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Brane-World charges

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ABSTRACT

As opposed to usual Einstein gravity in four dimensions, the Brane-World scenario allows the construction of a *local* density of gravitational energy (and also of momentum, of angular momentum, etc...). This is a direct consequence of the hypothesis that our universe is located at the boundary of a five-dimensional diffeomorphism invariant manifold.
We compute these Brane-World densities of charge using the Lanczos-Israel boundary conditions. To proceed, we implement an explicitely covariant generalization of the Hamiltonian procedure of Regge and Teitelboim given in a previous work.
We finally study two simple Brane-World examples.

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1 Introduction

The idea that our four-dimensional universe could be the boundary of a five-dimensional spacetime has been recently revived as an alternative to Kaluza-Klein compactification [1] (for similar suggestions, see also [2]). The classical theory in the bulk is nothing but ordinary Einstein gravity. A Brane-World is then located at one boundary (say at $r = 0$) and satisfies the Lanczos-Israel [3] junction conditions. Our results will be independent on what is located on the other side, say at $r = L$ or $r = \infty$.

As opposed to Kaluza-Klein prescription, gravity is allowed to propagate in the extra-dimension in the Brane-World scenarios. Therefore, the effective gravitational equations of motion are not the Einstein ones in one dimension less but the modified version derived in [4]. Another big difference concerns the definition of conserved charges associated with diffeomorphisms, and in particular the definition of energy. The main purpose of this manuscript is to study this problem in detail.

The "basic rule" for conserved charges in gauge invariant theories is the following: A conserved charge associated with a *gauge* symmetry in D dimensions behaves like a conserved charge associated with a *global* symmetry in $(D-1)$ dimensions. Let us make this more precise.

A global symmetry produces a well defined density of charge in a D-dimensional spacetime through the Noether construction. This density of charge is in fact given by the pullback of the corresponding Noether current on a Cauchy hypersurface Σ_t, namely $J^\mu t_\mu$ (with t_μ the normal to Σ_t).

For a gauge symmetry, this is not anymore true in general. In fact, the associated Noether current is generically not gauge invariant and then not well-defined locally. On the other hand, a gauge symmetry generates a local density of charge at *each boundary* (denoted by $\mathcal{H}_r$) of the D-dimensional spacetime. The Noether current is then replaced by a two index tensor, called *superpotential* $U^{\mu\nu} = -U^{\nu\mu}$. This tensor is in general only well-defined (that is covariant) at the boundary $\mathcal{H}_r$ considered. The density of charge (at a fixed time) is then given by the pullback of this superpotential on the closed manifold $\mathcal{B}_r = \mathcal{H}_r \cap \Sigma_t$, namely $U^{\mu\nu} t_\mu n_\nu$, with n_μ the normal to $\mathcal{H}_r$. The example of general relativity is well-known: we cannot give a well defined local density of energy in the bulk (the so-called energy-momentum pseudo-tensors are not covariant) but only at spatial infinity, the ADM mass.

Suppose now that the bulk spacetime has more boundaries, for instance an "isolated horizon" [5] or a Brane-World. Then, analogously to the ADM mass at spatial infinity, the bulk diffeomorphism symmetry allows the con-

struction of a local density of mass at *each* of these new boundaries in a completely independent way[2]. We can then talk about "holographic charges" [6].

For concrete calculations of these superpotentials, a precise knowledge of the boundary conditions is needed. For the brane-boundary example we are interested in, these are the Lanczos-Israel junction conditions. We can then compute the superpotential associated with the five-dimensional diffeomorphism invariance on this Brane-World[3]. Following the above "basic rule", the energy is now a *local density* in our four-dimensional boundary-universe. We compute a general expression for this density of energy[4] in any dimension D in section 4. This is the main result of this manuscript.

Note finally that some aspects of the Brane-World problem were studied by relativists in one dimension less. In fact, the situation is completely analogous to the case of an infinitely thin shell of dust embedded in a four-dimensional spacetime. Then, the results presented here could be also useful in this context.

In section 2, we recall the construction of conserved charges associated with gauge symmetries. In section 3 we fix the conventions and the notations for a D-dimensional gravity bounded by a Brane-World. We emphasize on the known relation between the Lanczos-Israel conditions and the variational principle. The charges due to the D-dimensional diffeomorphism invariance are computed in section 4. The final result namely equation (38) (together with (39)) is then discussed. The purpose of section 5 is to check if this result is modified by the introduction of scalar fields in the bulk, with first Dirichlet (5.1) and then Neumann (5.2) boundary conditions. We finish in section 6 with two simple Brane-World examples where our derived formula for the energy can be easily compared with the Hamiltonian result. With both method, we find that the *energy vanishes*.

Part of this work is also presented in [7].

[2]The notion of a local black hole mass computed on an isolated horizon is for instance defined in [5].

[3]As well as an ADM mass if the five-dimensional spacetime is also asymptotically flat at $r = \infty$. However this mass cannot be measured since we are not allowed to escape from the Brane-World.

[4]... and for all the other charges associated with the diffeomorphism symmetry, as for instance momentum or angular momentum.

2 Conserved charges in gauge theories

An Action invariant under one global symmetry produces one conserved charge. This charge is given by the integral of the Noether current on a Cauchy hypersurface Σ_t.

Assume now that the Lagrangian is invariant under a gauge (that is local) symmetry. Let us also denote by $\mathcal{B}_r$, $r = \{1, \ldots, n\}$ all the disconnected components of the boundary of a (partial) Cauchy hypersurface, namely $\partial\Sigma_t = \sum_{r=1}^{n} \mathcal{B}_r$. Then the conserved charges associated with the gauge symmetry (see [8, 9, 10, 6, 11] and references therein):

- can be computed on each $\mathcal{B}_r$ in a completely independent way by the formula,
$$Q_\xi = \int_{\mathcal{B}_r} U_\xi^{\mu\nu} d\Sigma_{\mu\nu}, \tag{1}$$
where $d\Sigma_{\mu\nu}$ is the volume element on $\mathcal{B}_r$;

- depend strongly on the boundary conditions imposed on $\mathcal{B}_r$.

The density $U_\xi^{\mu\nu}$, called superpotential, is antisymmetric in its upper indices and depends explicitely on the gauge parameter $\xi^a(x)$. We now recall the construction of $U_\xi^{\mu\nu}$.

