

Vertex Operators for the Supermembrane and Background Field Matrix Theory

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ABSTRACT

We derive the vertex operators that are expected to govern the emission of the massless $d = 11$ supermultiplet from the supermembrane in the light cone gauge. Our results immediately imply the linear coupling of matrix theory to an arbitrary supergravity background to all orders in anticommuting coordinates. Finally we address the definition of n -point tree level and one-loop scattering amplitudes. The resulting 3-point tree level amplitudes turn out to agree with $d = 11$ supergravity and are completely fixed by supersymmetry and the existence of a normalizable ground state.

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1 Introduction

One of the most pressing question in string theory to date is as to what the precise microscopic degrees of freedom of M theory are. On the face of it the 11 dimensional supermembrane [1] appears as a natural candidate for M theory, as it sits atop of the main contenders for a unified theory of quantum gravity: supergravity [2], superstrings [3] and matrix theory [4], which all are obtained through certain limits of the membrane model. This is obvious for the case of eleven dimensional supergravity, where one simply discards all internal excitations of the membrane, being left with a first quantized description of supergravity in form of the 11 dimensional superparticle. Similarly one reaches type IIA superstrings at the kinematic level through a procedure called double-dimensional reduction [5]. And finally matrix theory, a proposed candidate for light cone M theory, emerges as a finite N regularization of the light cone supermembrane. Despite these features the study of the fundamental supermembrane has not received much attention caused by the tremendous difficulties one encounters once one turns to a quantization of the model. Opposed to the particle and the string the membrane is a nonlinear interacting field theory, with a priori no well defined perturbative scheme in form of a sum over worldvolumes. Moreover, the model possesses a continuous spectrum, which, following the insights from the matrix theory proposal, should be interpreted as a second quantized feature. In order to make progress we believe that a starting point for a quantum treatment of the supermembrane should be to clarify what the sensible quantities or operators are whose expectation values one would like to compute. We here want to push forward the concept of membrane vertex operators, which are expected to govern the emission of the massless d=11 supermultiplet from the membrane world-volume. With these we are able to *define* scattering amplitudes in membrane theory, thus making the supermembrane more “computable”. Moreover, these operators immediately translate into the corresponding objects in matrix theory, thus yielding the linear order background field coupling of matrix theory to all orders in fermions. We shall show that the resulting 3-point amplitudes agree with d=11 supergravity, and comment at ongoing work to compute the membrane four graviton amplitude, which yields the famous \mathcal{R}^4 quantum correction to the d=11 supergravity action.

In analogy to superparticle and superstring theory the supermembrane vertex operators are naturally defined as the linear coupling of the supergravity background fields $g_{\mu\nu}$, $C_{\mu\nu\rho}$ and ψ_μ^α to the embedding coordinates $X^\mu(\xi)$ and $\theta_\alpha(\xi)$, where ξ parameterizes the three dimensional membrane world-volume. In principle these operators are deducible from the known background field action of the supermembrane in superspace [1]

$$S = \int d^3\xi \sqrt{-g[Z(\xi)]} + \frac{1}{6} \epsilon^{ijk} \pi_i^A \pi_j^B \pi_k^C B_{CBA}[Z(\xi)] \quad (1)$$

where E_M^A denotes the super-dreiundvierzig-bein, B_{CBA} the super three-form and we have $Z^A = (X^\mu(\xi), \theta^\alpha(\xi))$, $\pi_i^A = \frac{\partial Z^M}{\partial \xi^i} E_M^A$ and $g_{ij} = \pi_i^r \pi_j^s \eta_{rs}$. Obtaining the linearized action in components from (1), however, is a highly nontrivial task. To date this has only been achieved up to second order in anticommuting coordinates θ [6] through a process called gauge completion [7], i.e.

$$\begin{aligned} E_\mu^r &= e_\mu^r + 2\bar{\theta}\Gamma^r\psi_\mu + \bar{\theta}\Gamma^r\left[-\frac{1}{4}\hat{\omega}_\mu \cdot \Gamma + T_\mu \cdot \hat{F}\right]\theta + \dots + O(\theta^{32}) \\ E_\mu^\alpha &= \psi_\mu^\alpha - \frac{1}{4}\hat{\omega}_\mu^{rs}(\Gamma_{rs}\theta)^\alpha + (T_\mu \cdot \hat{F}\theta)^\alpha + \dots + O(\theta^{32}) \\ B_{\mu\nu\rho} &= C_{\mu\nu\rho} - 6\bar{\theta}\Gamma_{[\mu\nu}\psi_{\rho]} - 3\bar{\theta}\Gamma_{[\mu\nu}\left[-\frac{1}{4}\hat{\omega}_{\rho]} \cdot \Gamma + T_{\rho]} \cdot \hat{F}\right]\theta \dots + O(\theta^{32}) \end{aligned}$$

