \coth(\sigma/2)\gamma_\sigma. \quad (24)$$

This equation can be integrated to give

$$\phi = (\partial_{\sigma\sigma} - \coth(\sigma/2)\partial_\sigma) \int_{\tau_0}^{\tau} \gamma(\sigma, \tau') d\tau' - \gamma_\tau + c(\sigma),$$

where $c(\sigma)$ is an integration “constant” depending on σ . Let us introduce the function

$$\mathcal{F}(\sigma, \tau) = - \int_{\tau_0}^{\tau} \gamma(\sigma, \tau') d\tau'. \quad (25)$$

It is even, $\mathcal{F}(\sigma, \tau) = \mathcal{F}(\sigma, -\tau)$ and is supposed to obey the boundary condition $\mathcal{F}(\sigma, \tau_0) = 0$, where τ_0 is some arbitrary fixed point. Without loss of generality $c(\sigma)$ is absorbed into \mathcal{F} , which results only in a change of the boundary condition for \mathcal{F} . Thus,

$$\phi + \gamma = [\partial_{\tau\tau} - \partial_{\sigma\sigma} + \coth(\sigma/2)\partial_\sigma - \partial_\tau] \mathcal{F} \quad (26)$$

and the function \mathcal{F} plays the rôle of a *prepotential*.

Switching back to the original variables s, t we find

$$a_1 = \frac{s^4 t}{\lambda^3} \left[2\partial_t + \frac{s+t-1}{t} \partial_s + s\partial_{ss} + (s+t-1)\partial_{st} + t\partial_{tt} \right] \mathcal{F}(s, t), \quad (27)$$

$$a_3 = \frac{t^4 s}{\lambda^3} \left[2\partial_s + \frac{s+t-1}{s} \partial_t + s\partial_{ss} + (s+t-1)\partial_{st} + t\partial_{tt} \right] \mathcal{F}(s, t), \quad (28)$$

where the prepotential $\mathcal{F}(s, t)$ is an arbitrary symmetric function.

Some comments are due here. First of all, it is obvious that a_1 and a_3 admit the constant solution $a_1 = a_3 = A$. Computing $\gamma(\sigma, \tau)$ for this trivial solution and further integrating over τ we find the corresponding prepotential $\mathcal{F}_0(s, t)$:

$$\mathcal{F}_0(s, t) = \frac{A}{3} \frac{s^3 + t^3}{s^3 t^3} \lambda^3. \quad (29)$$

Secondly, the prepotential is not uniquely defined. The freedom in redefining \mathcal{F} without changing the amplitude can be easily found by solving the homogenous equation implied by (26). This is done by separating the variables. The resulting freedom is

$$\mathcal{F}(s, t) \rightarrow \mathcal{F}(s, t) + h(s, t), \quad (30)$$

where

$$h(s, t) = h_1 \left(st\rho^2 - \frac{1}{st\rho^2} - 2 \ln(st\rho^2) \right) + h_2 \quad (31)$$

and h_1 and h_2 are arbitrary constants.

Since the integrability condition (19) is already satisfied, we can now integrate, e.g., the first of the equations (8) for b_2 . We obtain

$$\begin{aligned} b_2(s, t) = & B - \frac{s^2 t^2}{\lambda^3} (s + t - 1) \left[s \partial_{ss} + (s + t - 1) \partial_{st} + t \partial_{tt} \right. \\ & \left. + \left(1 + \frac{2s}{s + t - 1} \right) \partial_s + \left(1 + \frac{2t}{s + t - 1} \right) \partial_t \right] \mathcal{F}(s, t), \end{aligned} \quad (32)$$

where B is a new integration constant. Under the replacement (30) the coefficients a_1 and a_3 and, therefore, eqs. (8) remain unchanged. However, the solution (32) is allowed to pick an additive constant. Indeed, we find that (30) leads to the shift $B \rightarrow B + h_1$. Finally, note that the constant B remains unchanged under the replacement $\mathcal{F}(s, t) \rightarrow \mathcal{F}(s, t) + \mathcal{F}_0(s, t)$.

Now we are in a position to find the implications of the global crossing symmetry conditions. Under the change $s \rightarrow s/t$, $t \rightarrow 1/t$ we find

$$a_1(s/t, 1/t) = \frac{s^4}{\lambda^3} \left[s \partial_{ss} + (s + t - 1) \partial_{st} + t \partial_{tt} \right] \mathcal{F}(s/t, 1/t). \quad (33)$$

Clearly, by choosing

$$\mathcal{F}(s/t, 1/t) = t \mathcal{F}(s, t) \quad (34)$$

we are able to satisfy the crossing symmetry relation $a_1(s/t, 1/t) = a_1(s, t)$. Note that neither $\mathcal{F}_0(s, t)$ nor $h(s, t)$ obey eq. (34). Therefore, in the following we represent the general solution of the superconformal Ward identities in the form

$$\mathcal{F}_0(s, t) + \mathcal{F}(s, t), \quad (35)$$

where $\mathcal{F}(s, t)$ satisfies the crossing symmetry relation (34). This requirement also fixes the freedom (30) in the prepotential.

Formulae (27), (28) and (32) can be further simplified to give

$$\begin{aligned}
a_1 &= \frac{s^4}{\lambda^3} \left[s\partial_{ss} + (s+t-1)\partial_{st} + t\partial_{tt} \right] \left(t\mathcal{F}(s,t) \right), \\
a_3 &= \frac{t^4}{\lambda^3} \left[s\partial_{ss} + (s+t-1)\partial_{st} + t\partial_{tt} \right] \left(s\mathcal{F}(s,t) \right), \\
b_2 &= \frac{s^2t^2}{\lambda^3} \left[s\partial_{ss} + (s+t-1)\partial_{st} + t\partial_{tt} \right] \left((1-s-t)\mathcal{F}(s,t) \right).
\end{aligned} \tag{36}$$

Now we note that the operator Δ has the following property. For any function $w(s,t)$,

$$\Delta \frac{w(s,t)}{\lambda^3} = \frac{1}{\lambda^3} \left[t\partial_{tt} + (s+t-1)\partial_{st} + s\partial_{ss} \right] w(s,t).$$

This allows us to move the factor $1/\lambda^3$ to the right through the differential operator in eqs. (36).