The general "recipe" goes as follows [9]: First, let us suppose that the gauge symmetry transformation laws of the fields[5] φ^i can be written as:

$$\delta_\xi \varphi^i = \partial_\mu \xi^a \Delta_a^{\mu i} + \xi^a \Delta_a^i. \tag{2}$$

We assume that these transformation laws contain no terms proportional to higher derivatives of the local gauge parameter $\xi^a(x)$. Naturally, the index ^{a} labels the set of gauge symmetries. The quantities $\Delta_a^{\mu i}(\varphi)$ and $\Delta_a^i(\varphi)$ given by the gauge symmetry considered are functionals of the fields (and their first derivatives).

The next step is then to construct the following tensor

$$W_\xi^\mu := \xi^a \Delta_a^{\mu i} \frac{\delta \mathcal{L}}{\delta \varphi^i} \tag{3}$$

where we used the definition (2) and the last term refers to the equations of motion of φ^i.

[5] The index ^{i} labels the set of all fields (even auxiliary) present in the Lagrangian.

Now comes a very important point: We assume that our theory has been rewritten in a *first order* form. That means, we require that both, the equations of motion $\frac{\delta\mathcal{L}}{\delta\varphi^i}$ and the transformation laws $\delta_\xi\varphi^i$ (2), to depend on the fields and at most their first derivatives[6]. We require these restrictions to give rigorous and general proofs to support the method proposed in [9]. For more details, see also [10, 11].

Then, an *arbitrary* variation of the superpotential (which defines the conserved charge through (1)) should satisfy:

$$\delta U_\xi^{\mu\nu} = -\delta\varphi^i \frac{\partial W_\xi^\mu}{\partial\partial_\nu\varphi^i} \tag{4}$$

The last step is then to "integrate" equation (4) *using the boundary conditions* on $\mathcal{B}_r$. By "integrate" we mean: using the imposed boundary conditions, we should be able to rewrite the rhs of (4) as $\delta(\textit{something})$.

The charge (1), with $U_\xi^{\mu\nu}$ satisfying (4), will be *conserved* if the boundary conditions on $\mathcal{B}_r$ are compatible with some variational principle [9, 10, 11].

The simplest examples of Yang-Mills, p-forms or Chern-Simons theories can be found in [9, 10]. The cases of gravity and supergravity at spatial infinity are treated in [6] and [12] respectively. The purpose of this manuscript is to study gravity bounded by a Brane-World.

For another approaches to compute superpotentials that do not emphasize the boundary conditions see [13, 14, 15]. For related works and references on conservation laws in field theories, see also [16].

3 Pure gravity bounded by a Brane-World

We consider a $(D-2)$-brane located at the boundary of a D-dimensional spacetime. In vacuum, with signature mostly plus, we start with the following action[7]:

$$\mathcal{S} = \int_{\mathcal{M}} \mathcal{L} := \int_{\mathcal{M}} \left(\frac{\sqrt{|g|}}{4\kappa^2} R - \sqrt{|g|}\Lambda - \partial_\mu S^\mu - \partial_\mu \left(\frac{n^\mu}{N^2} \mathcal{L}_{bra} \delta(\chi) \right) \right) \tag{5}$$

where

[6] That is, we assume that $\frac{\delta\mathcal{L}}{\delta\varphi^i}$ and $\delta_\xi\varphi^i$ *do not depend* on $\partial^2\varphi$, $\partial^3\varphi$, etc... In general, some auxiliary fields are needed to construct these *first order* formalisms.

[7] We use the convention $4\kappa^2 = 16\pi G$.

$$S^\mu := \frac{\sqrt{|g|}}{4\kappa^2}(\Gamma^\mu_{\rho\sigma} g^{\rho\sigma} - \Gamma^\sigma_{\rho\sigma} g^{\rho\mu}). \tag{6}$$

The Lagrangian (5) is pure gravity in D dimensions, with some special boundary terms which take into account the presence of the brane. We discuss this point in detail below.

The brane is a $(D-1)$-dimensional hypersurface (which is assumed time-like) embedded in the D-dimensional spacetime through the constraint:

$$\chi(x) = 0, \tag{7}$$

for some given bulk function χ.

The (spacelike) *in-going* vector normal to this brane is given by:

$$n_\mu := \partial_\mu \chi, \quad N := \sqrt{g^{\mu\nu} n_\mu n_\nu}, \quad \hat{n}_\mu := \frac{n_\mu}{N}. \tag{8}$$

Therefore, the Stokes theorem gives

$$\int_{\mathcal{M}} \partial_\mu \left(\Sigma^\mu \delta(\chi)\right) = -\int_{brane} n_\mu \Sigma^\mu. \tag{9}$$

In particular, the last term of equation (5) is nothing but $\int_{brane} \mathcal{L}_{bra}$. In the following, we will also assume that $\mathcal{L}_{bra}$ depends on the induced metric $h_{\mu\nu} := g_{\mu\nu} - \hat{n}_\mu \hat{n}_\nu$ and eventually on some brane-fields.

For simplicity, we choose the transverse D^{th} coordinate r such that

$$r = \chi(x). \tag{10}$$

The brane is then located at $r = 0$ and $n_\mu = \delta^r_\mu$. The transverse direction can be either a usual interval $r \in [0, L]$ or a semi-infinite one, $r \in [0, \infty)$. The orbifold case, namely $r \in S^1/\mathbb{Z}_2$, is up to a factor of two, analogous to the ordinary interval case. This can be taken into account by the replacements $\mathcal{L}_{bra} \to \mathcal{L}_{bra}/2$ (and then $T^{bra}_{\rho\sigma} \to T^{bra}_{\rho\sigma}/2$) in the following formulae, and in particular in the main result (38). We explain this point in detail in the Appendix.

We work with a first order formalism (namely the Palatini one) where the metric $g^{\mu\nu}$ and the connection $\Gamma^\rho_{\mu\nu}$ are assumed to be *independent fields*. Then, the scalar curvature in (5) is a functional of the metric, of the connection and its first derivatives, namely, $R = R(g, \Gamma, \partial\Gamma)$. Note that $\mathcal{L} = \mathcal{L}(g, \partial g, \Gamma)$ of (5) is the so-called Einstein Lagrangian which does not

depend on $\partial\Gamma$ and differs from the Hilbert one (namely the scalar curvature) by the surface term (6).