Note that these expansions in principle extend all the way up to order 32 in θ 's. We hence see that obtaining the complete expressions for the *covariant* vertex operators via superspace seems to be beyond reach². This motivates the transition to a light cone description of the supermembrane in general backgrounds for which one expects a substantial degree of simplification, as may be seen already the level of the flat space action. We shall see that here the θ expansion terminates at order four.

Imposing the light cone condition $X^+ = p^+ \tau$ and $\Gamma^+\Theta = 0$ by κ -symmetry the flat space action of the supermembrane reads [9]

$$\mathcal{L} = \frac{1}{2}(DX^a)^2 - \frac{1}{4}\{X^a, X^b\}^2 - i\theta D\theta - i\theta\gamma_a\{X^a, \theta\} \quad (2)$$

where $X^a = X^a(\tau, \sigma_1, \sigma_2)$ and $\theta^\alpha = \theta^\alpha(\tau, \sigma_1, \sigma_2)$ denote the transverse ($a = 1, \dots, 9$, $\alpha = 1, \dots, 16$) coordinates. We have used the covariant derivative $DX^a := \partial_\tau X^a - \{\omega, X^a\}$ along with the Poisson bracket $\{A, B\} := \epsilon^{ij} \partial_{\sigma_i} A \partial_{\sigma_j} B$. The gauge field ω entering the covariant derivative D stems from a residual symmetry of area preserving diffeomorphisms [10]

$$\delta X^a = \{\xi, X^a\}, \quad \delta\theta = \{\xi, \theta\}, \quad \delta\omega = \partial_\tau + \{\xi, \omega\} \quad (3)$$

The action (2) is moreover invariant under the set of supersymmetry transformations

$$\begin{aligned} \delta X^a &= -2\epsilon\gamma^a\theta & \delta\omega &= -2\epsilon\theta \\ \delta\theta &= iDX^a\gamma_a\epsilon - \frac{i}{2}\{X^a, X^b\}\gamma_{ab}\epsilon + \eta \end{aligned} \quad (4)$$

with 32 components, 16 from η and 16 from ϵ , a remnant of the 11 dimensional origin of the model. Viewing the parameters (σ_1, σ_2) as internal degrees of freedom the light cone supermembrane may be understood as a supersymmetric quantum mechanical system equipped with an infinite dimensional gauge group of area preserving diffeomorphisms.

²There exists an alternative and potentially more effective procedure via a normal coordinate expansion in superspace [8], which has so far not been applied to the eleven dimensional case.

2 Vertex Operator Construction

The graviton, three form and gravitino emission operators that we seek to construct take the general form

$$\begin{aligned}
V_h &= h_{ab} \int d\tau d^2\sigma \mathcal{O}^{ab}[X^a(\tau, \sigma_i), \theta(\tau, \sigma_i)] e^{ik \cdot X + ik_+ X^+} \\
V_C &= C_{abc} \int d\tau d^2\sigma \mathcal{O}^{abc}[X^a, \theta] e^{ik \cdot X + ik_+ X^+} \\
V_\psi &= \psi_a^\alpha \int d\tau d^2\sigma \mathcal{O}_\alpha^a[X^a, \theta] e^{ik \cdot X + ik_+ X^+}
\end{aligned} \tag{5}$$

with the polarizations $(h_{ab}, C_{abc}, \psi_a^\alpha)$ and momenta k_a and k_+ . The polarizations are subject to on-shell constraints, e.g. $k^a h_{ab} = h_{aa} = 0 = k^a C_{abc}$.³ The strategy for the explicit construction of the operators \mathcal{O}^{ab} , \mathcal{O}^{abc} and \mathcal{O}_α^a is rather simple. The (unknown) full background field action of the supermembrane transforms covariantly under supersymmetry as

$$\delta \mathcal{L}_{\text{full}}[X, \theta; h, C, \psi] = \mathcal{L}_{\text{full}}[X, \theta; \hat{\delta}h, \hat{\delta}C, \hat{\delta}\psi] \tag{6}$$

where δ denotes the supersymmetry variation of X^a, θ and ω of (4), whereas $\hat{\delta}$ are the induced light-cone supergravity variations of the background fields h_{ab}, C_{abc} and ψ_a^α whose precise expressions may be found in [11, 12]. Hence the vertex operators, being the linear background field couplings, must transform into each other under supersymmetry according to

$$\begin{aligned}
\delta V_h &= V_{\hat{\delta}\psi[h]} \\
\delta V_C &= V_{\hat{\delta}\psi[C]} \\
\delta V_\psi &= V_{\hat{\delta}h} + V_{\hat{\delta}C}
\end{aligned} \tag{7}$$