We thus obtain the complete solution for the coefficients of the four-point amplitude in terms of the prepotential $\mathcal{F}(s,t)$:

$$\begin{aligned}
a_1(s,t) &= A + s^4\Delta \left(\frac{1}{\lambda^3} t\mathcal{F}(s,t) \right), \\
a_2(s,t) &= A + \Delta \left(\frac{1}{\lambda^3} st\mathcal{F}(s,t) \right), \\
a_3(s,t) &= A + t^4\Delta \left(\frac{1}{\lambda^3} s\mathcal{F}(s,t) \right), \\
b_1(s,t) &= B + t^2\Delta \left(\frac{1}{\lambda^3} s(s-t-1)\mathcal{F}(s,t) \right), \\
b_2(s,t) &= B + s^2t^2\Delta \left(\frac{1}{\lambda^3} (1-s-t)\mathcal{F}(s,t) \right), \\
b_3(s,t) &= B + s^2\Delta \left(\frac{1}{\lambda^3} t(t-s-1)\mathcal{F}(s,t) \right).
\end{aligned} \tag{37}$$

In this form the crossing symmetry relation is most transparent, given that the operator Δ transforms as

$$\Delta_{s/t,1/t}(t^2w(s,t)) = t^4\Delta_{s,t}(w(s,t)) \tag{38}$$

for any function $w(s,t)$. Finally, note that λ is a symmetric function of s,t , however under $s \rightarrow s/t, t \rightarrow 1/t$ it transforms as $\lambda \rightarrow \lambda/t$. Thus, if we redefine \mathcal{F} as $\mathcal{F} \rightarrow \lambda^{-3}\mathcal{F}$, the new function obeys the crossing symmetry relation $\mathcal{F}(s/t, 1/t) = t^4\mathcal{F}(s,t)$.

4 Four-point amplitude from gauged supergravity

According to the duality conjecture for the (2,0) theory, in the supergravity regime the correlation functions of any 1/2 BPS operators and of their supersymmetry descendants can be computed from eleven-dimensional supergravity on an $AdS_7 \times S^4$ background. In this way many two- and three-point correlation functions have already been found [37]. Below we present the first example of a four-point amplitude of 1/2 BPS operators in this theory and subsequently analyze the leading terms of the underlying OPE. The operators whose amplitude we are going to find, have the lowest scaling dimension $\ell = 4$ and their dual supergravity scalars belong to the massless graviton multiplet of the $AdS_7 \times S^4$ compactification. Thus, for our present purposes it is enough to consider only the sector of the theory described by gauged seven-dimensional supergravity.

The gauged seven-dimensional $\mathcal{N} = 4$ supergravity was constructed in ref. [21] by gauging Poincaré supergravity. Alternatively, it can be obtained by compactifying eleven-dimensional supergravity on $AdS_7 \times S^4$ with a further Kaluza-Klein truncation to the massless graviton multiplet [38].

The bosonic sector of the theory consists of the metric $g_{\mu\nu}$, fourteen scalars parametrizing the coset space $SL(5, \mathbb{R})/SO(5)_c$, the $SO(5)_g$ Yang-Mills gauge fields A_μ^{IJ} and a five-plet of antisymmetric tensors $S_{\mu\nu\rho}^I$, $I, J = 1, \dots, 5$. The relevant part of the Lagrangian is (the metric is assumed to have Minkowskian signature)

$$e^{-1} \mathcal{L} = R + \frac{g^2}{8} (T^2 - 2T_{ij} T^{ij}) - \frac{1}{4} P_{\mu ij} P^{\mu ij} - \frac{1}{2} F_{\mu\nu}^{IJ} F_{IJ}^{\mu\nu}. \quad (39)$$

Here $F_{\mu\nu}^{IJ}$ is the field strength for A_μ^{IJ} . To describe the scalar manifold one introduces the vielbein $(S^{-1})^i_I \in SL(5, \mathbb{R})$, where $i = 1, \dots, 5$ is an $SO(5)_c$ index. Then $T_i^j = (SS^t)_i^j$ and $T = \text{Tr}(SS^t)$. The kinetic term is given by the matrix P_μ :

$$P_\mu = S \nabla_\mu S^{-1} - g S A_\mu S^{-1} + \nabla_\mu (S^{-1})^t S^t + g (S^{-1})^t A_\mu S^t,$$

where g is the Yang-Mills coupling. Note that in writing eq. (39) we omitted the part of the action depending on the antisymmetric fields since, as can be easily shown, the latter do not propagate in the AdS exchange graphs involving four external scalar fields.

To proceed, we choose the following natural parametrization for S : $S = e^\Lambda$ where Λ is a traceless symmetric 5×5 matrix. The scalar fields parametrizing Λ are dual to the 1/2 BPS operators \mathcal{O}^I of dimension $\ell = 4$ with the index I transforming under the irrep [02] of the R symmetry group $SO(5)$.

Since we are interested in the four-point amplitude of the operators \mathcal{O}^I , we decompose the Lagrangian in power series in Λ and then truncate it at the fourth order. The resulting expression

reads

$$\begin{aligned}
e^{-1}\mathcal{L} &= R - \left(\nabla_\mu \Lambda \nabla^\mu \Lambda + \frac{2}{3} \nabla_\mu \Lambda \Lambda^2 \nabla^\mu \Lambda - \frac{2}{3} \nabla_\mu \Lambda \Lambda \nabla_\mu \Lambda \Lambda + 2g \nabla^\mu \Lambda [\Lambda, A_\mu] \right) \\
&+ \frac{g^2}{8} \left(15 + 4\text{tr}\Lambda^2 - 8\text{tr}\Lambda^3 + 4(\text{tr}\Lambda^2)^2 - \frac{44}{3}\text{tr}\Lambda^4 \right) - \frac{1}{2} F_{\mu\nu}^{IJ} F_{IJ}^{\mu\nu}. \tag{40}
\end{aligned}$$

Obviously, to obtain the correct value $2\lambda = -(d-1)(d-2) = -30$ of the cosmological constant in $d = 7$, one has to set $g^2 = 16$.

To simplify the resulting expression, we perform the field redefinition $\Lambda \rightarrow \Lambda - \frac{2}{3}\Lambda^3$. It is also convenient to introduce another parametrization for the matrix of scalars and for the vector field:

$$\Lambda_{ij} = \frac{1}{2} C_{ij}^I s^I; \quad (A_\mu)_{ij} = C_{ij}^I A_\mu^I, \tag{41}$$

where C_{ij}^I and C_{ij}^I are traceless symmetric and antisymmetric matrices providing bases (an upper index) for the irreps [02] and [20], respectively. The normalization properties of these matrices are discussed in Appendix 2. We set out to work with the Euclidean version of the AdS metric which results in changing the overall sign of the Lagrangian.