The Euler-Lagrange variation of a total derivative vanishes. The surface terms in (5) then do not change the bulk equations of motion, which are given by:

$$\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} = \frac{\sqrt{|g|}}{4\kappa^2}G_{\mu\nu} + \frac{\sqrt{|g|}}{2}g_{\mu\nu}\Lambda \tag{11}$$

$$\frac{\delta\mathcal{L}}{\delta\Gamma^{\rho}_{\mu\nu}} = -\frac{1}{4\kappa^2}\nabla_\sigma\left(\sqrt{|g|}g^{\mu\nu}\delta^\sigma_\rho - \sqrt{|g|}g^{\sigma(\nu}\delta^{\mu)}_\rho\right). \tag{12}$$

where the Einstein tensor is as usual $G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. Note that for $D \geq 3$, the equation (12) is equivalent to $\nabla_\rho g^{\mu\nu} = 0$ which defines the connection in term of the metric and its first derivatives.

The surface term[8] $\partial_\mu S^\mu$ in (5) is needed in order to satisfy the variational principle and the junction conditions on the brane. In fact $\delta\mathcal{S} = 0 \Leftrightarrow (11) = (12) = 0$ is only true if the remaining boundary term vanishes. For the precise action (5) (and recalling equation (9)), this condition becomes:

$$\int_{brane}\left(-n_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu g^{\rho\sigma}}\delta g^{\rho\sigma} + \delta(\mathcal{L}_{bra})\right) = 0 \tag{13}$$

with

$$\frac{\partial\mathcal{L}}{\partial\partial_\mu g^{\rho\sigma}}\delta g^{\rho\sigma} = -\frac{1}{4\kappa^2}\left(\Gamma^\mu_{\rho\sigma}\delta(\sqrt{|g|}g^{\rho\sigma}) - \Gamma^\sigma_{\rho\sigma}\delta(\sqrt{|g|}g^{\mu\rho})\right) =: K^\mu_{\rho\sigma}\delta g^{\rho\sigma}, \tag{14}$$

and therefore,

$$K^\mu_{\rho\sigma} = -\frac{\sqrt{|g|}}{4\kappa^2}\left(\Gamma^\mu_{\rho\sigma} - \frac{1}{2}g_{\rho\sigma}\Gamma^\mu_{\alpha\beta}g^{\alpha\beta} - \Gamma^\alpha_{\alpha(\rho}\delta^\mu_{\sigma)} + \frac{1}{2}\Gamma^\alpha_{\beta\alpha}g^{\beta\mu}g_{\rho\sigma}\right). \tag{15}$$

Note that the equations (6) and (14), (15) imply the identities (to be used in the following section 4):

$$K^\mu_{\rho\sigma}g^{\rho\sigma} = \frac{D-2}{2}S^\mu, \tag{16}$$

$$K^\mu_{\rho\sigma}\delta g^{\rho\sigma} = -\delta S^\mu + \frac{\sqrt{|g|}}{4\kappa^2}g^{\rho\sigma}\left(\delta\Gamma^\mu_{\rho\sigma} - \delta\Gamma^\alpha_{\alpha\rho}\delta^\mu_\sigma\right). \tag{17}$$

[8]Together with equation (23), this surface term (6) is nothing but the Gibbons and Hawking [17] extrinsic curvature boundary term. In fact, using these equations we can check that $-n_\mu S^\mu = \sqrt{|h|}/(2\kappa^2)\mathcal{K}$, with $\mathcal{K} = h^{\mu\nu}\nabla_\mu\hat{n}_\nu$ the trace of the extrinsic curvature.

Let us now assume that the brane-matter Lagrangian $\mathcal{L}_{bra}$ depends on the pulled-back metric $(h_{\mu\nu} = g_{\mu\nu} - \hat{n}_\mu \hat{n}_\nu)|_{brane}$ (and possibly on other fields) but not on the connection. The condition (13) together with the definition (14) then implies two equations:

$$\int_{brane} \delta g^{\rho\sigma} \left(-n_\mu K^\mu_{\rho\sigma} - \frac{\sqrt{|g|}}{2} T^{bra}_{\rho\sigma} \right) = 0 \tag{18}$$

$$\int_{brane} \delta \mathcal{L}_{bra}|_{\delta g=0} = 0 \tag{19}$$

where

$$T^{bra}_{\rho\sigma} := -\frac{2}{\sqrt{|g|}} \frac{\delta \mathcal{L}_{bra}}{\delta g^{\rho\sigma}} \tag{20}$$

denotes the brane-matter energy-momentum tensor which satisfies $T^{bra}_{\rho\sigma} n^\sigma = 0$.

As we recall in the Appendix, the boundary equation (18) is equivalent to the Lanczos-Israel [3] junction condition:

$$-\frac{1}{2\kappa^2} \mathcal{K}_{\mu\nu} = T^{bra}_{\mu\nu} - \frac{1}{D-2} h_{\mu\nu} T^{bra} \tag{21}$$

with $\mathcal{K}_{\mu\nu} := h^\rho{}_{(\mu} h^\sigma{}_{\nu)} \nabla_\rho \hat{n}_\sigma$ the extrinsic curvature on the brane, $T^{bra} := h^{\rho\sigma} T^{bra}_{\rho\sigma}$ and $2\kappa^2 = 8\pi G$.

The relation between the variational principle (together with the boundary term (6)) and the junction condition (21) was first realized by Hayward and Louko [18] using the Hamiltonian formalism, for a shell of dust in four-dimensional gravity (for more recent work, see [19] and also [20, 21, 22] in the Brane-World context).

On the other hand, the equation (19) generates the equations of motion of the Brane-World fields.

4 The Brane-World charges

We assumed that the D-dimensional manifold is bounded by one brane. Following section 2, we can therefore compute the superpotential associated with the (D-dimensional) diffeomorphism invariance, at the boundary where this Brane-World is located.

With the Brane-World scenario in mind, we also assume that the D-dimensional metric is

$$ds^2 = e^{2A(r)} h_{ij}(x) dx^i dx^j + dr^2, \tag{22}$$

with the label i running from 0 to $(D-2)$ and $h_{ij}(x)$ some "World" metric.