So for example under the linear part of the supersymmetry transformations of (4) $\delta X^a = 0$ and $\delta\theta = \eta$ the vertex operator V_ψ transforms into a sum of the graviton and three-form vertices, whose polarizations are then given by $\delta h_{ab} = -\tilde{\psi}_{(a}\gamma_{b)}\eta$ and $\delta C_{abc} = \frac{3}{2}\tilde{\psi}_{[a}\gamma_{bc]}\eta$ respectively, the linearized form of the supergravity transformations parametrized by the same η entering (4).

It turns out that this requirement of covariance under supersymmetry completely determines the form of (5). The results read [12]

$$V_h = h_{ab} \left[DX^a DX^b - \{X^a, X^c\} \{X^b, X^c\} - i\theta\gamma^a \{X^b, \theta\} \right]$$

³A derivation of this may be found in [11]. Note that in the above we are suppressing all terms coupling to k_- , thus effectively setting $k_- = 0$ in order to decouple X^- in the expressions, a complicated function in X^a and X^+ . This is standard practice in light cone string theory, albeit questionable, as with it all transverse momenta become complex. A cleaner way to state this point is that one consistently ignores all terms coupling to k_- in the computations, assuming that they work out by themselves.

$$\begin{aligned}
& -2DX^a R^{bc} k_c - 6\{X^a, X^c\} R^{bcd} k_d + 2R^{ac} R^{bd} k_c k_d \Big] \\
V_C = & -C_{abc} DX^a \{X^b, X^c\} + F_{abcd} \left[(DX^a - \frac{2}{3}R^{ae} k_e) R^{bcd} \right. \\
& \left. - \frac{1}{2}\{X^a, X^b\} R^{cd} - \frac{1}{96} \{X^e, X^f\} \theta \gamma^{abcdef} \theta \right] \\
V_\psi = & \psi_a \left[(DX^a - 2R^{ab} k_b + \gamma_c \{X^c, X^a\}) \theta \right] \\
& + \tilde{\psi}_a \left[\gamma \cdot DX (DX^a - 2R^{ab} k_b + \gamma_c \{X^c, X^a\}) \theta \right. \\
& + \frac{1}{2} \gamma_{bc} \{X^b, X^c\} (DX^a - \{X^a, X^d\} \gamma^d) \theta \\
& + 8\gamma_b \theta \{X^b, X^c\} R^{cad} k_d + \frac{5}{3} \gamma_{bc} \theta \{X^b, X^c\} R^{ad} k_d \\
& + \frac{4}{3} \gamma_{bc} \theta (\{X^a, X^b\} R^{cd} + \{X^c, X^d\} R^{ab}) k_d \\
& + \frac{2}{3} i (\gamma_b \theta \{X^a, \theta\} \gamma^b \theta - \theta \{X^a, \theta\} \theta) \\
& \left. + \frac{8}{9} \gamma^b \theta R^{ac} R^{bd} k_c k_d \right] \tag{8}
\end{aligned}$$

where $R^{ab} = \frac{1}{4} \theta \gamma^{ab} \theta$ and $R^{abc} = \frac{1}{12} \theta \gamma^{abc} \theta$, suppressing the overall $\exp(ik \cdot X)$ of (5). The vertices for $h_{+a}, h_{++}, C_{+ab}, \psi_+$ and $\tilde{\psi}_+$ are also known; ψ and $\tilde{\psi}$ are the light-cone decompositions of the gravitino [11].

These results are subject to three stringent consistency checks. Firstly, the vertex operators are invariant under the following background field symmetries:

$$\begin{aligned}
\delta h_{ab} &= k_{(a} \xi_{b)} && \text{(coordinate transformations)} \\
\delta C_{abc} &= k_{[a} \xi_{bc]} && \text{(tensor gauge transformations)} \\
\delta \psi_a &= k_a \epsilon && \text{(field independent SUSY)}
\end{aligned}$$

Secondly, the point particle limit of the membrane vertices (8), which amounts to simply dropping the terms involving the Poisson brackets $\{.,.\}$ yields the d=11 superparticle vertex operators of Green, Gutperle and Kwon [11].