Let us mention the issue of the overall normalization of the gravity action. We normalize the action of eleven-dimensional supergravity as $S = \frac{1}{2k_{11}^2} \int \sqrt{g} R + \dots$, where k_{11} is the eleven-dimensional Newton constant: $\frac{1}{2k_{11}^2} = \frac{2N^3}{\pi^5}$. For the $AdS_7 \times S^4$ solution with the radii $R_{AdS_7} = 1$ and $R_{S^4} = 1/2$ the reduction to seven dimensions yields $\frac{1}{2k_7^2} = \frac{N^3}{3\pi^3}$.

Thus, in the sequel we will work with the action

$$S(s) = \frac{N^3}{3\pi^3} \int_{AdS_7} \sqrt{g} \mathcal{L}, \tag{42}$$

where, after the manipulations described above the Lagrangian acquires the form

$$\begin{aligned}
\mathcal{L} &= \frac{1}{4} (\nabla_\mu s^I \nabla^\mu s^I - 8s^I s^I) + 2C_{I_1 I_2 I_3} s^{I_1} s^{I_2} s^{I_3} \\
&- \frac{1}{4} T_{\mu\nu} \phi^{\mu\nu} - \mathcal{L}_2(\phi_{\mu\nu}) + 4J_{\mu; I} A^{\mu; I} + \frac{1}{2} F_{\mu\nu}^I F^{\mu\nu; I} \\
&- \frac{1}{2^4} C_{I_1 I_2 I_3 I_4} \nabla_\mu (s^{I_1} s^{I_2}) \nabla^\mu (s^{I_3} s^{I_4}) - \frac{1}{2} s^{I_1} s^{I_1} s^{I_2} s^{I_2} + \frac{5}{2} C_{I_1 I_2 I_3 I_4} s^{I_1} s^{I_2} s^{I_3} s^{I_4}. \tag{43}
\end{aligned}$$

Here we introduced the currents

$$\begin{aligned}
T_{\mu\nu} &= \nabla_\mu s^I \nabla_\nu s^I - \frac{1}{2} g_{\mu\nu} (\nabla_\rho s^I \nabla^\rho s^I - 8s^I s^I), \\
J_\mu^I &= T^{I_1 I_2 I} s^{I_1} \nabla_\mu s^{I_2}
\end{aligned}$$

obeying an on-shell conservation law. The tensors $C^{I_1 \dots I_n}$ were already introduced in Section 2.2, and $T^{I_1 I_2 I_3} = C_{ij}^{I_1} C_{jk}^{I_2} C_{ki}^{I_3}$ is antisymmetric in the indices I_1, I_2 . In what follows we use

a more concise notation, e.g., $C_{I_1 I_2 I_3 I_4} \equiv C_{1234}$. In eq. (43) $\mathcal{L}_2(\phi_{\mu\nu})$ stands for the standard quadratic Lagrangian of the graviton $\phi_{\mu\nu}$.

Now it is straightforward, although rather tedious to compute the on-shell value of the above action subject to the Dirichlet boundary conditions. Like in the case of gauged $d = 5$ supergravity, we have to evaluate the exchange graphs ⁸ describing quartic scalar interactions [20, 40]. We omit the details of the computation since they are similar to those of ref. [20]. We present only the final result for the four-point amplitude of the canonically normalized 1/2 BPS operators which is found by varying the on-shell action with respect to the boundary data for the scalars:

$$\begin{aligned} \langle \mathcal{O}^{I_1}(x_1) \cdots \mathcal{O}^{I_4}(x_4) \rangle &= \frac{\delta^{12} \delta^{34}}{x_{12}^8 x_{34}^8} \\ &+ \frac{2^5 \cdot 3^3}{\pi^3 N^3} \left[C_{1234}^+ A_{1234}^+ + \delta^{12} \delta^{34} A_{1234}^0 + C_{1234}^- A_{1234}^- \right] + t + u. \end{aligned} \quad (44)$$

Here $C_{1234}^\pm = \frac{1}{2}(C_{1234} \pm C_{2134})$. We exhibit explicitly only the expression in the s-channel, the t-channel is obtained by replacing $1 \leftrightarrow 4$, and the u-channel by $1 \leftrightarrow 3$. The first term in (44) and its t- and u-counterparts represent the contributions of the disconnected AdS graphs. The coefficients $A^{\pm,0}$ are obtained in terms of the D -functions defined in Appendix 2 and read

$$\begin{aligned} A_{1234}^+ &= \frac{1}{2x_{34}^4} D_{4422} + \frac{1}{x_{34}^2} D_{4433} - \frac{7}{2} D_{4444} + 4x_{34}^2 D_{4455}, \\ A_{1234}^0 &= -\frac{1}{6x_{34}^4} D_{4422} + \frac{1}{18} \left(\frac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2} + \frac{x_{14}^2 x_{23}^2}{x_{12}^2 x_{34}^2} - \frac{25}{4} \right) \frac{1}{x_{34}^2} D_{4433} \\ &+ \frac{1}{8} \left(\frac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2} + \frac{x_{14}^2 x_{23}^2}{x_{12}^2 x_{34}^2} + \frac{31}{4} \right) D_{4444} + \frac{1}{2} \left(\frac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2} + \frac{x_{14}^2 x_{23}^2}{x_{12}^2 x_{34}^2} - 1 \right) x_{34}^2 D_{4455}, \\ A_{1234}^- &= \frac{1}{x_{12}^2 x_{34}^2} \left[(x_{14}^2 x_{23}^2 - x_{13}^2 x_{24}^2) D_{4444} + \frac{5}{6} (x_{24}^2 D_{3434} - x_{14}^2 D_{4334} - x_{23}^2 D_{3443} + x_{13}^2 D_{4343}) \right] \\ &+ \frac{1}{x_{12}^4 x_{34}^4} \left[\frac{1}{9} (x_{14}^2 x_{23}^2 - x_{13}^2 x_{24}^2) D_{3333} + \frac{1}{18} (x_{24}^2 D_{2323} - x_{14}^2 D_{3223} - x_{23}^2 D_{2332} + x_{13}^2 D_{3232}) \right]. \end{aligned} \quad (45)$$

Having found the four-point amplitude, we first check if it obeys the superconformal Ward identities (8) derived in Section 3. Rewriting the amplitude (44) in the form (5), we make the following identification:

$$\begin{aligned} a_1(s, t) &= 1 + \frac{2^5 \cdot 3^3}{\pi^3 N^3} x_{12}^8 x_{34}^8 A_{1234}^0, \\ b_2(s, t) &= \frac{2^4 \cdot 3^3}{\pi^3 N^3} x_{12}^4 x_{14}^4 x_{23}^4 x_{34}^4 (A_{1234}^+ + A_{3214}^+ + A_{1234}^- + A_{3214}^-). \end{aligned} \quad (46)$$

⁸The exchange AdS graphs are reduced to the contact ones by using the technique developed in ref. [39].