Then, with the Brane-World ansätze (10) and (22), the normal vector (8) takes the simple form:

$$n_\mu = \hat{n}_\mu = \delta^r_\mu, \quad n^\mu = \hat{n}^\mu = \delta^{\mu r}, \quad N = 1. \tag{23}$$

The diffeomorphism symmetry parameter ξ^μ is not arbitrary but has to be compatible with the boundary conditions:

- First, it should leave the constraint (7) unchanged:

$$\mathcal{L}_\xi \chi = \xi^\mu \partial_\mu \chi = 0 \quad \textit{on the brane}, \tag{24}$$

with $\mathcal{L}_\xi$ the Lie derivative. Note that equation (24) together with (8) implies:

$$\xi^\mu n_\mu = 0, \quad \mathcal{L}_\xi \hat{n}_\mu = -\hat{n}_\mu \xi^\rho \partial_\rho (\ln N) \quad \textit{on the brane}. \tag{25}$$

Therefore, the diffeomorphism parameter ξ^μ should be tangent to the brane. For our simple ansätze (22) and (23), this becomes:

$$\xi^r = 0, \quad \mathcal{L}_\xi n_\mu = 0 \quad \textit{on the brane}. \tag{26}$$

- Second, the boundary condition (18) should be satisfied by the transformed fields $\tilde{\varphi}^i = \varphi^i + \mathcal{L}_\xi \varphi^i$, where φ^i goes for all the fields of our theory. This condition leads to:

$$\xi^\nu \partial_\nu \left(n_\mu K^\mu_{\rho\sigma} - T^{bra}_{\rho\sigma} \right) \Big|_{brane} = 0. \tag{27}$$

However, ξ^μ is tangent to the brane by equation (25). Therefore, the last condition (27) is automatically satisfied from the junction condition (18).

- Third, the ansatz (22) should be unchanged. This, together with (26), implies:

$$\mathcal{L}_\xi g_{\mu r} = 0 \Rightarrow -\partial_r \xi^\mu = \mathcal{L}_\xi n^\mu = 0. \tag{28}$$

This last condition could be relaxed in more general cases where the metric is not required to be of the form (22).

In the following, we assume that ξ^μ satisfies both (26) and (28).

The general variation of the superpotential associated with diffeomorphisms, for any D-dimensional pure gravity and for any boundary condition was given in [6] in differential forms (where calculations are easier). We translate here in components the steps of this calculation. There is however an important point: we cannot use in a straightforward way the method summarized in section 2. In fact, in the Palatini formalism, the transformation law (under a diffeomorphism) of the connection contains *second* derivatives of the gauge parameter, namely $\delta_\xi \Gamma = \partial^2 \xi + \textit{other}$. Therefore, it is not of the form (2). This technical problem is cured using the so-called Affine-$GL(D, \mathbb{R})$ gravity [8, 6] (see also the discussion in [7]). Then, we are not going to re-compute but *translate* to components the results of [6] for illustrative purposes.

In particular, the W^μ_ξ tensor of (3) is given by[9]:

$$W^\mu_\xi = -2\xi^\rho g^{\sigma\mu} \frac{\delta \mathcal{L}}{\delta g^{\rho\sigma}} + \nabla_\rho \xi^\sigma \frac{\delta \mathcal{L}}{\delta \Gamma^\sigma_{\mu\rho}}. \tag{29}$$

A very similar expression exists for the vielbein formalism (in its first order form), namely:

$$W^\mu_\xi = \xi^a \frac{\delta \mathcal{L}}{\delta e^a_\mu} + D_b \xi^a \frac{\delta \mathcal{L}}{\delta \boldsymbol{\omega}^a_{\mu b}} \tag{30}$$

where the internal Lorentz indices are denoted by Latin letters, and $\xi^a := \xi^\rho e^a_\rho$ and $D_b \xi^a := e^\rho_b (\partial_\rho \xi^a + \boldsymbol{\omega}^a_{\rho c} \xi^c)$.

The similarity between (29) and (30) can be explained by their common origin in the Affine-$GL(D, \mathbb{R})$ gravity. Again, following the results of the work [6], the variation of the superpotential due to the diffeomorphism invariance of the gravitational theory is given by,

$$\begin{aligned} \delta U^{\mu\nu}_\xi &= \frac{1}{4\kappa^2} \left(2\ \nabla_\sigma \xi^\tau \delta \left(\sqrt{|g|} g^{\sigma[\mu} \delta^{\nu]}_\tau \right) + 6\ \delta \left(\Gamma^\tau_{\rho\sigma} \right) \sqrt{|g|} g^{\sigma[\mu} \delta^\nu_\tau \xi^{\rho]} \right) && (31) \\ &= \frac{1}{4\kappa^2} \left(2\ D^a \xi^b \delta \left(|e|\, e^{[\mu}_a e^{\nu]}_b \right) + 6\ \delta \left(\omega^{ab}_\rho \right) |e|\, e^{[\mu}_a e^\nu_b \xi^{\rho]} \right) && (32) \end{aligned}$$

[9] The fact that the tensor (29) contains first derivatives of the gauge parameter ξ^μ while it does not in the definition (3) is not a contradiction. As we have mentioned, formula (29) cannot be used in the Palatini formalism due to the presence of second derivatives of the gauge parameter in the transformation law of the connection. The rigorous way to proceed is then to use the Affine-$GL(D, \mathbb{R})$ formalism [8, 6] which gives the result (29).

in the Palatini and vielbein formalisms respectively.

The equations (31) and (32) are valid when pulled-back on *any* $\mathcal{B}_r$ (and for *any* boundary condition compatible with the variational principle, see section 2). In particular, they were used in [6] to derive the KBL superpotential [23] at spatial infinity.

Now, assuming $\delta\xi^\rho = 0$ and using the identity (17), the equation (31) can be rearranged as[10]:

$$\delta U_\xi^{\mu\nu} = \delta\left({}^{Ko}U_\xi^{\mu\nu} + S^\mu\xi^\nu - S^\nu\xi^\mu \right) + \delta g^{\rho\sigma} K^\mu_{\rho\sigma}\xi^\nu - \delta g^{\rho\sigma} K^\nu_{\rho\sigma}\xi^\mu \tag{33}$$

where we have defined

$${}^{Ko}U_\xi^{\mu\nu} := \frac{\sqrt{|g|}}{4\kappa^2}(\nabla^\mu\xi^\nu - \nabla^\nu\xi^\mu), \tag{34}$$

for the (1/2) Komar superpotential [24].