Finally, a stronger check is to perform a double dimensional reduction [5] of the membrane vertices, which should reduce them to the type IIA superstring vertex operators. This reduction procedure is performed by wrapping the σ_2 coordinate of the membrane around a target space circle along X^9

$$X^a(\tau, \sigma_1, \sigma_2) \rightarrow \begin{pmatrix} X^i(\tau, \sigma_1) \\ X^9 = \sigma_2 \end{pmatrix} \quad i = 1, \dots, 8 \tag{9}$$

along with

$$\theta_\alpha \rightarrow \begin{pmatrix} S_a(\tau, \sigma_1) \\ \tilde{S}_{\dot{a}}(\tau, \sigma_1) \end{pmatrix} \quad a, \dot{a} = 1, \dots, 8 \tag{10}$$

Quite remarkably under this description the vertex operators of (8) factorize into left and right moving contributions. Let us demonstrate this for the graviton vertex reduction. Under (9) and (10) we have

$$\{X^i, X^j\} = 0 \quad \{X^i, X^9\} = \partial_{\sigma_1} X^i \quad \omega = 0$$

$$\theta\gamma^{ij}\theta = S\Gamma^{ij}S + \tilde{S}\Gamma^{ij}\tilde{S} \quad \theta\gamma^{ij9}\theta = S\Gamma^{ij}S - \tilde{S}\Gamma^{ij}\tilde{S} \quad (11)$$

where Γ^i are the standard SO(8) Γ -matrices. Then

$$\begin{aligned} V_h|_{\text{DDR}} &\rightarrow h_{ij} \left[\partial_0 X^i \partial_0 X^j - \partial_1 X^i \partial_1 X^j - \frac{1}{2} \partial_0 X^i (S\Gamma^{jm}S \right. \\ &\quad \left. + \tilde{S}\Gamma^{jm}\tilde{S}) k_m + \frac{1}{2} \partial_1 X^i (S\Gamma^{jm}S - \tilde{S}\Gamma^{jm}\tilde{S}) k_m + \frac{1}{4} S\Gamma^{im}S \tilde{S}\Gamma^{jn}\tilde{S} k_m k_n \right] \\ &= h_{ij} \left(\partial_+ X^i - \frac{1}{2} S\Gamma^{im}S k_m \right) \left(\partial_- X^j - \frac{1}{2} \tilde{S}\Gamma^{jn}\tilde{S} k_n \right) \end{aligned}$$

which is nothing but the IIA graviton vertex.

3 Matrix Theory in Background Fields

The obtained results may be directly translated to matrix theory, which emerges from a supersymmetry preserving ‘‘discretization’’ of the membrane spacesheet. For this one replaces the infinite dimensional gauge group of area preserving diffeomorphisms by the large N limit of $SU(N)$ [10]. This well known prescription here amounts to the replacements

$$\begin{aligned} X^a(\tau, \sigma^1, \sigma^2) &\rightarrow \mathbf{X}_{mn}^a(\tau) & \theta^\alpha(\tau, \sigma^1, \sigma^2) &\rightarrow \Theta_{mn}^\alpha(\tau) & m, n &= 1, \dots, N \\ \{.,.\} &\rightarrow i[.,.] & \frac{1}{4\pi} \int d^2\sigma(\dots) &\rightarrow \frac{1}{N} \text{STr}[\dots] \end{aligned} \quad (12)$$

STr denotes the symmetrized trace whose introduction becomes necessary due to ordering ambiguities for the composite operators that we have been discussing. The STr prescription guarantees that all manipulations performed in section 2 for the continuous membrane model go through for the matrix model as well. In principle there may exist a less symmetric ordering that also works, but the differences to the STr prescription will be subleading in N .