Now we recall that all D -functions appearing in (45) can be expressed as derivatives of D_{2222} with respect to x_{ij}^2 . On the other hand, the function D_{2222} itself is given by

$$D_{2222} = \frac{\pi^3}{2x_{12}^2 x_{13}^2 x_{24}^2 x_{34}^2} (s\partial_s)^2 \Phi(s, t), \quad (47)$$

where the function $\Phi(s, t)$, introduced in ref. [41], admits the following explicit representation in terms of logarithms and dilogarithms:

$$\Phi(s, t) = \frac{1}{\lambda} \left(2(\text{Li}_2(-\rho s) + \text{Li}_2(-\rho t)) + \ln \frac{t}{s} \ln \frac{1 + \rho t}{1 + \rho s} + \ln(\rho s) \ln(\rho t) + \frac{\pi^2}{3} \right). \quad (48)$$

In this way we therefore obtain a representation for the coefficients $A^{\pm,0}$ in terms of certain differential operators in the variables s, t acting on $\Phi(s, t)$, which is given in Appendix 3. Such a representation proves useful, since the derivatives $\partial_s \Phi(s, t)$ and $\partial_t \Phi(s, t)$ are again expressed in a simple manner via $\Phi(s, t)$. Using the formulae (70)-(72) together with (46), we have verified that the supergravity amplitude we found does indeed obey the superconformal Ward identities (8). According to our general considerations from Section 3, this means that a prepotential of the type (37) should exist.

At first sight, the problem of finding the prepotential corresponding to the supergravity solution (45) looks extremely complicated, because one needs to perform the integral (25) whose integrand involves $\Phi(s, t)$. To solve this problem we make the assumption that \mathcal{F} (25), written in terms of the variables s, t , has the structure $q_1(s, t)\Phi(s, t) + q_2(s, t)$, where q_1 and q_2 are two unknown symmetric functions. Then using the fact that the derivatives of Φ are again expressed via Φ and by trial and error we were able to find these unknown functions. The final answer is surprisingly simple

$$\mathcal{F}(s, t) = \frac{\lambda^3}{2N^3 st} (1 - s\partial_s)(1 - t\partial_t)(2 + s\partial_s + t\partial_t)(1 + s\partial_s + t\partial_t)(st\partial_{st})\Phi(s, t). \quad (49)$$

We can now directly verify that substituting (49) in eqs. (37) reproduces exactly the coefficients a_i of the four-point amplitude. To reproduce b_i from the prepotential (49) as well, we found that a particular value of the integration constant B is required, namely $B = \frac{1}{N^3}$. Since $\Phi(s, t)$ obeys the crossing symmetry relation $\Phi(s/t, 1/t) = t\Phi(s, t)$, one can prove that the same relation holds for the prepotential $\mathcal{F}(s, t)$, in accord with our previous considerations.

The form (49) suggests that it can be recast in terms of the so-called \bar{D} -functions that are defined in Appendix 3. Indeed, it is easy to see that the following formula holds

$$\mathcal{F}(s, t) = \frac{240}{N^3} \frac{\lambda^3}{st^2} \bar{D}_{7333}(s, t). \quad (50)$$

This time the crossing symmetry relations for the prepotential follow from the ones for the corresponding \bar{D} -function, eqs. (68) and (69) from Appendix 3. Such an elegant form of the prepotential suggests that there may exist a much simpler way of extracting it from the supergravity solution.

Thus we have completely unraveled the structure of the supergravity solution. It consists of the prepotential (50) supplemented with the following integration constants A, B :

$$A = 1, \quad B = \frac{1}{N^3}. \quad (51)$$

The comparison of this result with the one provided by the free theory has already been discussed in Section 2.3.

Finally, we establish a non-trivial relation between the $d = 6$ prepotential (49) and its four-dimensional analogue (15). The function $F(s, t)$ from the solution (15) can be written in a form similar to (49):

$$F(s, t) = -\frac{4}{N^2}(1 + s\partial_s + t\partial_t)(st\partial_{st})\Phi(s, t). \quad (52)$$

Therefore, comparing with (49) we obtain the following relation

$$\mathcal{F}(s, t) = \mathcal{D}_{s,t}F(s, t), \quad (53)$$

where

$$\mathcal{D}_{s,t} = -\frac{1}{8N} \frac{\lambda^3}{st} (1 - s\partial_s)(1 - t\partial_t)(2 + s\partial_s + t\partial_t)$$

is a symmetric third-order differential operator. One can easily check that it satisfies the commutation relation $\mathcal{D}_{s/t, 1/t} \cdot t = t \cdot \mathcal{D}_{s,t}$ which makes the crossing symmetry relation for $\mathcal{F}(s, t)$ obvious. Thus, we conclude that at large N the dynamical properties of the stress-tensor multiplet in (2,0) theory are inherited from those of the $d = 4$ $\mathcal{N} = 4$ theory.

The last observation suggests another non-trivial test of the original AdS/CFT duality conjecture for the $d = 4$ $\mathcal{N} = 4$ theory. In perturbation theory the $d = 4$ function $F(s, t)$ appears as a series $F(s, t) = \frac{1}{N^2}F_1(s, t; g) + \mathcal{O}\left(\frac{1}{N^4}\right)$, where $g = g_{YM}^2 N$ is the t'Hooft coupling. If we assume that $F_1(s, t; g)$ interpolates smoothly between the free theory ($g = 0$) and the theory dual to the corresponding supergravity ($g = \infty$), then formula (53) provides a smooth deformation connecting the free (2,0) theory and its supergravity dual. However the OPE of the 1/2 BPS operators in the free (2,0) theory contains the Konishi-like multiplets while the corresponding supergravity OPE does not. Therefore, the decoupling of the *protected* Konishi-type multiplets along this particular deformation flow induced by the $\mathcal{N} = 4$ theory should take place only due to the vanishing of their OPE coefficients. Here we should recall the $d = 4$ case, where the Konishi-like multiplets decouple because their conformal dimensions tends to infinity. Thus, the known one- and two-loop results for $F_1(s, t; g)$ could be analyzed to see whether under (53) the protected Konishi-type multiplets emerge or not. Actually, it would be very interesting to understand how the supermultiplets arising in the OPE of the $\ell = 2$ 1/2 BPS operators of the $\mathcal{N} = 4$ $d = 4$ theory are rearranged under (53), as well as to clarify the meaning of the operator $\mathcal{D}_{s,t}$ both on the AdS and the CFT sides.