For the Brane-World boundary, we contract equation (33) along t_μ and n_ν, with t_μ some timelike co-vector (tangent to the brane). With the condition (25), the equation (33) *on the brane* becomes:

$$\begin{aligned} \delta\left(U_\xi^{\mu\nu}\ t_\mu n_\nu\right) &= \delta\left({}^{Ko}U_\xi^{\mu\nu}\ t_\mu n_\nu - n_\nu S^\nu\ t_\mu\xi^\mu \right) - \delta g^{\rho\sigma} n_\nu K^\nu_{\rho\sigma}\ t_\mu\xi^\mu \\ &= \delta\left({}^{Ko}U_\xi^{\mu\nu}\ t_\mu n_\nu + \frac{\sqrt{|g|}}{D-2}T^{bra}\ t_\mu\xi^\mu - \mathcal{L}_{bra}\ t_\mu\xi^\mu \right) \end{aligned} \tag{35}$$

where we used equations (13), (14), (16) and (18).

Therefore, the superpotential is (up to some global constant):

$$U_\xi^{\mu\nu}\ t_\mu n_\nu = {}^{Ko}U_\xi^{\mu\nu}\ t_\mu n_\nu + \left(\frac{\sqrt{|g|}}{D-2}T^{bra} - \mathcal{L}_{bra}\right)\ t_\mu\xi^\mu + C^t. \tag{36}$$

The above number C^t can be fixed by imposing the vanishing of the superpotential for some reference solution. It just defines the zero point energy. We will set $C^t = 0$ in the following.

[10] Schematically, the calculation from equation (31) to equation (33) goes as follows: the first term of equation (31) plus two of the six $\delta\Gamma$'s gives $\delta({}^{Ko}U_\xi^{\mu\nu})$. The four remaining $\delta\Gamma$'s give $(\delta g^{\rho\sigma}K^\mu_{\rho\sigma} + \delta S^\mu)\xi^\nu - \mu \leftrightarrow \nu$ using the identity (17).

The expression (36) can be simplified. If fact, using again the conditions (26) and (28), the (1/2) Komar superpotential (34) along t_μ and n_ν can be rewritten as[11]

$$ {}^{Ko}U_\xi^{\mu\nu} t_\mu n_\nu := -\frac{\sqrt{|g|}}{2\kappa^2} t^\mu \xi^\nu \mathcal{K}_{\mu\nu}, \tag{37} $$

with $\mathcal{K}_{\mu\nu}$ the extrinsic curvature on the brane.

Finally, using once again the boundary conditions (21), the *total* superpotential (36) simplifies considerably:

$$ \begin{aligned} U_\xi^{\mu\nu}\ t_\mu n_\nu &= \sqrt{|g|} t^\mu \xi^\nu T_{\mu\nu}^{bra} - \mathcal{L}_{bra} t_\mu \xi^\mu \\ &= \sqrt{|g|} t^\mu \xi^\nu \hat{T}_{\mu\nu}^{bra}, \end{aligned} \tag{38} $$

where we have defined:

$$ \hat{T}_{\mu\nu}^{bra} := -2 \frac{\delta}{\delta g^{\mu\nu}} \left(\frac{\mathcal{L}_{bra}}{\sqrt{|g|}} \right). \tag{39} $$

The quantity (38) gives the density of charge at a fixed time depending on the vector ξ^μ used (for instance mass, momentum or angular momentum). The expression (39) is "almost" the energy momentum tensor on the brane (compare with (20)). However, the differences are quite important:

- The tensor (39) will *vanish* for any Brane-Lagrangian which depends on the metric only through an overall $\sqrt{|g|}$ factor. See for instance the examples of section 6.

- The expression (39) gives the total energy (and other charges as linear and angular momentum), including the *gravitational contribution*. In the Brane-World scenario, the (gravitational) energy is then a *local density*. The total conserved charge is simply the integral of (38) on a $(D-2)$-dimensional Cauchy hypersurface on the brane at fixed time (namely, on $\mathcal{B}_0$ in the notation of section 2).

- The positivity of energy would require some modified *energy condition*, namely $\hat{T}_{\mu\nu}^{bra} t^\mu \xi^\nu \geq 0$, with ξ^μ some timelike vector.

[11] If we relax the constraint (28), that is, we do not require the ansatz (22), a new term appears in the rhs of equation (37), namely $\sqrt{|g|}/(4\kappa^2) t_\mu \mathcal{L}_\xi n^\mu$. This could be relevant for more general situations.

Note also that the Nester superpotential [25] was used in previous works [26] in order to define the gravitational energy on a Brane-World (Domain Wall). However, this superpotential[12] is associated with *asymptotically flat* boundary conditions and not with the Lanczos-Israel ones (21).

It would also be interesting to compare our result (39) with the effective energy-momentum stress tensor derived (at the linearized level) in [27][13].

Finally, it should be possible to recover the result (38) using the Hamiltonian formalism together with the Regge and Teitelboim prescription [28].

5 The bulk scalar fields

Let us now introduce some scalar fields ϕ^I in the bulk. The corresponding action is

$$\mathcal{L}_{sc} := \sqrt{|g|}\left(-\Pi_I^\mu \partial_\mu \phi^I + \frac{1}{2} g_{\mu\nu}\ G^{IJ}(\phi)\Pi_I^\mu \Pi_J^\nu - V(\phi)\right), \tag{40}$$

with $G_{IJ}(\phi)$ some given moduli space metric. Remember that we are using exclusively first order formalisms; that is why we introduced the auxiliary fields Π_I^μ.