Hence the weak (i.e. linear coupling) background field action of matrix theory is now known to *all* orders in Θ and derivatives $\partial/\partial\mathbf{X}^a$

$$\mathcal{S}_{\text{MT}} = \int d\tau \left(\mathcal{L}_0 + V_{h(X)} + V_{C(X)} + V_{\Psi(X)} \right) \quad (13)$$

where e.g. the graviton coupling takes the form

$$\begin{aligned} V_{h(X)} &= \text{STr} \left[\left\{ \dot{\mathbf{X}}^a \dot{\mathbf{X}}^b + [\mathbf{X}^a, \mathbf{X}^c] [\mathbf{X}^b, \mathbf{X}^c] + \Theta \gamma^a [\mathbf{X}^b, \Theta] \right. \right. \\ &\quad \left. \left. - \frac{1}{2} \dot{\mathbf{X}}^a (\Theta \gamma^{bc} \Theta) \frac{\partial}{\partial \mathbf{X}^c} - \frac{1}{2} i [\mathbf{X}^a, \mathbf{X}^c] (\Theta \gamma^{bcd} \Theta) \frac{\partial}{\partial \mathbf{X}^d} \right. \right. \\ &\quad \left. \left. + 2(\Theta \gamma^{ac} \Theta) (\Theta \gamma^{bd} \Theta) \frac{\partial}{\partial \mathbf{X}^c} \frac{\partial}{\partial \mathbf{X}^d} \right\} h_{ab}(\mathbf{X}) \right] \end{aligned} \quad (14)$$

Note that we have performed a Fourier transformation of (8) to target configuration space. These results agree with and go beyond the explicit matrix theory current calculations of Taylor and V.Raamsdonk [13], which so far have been performed up to order $o(\theta^2)$ and linear order in $\partial/\partial\mathbf{X}^a$.

4 Scattering Amplitudes

We now turn to the discussion of three point tree level scattering amplitudes. For this it is advantageous to work in the framework of the finite N matrix theory. In order to define a tree level amplitude we first split off the center of mass degrees of freedom of the matrices by writing

$$\mathbf{X}^a = x^a \mathbb{1} + \hat{\mathbf{X}}^a \quad \Theta^a = \theta^a \mathbb{1} + \hat{\Theta}^a \quad (15)$$

with traceless matrices $\hat{\mathbf{X}}^a$ and $\hat{\Theta}$. An asymptotic 1-graviton state in matrix theory is then given by

$$|\text{IN}\rangle\rangle = |k_1, h_1\rangle_{x,\theta} \otimes |\text{GS}\rangle_{\hat{\mathbf{X}}, \hat{\Theta}}^{\text{SU}(N)} \quad (16)$$

where $|k_1, h_1\rangle$ is the graviton state of the superparticle [14, 11] and $|\text{GS}\rangle$ denotes the *exact* $\text{SU}(N)$ normalized zero energy groundstate, whose explicit form is unknown but is known to exist [15]. The tree level three point amplitude is then defined by

$$\mathcal{A}_{3\text{-point}} = \langle\langle 1 | V_2 | 3 \rangle\rangle \quad (17)$$

where one inserts the graviton vertex operator

$$V_2 = h_{ab}^{(2)} \frac{1}{N} \text{STr} \left[(p^a p^b + 2p^a \hat{\mathbf{P}}^b + \hat{\mathbf{P}}^a \hat{\mathbf{P}}^b + [\hat{\mathbf{X}}^a, \hat{\mathbf{X}}^c] [\hat{\mathbf{X}}^b, \hat{\mathbf{X}}^c]) e^{ik \cdot \hat{\mathbf{X}}} \right] e^{ik \cdot x} + \text{fermions} \quad (18)$$

One may wonder how one could ever evaluate (17) upon inserting (18) without the knowledge of $|\text{GS}\rangle$. The first contribution to (17) takes the form

$$\langle k_1, h_1 | p^a p^b e^{ik \cdot x} | k_3, h_3 \rangle h_{ab}^{(2)} \langle \text{GS} | \text{STr} e^{ik \cdot \hat{\mathbf{X}}} | \text{GS} \rangle \quad (19)$$

Now by $\text{SO}(9)$ covariance $\langle \text{GS} | \text{STr} e^{ik \cdot \hat{\mathbf{X}}} | \text{GS} \rangle = N$, as the only $\text{SO}(9)$ scalar it could depend on would be k^2 which vanishes on shell. It must then be a constant which is fixed to be N by considering the $k^a \rightarrow 0$ limit. Remarkably the remaining two terms in (17) upon inserting (18) vanish by a combination of $\text{SO}(9)$ covariance and on-shell arguments:

$$h_{ab} \langle \text{GS} | \text{STr} \hat{\mathbf{P}}^b e^{ik \cdot \hat{\mathbf{X}}} | \text{GS} \rangle \sim k^b h_{ab} = 0 \quad (20)$$

$$h_{ab} \langle \text{GS} | \text{STr} \left[(\hat{\mathbf{P}}^a \hat{\mathbf{P}}^b + [\hat{\mathbf{X}}^a, \hat{\mathbf{X}}^c] [\hat{\mathbf{X}}^b, \hat{\mathbf{X}}^c]) e^{ik \cdot \hat{\mathbf{X}}} \right] | \text{GS} \rangle \sim (k^a k^b + c \delta^{ab}) h_{ab} = 0$$