5 Operator Product Expansion

In Section 2.1 we have already presented the general kinematical restrictions on the OPE content of two 1/2 BPS operators $\mathcal{D}(4;000;02)$. At the level of the four-point amplitude these restrictions are encoded in the solution (37) of the superconformal Ward identities. In principle, one should be able to obtain the conformal partial wave expansion of the four-point amplitude (37) with an arbitrary function $\mathcal{F}(s,t)$ and to restore all the information about the OPE which was obtained from solving the kinematical constraints.⁹ However, the presence of the second-order differential operator Δ in (37) complicates the analysis considerably and we have not yet found an easy way to do it. Therefore, in this paper we confine ourselves to the study of the conformal partial wave expansion of the particular supergravity amplitude (44).

The leading terms in the double OPE arising in the short-distance limit $x_1 \rightarrow x_2$, $x_3 \rightarrow x_4$ can be found as follows. First we project the four-point amplitude (44) on the different R symmetry channels (the necessary projectors are given in Appendix 2). Then we replace the D -functions by their series representation (with powers and logs). The series representation of an arbitrary D -function was worked out in detail ref. [14]. In particular, the short distance limit under consideration is naturally described in terms of the variables:

$$v = \frac{s}{t}, \quad Y = 1 - \frac{1}{t},$$

such that $v \rightarrow 0$, $Y \rightarrow 0$. The leading term

$$v^{\frac{\tau}{2}} F(Y)$$

in the conformal partial wave amplitude (CPWA) expansion of the four-point amplitude corresponds to the contributions of all operators of twist $\tau = \ell - s$. A logarithmic term of the form $v^{\frac{\tau}{2}} Y^s \ln v$ signals an anomalous dimensions for an operator of twist τ and spin s . In the sequel we will work out in detail *only the leading terms* of the conformal partial wave expansion for (44) for the singlet (unprotected) R symmetry channel and briefly comment on what we have found in the remaining (protected) channels.

5.1 Projection on the singlet

The projection of the connected part of the four-point supergravity amplitude on the R symmetry singlet channel can be schematically written in the form

$$\langle \mathcal{O} \cdots \mathcal{O} \rangle_{[00]} = \frac{\delta^{12} \delta^{34}}{N^3 x_{12}^8 x_{34}^8} \left[\frac{12}{175} v^2 F_2(Y) + \frac{12}{175} v^3 F_3(Y) + v^4 \log v G_4(Y) \right], \quad (54)$$

where the functions $F_2(Y)$ and $F_3(Y)$ coincide with the canonically normalized CPWA of a second-rank tensor with $\ell = 6$. In particular, the corresponding power series expansions start

⁹For the $d = 4$ $\mathcal{N} = 4$ theory the corresponding conformal partial wave analysis was performed in ref. [17].

as follows:

$$F_2(Y) = \frac{1}{4}Y^2 + \frac{1}{2}Y^3 + \dots, \quad F_3(Y) = -\frac{1}{6} - \frac{1}{4}Y + \dots.$$

This leading operator is nothing but the stress tensor. Since the $\log v$ term appears only at order v^4 , we conclude that the stress tensor keeps its canonical dimension. The function $G_4(Y) = -\frac{12}{7} + O(Y)$ which implies that the first operator receiving anomalous dimension ℓ_a of order N^{-3} is a scalar of approximate dimension $\ell = 8$. Taking into account the disconnected part of the supergravity amplitude, one finds

$$\ell_a = -\frac{24}{N^3}.$$

This perfectly agrees with the classifications of the UIRs presented in Section 2.1: The superconformal primary operator of canonical dimension $\ell = 8$ lies beyond the unitarity bound of the continuous series A and is allowed to acquire an anomalous dimension in a non-trivial interacting theory. With the help of the techniques developed in refs. [42, 19] the calculation of the anomalous dimension could be extended to the higher supermultiplets occurring in the R symmetry singlet channel. It is then of interest to see how these anomalous dimensions are related to those of the $\mathcal{N} = 4$ theory.

It is also instructive to make the comparison with the free theory of η (2,0) tensor multiplets. One tensor multiplet comprises five scalars, two Weyl fermions and a two-form with a self-dual field strength. Assuming the free form of the propagator $\frac{\delta^{ij}\delta_{ab}}{\pi^3 x_{12}^4}$ for the scalars ϕ_a^i , where $i = 1, \dots, 5$ and $a = 1, \dots, \eta$, the canonically normalized BPS operator under consideration is of the form $\mathcal{O}^I = (2\eta)^{-1/2} \pi^3 C_{ij}^I : \phi_a^i \phi_a^j :$. The leading terms of the corresponding free OPE in the R symmetry singlet channel are

$$\mathcal{O}^{I_1}(x_1)\mathcal{O}^{I_2}(x_2) = \delta^{I_1 I_2} \left[\frac{1}{x_{12}^8} + \left(\frac{8}{5\eta} \right)^{1/2} [K] - \frac{4\pi^3}{5\eta} \frac{x_{12}^\mu x_{12}^\nu}{x_{12}^4} [T_{\mu\nu}^s] + \dots \right]$$

where $K = \pi^3(10\eta)^{-1/2} : \phi_a^i \phi_a^i :$ is a canonically normalized bilinear operator which we can call a ‘‘Konishi-type scalar’’ and

$$T_{\mu\nu}^s = \partial_\mu \phi_a^i \partial_\nu \phi_a^i - \frac{1}{5} \partial_\mu \partial_\nu (\phi_a^i \phi_a^i) - \frac{1}{10} \delta_{\mu\nu} (\partial_\rho \phi_a^i \partial_\rho \phi_a^i) \quad (55)$$

is the stress-tensor of 5η free scalars normalized as $\langle T^s T^s \rangle = \frac{6}{\pi^6} \eta$. The brackets $[\dots]$ denote the contribution of the conformal block of the corresponding primary operator. It is easy to see, however, that in the free theory the operator T^s can be written as a sum of three operators $T_{\mu\nu}$, $K_{\mu\nu}$ and $\Sigma_{\mu\nu}$ which belong to different supermultiplets:

$$T_{\mu\nu}^s = \frac{1}{14} T_{\mu\nu} + \frac{25}{42} K_{\mu\nu} + \frac{1}{3} \Sigma_{\mu\nu}. \quad (56)$$

The operators on the right-hand side are orthogonal to each other, i.e., the mixed two-point functions vanish. $T_{\mu\nu}$ is the stress tensor of η copies of the (2,0) theory, $K_{\mu\nu}$ belongs to the Konishi-type multiplet and $\Sigma_{\mu\nu}$ is the leading component of a new current multiplet.