The equations of motion derived from (40) are:

$$\frac{\delta \mathcal{L}_{sc}}{\delta \phi^I} = \partial_\mu\left(\sqrt{|g|}\Pi_I^\mu\right) + \frac{\sqrt{|g|}}{2} g_{\mu\nu} G_{,I}^{JK} \Pi_J^\mu \Pi_K^\nu - \sqrt{|g|} V_{,I} \tag{41}$$

$$\frac{\delta \mathcal{L}_{sc}}{\delta \Pi_I^\mu} = \sqrt{|g|}\left(g_{\mu\nu} G^{IJ} \Pi_J^\nu - \partial_\mu \phi^I\right) \tag{42}$$

$$\frac{\delta \mathcal{L}_{sc}}{\delta g^{\mu\nu}} = -\frac{\sqrt{|g|}}{2} \Pi_{I\mu} G^{IJ} \Pi_{J\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{L}_{sc}. \tag{43}$$

We will study two kind of boundary conditions for the scalar fields:

1. The Dirichlet condition together with the assumption that the brane Lagrangian $\mathcal{L}_{bra}$ (of equation (5)) depends *neither* on the scalar fields ϕ^I *nor* on the auxiliary ones Π_I^μ.

2. The Neumann condition together with the assumption that $\mathcal{L}_{bra}$ can depend on the scalar fields ϕ^I (but *not* on Π_I^μ).

[12] The Nester superpotential is nothing but a covariant expression for the ADM mass. Moreover, it appears naturally in any supergravity in the commutation of two supercharges (also defined at *spatial infinity*), see for instance [12] and references therein.

[13] I thank S.B. Giddings for pointing out this suggestion.

5.1 The Dirichlet boundary conditions

We would like to impose some Dirichlet boundary conditions on the scalar fields, that is, $\phi^I = \phi_0^I =$*constant* on the brane. This can be implemented at the variational principle level by adding the following surface term to the action (40)

$$\partial_\mu \left(\sqrt{|g|} \Pi_I^\mu (\phi^I - \phi_0^I) \right), \tag{44}$$

which of course does not modify the equations of motion (41), (42) and (43).

The variational principle, namely $\delta \int ((40) + (44)) = 0 \Leftrightarrow (41) = (42) = (43) = 0$, is satisfied if and only if

$$- \int_{\text{brane}} \delta \left(\sqrt{|g|} n_\mu \Pi_I^\mu \right) (\phi^I - \phi_0^I) = 0 \tag{45}$$

which are our imposed Dirichlet boundary conditions.

The purpose is now to prove that the result (38) is not modified by the presence of the scalar Lagrangian (40)+(44).

Under one diffeomorphism, the scalar fields transform as:

$$\delta_\xi \phi^I = \xi^\rho \partial_\rho \phi^I \tag{46}$$

$$\delta_\xi \Pi_\mu^I = \xi^\rho \partial_\rho \Pi_\mu^I + \partial_\mu \xi^\rho \Pi_\rho^I \tag{47}$$

Then, the tensor W_ξ^μ of (29) receives a new contribution (see definition (3)), namely:

$${}_{sc}W_\xi^\mu = -2\xi^\rho g^{\sigma\mu} \frac{\delta \mathcal{L}_{sc}}{\delta g^{\rho\sigma}} - \Pi_I^\mu \xi^\rho \frac{\delta \mathcal{L}_{sc}}{\delta \Pi_I^\rho}. \tag{48}$$

Following equation (4) the variation of the superpotential (31) receives a "scalar fields" contribution:

$$\begin{aligned} \delta\ {}_{sc}U_\xi^{\mu\nu} &= 2\xi^\rho g^{\sigma\mu} \left(\frac{1}{2} g_{\rho\sigma} \sqrt{|g|} \Pi_I^\nu \right) \delta\phi^I - \Pi_I^\mu \xi^\nu \sqrt{|g|} \delta\phi^I \\ &= (\xi^\mu \Pi_I^\nu - \xi^\nu \Pi_I^\mu) \sqrt{|g|} \delta\phi^I. \end{aligned} \tag{49}$$

Again, the result (49) is valid for *any* boundary condition and then will also be used in the next subsection for Neumann boundary conditions. Note

that the manifest antisymmetry in μ and ν was guaranteed by a theorem [9].

As in the previous section 4, we pullback equation (49) on the brane and use the boundary conditions (45) (and the condition (25)) in order to "integrate" it:

$$\begin{aligned} \delta\left({}_{sc}U_{\xi}^{\mu\nu}\ t_{\mu}n_{\nu}\right) &= \sqrt{|g|}n_{\nu}\Pi_{I}^{\nu}\ t_{\mu}\xi^{\mu}\delta\phi^{I} \\ &= \delta\left(\sqrt{|g|}n_{\nu}\Pi_{I}^{\nu}\ t_{\mu}\xi^{\mu}\left(\phi^{I}-\phi_{0}^{I}\right)\right)=0. \end{aligned} \tag{50}$$

Then, the superpotential (38) is not modified by the scalar fields with Dirichlet boundary conditions (45). Note that the same is true for asymptotically flat spacetimes: The formula for the ADM mass at spatial infinity remains unchanged if some scalar fields with Dirichlet boundary conditions are included in the bulk.

5.2 The Neumann boundary conditions

Let us turn now to the case of Neumann boundary conditions on the scalar fields. We first add to the Lagrangian (40) the following surface term:

$$\partial_{\mu}\left(\Pi_{0I}^{\mu}\phi^{I}\right) \tag{51}$$

with Π_{0I}^{μ} some given constant (possibly zero).

We now assume that $\mathcal{L}_{bra}=\mathcal{L}_{bra}(\phi)$. This introduces a coupling with our pure gravity model of section 4. The variational principle associated with (5)+(40)+(51) is satisfied if and only if the following condition on the boundary (brane) holds:

$$\int_{brane}\left(-n_{\mu}K_{\rho\sigma}^{\mu}\delta g^{\rho\sigma}+\delta(\mathcal{L}_{bra})+\sqrt{|g|}n_{\mu}\Pi_{I}^{\mu}\delta\phi^{I}-\delta(n_{\mu}\Pi_{0I}^{\mu}\phi^{I})\right)=0. \tag{52}$$

The equation (52) implies then together with (18-19) another "junction equation" for the scalar fields on the brane:

$$\sqrt{|g|}n_{\mu}\Pi_{I}^{\mu}-n_{\mu}\Pi_{0I}^{\mu}=-\frac{\partial\mathcal{L}_{bra}}{\partial\phi^{I}}. \tag{53}$$

This is the usual Neumann boundary condition which fixes the normal derivative of the scalar fields. In many case the constant Π_{0I}^{μ} is set to zero. We will however consider the general case.