But as the first correlator in (19) is nothing but the bosonic contribution to the 3-point d=11 *superparticle* amplitude [11] and as the fermionic terms work out in a similar fashion we see that our 3-point tree level amplitude

$$\langle\langle 1 | V_2 | 3 \rangle\rangle = \langle 1 | V_2 | 3 \rangle_{x,\theta} \langle \text{GS} | \text{GS} \rangle \quad (21)$$

agrees with the 3-point amplitude of d=11 supergravity!

Clearly the next step would be to study n -point tree level amplitudes which should be given by

$$\mathcal{A}_{n\text{-point}} = \langle\langle 1 | V_2 \Delta V_3 \Delta \dots \Delta V_{n-1} | n \rangle\rangle \quad (22)$$

where Δ denotes the propagator $1/(\frac{1}{2}p_0^2 + \hat{\mathbf{H}})$ built from the *interacting* membrane Hamiltonian $\hat{\mathbf{H}}$. However, now we expect the details of the groundstate $|\text{GS}\rangle$ to enter the computation. Developing some perturbative scheme for calculating (22) would be highly desirable, but is conceivably very complicated as it must involve an expansion in both the propagator Δ and the groundstate $|\text{GS}\rangle$.

Instead we shall briefly comment on ongoing attempts to compute loop amplitudes within this scenario. Here, led by the formalism in light cone superstring and superparticle theory, we propose to define a membrane n -point one-loop amplitude by the expression

$$\mathcal{A}_{1\text{-loop}, n\text{-point}} = \int d^{11}p_0 \text{Tr}(\Delta V_1 \Delta V_2 \Delta V_3 \Delta V_4 \dots \Delta V_n) \quad (23)$$

where the trace is over the Hilbert space of $\hat{\mathbf{H}}$. Again this appears as a daunting task, however the zero mode sector of (23) already yields some amount of information. In particular the trace over the fermionic zero mode θ of (15) tells us that all 2 and 3 particle amplitudes vanish at one loop, as at least four vertex operators (≤ 16 θ 's) are needed to saturate the fermion zero mode trace

$$\text{Tr}(\theta_{\alpha_1} \dots \theta_{\alpha_N})_{\theta} = \delta_{N,16} \epsilon^{\alpha_1 \dots \alpha_{16}} \quad (24)$$

In the pure graviton sector the first non-vanishing amplitude is then the 4-graviton amplitude whose leading momentum dependence is given by

$$\mathcal{A}_{4h} = \epsilon^{\alpha_1 \dots \alpha_{16}} \gamma_{\alpha_1 \alpha_2}^{a_1 a_2} \dots \gamma_{\alpha_{15} \alpha_{16}}^{a_{15} a_{16}} R_{a_1 a_2 a_3 a_4}^{(1)} \dots R_{a_{13} a_{14} a_{15} a_{16}}^{(4)} \int d^{11}p_0 \text{Tr}' \Delta^4. \quad (25)$$

We thus see the emergence of the expected \mathcal{R}^4 term [16] in the kinematical sector, but it remains to be seen what can be said about the remaining trace. Potentially BPS arguments could here come to ones aid [17].

5 Outlook

In this talk we have constructed the supermembrane vertex operators in the light cone gauge, which hopefully provide us with a new tool in the study of quantum M theory. We have demonstrated their reduction to the corresponding vertices of the d=11 superparticle and d=10 type IIA strings. Moreover they yield complete weak background field matrix theory action. It would be interesting to clarify their relevance for the related IKKT matrix model [18] and matrix string theory [19]. N-point tree level and 1-loop amplitudes were defined for the model and

it was argued that the resulting 3-point tree level amplitudes agree with d=11 supergravity.

Clearly there remains a host of open questions. On the conceptual side one may ask where the multiparticle interpretation of the membrane/matrix theory emerges in the outlined formalism. After all our construction proceeds in complete analogy with the first quantized particle and string theories. On the technical side progress is clearly necessary for the evaluation of higher loop amplitudes, both at tree and one loop level.

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