In fact, the operators K and $\Sigma_{\mu\nu}$ are the first two operators from an infinite tower of Konishi-type currents arising in the singlet channel of the free OPE, all of them having twist $\tau = 4$. However, as we have shown above, the only operator of $\tau = 4$ contributing to the CPWA expansion of the supergravity four-point amplitude and thus to the OPE, is the stress tensor. Therefore, all the Konishi-type currents are absent in the supergravity OPE. Unlike the $d = 4$ $\mathcal{N} = 4$ theory, in $d = 6$ unitarity puts all of these currents in the isolated series B of UIRs, so they cannot develop an anomalous dimension in the interacting theory. We therefore arrive at our conclusion about the absence of a superconformal theory smoothly interpolating between the free theory and the one described by the supergravity dual. Apart from this, the free and the supergravity-dual (2,0) theories have exactly the same features as their counterparts in $d = 4$. In particular, the same type of splitting (56) for T^s occurs in $d = 4$ [16, 13, 14], which merely reflects the similar structure of the supersymmetry algebras in $d = 4$ and $d = 6$.

Finally, we comment once more on the relationship of our results with those obtained in refs. [32–34]. If we substitute the splitting (56) into the free OPE, then the coefficient in front of the stress tensor, which equals $C_{\mathcal{O}\mathcal{O}T}/C_T$, becomes $\frac{2\pi^3}{35\eta}$. Here $C_T = \langle TT \rangle$ is the coefficient of the two-point function of the stress tensor and $C_{\mathcal{O}\mathcal{O}T}$ is the normalization constant of the three-point function of two scalars \mathcal{O} with the stress tensor. According to ref. [34], one has $C_T = \frac{84}{\pi^6}\eta$ and, therefore, we get $C_{\mathcal{O}\mathcal{O}T} = \frac{24}{5\pi^3}$. The same value also follows from the conformal Ward identity relating the three-point function $\langle \mathcal{O}\mathcal{O}T \rangle$ to the two-point function $\langle \mathcal{O}\mathcal{O} \rangle$. Hence, the coefficient in front of the canonically normalized CPWA of the stress tensor in the CPWA expansion of the four-point amplitude turns out to be $C_{\mathcal{O}\mathcal{O}T}^2/C_T = \frac{48}{175\eta}$. If we want to match it with the supergravity result (54), i.e., with the value $\frac{12}{175N^3}$, we should choose the number of free multiplets to be $\eta = 4N^3$. This is of course a manifestation at the OPE level of the equality between the free and the supergravity dual integration constants A, B .

5.2 Projection on [02]

This projection gives

$$\langle \mathcal{O} \cdots \mathcal{O} \rangle_{[02]} = \frac{C_{\mathcal{J}_{[02]}}^{12} C_{\mathcal{J}_{[02]}}^{34}}{N^3 x_{12}^8 x_{34}^8} \left[\frac{27}{10} v^2 F_2(v) + \frac{27}{10} v^3 F_3(Y) + v^4 F_4(Y) + v^4 \log v G_4(Y) \right], \quad (57)$$

Here and in what follows $C_{\mathcal{J}}^{12}$ denote the orthonormal Clebsh-Gordon coefficient for an irrep \mathcal{J} appearing in the tensor product $[02] \times [02]$ (recall (4)). In (57) the functions

$$F_2(Y) = 1 + Y + \cdots, \quad F_3(Y) = \frac{2}{5} + \frac{3}{5}Y + \cdots$$

represent the contribution of the canonical CPWA of the $\ell = 4$ scalar which is nothing but the 1/2 BPS primary operator \mathcal{O} . We also find $F_4(Y) = \frac{97}{175} + O(Y)$, where the first term receives, in particular, a contribution from a scalar of free field dimension $\ell = 8$. On the other hand, the

function $G_4(Y)$ has the form

$$G_4(Y) = -\frac{27}{7}Y^2 + O(Y^3).$$

Since G_4 does not contain a constant term, we conclude that the scalar of free field dimension 8 transforming in the [02] does not receive corrections to its free field dimension. According to the classification of UIRs, this scalar gives rise to a semishort multiplet from the isolated series B. Recall that in the $d = 4$ case the corresponding operator is also a protected semishort multiplet, however, there unitarity puts it at the bound of the continuous series A. Its protection can be understood as a consequence of the three-point function selection rules. The first operator in (57) receiving an anomalous dimension is a second-rank tensor of approximate dimension 10.

5.3 Projection on [20]

This projection gives

$$\langle \mathcal{O} \cdots \mathcal{O} \rangle_{[20]} = \frac{C_{\mathcal{J}_{[20]}^{12}}^{12} C_{\mathcal{J}_{[20]}^{34}}^{34}}{N^3 x_{12}^8 x_{34}^8} \left[\frac{7}{10} v^2 F_2(v) + \frac{7}{10} v^3 F_3(Y) + v^4 \log v G_4(Y) \right],$$

where

$$F_2(Y) = Y + \frac{3}{2}Y^2 + \cdots, \quad F_3(Y) = \frac{3}{7}Y + \frac{6}{7}Y^2 + \cdots$$

are precisely the contributions of the CPWA of the $\ell = 5$ R symmetry current. The function $G_4(Y) = -8Y + O(Y^2)$, therefore the first operator in the [20] receiving an anomalous dimension is a vector of approximate dimension 9.

5.4 Projection on [40]

This projection gives

$$\langle \mathcal{O} \cdots \mathcal{O} \rangle_{[40]} = \frac{C_{\mathcal{J}_{[40]}^{12}}^{12} C_{\mathcal{J}_{[40]}^{34}}^{34}}{N^3 x_{12}^8 x_{34}^8} \left[v^4 F_4(v) + v^5 F_5(Y) + v^5 \log v G_5(Y) \right].$$

Therefore, all traceless symmetric rank- $2k$ tensors of twist 8 transforming in the [40] are non-renormalized. The explicit form of the function G_5 shows that the first operator acquiring an anomalous dimension is a scalar of approximate dimension 10.