Now, the variation of the total superpotential is just the sum of (33) and (49). When pulled-back on the brane, this equation gives:

$$\begin{aligned}\delta\left({}_{tot}U_{\xi}^{\mu\nu}\ t_{\mu}n_{\nu}\right) &= \delta\left({}^{Ko}U_{\xi}^{\mu\nu}\ t_{\mu}n_{\nu} - n_{\nu}S^{\nu}\ t_{\mu}\xi^{\mu}\right) - \delta g^{\rho\sigma}n_{\nu}K_{\rho\sigma}^{\nu}\ t_{\mu}\xi^{\mu} \\ &\quad + \sqrt{|g|}n_{\nu}\Pi_{I}^{\nu}\ t_{\mu}\xi^{\mu}\delta\phi^{I}. \end{aligned} \tag{54}$$

Using the boundary condition (52), we find that:

$$ {}_{tot}U_{\xi}^{\mu\nu}\ t_{\mu}n_{\nu} = U_{\xi}^{\mu\nu}\ t_{\mu}n_{\nu} + n_{\nu}\Pi_{0I}^{\nu}\phi^{I}\ t_{\mu}\xi^{\mu} \tag{55}$$

with $U_{\xi}^{\mu\nu}$ given by (38). Then, the total superpotential is modified only if Π_{0I}^{ν} is non-zero.

6 Applications

The purpose of this last section is to illustrate the formula (38) with some Brane-World examples. For concreteness, we will start with the following five-dimensional metric (the extension to higher dimensions is straightforward):

$$ds^2 = e^{2A(r)}\eta_{ij}dx^i dx^j + dr^2, \tag{56}$$

with $\eta_{ij} = \{-,+,+,+\}$.

We mostly follow the conventions of [29] (the signature of the metric is, however, inverted). We then fix $\kappa^2 = 1$ and $\Lambda = 0$ in the Lagrangian (5).

We also include one bulk scalar field $\phi(r)$, with Neumann boundary conditions (53) together with $\Pi_{0I}^{\mu} = 0$ (see subsection 5.2). Finally, we choose

$$\mathcal{L}_{bra} = -\frac{\sqrt{|g|}}{2}\lambda(\phi). \tag{57}$$

The brane is located at $r = 0$. The r-direction is assumed to be $S^1/\mathbb{Z}_2$ instead of an ordinary interval. This explains the factor one-half in (57); see Appendix for more details.

The equations of motion (11), (12) and (41), (42) and (43), together with the Brane-World ansätze (56), (57), reduce to:

$$A'' = -\frac{2}{3}\phi'^2 \tag{58}$$

$$A'^2 = -\frac{1}{3}V(\phi) + \frac{1}{6}\phi'^2 \tag{59}$$

$$\frac{\partial V}{\partial \phi} = 4A'\phi' + \phi'' \tag{60}$$

where the prime goes for differentiation with respect to r.

The junction (or boundary) conditions (18) (or (21)) and (53) on the brane simplify to (see also [29]):

$$A' = -\frac{1}{3}\lambda \tag{61}$$

$$\phi' = \frac{1}{2}\frac{\partial \lambda}{\partial \phi}. \tag{62}$$

Now, it is straightforward to realize that the expression (38) (together with (39)) *vanishes*[14] with the ansatz (57):

$$U_\xi^{\mu\nu} t_\mu n_\nu = 0. \tag{63}$$

In particular, the energy vanishes. In previous works [30, 29, 31] the energy for the simple model (56-57) was defined as minus the Lagrangian (5)+(40), *without* the surface term (6). In the static configuration (56) we expect to find an agreement between the energy and $-\mathcal{L}$. This is indeed the case when this boundary term $\partial_\mu S^\mu$ (6) is properly taken into account. Using the equations (56-57) in (5)+(40), we find:

$$\begin{aligned} \mathcal{H} = -\mathcal{L} &= e^{4A}\left(5A'^2 + 2A'' + \frac{1}{2}\phi'^2 + V(\phi)\right) - \frac{1}{2}\partial_r\left(e^{4A}\lambda\right) \\ &\quad -2\partial_r\left(e^{4A}A'\right) \end{aligned} \tag{64}$$

In the second line of (64) we isolated the contribution of $\partial_\mu S^\mu$. In the first line we just recovered the proposal of [30, 29, 31] for the energy.

Now using the equations of motion (59) to replace $V(\phi)$ and then (58) to eliminate ϕ', it is easy to check that the first term in the rhs of (64) gives

[14] In fact, (38) vanishes for any ξ^μ.

$\partial_r(A'e^{4A})/2$. Then, using the boundary condition (61), we find that the total Hamiltonian $\mathcal{H}$ indeed vanishes on-shell, in agreement with (63). The vanishing of the energy in the supersymmetric extension was also pointed out in [32].

The inclusion of the boundary term[15] $\partial_\mu S^\mu$ (6) in the gravitational Lagrangian is quite important since it ensures that the variational principle is satisfied. This criterion allows us to fix the surface term ambiguity in the Lagrangian and cannot be neglected. The gravitational Lagrangian of a static solution will always be on-shell a boundary term. If this boundary term is not fixed by some appropriate criterion, as the variational principle, then the Lagrangian (and so the energy) becomes completely arbitrary.

We will finish with another example. The "Minkowski case" (56) can be generalized to the de Sitter one by using

$$dS_4 : h_{ij}dx^i dx^j = -dt^2 + e^{2\sqrt{\Lambda}t}(dx_1^2 + dx_2^2 + dx_3^2) \tag{65}$$

in the general ansatz (22).

Since the brane Lagrangian is still the simple expression (57), the equation (38) again predicts the vanishing of the energy (63). Now, this cannot be directly compared to minus the Lagrangian $-\mathcal{L}$ because the metric (65) is no longer static. In fact, a direct calculation gives (on-shell)

$$\begin{aligned} -\mathcal{L} &= \frac{1}{2}\partial_r\left((A' - \lambda(\phi))e^{4A+3\sqrt{\Lambda}t}\right) - \frac{3}{2}\bar{\Lambda}e^{4A+3\sqrt{\Lambda}t} \\ &\quad + \frac{3}{2}\partial_t\left(\sqrt{\bar{\Lambda}}e^{2A+3\sqrt{\Lambda}t}\right) - 2\partial_r\left(A'e^{4A+3\sqrt{\Lambda}t}\right) \end{aligned} \tag{66}$$

where both terms in the second line come from the additional surface term $\partial_\mu S^\mu$ (6).

