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ON EXCEPTIONAL NON-RENORMALIZATION PROPERTIES OF $\mathcal{N} = 4$ SYM$_4$

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Abstract

We discuss non-renormalization properties of some composite operators in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory.

Recently considerable attention was attracted to the $\mathcal{N} = 4$ super Yang-Mills theory basically due to the prominent role it plays among the models realizing the holographic AdS/CFT duality [1]. In the superconformal phase the dynamics of the gauge theory is encoded in the correlation functions of the composite gauge invariant operators, which might exhibit in general a non-trivial behavior under the RG flow. In particular it is of great interest to determine the 4-point correlation functions (both holographic and weak coupling) of the $\mathcal{N} = 4$ supercurrent (stress-tensor) multiplet L and its OPE; the latter contains an information about many other composite operators present in the theory.

A superconformal primary operator generating L is a scalar O^I of dimension 2 transforming in the irrep **20** of the R-symmetry group $SU(4)$, $I = 1, \ldots, 20$. Presently both the holographic [2] and the weak coupling [3] 4-point correlators of O^I and their OPE studies [4, 5, 6] are available.[1] Surprisingly composite operators[2] with vanishing anomalous dimensions were found [4] though naively unitarity allows the latter to appear in quantum interacting theory.

This note is based on the paper [6] and reviews a statement that the OPE of two primary operators from the multiplet L can contain superconformal

[1]For studies of other correlation functions from stress-tensor multiplet see e.g. [7].

[2]They saturate the bound of the so-called series A) of unitary irreps of $SU(2,2|4)$ and transform non-trivially under R-symmetry [8].

primary operators with a non-vanishing anomalous dimension *only* in the singlet of $SU(4)$.

It was found non-perturbatively [9] that the "quantum" part of the four-point function of O^I comprising all possible quantum corrections to the free-field result is given by a *single* function $F(v,u)$ of conformal cross-ratios, which we choose to be $v = \frac{x_{12}^2 x_{34}^2}{x_{14}^2 x_{23}^2}$ and $u = 1 - \frac{x_{13}^2 x_{24}^2}{x_{14}^2 x_{23}^2}$. Under $SU(4)$ the product of two O^I decomposes as $\mathbf{20}\times\mathbf{20} = \mathbf{1}+\mathbf{20}+\mathbf{105}+\mathbf{84}+\mathbf{15}+\mathbf{175}$. The "quantum" part of the four-point function of the operators O^I projected on different irreps is

$$\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle_i = \frac{1}{x_{12}^4 x_{34}^4} P_i(v,u) \frac{vF(v,u)}{(1-u)^2} \, , \tag{1}$$

where $P_i(v,u)$ are certain polynomials [4, 5]. Every irrep i of $SU(4)$ in the OPE of two O^I represents a contribution from an infinite tower of operators $O^i_{\Delta,l}$, where Δ is the conformal dimension of the operator, l is its Lorentz spin. The corresponding contribution to the four-point function can then be represented as an expansion of the type

$$\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle_i = \sum_{\Delta,l} a^i_{\Delta,l} \mathcal{H}_{\Delta,l}(x_{1,2,3,4}) \, . \tag{2}$$

Here $\mathcal{H}_{\Delta,l}(x_{1,2,3,4})$ denotes the (canonically normalized) Conformal Partial Wave Amplitude (CPWA) for the exchange of an operator $O^i_{\Delta,l}$ and $a^i_{\Delta,l}$ is a normalization constant. We treat the CPWA as a double series of the type

$$\mathcal{H}_{\Delta,l} = \frac{1}{x_{12}^4 \, x_{34}^4} \, v^{\frac{h}{2}} \sum_{n,m=0}^{\infty} c^{\Delta,l}_{nm} v^n u^m \, , \tag{3}$$

where the dimension Δ was split into a canonical part Δ_0 and an anomalous part h: $\Delta = \Delta_0 + h$. Assigning the grading parameter $T = 2n+m$ to the monomial $v^n u^m$ one can show that the monomials in (3) with the lowest value of T have $T = \Delta_0$, where Δ_0 is the canonical (free-field) dimension of the corresponding operator.

Comparing (1) and (2) one finds, within every fractional power $v^{\frac{h}{2}}$, the following compatibility conditions

$$P_i \sum_{\Delta,l} a^j_{\Delta,l} \mathcal{H}_{\Delta,l}(x_{1,2,3,4}) = P_j \sum_{\Delta,l} a^i_{\Delta,l} \mathcal{H}_{\Delta,l}(x_{1,2,3,4}) \tag{4}$$

which hold for all pairs. Here the sums are taken over operators which have the same h. Thus, eqs. (4) imply non-trivial relations between the CPWAs of primary operators belonging to the same supersymmetry multiplet(s) with anomalous dimension h. Only one of these primary operators is the superconformal primary operator, i.e., it generates under supersymmetry the whole multiplet, while the others are its descendents.

Now we see that a superconformal primary operator appears only in the singlet of $SU(4)$. Indeed, let us choose in (4) the irrep j to be the singlet. The polynomial $P_{\mathbf{1}}$ is distinguished from the other P_i's by the presence of a constant term. Suppose that a superconformal primary operator with a canonical dimension Δ_0 contributes to the OPE and transforms in some irrep i which is not a singlet. Due to the constant in $P_{\mathbf{1}}$, the lowest-order monomials on the r.h.s. of (4) would have $T = 2n+m = \Delta_0$. Clearly, all the other P_i's always raise the T-grading by at least unity. The lowest dimension operator with canonical dimension Δ_0' in the singlet would have the lowest terms with at least $T = \Delta_0 - 1$ (or lower) to saturate (4). Hence, Δ_0' is always lower then Δ_0, and therefore the corresponding operator cannot be a supersymmetry descendent of an operator in the irrep i. This shows that anomalous superconformal primary operators are occure in the singlet of the R-symmetry group.

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