5.5 Projection on [04]

This projection gives

$$\langle \mathcal{O} \cdots \mathcal{O} \rangle_{[04]} = \frac{C_{\mathcal{J}_{[04]}^{12}}^{12} C_{\mathcal{J}_{[04]}^{34}}^{34}}{N^3 x_{12}^8 x_{34}^8} \left[v^4 F_4(v) + v^5 F_5(Y) + v^6 \log v G_6(Y) \right].$$

Such a structure shows that all rank- $2k$ tensors of twists 8 and 10 are non-renormalized.¹⁰ The first operator receiving an anomalous dimension is a scalar of approximate dimension 12.

5.6 Projection on [22]

This projection gives

$$\langle \mathcal{O} \cdots \mathcal{O} \rangle_{[22]} = \frac{C_{\mathcal{J}_{[22]}}^{12} C_{\mathcal{J}_{[22]}}^{34}}{N^3 x_{12}^8 x_{34}^8} \left[v^4 F_4(v) + v^5 F_5(Y) + v^5 \log v G_5(Y) \right].$$

The function $F_4(v)$ comprises contributions of rank- $(2k+1)$ tensors of $\tau = 9$. Since the term $v^4 \log v$ is absent, all these tensors are non-renormalized. The first operator with anomalous dimension turns out to be a vector of approximate dimension 11.

This completes our OPE analysis. We see that there are several towers of traceless symmetric tensors in the irreps [40], [04] and [22] with vanishing anomalous dimensions. This is again in complete agreement with the classification of the UIRs, because the corresponding superconformal primary operators belong to the isolated series D, i.e., they are BPS short. Operators with anomalous dimensions in the first five channels of (4) are supersymmetry descendents of the superconformal primary operators in the R symmetry singlet. As to the OPEs in the supergravity regime, we conclude that the $d = 4$ and $d = 6$ theories have an identical structure though their relation with the free field limit is of completely different nature.

Acknowledgements

We are grateful to F. Bastianelli, S. Ferrara, P. Fré, S. Frolov, S. Minwalla, A. Petkou, K. Skenderis, S. Theisen and A. Tseytlin for useful discussions. G. A. would like to thank the CERN Theory Division, where part of this work was done, for the kind hospitality. G. A. was supported by the DFG and by the European Commission RTN programme HPRN-CT-2000-00131, and in part by RFBI grant N99-01-00166 and by INTAS-99-1782.

6 Appendix 1

For completeness here we recall the general solution of the $d = 4$ Ward identities [25]. The integrability condition for the system (7) reads

$$[s\partial_{ss} + t\partial_{tt} + (s+t-1)\partial_{st} + 2\partial_s + 2\partial_t] \left(\frac{a_1}{s} - \frac{a_3}{t} \right) = 0. \quad (58)$$

¹⁰Note that the channel [04] contains *two* towers of protected tensors. The same behavior occurs for the irrep [040] in the $d = 4$ theory [13].

This equation can be integrated to give

$$a_1(s, t) = \frac{s}{\lambda}[h(\rho s) - h(\rho t)] + sF(s, t), \quad (59)$$

$$a_3(s, t) = \frac{t}{\lambda}[h(\rho t) - h(\rho s)] + tF(s, t). \quad (60)$$

Here $F(s, t)$ is an *a priory* arbitrary symmetric function and h is a function of a single variable. In particular, the constant (free) solution $a_1 = a_3 = A$ corresponds to

$$h_0(\rho s) = \frac{A}{2} \left(\rho s + \frac{1}{\rho s} \right), \quad F_0(s, t) = \frac{A}{2} \frac{s+t}{st}. \quad (61)$$

Integrating the equation for $b_2(s, t)$ one gets

$$b_2(s, t) = B - [h(\rho t) + h(\rho s)] + (1 - s - t)F(s, t). \quad (62)$$

Shifting the solution by the free values, $h \rightarrow h + h_0$, $F \rightarrow F + F_0$ leaves $b_2(s, t)$ unchanged. Separating the trivial solution and regarding the remaining h and F as independent functions, the crossing symmetry relation for the four-point amplitude takes the form (9) for F together with the following condition on h :

$$[h(\rho s) + h(\rho t)]_{s \rightarrow s/t, t \rightarrow 1/t} = \text{const}. \quad (63)$$

This completes our discussion of the $d = 4$ Ward identities.

7 Appendix 2

The matrices C_{ij}^I and $C_{m;l}^I$ introduced in eq. (41) are subject to the following normalization condition:

$$\sum_I C_{ij}^I C_{kl}^I = \frac{1}{2} \delta_{ik} \delta_{jl} + \frac{1}{2} \delta_{il} \delta_{jk} - \frac{1}{5} \delta_{ij} \delta_{kl}, \quad \sum_I C_{i;j}^I C_{k;l}^I = \frac{1}{2} (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{kj}). \quad (64)$$

The projectors $P_{1234}^{\mathcal{J}}$ projecting the four-point amplitude (5) on the contributions of different R symmetry irreps \mathcal{J} are constructed by using the technique of ref. [13]. We find the following formulae:

$$\begin{aligned} P_{1234}^{[00]} &= \frac{1}{196} \delta_{12} \delta_{34}, \\ P_{1234}^{[02]} &= \frac{10}{189} \left(C_{1234}^+ - \frac{1}{5} \delta_{12} \delta_{34} \right), \\ P_{1234}^{[20]} &= -\frac{2}{35} C_{1234}^-, \\ P_{1234}^{[40]} &= \frac{1}{105} \left(\delta_{13} \delta_{24} + \delta_{14} \delta_{23} + \frac{1}{6} \delta_{12} \delta_{34} - 2C_{1324} - \frac{4}{3} C_{1234}^+ \right), \\ P_{1234}^{[04]} &= \frac{1}{330} \left(\delta_{13} \delta_{24} + \delta_{14} \delta_{23} + \frac{8}{63} \delta_{12} \delta_{34} + 4C_{1324} - \frac{16}{9} C_{1234}^+ \right), \\ P_{1234}^{[22]} &= \frac{1}{162} \left(\delta_{13} \delta_{24} - \delta_{14} \delta_{23} + \frac{8}{7} C_{1234}^- \right). \end{aligned}$$

In particular, the projectors are normalized to obey the condition $(P_D)^2 = 1/\nu_{\mathcal{J}}$, where $\nu_{\mathcal{J}}$ is the dimension of the representation \mathcal{J} :

$$\dim[00] = 1, \quad \dim[02] = 14, \quad \dim[20] = 10, \quad \dim[04] = 55, \quad \dim[40] = 35, \quad \dim[22] = 81.$$