Again, the total boundary term at $r = 0$ vanishes using the junction condition (61). Since the metric (65) is not static, the total Hamiltonian receives a non-vanishing contribution from the $\pi\dot{g}$ term. An explicit calculation gives:

$$\begin{aligned} \mathcal{H} &= \pi_{\mu\nu}\dot{g}^{\mu\nu} - \mathcal{L} \\ &= -3\bar{\Lambda}e^{4A+3\sqrt{\Lambda}t} - \mathcal{L} = 0 \end{aligned} \tag{67}$$

[15] Remember that the boundary term $\partial_\mu S^\mu$ is equivalent to the Gibbons and Hawking term using the ansatz (23).

where the canonical momenta is defined as usual by

$$\pi_{\mu\nu} := \frac{\partial \mathcal{L}}{\partial \dot{g}^{\mu\nu}} = \frac{\sqrt{|g|}}{4\kappa^2}\left(\tilde{\mathcal{K}}_{\mu\nu} - h_{\mu\nu}\tilde{\mathcal{K}}\right), \tag{68}$$

with now $\tilde{\mathcal{K}}_{\mu\nu} = \nabla_{(\mu}\hat{t}_{\nu)}$ (with $\hat{t}_\mu = \delta^0_\mu e^A$) the extrinsic curvature on a Cauchy hypersurface.

Then, the complete Hamiltonian (67) indeed vanishes, again in agreement with our straightforward result (63). A similar calculation can be repeated for the anti-de Sitter case.

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Appendix: The Lanczos-Israel equation from the variational principle

We recall in this appendix the equivalence between the "variational principle" equation (18) and the Lanczos-Israel junction conditions [3]. This result, first realized by Hayward and Louko [18], can also be found in [20, 21, 19, 22].

The starting point is equation (18) (together with equation (15)):

$$\frac{1}{2\kappa^2}\left(n_\mu \Gamma^\mu_{\rho\sigma} - \frac{1}{2} g_{\rho\sigma} n_\mu \Gamma^\mu_{\alpha\beta} g^{\alpha\beta} - \Gamma^\alpha_{\alpha(\rho} n_{\sigma)} + \frac{1}{2}\Gamma^\alpha_{\beta\alpha} n^\beta g_{\rho\sigma}\right) = T^{bra}_{\rho\sigma}. \tag{69}$$

If we contract equation (69) with $g^{\rho\sigma}$ and put it back into (69), we obtain:

$$\frac{1}{2\kappa^2}\left(n_\mu \Gamma^\mu_{\rho\sigma} - \Gamma^\alpha_{\alpha(\rho} n_{\sigma)}\right) = T^{bra}_{\rho\sigma} - \frac{1}{D-2} g_{\rho\sigma} T^{bra}. \tag{70}$$

We can project the above equation with $h^\rho_\mu h^\sigma_\nu$ (remember that $h^\rho_\mu = \delta^\rho_\mu - \hat{n}^\rho \hat{n}_\mu$) and use the fact that $n_\mu = \hat{n}_\mu$ is constant[16] (see equation (23)) to rewrite it as:

$$-\frac{1}{2\kappa^2}\mathcal{K}_{\mu\nu} = T^{bra}_{\mu\nu} - \frac{1}{D-2} h_{\mu\nu} T^{bra}, \tag{71}$$

[16] The result (71) can also be recovered by adding the Gibbons and Hawking [17] surface term (instead of (6)) to the gravitational Lagrangian [18, 20].

with $\mathcal{K}_{\mu\nu} := \nabla_{(\mu} n_{\nu)}$ the extrinsic curvature on the brane.

If the brane is locate in the bulk (not at one boundary), the same variational principle argument modifies (71) to [18]:

$$-\frac{1}{2\kappa^2}\left(\mathcal{K}^+_{\mu\nu} - \mathcal{K}^-_{\mu\nu}\right) = T^{bra}_{\mu\nu} - \frac{1}{D-2} h_{\mu\nu} T^{bra}, \tag{72}$$

which are the usual Lanczos-Israel junction conditions [3] (remember that $2\kappa^2 = 8\pi G$).

In this manuscript, we worked with one brane located at the boundary $r = 0$. If we consider the orbifold $S^1/\mathbb{Z}_2$ instead of an ordinary interval, the equation (72) has to be completed with $\mathcal{K}^+_{\mu\nu} = -\mathcal{K}^-_{\mu\nu} = \mathcal{K}_{\mu\nu}$. The net result is that equation (71) is modified by a factor of two (see section 6 for examples):

$$-\frac{1}{2\kappa^2}\mathcal{K}_{\mu\nu} = \frac{1}{2}\left(T^{bra}_{\mu\nu} - \frac{1}{D-2} h_{\mu\nu} T^{bra}\right). \tag{73}$$

Note that the rhs of the junction condition (53) for the scalar fields is also modified by a factor 1/2.

We can then repeat the calculation of the superpotential using now the modified boundary conditions (73) in order to "integrate" equation (33). It is straightforward to check that the final superpotential on the orbifold is one half the one on the plain interval:

$${}_{(S^1/\mathbb{Z}_2)}U^{\mu\nu}_{\xi} t_\mu n_\nu = \frac{1}{2} U^{\mu\nu}_{\xi} t_\mu n_\nu. \tag{74}$$

The simplest way to take into account this 1/2 factor which appears in the orbifold case is to make the replacement $\mathcal{L}_{bra} \to \mathcal{L}_{bra}/2$ (and then $T^{bra}_{\rho\sigma} \to T^{bra}_{\rho\sigma}/2$) in all the above formulas. In other words, the orbifold case can be treated using the plain segment equations together with a rescalling of the brane Lagrangian by one half.

We would like to emphasize that this factor of one half is given by the manifold considered. It is therefore an extra imput to be adjusted according to the physical problem studied.

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