When working out the action of the projection operators on the four-point amplitude (5), the following contractions prove helpful:

$$\begin{aligned} C_{1234}C_{1234} &= \frac{3199}{50}, & C_{1234}^+C_{1234}^+ &= \frac{6671}{200}, & C_{1234}^+C_{1324} &= \frac{273}{100}, \\ C_{2134}C_{1234} &= \frac{273}{100}, & C_{1234}^-C_{1234}^- &= \frac{245}{8}, & C_{1234}^-C_{1324} &= 0, \\ C_{1122} &= \frac{196}{5}, & C_{1212} &= \frac{21}{5}. \end{aligned} \tag{65}$$

8 Appendix 3

The D -functions related to the space AdS_7 can be defined by the formula

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(x_1, x_2, x_3, x_4) = \int \frac{d^6 w dw_0}{w_0^7} \prod_i K_{\Delta_i}(x_i, w), \tag{66}$$

where $K_{\Delta}(x, w) = \left(\frac{w_0}{w_0^2 + (\vec{w} - x)^2}\right)^{\Delta}$ and the integral is taken over the seven-dimensional space parametrized by $w = (w_0, \vec{w})$, \vec{w} being a six-dimensional vector.

We also define the \bar{D} -functions [13] for dimension d which are viewed here as functions of the conformal cross-ratios s, t

$$\begin{aligned} D_{\Delta_1\Delta_2\Delta_3\Delta_4}(x_1, x_2, x_3, x_4) &= \\ &= \frac{\pi^{\frac{d}{2}} \bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}(s, t)}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2-\Delta_3-\Delta_4}{2}} (x_{13}^2)^{\frac{\Delta_1+\Delta_3-\Delta_2-\Delta_4}{2}} (x_{23}^2)^{\frac{\Delta_2+\Delta_3+\Delta_4-\Delta_1}{2}} (x_{14}^2)^{\Delta_4}}. \end{aligned} \tag{67}$$

These functions have the following transformation properties

$$\bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}(t, s) = \left(\frac{t}{s}\right)^{\frac{1}{2}(\Delta_1-\Delta_2-\Delta_3-\Delta_4)} \bar{D}_{\Delta_1\Delta_4\Delta_3\Delta_2}(s, t), \tag{68}$$

$$\bar{D}_{\Delta_1\Delta_2\Delta_3\Delta_4}\left(\frac{s}{t}, \frac{1}{t}\right) = t^{\frac{1}{2}(\Delta_1-\Delta_2-\Delta_3-\Delta_4)} \bar{D}_{\Delta_1\Delta_2\Delta_4\Delta_3}(s, t), \tag{69}$$

which can be easily proven by using, e.g., the Feynman parameter representation.

We find the following representations for the coefficients $A^{\pm,0}$ of the four-point amplitude in

terms of differential operators acting on $\Phi(s, t)$:

$$\begin{aligned}
A_{1234}^0 &= \frac{\pi^3}{(x_{12}^2 x_{34}^2)^3 x_{13}^2 x_{24}^2} \left[-\frac{1}{2^3 \cdot 3^3} (2 - s\partial_s)(1 - s\partial_s)(s\partial_s)^2 \right. \\
&+ \frac{1}{2^5 \cdot 3^3} \left(\frac{1}{s} + \frac{t}{s} - \frac{25}{4} \right) (2 - s\partial_s)(1 - s\partial_s)^2 (s\partial_s)^2 \\
&+ \frac{1}{2^5 \cdot 3^3} \left(\frac{1}{s} + \frac{t}{s} + \frac{31}{4} \right) (2 - s\partial_s)^2 (1 - s\partial_s)^2 (s\partial_s)^2 \\
&\left. + \frac{5}{2^7 \cdot 3^3} \left(\frac{1}{s} + \frac{t}{s} - 1 \right) (3 - s\partial_s)(2 - s\partial_s)^2 (1 - s\partial_s)^2 (s\partial_s)^2 \right] \Phi(s, t), \tag{70}
\end{aligned}$$

$$\begin{aligned}
A_{1234}^+ &= \frac{\pi^3}{(x_{12}^2 x_{34}^2)^3 x_{13}^2 x_{24}^2} \left[\frac{1}{2^3 \cdot 3^2} (2 - s\partial_s)(1 - s\partial_s)(s\partial_s)^2 \right. \\
&+ \frac{1}{2^4 \cdot 3} (2 - s\partial_s)(1 - s\partial_s)^2 (s\partial_s)^2 - \frac{7}{2^3 \cdot 3^3} (2 - s\partial_s)^2 (1 - s\partial_s)^2 (s\partial_s)^2 \\
&\left. + \frac{5}{2^4 \cdot 3^3} (3 - s\partial_s)(2 - s\partial_s)^2 (1 - s\partial_s)^2 (s\partial_s)^2 \right] \Phi(s, t), \tag{71}
\end{aligned}$$

$$\begin{aligned}
A_{1234}^- &= \frac{\pi^3}{(x_{12}^2 x_{34}^2)^3 x_{13}^2 x_{24}^2} \left[\frac{1}{2^2 \cdot 3^3} \left(\frac{t}{s} - \frac{1}{s} \right) (2 - s\partial_s)^2 (1 - s\partial_s)^2 (s\partial_s)^2 \right. \\
&+ \frac{5}{2^4 \cdot 3^2} (1 + s\partial_s + 2t\partial_t)(1 - s\partial_s)^2 (s\partial_s)^2 \\
&\left. + \frac{1}{2^4 \cdot 3^2} \left(\frac{t}{s} - \frac{1}{s} \right) (1 - s\partial_s)^2 (s\partial_s)^2 + \frac{1}{2^3 \cdot 3^2} (1 + s\partial_s + 2t\partial_t)(s\partial_s)^2 \right] \Phi(s, t). \tag{72}
\end{aligned}$$

Further simplification is achieved by successive use of the identities [25]

$$\begin{aligned}
\partial_s \Phi(s, t) &= \frac{1}{\lambda^2} \left(\Phi(s, t)(1 - s + t) + 2 \ln s - \frac{s + t - 1}{s} \ln t \right), \\
\partial_t \Phi(s, t) &= \frac{1}{\lambda^2} \left(\Phi(s, t)(1 - t + s) + 2 \ln t - \frac{s + t - 1}{t} \ln s \right).
\end{aligned}$$

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